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Implications of a visco-elastic model of the lithosphere for calculating yield strength envelopes

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Abstract

The dominant deformation mechanism in the ductile part of the lithosphere is creep. From a mechanical point of view, creep can be modelled as a viscous phenomenon. On the other hand, yield-strength envelopes (YSEs), commonly used to describe lithosphere rheology, are constructed supposing creep to be mechanically plastic (note that the meaning of the term "plastic" used in mechanics and material sciences are different). Such rheological models are simple but can lead to internal inconsistencies. However, evaluating the "strength" of the lithosphere using a viscous rheology requires incorporation of the time-dependence of stresses, strains, and strain rates and also the dependence of the bulk strain rate on the total applied force. The two approaches are compared by computing stress distributions in the lithosphere for given structure, mineralogy, geotherm, and applied forces using two methods in which creep is modelled as "plastic" and "viscous" respectively. The results demonstrate the importance of the bulk strain rate of the lithosphere in determining stress distribution. Further, the bulk strain rate is not independent of the total applied force although this is an underlying assumption of the YSE-based (plastic rheology) approach. For typical plate boundary forces and normal geotherms, appropriate bulk strain rates are low, about $10^{-17}\ s^{-1}$, indicating that lithospheric "strength" computed from YSEs using a constant bulk strain rate in the range of 10^{-16} to $10^{-14}\ s^{-1}$ is overestimated by one-half to one order of magnitude. Stresses using the YSE (plastic) approach can be significantly over- or under-estimated due to an inappropriate choice of bulk strain rate for the tectonic setting and duration of loading of forces under consideration.

Keywords: Rheology; Yield-strength envelope; Lithosphere; Stress; Strain rate; Strength

1. Introduction

The concept of a lithospheric "yield-strength envelope" (YSE) is well known in many facets of geophysical sciences (cf. Ranalli, 1995, 1997). It has been widely used in models of lithosphere deformation, such as those of sedimentary basin formation or orogen development, in which lithospheric strength—or resistance to deformation under defined loading conditions—is paramount. Several examples include Stephenson et al. (1990), Burov and Diament (1995), Fernandez and Ranalli (1997), and Lankreijer et al. (1999). The YSE concept has also been central to most quantitative estimations of stress-state in the lithosphere, including the relationship of stress-state to observed seismicity (e.g.

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Cloetingh and Banda, 1992; Liu and Zoback, 1997), and to estimates of the strength of the lithosphere (e.g. Ranalli, 2003; Burov, 2003; Cloetingh et al., 2006).

Goetze and Evans (1979) introduced the YSE as an estimate of the upper limit of lithospheric stress (greatest possible stress given sufficient force) as a function of depth. Goetze and Evans based their calculations on experimental laboratory studies of rock mechanics, in which three main types of rock deformation are typically observed: (1) elastic, (2) brittle failure, and (3) creep. Typical experimental results can be found in Carter and Kirby (1978). In terms of mechanical theory, these kinds of observed rheological behaviours can be classified, respectively, as follows:

(1) "Elasticity"—with "elastic" deformation, being instantaneous upon loading, perfectly recoverable upon removal of the loading stress, with strain that is a linear function of stress.
(2) "Plasticity"—with "plastic" deformation, occurring only when the loading stress overcomes a critical yield value (known as the "yield strength" or "yield stress" or "critical stress"), being instantaneous (i.e. is independent of time) upon reaching the critical stress and non-recoverable upon removal of the load. Plastic deformation that results in discontinuous strain (e.g. fracturing and faulting) can be characterised as "brittle" and plastic deformation that is spatially continuous as "ductile".
(3) "Viscosity"—with "viscous" deformation or "creep", also non-recoverable but accumulating during the time in which a finite loading stress is applied (being time-dependent, therefore), with the strain rate during this time being a function of the stress, there being no ("critical") stress limit below which the deformation does not occur.[1]

In the numerous modelling and other studies in which the YSE concept has been utilised, it is generally considered to represent the profile of maximum rock strength (yield strength) with depth, this being the lesser of either "brittle strength" or "creep strength" calculated at each depth for some model of the physical properties of the lithosphere (composition, temperature, pressure) at that depth. The computed "brittle strength" is based on a brittle failure criterion such as the Coulomb-Navier failure criterion or Byerlee's law (e.g. Ranalli, 1995) whereas the computed "creep strength" is based on one of a number of laboratory-derived creep laws such as "power-law" creep or "Dorn creep" (e.g. Ranalli, 1995). As such, "creep strength" is taken to be equivalent to "creep stress"—the stress required to maintain a given rate of steady-state viscous deformation (i.e. strain rate).

Deformation at any stress below the calculated maximum rock strength at any depth in a YSE model is considered to be elastic. Stresses reaching the maximum rock strength at depths where it is defined by a brittle yield criterion are assumed to lead to brittle deformation. Stresses reaching the maximum rock strength at depths where it is defined by a creep law are assumed to lead to steady-state viscous deformation at a pre-defined bulk strain rate. In either case, therefore, plastic deformation is considered to "begin" and the stress level cannot increase any further such that it exceeds the defined critical yield value. Thus, in a YSE model, creep is considered to be a "plastic" deformation process (in the "mechanical" sense of having a yield stress below which it does not occur), whereas the laboratory data—on which the actual YSE models are based—demonstrate it to be "viscous".

The assessment of the validity of such an approximation—using viscous rheological laws to approximate a yield stress at some given strain rate—is the main objective of the present paper. In so doing, it is necessary to derive the stress state of the lithosphere in two ways: one in which deformation in the ductile part of the lithosphere is treated as truly viscous in nature, with no yield stress, and one in which deformation in the ductile part of the lithosphere is treated, as in many conventional applications, as plastic. Models with the viscous representation of creep will be referred to as to V-type models and models with plastic creep as P-type models. Neither model is "new". The general principles of the V-type model are similar to those developed by Kusznir (1982) and De Rito et al. (1986) whereas, as already mentioned above, the P-type model is based on the YSE construction according to Goetze and Evans (1979), utilised subsequently in many different studies.

Despite the wide appeal of the concepts inherent in both approaches to modelling lithosphere rheology, little attention has been given to directly comparing them. One partial exception is the work of Porth (2000) who developed

[1] In terms of the theoretical rheological descriptions of rock deformation, therefore, brittle failure observed in the laboratory is a form of "plasticity". However, in material sciences terminology, the words "plasticity" and "plastic flow" are used to describe inelastic deformation without rupture, as opposed to brittle deformation (inelastic deformation with rupture). In these terms, creep is a kind of plastic rock flow, an obvious source of confusion in respect of the all-important presence or absence of a critical or yield strength (stress) for a given material. Here, the word "plastic" is used in its "mechanical" (presence of a yield stress) rather than "viscous" (no yield stress) sense.

a visco-plastic model in which applied forces were dependent upon deformation rates, focusing on how the balance between driving and resistive forces plays an important role in controlling intraplate deformation. Here, the focus is more on the systematic differences in the implications of the P-type (YSE) and V-type approaches, a difference which suggests the improper use of YSE models in their present, commonly accepted form. It will be shown that the P-type model often gives results that are internally inconsistent—or are inconsistent with what is known of the tectonic or geodynamic setting of the modelled region. This is because bulk strain rate and total applied force are interdependent and cannot be introduced into such models as independent parameters. A way to avoid this defect will be described and its implications for the YSE technique in various geodynamical applications will be discussed.

2. Description of the models

2.1. Composition and temperature of the lithosphere

The rheological models were derived for representative lithosphere geotherms calculated by the inversion of a range of surface heat flow values, meant to be generally characteristic of cold, intermediate, and hot continental lithosphere. This was done by solving the general heat transfer equation using standard methods for a 200 km thick continental lithosphere consisting of three compositional layers (upper crust, lower crust, and mantle). Heat production was assumed to be exponentially distributed in the crust, decreasing with depth. All adopted parameter values are presented in Table 1.

2.2. Stress and strain

It is assumed that deformation of the lithosphere is uniform plane-strain compressional and tensional deformation only; that is, lithospheric flexure is excluded from consideration. The horizontal axis is the x-axis and the vertical axis is the z-axis but subscripts are not written; ε and σ are meant as the xx components of their respective tensors. Only deviatoric stresses and strains are considered. Further, it is assumed that principal stresses are parallel to the x- and z-axes and that vertical deviatoric stress is equal to zero. The latter is the common with the theory of thin plates and is implicitly adopted in the whole generation of YSE-based models. As usual in geological applications, positive stress/strain is taken to be compressive and negative to be extensional.

A distinction is made between local strain rate at a given point in the lithosphere and the bulk strain rate of the lithosphere as a whole. The latter is defined as the average local strain rate on the vertical co-ordinate. Instantaneous strain rate, at a given moment of time, and time-averaged strain rate are similarly distinguished. When deformation is

Table 1
Numerical values of the parameters adopted in the calculations

Parameter	Upper crust	Lower crust	Mantle	Units
Layer thickness	20	20	–	km
Density	2700	2900	3300	kg/m^3
Young's modulus (E)	70	70	90	GPa
Poisson's coefficient (ν)	0.25	0.25	0.25	–
Thermal conductivity	2.7	3.0	3.5	W/m K
Heat capacity	1050	1050	1050	J/kg K
Heat production rate	2.0	2.0	0	10^{-6} W/m^3
Decay rate	9	9	–	10^3 m
Representative mineralogy	Dry Simpson quartzite	Maryland diabase	Dry olivine	
Power factor (N)	2.72	3.05	3.6	–
Activation energy (E_p)	134	276	530	10^3 J/mol
Pre-exponential factor (A_p)	6.03×10^{-24}	3.16×10^{-20}	7.2×10^{-18}	1/s Pa^N
Activation energy (E_{plb})	–	–	535	10^3 J/mol
Pre-exponential factor (A_{plb})	–	–	5.7×10^{11}	1/s
Flow stress at 0 K (σ_{plb})	–	–	8.5	GPa

Crustal rheological parameters are after Carter and Tsenn (1987) and dry olivine after Tsenn and Carter (1987).

homogeneous with depth, the lithosphere's bulk strain is equal to the local total strain and bulk strain rate is equal to the total strain rate at each point.

As conventionally adopted for the analysis of the mechanics of plasticity and visco-elasticity (Leonov and Prokunin, 1994), total strain ε is assumed to consist of two parts: recoverable elastic strain ε_e and non-recoverable (or residual) inelastic strain ε_r, viz.:

$$\varepsilon = \varepsilon_e + \varepsilon_r. \tag{1}$$

Given the assumption of homogeneous deformation with depth, the total strain is obviously depth-independent. The stress is determined by the elastic component of strain:

$$\sigma = \frac{E}{1-\nu^2}\varepsilon_e, \tag{2}$$

where E is Young's module and ν is Poisson's ratio.

The viscous and/or plastic deformation represents a transformation of (recoverable) elastic strain into (non-recoverable) residual strain ($\varepsilon_e \to \varepsilon_r$), described by appropriate constitutive (rheological) equations.

2.3. Constitutive rheological equations

In the "plastic creep" (P-type) model, it is implied that material cannot support stresses higher than the yield stress. For lower stresses, deformation is assumed to be completely elastic and, when stress reaches the yield stress, plastic flow begins. In the plastic regime the stress (and, therefore, the elastic component of strain) stays constant (equal to the yield value) and all further deformation is achieved by non-recoverable plastic strain (the residual component of strain). In agreement with commonly used YSE models (e.g. Goetze and Evans, 1979), the "yield stress" is equal to:

$$\sigma_{\text{yield}} = \text{sign}(\dot{\varepsilon})\begin{cases} \left(|\dot{\varepsilon}|A_p \exp\left(\dfrac{E_p}{RT}\right)\right)^{1/N} & \text{(below the PLB threshold)} \\ \sigma_{\text{plb}}\left(1-\sqrt{\dfrac{RT}{E_{\text{plb}}}\ln\left(\dfrac{A_{\text{plb}}}{|\dot{\varepsilon}|}\right)}\right) & \text{(above the PLB threshold)} \end{cases}, \tag{3}$$

where $\dot{\varepsilon}_r$ is the residual strain rate, R the universal gas constant; T the temperature (K), and A_p, E_p, N, A_{plb}, E_{plb}, and σ_{plb} are experimentally-derived, mineralogy-dependent parameters (further defined and numerical values listed in Table 1). The power-law breakdown (PLB) threshold is 200 MPa (Tsenn and Carter, 1987), below which rock deformation occurs under a power-law regime and above which it occurs in a PLB regime. The strain rate $\dot{\varepsilon}$ is assumed to be homogeneous throughout the lithosphere (i.e. z-independent). The values $\dot{\varepsilon} \sim 10^{-14}$–$10^{-16}\,\text{s}^{-1}$ are usually adopted on the basis of observations in areas of recent active deformation (Goetze and Evans, 1979; Carter and Tsenn, 1987).

In the "viscous creep" (V-type) case, deformation is described by the empirically derived equations for power-law creep (e.g. Carter and Tsenn, 1987; Kohlstedt et al., 1995):

$$\dot{\varepsilon}_r = \text{sign}(\sigma)\begin{cases} \sigma^N A_p \exp\left(\dfrac{E_p}{RT}\right) & \text{(below the PLB threshold)} \\ A_{\text{plb}} \exp\left(-\dfrac{E_{\text{plb}}}{RT}\left(1-\dfrac{\sigma}{\sigma_{\text{plb}}}\right)^2\right) & \text{(above the PLB threshold)} \end{cases}, \tag{4}$$

where σ is deviatoric stress, with the same PLB threshold determination as in P-type models.

Obviously, the constitutive equations for P-type and V-type models written above are transformations of each other, with one exception: in the former, strain rate is the residual strain rate while, in the latter, it is the total strain rate.

Brittle deformation is described by Byerlee's law (Byerlee, 1978) rewritten in terms of principal stresses for faults dipping at 60° (Ranalli, 1995), with friction coefficient $\mu = 0.75$ and pore-fluids factor $\lambda = 0.35$. It is, in mechanical terms, a plastic deformation with yield stress,

$$\sigma_{\text{yield}} = \begin{cases} -0.5\rho g z & \text{(for extension)} \\ 2\rho g z & \text{(for compression)} \end{cases}, \tag{5}$$

and is considered as such in both types of models discussed here.

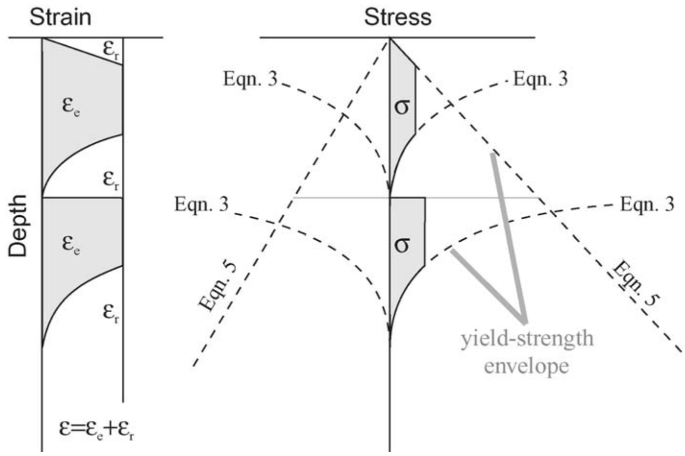


Fig. 1. Schematic illustration of the "plastic" (P-type) model. Strain with depth is shown at the left; it comprises an elastic component ε_e (shaded part) and a non-elastic (residual) component (ε_r), the sum of which (total strain) is constant with depth. The elastic strain is defined by the stress, which does not exceed σ_{yield} as defined by Eqs. (5) and (6).

2.4. "Plastic" (P-type) algorithm

In the framework of the P-type model, stresses should not exceed either the brittle yield stress (Eq. (5)) or the creep "yield stress" (Eq. (3)), whichever is less. In terms of its numerical implementation, where and whenever stress exceeds the yield stress, plastic flow (either brittle or ductile) effects a stress reduction (or relaxation) back to the yield limit (Fig. 1). The local elastic strain (shaded part in Fig. 1) can, therefore, be written as:

$$\varepsilon_e = \begin{cases} \varepsilon & \left(\text{when } \varepsilon \dfrac{E}{1-\nu^2} \le \sigma_{yield}\right) \\ \sigma_{yield} \dfrac{1-\nu^2}{E} & \left(\text{when } \varepsilon \dfrac{E}{1-\nu^2} > \sigma_{yield}\right) \end{cases}. \tag{6}$$

The residual strain is equal to the difference between total and elastic strains (Eq. (1)). In the "plastic" algorithm, the value of total strain ε was found such that the integral of the resultant lithospheric stress distribution would be equivalent to the applied force. The former can never exceed the integral of the lithosphere strength envelope because this represents the integrated strength of the lithosphere.

The "plastic" model is time-invariant. Although strain rate $\dot{\varepsilon}$ appears in the constitutive equations (Eq. (3)), it is an input parameter of the model, like applied force. It cannot be calculated and it is not correlated with the calculated lithospheric strains, in spite of the force, which is linked directly to the resultant calculated stresses.

Adopted strain rates in P-type models are typically in the range $\dot{\varepsilon} \sim 10^{-14}$–$10^{-16}\,\mathrm{s}^{-1}$, based on two types of observations (Carter and Tsenn, 1987): deformation of recently tectonically active zones (e.g. Whitten, 1956; Gilluly, 1972; Fletcher and Hallet, 1983) and rheomorphic (or paleopiezometric) studies (e.g. Mercier, 1980; Ave Lallemant et al., 1980). Strain rates in stable areas, such as in the interior of continents might be somewhat less. For example, Artyushkov (1983) estimated that the strain rate associated with self-gravitational forces loading stable continents for periods greater than 1000 Myr must be less than $10^{-18}\,\mathrm{s}^{-1}$. Furthermore, paleopiezometric data for cratonic areas suggest strain rates as low as $10^{-18}\,\mathrm{s}^{-1}$ (cf. Mercier, 1980).

2.5. "Viscous" (V-type) algorithm

The "viscous" model is time-dependent and the total force applied to lithosphere is assumed constant with time, viz.:

$$F = \int \sigma \, dz = \text{constant in time}, \tag{7}$$

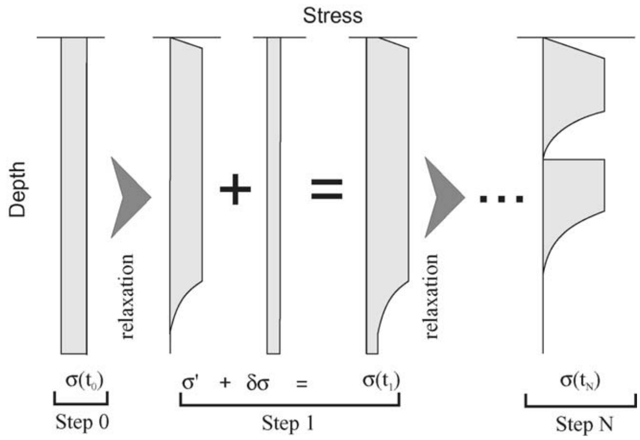


Fig. 2. Schematic illustration of the "viscous" (V-type) model. Step 0: initial (elastic) stress $\sigma(t_0)$ arising from a constant force boundary condition. Step 1: initial stress after relaxation σ', according to Eq. (4); stress $\delta\sigma$ added to σ' (giving $\sigma(t_1)$), compensating the implicit force reduction due to the relaxation (i.e. difference between $\sigma(t_0)$ and σ') in order to maintain the constant force boundary condition.

and an explicit finite-difference algorithm with an adaptive time step has been used to calculate the stress evolution. How the calculations proceed is schematically illustrated in Fig. 2. The initial stress distribution is elastic and uniform with depth, viz.:

$$\sigma = \frac{E}{1-\nu^2}\varepsilon_e = \frac{E}{1-\nu^2}\varepsilon = \frac{F}{\Delta z} = \text{constant in depth.} \tag{8}$$

Subsequently, two calculations are made at each time step. First, the stress relaxation (i.e. transition of elastic strain into residual strain) is determined according to Eq. (4) and, second, the stress lost through relaxation is reconstituted in order to maintain the total integrated force, which is, by definition, constant in time. This is seen as "Step 1" in Fig. 2, with the incremental stress being:

$$\delta\sigma = \frac{F - \int \sigma' \,\mathrm{d}z}{\Delta z}, \tag{9}$$

with σ' being the stress after the relaxation phase. This process is repeated at time steps chosen to be much shorter than the Maxwellian visco-elastic relaxation at the weakest point in the lithosphere according to its viscosity–depth distribution, thus ensuring numerical stability. A less stringent constraint was adopted for those time steps in which strain rate variations with depth were insignificant:

$$\mathrm{d}t = 0.01\left|\frac{P}{\Delta z\,\delta\varepsilon_r}\frac{1-\nu^2}{E}\right|, \tag{10}$$

where $\delta\varepsilon_r$ is the bulk strain increase during the previous time step (even in such a case, the initial time step is less than 1 s).

Total strain rate in the viscous model, like strain, comprises an elastic component $\dot{\varepsilon}_e$ and a non-elastic, residual, component $\dot{\varepsilon}_r$. The "elastic strain rate" is locally non-zero because of relaxation of elastic stresses with time although the depth integral of $\dot{\varepsilon}_e$ is zero given the constant force boundary condition. The instantaneous bulk strain rate in the viscous constant force model is:

$$\dot{\varepsilon}_{\mathrm{bulk}}(t) = \frac{1}{\Delta z}\int(\dot{\varepsilon}_e + \dot{\varepsilon}_r)\,\mathrm{d}z = \frac{1}{\Delta z}\int \dot{\varepsilon}_r\,\mathrm{d}z, \tag{11}$$

with the time-averaged bulk strain rate determined from the total strain:

$$\langle\dot{\varepsilon}_{\mathrm{bulk}}\rangle = \frac{\varepsilon_{\mathrm{bulk}}}{\Delta t}. \tag{12}$$

The V-type model differs from the P-type model in two significant ways: (1) stresses and strains are time-dependent and (2) strain rate is not independent but rather is a function of the applied force. Since the V-type model implicitly accounts also for brittle deformation (as a consequence of the imposed stress relaxation at each time step), it might better be called a non-linear elasto-visco-plastic model (analogously, the P-type model could be referred to as elasto-plastic).

3. Model results

The stress distribution in the lithosphere under compression for selected P-type and V-type models is shown in Fig. 3, calculated for different thermal regimes and applied plate-tectonic forces, as indicated. All rheological and material parameters are otherwise the same (cf. Table 1). Results for extension in the lithosphere are qualitatively similar but are not discussed further.

3.1. P-type model results

For each compositional layer, the P-type stress distributions consist of up to three segments, from top to bottom (e.g. Fig. 3a, black lines): one where stress increases linearly with depth (Byerlee's law; Eq. (5)), one in which stress is constant with depth (the elastic "core" of the layer, related to the applied force), and one in which stress decreases non-linearly with depth (creep law; Eq. (3)). With decreasing strain rate, for given force and geotherm, the thickness of the elastic core of each layer decreases. As temperatures in the lithosphere increase (i.e. higher heat flow), for given force and strain rate (e.g. Fig. 3a and c), the thickness of the elastic core also decreases, and in fact vanishes in some cases (e.g. Fig. 3c and d, at slower strain rates). As the applied force increases, for given strain rate and geotherm (e.g. Fig. 3a and b), the stress levels within the elastic cores increase, with a complementary decrease in the core thickness. Thus, increasing applied force can also have the effect of causing the elastic core to vanish.

A P-type model stress distribution in which there remain no elastic cores of layers (e.g. Fig. 3d; $\dot{\varepsilon} = 4 \times 10^{-14}\ \mathrm{s}^{-1}$) is equivalent to a YSE for the same model parameters (excluding applied force, which is not relevant to the calculation of a YSE).

3.2. V-type model results

The V-type stress distributions (thicker grey lines in Fig. 3), are similar in shape to the P-type stress distributions for the chosen periods of loading (∼1–10 Myr), but these are not dependent on a given strain rate. For the V-type model, bulk strain rate (Eq. (11)) decreases through time as a result of stress relaxation in the ductile regimes of the

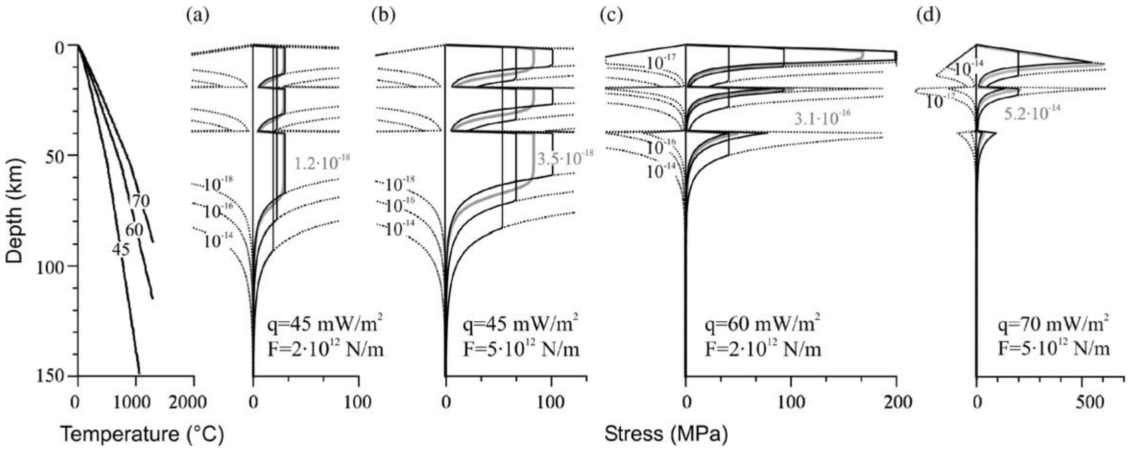


Fig. 3. Comparison of lithosphere stress profiles calculated for the V-type (thicker grey lines) and P-type (thinner black lines) models, for different geotherms and applied forces. (a) $F = 2 \times 10^{12}\ \mathrm{N\,m^{-1}}$ and surface heat flow $q = 45\ \mathrm{mW\,m^{-2}}$; (b) $F = 5 \times 10^{12}\ \mathrm{N\,m^{-1}}$ and surface heat flow $q = 45\ \mathrm{mW\,m^{-2}}$; (c) $F = 2 \times 10^{12}\ \mathrm{N\,m^{-1}}$ and surface heat flow $q = 60\ \mathrm{mW\,m^{-2}}$; (d) $F = 5 \times 10^{12}\ \mathrm{N\,m^{-1}}$ and surface heat flow $q = 70\ \mathrm{mW\,m^{-2}}$. The P-type model stresses are calculated for various strain-rates, as labelled, with corresponding yield-strength envelopes plotted as dashed lines. The V-type model stresses shown are those in all cases developed after 1 Myr of loading, with the total strain rate (grey font) occurring at this time also indicated. Geotherms corresponding to the indicated surface heat flow values are shown at the left (parameters listed in Table 1).

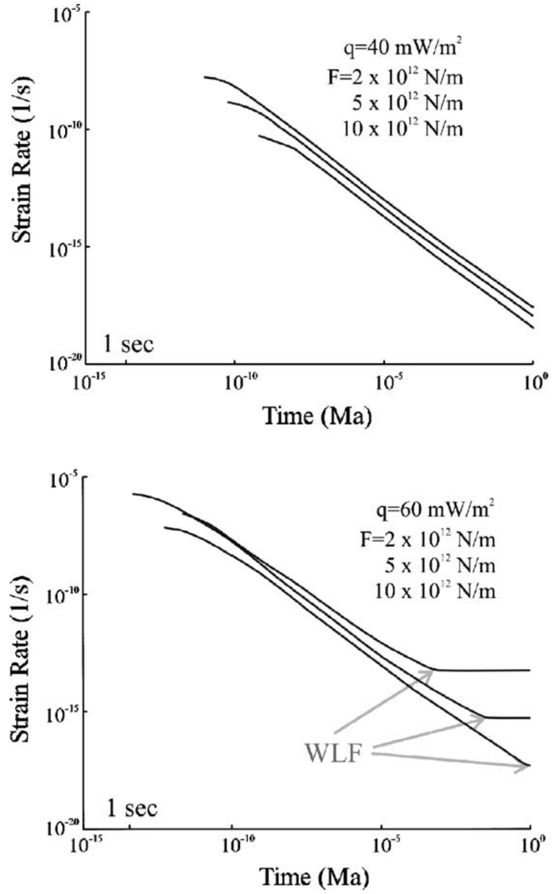


Fig. 4. Evolution of strain rate through time in the V-type model for two different heat flow values and various applied forces (higher force, higher strain rate). The lithosphere reaches a critical stress regime when the strain rate becomes constant in time (labelled WLF—whole lithospheric failure).

lithosphere and complementary amplification in the elastic "cores" of layers. Strain rate reduction through time for different surface heat flows (geotherms) and applied forces and is shown in Fig. 4.

V-type models in which there remains an elastic core in one or more layers (Fig. 3a–c) can be considered to be "not broken", using the terminology of Kusznir (1982) and Kusznir and Park (1984, 1987); here, this is referred to the "sub-critical" stress regime. When all of the brittle and ductile parts of the lithosphere stress distribution coalesce (i.e. all elastic cores vanish), such as in the model shown in Fig. 3d (and almost in Fig. 3c), the lithosphere can be considered to be "broken" (i.e. Kusznir). Here, this is called the "critical" stress state. Kusznir referred to this as the stress state at which whole lithospheric failure (WLF) would occur. In time, under a constant applied force, the critical stress state is eventually reached when the strain rate stops decreasing and becomes constant. These are labelled WLF in Fig. 4 [strictly speaking, WLF, as indicated here, is not exactly the same as defined by Kusznir and Park (1987) who defined, rather, a critical force being the force inducing a certain amount of strain (10%) at a given time (1 Myr). The difference between these two definitions is evident at low forces, where even a "broken" lithosphere remains essentially undeformed for long periods of geological time because of the low strain rates at which WLF occurs].

The critical lithospheric stress state (WLF) is reached at different times (since loading) and different strain rates depending on the magnitude of the applied force and the temperature structure of the lithosphere (i.e. geotherm). This is illustrated in Figs. 5 and 6. Fig. 5 plots the calculated strain rate after 1 Myr of loading, the shaded area indicating the "critical" force–geotherm space in which WLF has occurred by this time. Fig. 6a shows the duration of loading required to reach WLF as a function of applied force and heat flow. The higher the heat flow, the lower the necessary applied force to achieve WLF in the same amount of time. Fig. 6b shows the strain rate occurring at WLF (referred to as "critical strain rate") as a function of applied force and heat flow, for whatever duration of time that was necessary

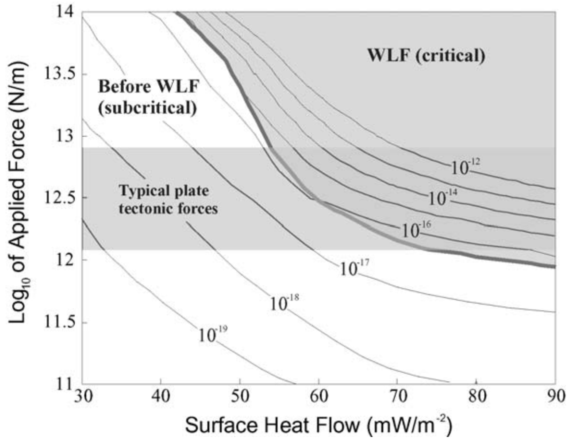


Fig. 5. Calculated strain rates in the V-type model after 1 Myr of loading as a function of applied force and heat flow; the shaded region is where the stress state has become critical by this time, where WLF can be considered to have occurred. The horizontal shaded band indicates the range of typical plate tectonic forces in the lithosphere (e.g. Turcotte and Schubert, 1982).

to achieve it (which can be read directly from the corresponding point in Fig. 6a) [note that the state of stress in the lithosphere occurring at the time at which WLF is achieved is not necessarily a "final" stress-state that cannot be exceeded. A higher applied force for the same model leads to higher stresses in a stress distribution that remains one of WLF (Fig. 7)].

4. Discussion

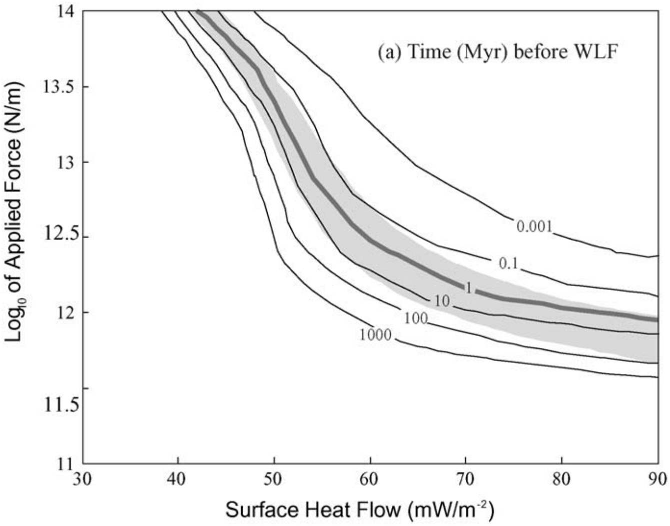
4.1. Comparison of P-type and V-type model results

The stress distributions in the lithosphere calculated for the P-type and V-type models, which both assume a uniform strain rate in the lithosphere, are broadly similar (Fig. 3). However, strain rates in the P-type models are assumed and are time-invariant whereas strain rates in the V-type models evolve with time. Nevertheless, when stress distributions are similar, for given forces and heat flows, the strain rates are also similar. It follows that strain rate in P-type models is not independent but should be considered as a function of applied force and thermal regime. This is, of course, mathematically impossible to do within the framework of the P-type model but the V-type model results do afford some insight into the ranges of strain rates that are appropriate for P-type models for different tectonic regimes (forces and geotherms).

Kusznir (1982) described the mechanism that controls strain rate and stress distribution in the lithosphere prior to WLF in the V-type model. It is one of stress redistribution from ductile deformation zones into elastic ones, as determined by stress relaxation in the former and stress amplification in the latter given the requirement of a constant applied boundary force. A stable equilibrium regime of deformation occurs when, at each point within the lithosphere, the stress reduction effected by viscous relaxation equals the stress increase effected by the applied force. The stress distribution represented by a YSE (Eq. (3)) is, in effect, such an equilibrium stress distribution in the framework of the V-type model in a state of WLF.

Furthermore, it can be demonstrated mathematically that, prior to reaching a state of WLF, the stress distribution in the V-type model will always tend towards an equilibrium state. That is, if stress at some point is ever higher than the equilibrium stress then it will always decrease with time because of relaxation and, conversely, if stress at some point is ever less than the equilibrium stress then it will always increase with time. This is demonstrated in Fig. 8 for the particular stress distribution in Fig. 3b. The total strain rate $\dot{\varepsilon}_e + \dot{\varepsilon}_r$ is uniform with depth (indicated by the strain rate shown in Fig. 3b) but $\dot{\varepsilon}_e$ and $\dot{\varepsilon}_r$ each vary with depth. Stresses in the ductile regions decrease because they are higher than the equilibrium stresses (the YSE in the left panel) whereas stresses in the elastic regions increase because they are lower.

It follows that one output of the V-type model is the bulk strain rate appropriate to a P-type model given an applied force, geotherm, and duration of loading based on other observational data. In practice, P-type models are typically



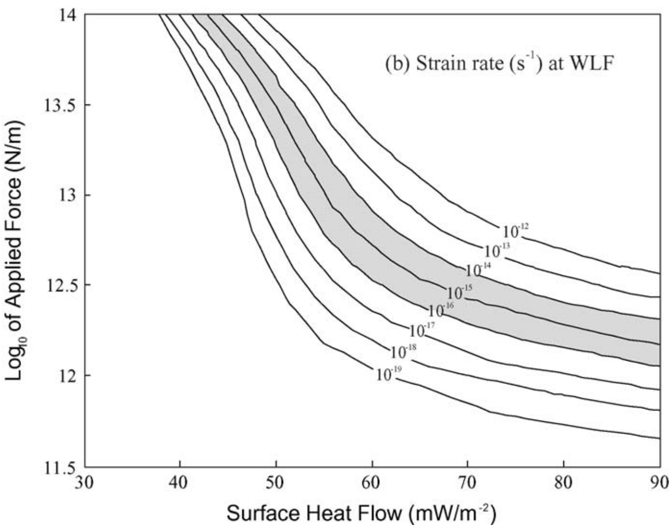


Fig. 6. (a) Duration of loading (in Myr) required to reach WLF and (b) critical strain rates (in s^{-1}) occurring at WLF in the V-type model as functions of applied force and heat flow. The 1 Myr isochron in (a) corresponds to the left boundary of the shaded (critical) area in Fig. 5. The duration of loading necessary to achieve the critical strain rates in (b) can be read at the corresponding point in (a). The shaded regions in (a) and (b) correspond to critical strain rates in the range of 10^{-14} to 10^{-16} s^{-1}.

constructed without taking into account any explicit knowledge of the duration of loading and bulk strain rates are chosen in the range of 10^{-14} to 10^{-16} s^{-1}. The V-type model results summarised in Fig. 5, however, which have been calculated with a duration of loading of only 1 Myr indicate strain rates in sub-critical ("unbroken") lithosphere, for typical plate tectonic forces, that are only 10^{-17} s^{-1} and less. For longer periods of loading, which might very well be appropriate for some tectonic settings, even lower bulk strain rates are indicated for lithosphere that has not reached WLF (cf. Fig. 6).

4.2. Calibration of the P-type model and YSEs

As discussed in the Section 4.1, the proper choice of strain rate in the P-type model gives a reasonable estimation of lithospheric stress distribution. This strain rate depends not only on the intrinsic parameters of the lithosphere, such as those controlled by its thermal state, but also on the imposed force and on time. It follows that the adopted strain rate in a P-type model aimed at calculating a lithospheric stress distribution should be calibrated according to

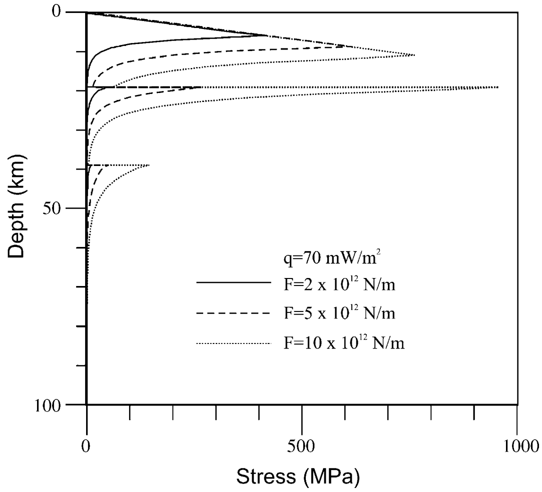


Fig. 7. Stresses developed in a V-type model in WLF at different applied forces and surface heat flow $q = 70\,\mathrm{mW\,m^{-2}}$. The dashed line ($F = 5 \times 10^{12}\,\mathrm{N\,m^{-1}}$) is the same as in Fig. 3d.

applied forces and duration of loading, even though the latter is not a parameter in the P-type model. This contrasts with many published studies in which stresses have been calculated using an arbitrarily adopted strain rate (e.g. Burov and Diament, 1995; Ershov, 1999). However, given some estimate of duration of loading appropriate for the problem under study, a "calibrated" P-type stress distribution can be calculated using the method described in Appendix A for choosing a more the relevant strain rate.

This is also true for calculating YSEs for the same reasons. From the physical point of view, as explained above, the YSE represents the equilibrium state of lithospheric deformation. In the framework of the P-type model, the strain rate value appropriate for calculating a YSE can be found only for a V-type model in the critical (WLF) regime of lithospheric deformation. This will, of course, be the strain rate that produces a YSE that, integrated with depth, is equivalent to the

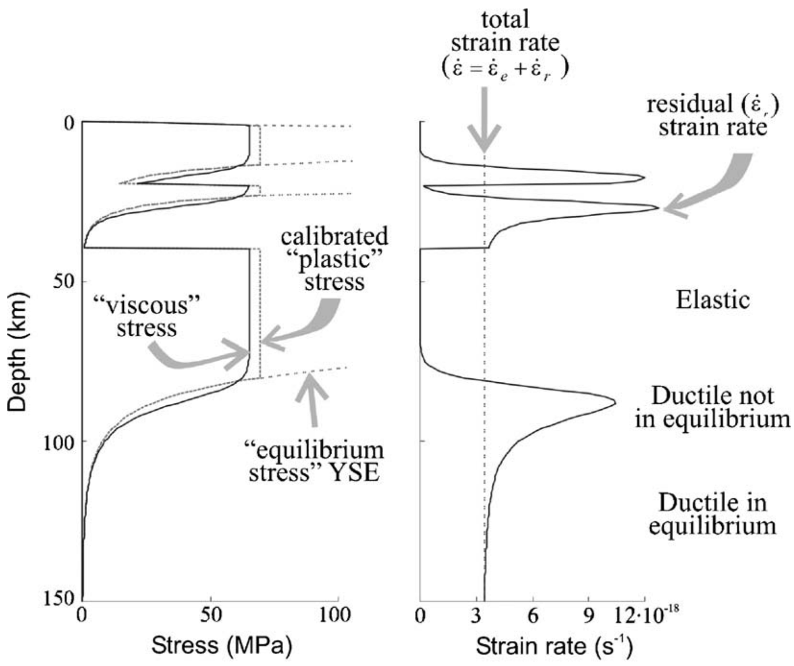


Fig. 8. Comparison of stresses for the calibrated P-type model and stresses for the V-type model (left) and the strain rate distribution with depth in the V-type model with $F = 5 \times 10^{12}\,\mathrm{N\,m^{-1}}$ and surface heat flow $q = 45\,\mathrm{mW\,m^{-2}}$ after 1 Myr.

applied force. A YSE calculated using a properly chosen strain rate is hereafter referred to a "calibrated" YSE (CYSE). The properly chosen strain rate can be determined relatively simply, evaluating YSEs iteratively, but this provides no way of determining when, or even if, the critical stress regime represented by the YSE will indeed ever be reached in the lithosphere under study (cf. Fig. 6). Appendix A presents a simple method for finding the calibrated strain rate, without the requirement of determining the true viscous strain rate at WLF. Fig. 6 shows that not only are appropriately chosen strain rates (for calculating CYSEs) less than $10^{-16}\,\mathrm{s}^{-1}$ for a wide range of applied forces and heat flows, but the time necessary to achieve these are of very great duration.

One way in which YSEs have been utilised is in characterising rheological changes with depth in the lithosphere. This role can be given to the CYSE, which represents the stress distribution at the moment of WLF. Nevertheless, it is important to remember that it does not really represent yield-strength: at lower forces (and therefore strain rates), the stresses in the ductile part will be lower, at higher forces (and strain rates), they will be higher (cf. Fig. 7). Since the calculated stress distribution has more to do with changes in the rheological properties of rocks with depth than with their yield stresses, the term "rheological profile" (Ranalli, 1995) rather than "yield-strength envelope" is preferred.

4.3. Strength of the lithosphere

The results presented above have important geodynamic implications for using YSEs in qualitatively comparing the strength of the lithosphere in different tectonic regions (e.g. Cloetingh and Burov, 1996; Cloetingh et al., 2006). In the framework of the P-type model, the total lithospheric strength (TLS) can be defined as the integral of the YSE (Ranalli, 1995). In the framework of the V-type model, TLS can be analogously defined as the minimum force necessary to induce WLF at some time. For any given set of model parameters, this force is intrinsically linked with a certain strain rate, which is, by definition, the calibrated strain rate used in calculating the CYSE.

As already discussed, calibrated strain rates are typically less than those adopted in the literature for the calculation of YSEs and it follows that TLS for the former will be less. As seen in Fig. 5, typical bulk strain rates of the non-broken lithosphere in the vicinity of WLF (calibrated strain rates therefore) are in the range of 10^{-17} to $10^{-16}\,\mathrm{s}^{-1}$, for moderately hot and hot lithosphere, and greater than $10^{-16}\,\mathrm{s}^{-1}$ for normal lithosphere ($q < \sim 55\,\mathrm{mW\,m}^{-2}$) given typical plate-tectonic forces in the range of 10^{12} to $10^{13}\,\mathrm{N\,m}^{-1}$. This range of strain rate values does overlap with the range of those used in many published studies (10^{-16} to $10^{-14}\,\mathrm{s}^{-1}$) but, at typical plate-tectonic force levels (e.g. Turcotte and Schubert, 1982), the calibrated strain rates are one to three orders of magnitude lower. Given that three orders difference in strain rate is about equivalent to about one order difference in stress and TLS (because $\dot{\varepsilon} \propto \sigma^N$ and N is about 3 for the mantle), TLS could therefore be overestimated by about one-half to one orders of magnitude.

4.4. Bulk rheology of the lithosphere

By "bulk rheology" of the lithosphere, it is meant the non-flexural response to applied forces of the lithosphere as a whole. For the V-type lithosphere considered here the dependence on force is non-linear and is characterised by strengthening with increasing applied force (i.e. "bulk" effective viscosity increases with increasing stress). It follows that applied force may still increase even when the lithosphere is in a state of WLF (cf. Fig. 5). This avoids some problematic implications of models in which lithosphere is characterised in terms of a standard YSE. In such models the lithosphere is considered to be unable to support forces larger than the TLS. However, this appears to inconsistent with observations that suggest that large tectonic stresses can be transferred into the deep interiors of continents through weak zones (such as rifts and active orogenic belts) at their margins and elsewhere (cf. Ziegler et al., 1998). For the V-type model, however, this is explicable in terms of the implicit strengthening of the lithosphere.

5. Conclusions

Creep in the lithosphere is viscous in terms of basic mechanical–rheological models. The description of creep in terms of a plastic rheology (perhaps having its origin with the term "plastic" as used in the material sciences, where it means a type of deformational mechanism) in calculating YSEs without further analysis can lead to internal inconsistencies. Taking into account the implications of adopting a viscous rheology in place of a plastic one in evaluating the "strength" of the lithosphere, it is necessary to incorporate the time-dependence of stresses, strains and strain rates and also the dependence of the bulk strain rate on the total applied force.

The main conclusions are as follows:

(1) Total applied force and bulk strain rate are not independent. In order to calculate a strength profile (YSE), given other thermo-compositional parameters, it is first necessary to calculate a bulk strain rate appropriate for the applied forces and duration of loading of the specific problem under consideration.
(2) For "typical" plate boundary forces and "normal" geotherms, appropriate bulk strain rates are very low, about $10^{-17}\,s^{-1}$ or less, and loading durations can be very long. Lithospheric "strength" computed using a constant bulk strain rate in the range of 10^{-16} to $10^{-14}\,s^{-1}$ is overestimated by one-half to one order of magnitude. However, it is noted that the uncertainties in choosing rheological and thermo-compositional model parameters are, in general, considerable. As such, any discrepancies in calculating lithosphere strength assuming plastic rheology in place of viscous, although probably not negligible, may not be fundamental given the rheological and thermo-compositional uncertainties.
(3) In many—perhaps most—cases of "typical" plate boundary forces and "normal" geotherms, WLF as predicted by the V-type model will not be approached during a geologically reasonable loading time (Fig. 6a). Therefore, stresses calculated by means of a non-calibrated YSE (implicitly assuming independence of bulk strain rate and force) can be significantly over- or under-estimated, in contrast to the lithospheric "strength". Stresses computed by means of a CYSE (i.e. when bulk strain rate is correlated with applied force and duration of loading), however, are close to the stresses computed in the framework of the V-type ("viscous creep") model that corresponds with available laboratory data. It is therefore possible to use the CYSE to estimate a more realistic lithospheric stress distribution, at least for homogeneous deformation of lithosphere.

Finally, it is noted that the results presented here do not apply in the case of a flexed lithosphere. The CYSE of the flexed lithosphere will, in general, be different from the CYSE of uniformly deformed lithosphere when the change of plate curvature with time is not zero.

Acknowledgements

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Appendix A. Calculation of calibrated strain rates

To calculate stress in the lithosphere using the V-type method described above is computationally very expensive. However, the relatively simple shape of the strain rate versus time curve (Fig. 4) allows the possibility of using some approximate methods, which are discussed below.

The strain rate dependence on time may be approximated by two linear segments in a log–log graph (Fig. A.1). The first segment corresponds to strain rates in the sub-critical stress regime with strain rate decreasing with time; the second corresponds with the critical regime with strain rate constant with time. The latter can easily be estimated by choosing a strain rate that equates the integral of the YSE and the total applied force; i.e. by finding the solution to:

$$\int \sigma_{\mathrm{yield}}(\dot{\varepsilon})\,\mathrm{d}z = F.$$

An estimate of strain rate at two points in time is all that is necessary to approximate the sub-critical part of the curve. For the V-type model rheology, relaxation after the instantaneous application of loading under the constant total strain condition can be written as:

$$\dot{\varepsilon} = \dot{\varepsilon}_{\mathrm{e}} + \dot{\varepsilon}_{\mathrm{r}} = \frac{\sigma}{E} + C\sigma^{N} = 0$$

where

$$C = A_{\mathrm{p}} \exp\left(\frac{E_{\mathrm{p}}}{RT}\right).$$

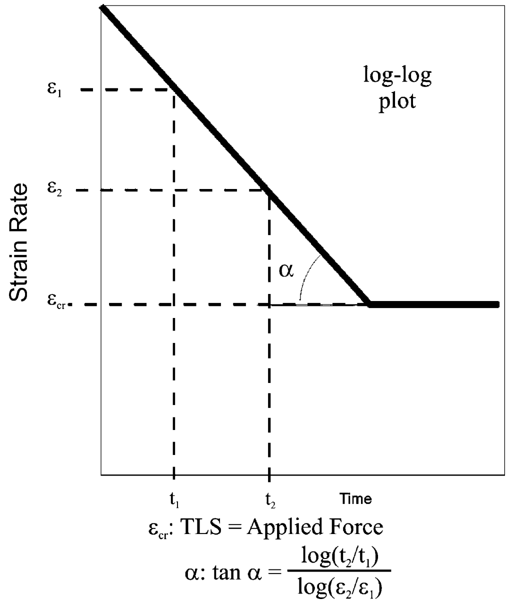

Fig. A.1. Simplified strain-rate vs. time dependence in the V-type model.

Solving for stress as a function of time gives:

$$\sigma(t) = \left(\sigma_0^{1-N} + (N-1)CEt\right)^{1/(1-N)}$$

where σ_0 is initial stress:

$$\sigma_0 = \frac{F}{\Delta z}.$$

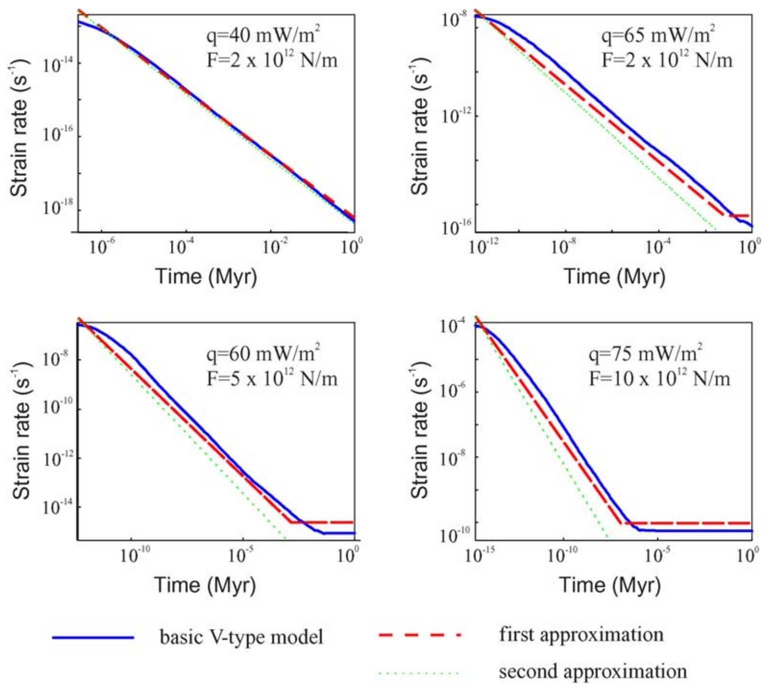

Fig. A.2. Strain rate vs. time dependencies in the V-type model, with approximations as discussed in the text for different applied forces and surface heat flows.

This allows stress to be determined as a function of depth:

$$\sigma(t, z) = \left(\sigma_0^{1-N} + (N-1)C(z)Et\right)^{1/(1-N)},$$

and the bulk strain rate as:

$$\langle \dot{\varepsilon}_{\rm r} \rangle = \frac{1}{\Delta z} \int \dot{\varepsilon}_{\rm r}\, {\rm d}z = \frac{1}{\Delta z} \int C\sigma^N\, {\rm d}z.$$

Thus, strain rate can be determined at two times and the slope of the line in Fig. A.1 estimated. A major weakness inherent to this method is that total applied force is not conserved. A better, but less computationally efficient, approximation can be made by letting σ_0 change in time to simulate stress relaxation:

$$\sigma(t, z) = \left(\sigma_0(t)^{1-N} + (N-1)C(z)Et\right)^{1/(1-N)},$$

in order to equalise the integral of lithospheric stresses and total applied force; that is, $\sigma_0(t)$ is found to satisfy the equation:

$$\int \sigma(t, z)\, {\rm d}z = F.$$

The accuracy of these approximations is illustrated in Fig. A.2.

References

Artyushkov, E.V., 1983. Geodynamics. Elsevier, Amsterdam, 312 pp.

Ave Lallemant, H.V., Mercier, J-C.C., Carter, N.L., Ross, J.V., 1980. Rheology of the upper mantle: inferences from peridotite xenoliths. Tectonophysics 70, 85–113.

Burov, E.B., 2003. The upper crust is softer than dry quartzite. Tectonophysics 361, 321–326.

Burov, E.B., Diament, M., 1995. The effective elastic thickness of continental lithosphere: what does it really mean? J. Geophys. Res. 100, 3905–3927.

Byerlee, J.D., 1978. Friction of rock. Pure Appl. Geophys. 116, 615–626.

Carter, N.L., Kirby, S.H., 1978. Transient creep and semibrittle behaviour of crystalline rocks. Pure Appl. Geophys. 116, 807–839.

Carter, N.L., Tsenn, M.C., 1987. Flow properties of continental lithosphere. Tectonophysics 136, 27–63.

Cloetingh, S., Banda, E., 1992. Europe's lithosphere: physical properties, mechanical structure. In: Blundell, D., Freeman, R., Mueller, S. (Eds.), A Continent Revealed: The European Geotraverse. Cambridge University Press, Cambridge, pp. 80–91.

Cloetingh, S., Burov, E.B., 1996. Thermo-mechanical structure of European continental lithosphere: constraints from rheological profiles and EET estimates. Geophys. J. Int. 124, 695–723.

Cloetingh, S., Cornu, T., Ziegler, P.A., Beekman, F., The Environmental Tectonics (ENTEC) Working Group, 2006. Neotectonics and intraplate continental topography of the northern Alpine Foreland. Earth-Sci. Rev. 74, 127–196.

De Rito, R.F., Cozzarelli, F.A., Hodge, D.S., 1986. A forward approach to the problem of non-linear viscoelasticity and the thickness of the mechanical lithosphere. J. Geophys. Res. 91, 8295–8313.

Ershov, A.V., 1999. Effective middle surface of lithosphere. Earth Planet. Sci. Lett. 173, 129–141.

Fernandez, M., Ranalli, G., 1997. The role of rheology in extensional basin formation modelling. Tectonophysics 282, 129–145.

Fletcher, F.C., Hallet, B., 1983. Unstable extension of the lithosphere: a mechanical model for basin-and-range structure. J. Geophys. Res. 88, 7446–7457.

Gilluly, J., 1972. Tectonics involved in the evolution of mountain ranges. In: Robertson, E.C. (Ed.), The Nature of the Solid Earth. McGraw-Hill, New York.

Goetze, C., Evans, B., 1979. Stress and temperature in the bending lithosphere as constrained by experimental rock mechanics. Geophys. J. Roy. Astron. Soc. 59, 463–478.

Kohlstedt, D.L., Evans, B., Mackwell, S.J., 1995. Strength of the lithosphere: constraints imposed by laboratory experiments. J. Geophys. Res. 100, 17587–17602.

Kusznir, N.J., 1982. Lithosphere response to externally and internally derived stresses: a visco-elastic stress guide with amplification. Geophys. J. Roy. Astron. Soc. 70, 399–414.

Kusznir, N.J., Park, R.G., 1984. Intraplate lithosphere deformation and the strength of the lithosphere. Geophys. J. Roy. Astron. Soc. 79, 513–538.

Kusznir, N.J., Park, R.G., 1987. The extensional strength of the continental lithosphere: its dependence on geothermal gradient and crustal composition and thickness. In: Coward, M.P., Dewey, J.F., Hanckock, P.L. (Eds.), Continental Extensional Tectonics, vol. 28. Geological Society, London, pp. 35–52 (Special Publications).

Lankreijer, A., Bielik, M., Cloetingh, S., Majcin, D., 1999. Rheology predictions across the western carpathians, Bohemian massif, and the Pannonian basin: implications for tectonic scenarios. Tectonics 18, 1139–1153.

Leonov, A.I., Prokunin, A.N., 1994. Non-Linear Phenomena in Flows of Visco-Elastic Polymer Fluids. Chapman and Hall, London, p. 475.

Liu, L., Zoback, M.D., 1997. Lithospheric strength and intraplate seismicity in the New Madrid seismic zone. Tectonics 16, 585–595.

Mercier, J.-C.C., 1980. Magnitude of continental lithosphere stresses inferred from rheomorphic petrology. J. Geophys. Res. 85, 6293–6303.

Porth, R., 2000. A strain-rate-dependent force model of lithospheric strength. Geophys. J. Int. 141, 647–660.

Ranalli, G., 1995. Rheology of the Earth, 2nd ed. Chapman and Hall, London, p. 415.

Ranalli, G., 1997. Rheology and deep tectonics. Ann. Geofis. XI, 671–680.

Ranalli, G., 2003. How soft is the crust? Tectonophysics 361, 319–320.

Stephenson, R.A., Ricketts, B.D., Cloetingh, S.A., Beekman, F., 1990. Lithosphere folds in the Eurekan Orogen, Arctic Canada? Geology 18, 603–606.

Tsenn, M.C., Carter, N.L., 1987. Upper limits of power law creep of rock. Tectonophysics 136, 1–26.

Turcotte, D.L., Schubert, G., 1982. Geodynamics—Applications of Continuum Physics to Geological Problems. John Wiley and Sons, New York, p. 450.

Whitten, C.A., 1956. Crustal movements in California and Nevada. Eos 37, 393.

Ziegler, P.A., van Wees, J.-D., Cloetingh, S., 1998. Mechanical controls on collision-related compressional intraplate deformation. Tectonophysics 300, 103–129.