Available online at www.sciencedirect.com

SCIENCE DIRECT®

Earth and Planetary Science Letters 243 (2006) 383 – 389

EPSL

www.elsevier.com/locate/epsl

Detection of a gravitational oscillation in length-of-day

Jon E. Mound [a,*], Bruce A. Buffett [b]

[a] *Department of Earth and Planetary Sciences, Harvard University, Cambridge, MA 02138, United States*
[b] *Department of Geophysical Sciences, University of Chicago, Chicago, IL 60637, United States*

Received 18 October 2005; received in revised form 17 January 2006; accepted 22 January 2006
Available online 28 February 2006
Editor: R.D. van der Hilst

Abstract

A high resolution length-of-day data set spanning approximately the last 42 years is found to contain an oscillation with a period of 5.8 ± 0.8 years. This oscillation is interpreted to represent a normal mode of the Earth that arises due to gravitational coupling between the mantle and inner core. In order to match the observed period of the oscillation, the strength of the gravitational coupling between the mantle and inner core must be $\sim 3.0 \times 10^{20}$ N m. This coupling constant depends on the heterogeneous density distribution of the mantle, including flow-induced deformation of the core–mantle boundary. The existence of this normal mode requires that the relaxation time of the inner core be sufficiently long (on the order of a few years or more) that a perturbation from equilibrium results predominantly in solid-body rotation, not viscous deformation, of the inner core. The inferred lower bound for the viscosity of the inner core is of the order of 10^{17} Pa s.

Keywords: length-of-day; core–mantle coupling; inner core; mantle structure

1. Introduction

Length-of-day (LOD) is observed to vary on a wide range of timescales (e.g. [1]); interannual LOD variations are of particular interest in this paper. Variations in LOD correspond to changes in the rotation rate of the solid Earth and hence the angular momentum of the mantle. If no external forcings exist, then conservation of angular momentum implies that LOD variations correspond to angular momentum exchange between the mantle and some other region within the Earth. For example, changes in LOD are known to be well correlated with variations in atmospheric angular momentum at a number of frequencies (see e.g. [2–5]).

An LOD variation with a period near 6 yr has been inferred from spectral analysis of approximately 50 yr of high resolution data; this oscillation is not explained by observationally constrained estimates of angular momentum exchange with the atmosphere, oceans, and continental water reservoirs [6,7] and thus likely represents an interaction with the core. The possibility remains that the observed LOD variation arises due to unrecognised surface mass transport, atmospheric or oceanic effects. One such possibility is ocean circulation change related to the El Nino Southern Oscillation (ENSO). However, spectra of ENSO variability during the twentieth century have interannual power at a period of approximately 4.5 yr (as opposed to the observed 6-yr

---

* Corresponding author.
E-mail address: mound@geophysics.harvard.edu (J.E. Mound).

0012-821X/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.epsl.2006.01.043

LOD signal) and the ENSO effect on LOD appears to occur primarily through changes in atmospheric angular momentum [8,9]. An atmosphere corrected LOD time series [10] correlates poorly (correlation coefficients of the order of 0.2–0.3) with ENSO indices [11–14], further suggesting that the observed LOD oscillation is unrelated to oceanic ENSO effects. Other changes in ocean circulation may influence interannual LOD; however, models suggest that oceanic contributions to LOD on longer than yearly timescales are small [R. Holme, personal communication].

At decadal periods LOD has been shown to correlate with changes in core angular momentum, as determined from core flow models constrained by observations of geomagnetic field variations (e.g. [15–18]). These fluctuations in core angular momentum are attributed to a class of normal modes termed torsional oscillations [19]. Torsional oscillations are standing waves in the fluid core, the period and spatial structure of the normal modes being controlled primarily by the strength of the radial magnetic field within the outer core. The period of the fundamental torsional oscillation normal mode is expected to be on the order of several decades [19–21] and in principle an infinite number of higher harmonics exist. A suite of interannual torsional oscillations may therefore be expected [22–24].

We find that a suite of interannual torsional oscillations cannot, by themselves, account for the observed ~6 yr LOD oscillation. Instead, we propose that this oscillation arises due to a rotational normal mode associated with gravitational coupling between the mantle and inner core [25,22,24]. The mantle–inner core gravitational (MICG) mode arises due to the non-hydrostatic shape of the mantle and inner core [25]. The strength of the gravitational coupling between the mantle and inner core, and thus the period of oscillation, depends on the heterogeneous distribution of mass in the mantle, including the flow-induced deformation of the CMB. The mantle density field controls the shape of gravitational equipotential surfaces inside the core and hence the shape of the inner core. Detection of the MICG mode in the LOD data set can therefore lead to a constraint on the non-hydrostatic structure of the mantle.

Detection and interpretation of the MICG mode require that it be distinguishable from the expected suite of interannual torsional oscillations. The last ~42 yr of high resolution LOD data have been recently reanalysed, correcting for atmospheric effects [10]. This new LOD time series provides a cleaner isolation of the interannual LOD oscillation than earlier studies. We are, therefore, able to better determine its period and constrain our model of the gravitational coupling between the mantle and inner core.

We have developed a model of the torsional oscillations of the core–mantle system designed to investigate variations in the magnetic field and LOD arising from decadal period flows in the outer core [23,24]. The model includes gravitational coupling between the mantle and inner core and thus predicts the presence of the MICG normal mode as well as a suite of interannual torsional oscillation normal modes. As discussed above, the detection of the MICG mode is based on several decades of data which reveal a single peak; a suite of torsional oscillation normal modes near this period are not seen in the observationally derived LOD spectrum. However, the model results correspond to ideal, non-time-limited, spectra and thus should not be compared directly to the physical, limited duration, times series of LOD data. In particular, spectral spreading reduces the frequency resolution of time-limited data and, as discussed below, does not allow for the resolution of individual interannual torsional oscillations. Inclusion of both the MICG normal mode and the effect of spectral spreading in the model enables us to closely match the observations.

2. Methods

The atmosphere corrected LOD time series that we use consists of daily values spanning 41.95 yr. In the study describing this data set [10], the authors smoothed the data by taking a 1 yr running average; the resulting time series was further smoothed, to varying degrees, by determining two spline fits. Simple, visual inspection of the smoothed time series, in particular the time derivative of the smoothed data, reveals the presence of an approximately 6 yr oscillation (see Figs. 2 and 3 of [10]). In order to investigate this oscillation we determine the spectrum of the LOD time series using the atmosphere corrected, but unsmoothed, LOD time series (as discussed above, the interannual, oceanic contribution to this LOD time series is unlikely to be important). After removal of a linear trend, we apply a cosine taper, and pad with zeroes to extend the LOD time series before taking an FFT; we then multiply by $i\omega$ in the frequency domain, which is equivalent to differentiation in the time domain and accentuates the higher frequency components. Since changes in LOD correspond to variations in angular momentum, the derivative of the LOD signal corresponds to a time series of the torque applied to the mantle.

Our model of angular momentum exchange within the core–mantle system was developed for the study of torsional oscillation normal modes which typically have periods on the order of several decades [23,24]. In this model the torsional oscillation normal modes are found by solving an eigenvalue problem developed from the equations describing conservation of angular momentum. Use of the appropriate orthogonality condition leads to a Green's function formulation that allows the response of the system to any arbitrary forcing to be computed [23]. The system response can also be found directly when a periodic forcing is assumed [24]. Here, we use the model to calculate the response of the core–mantle system to a forcing that is distributed in both space and time. The forcing consists of a component distributed throughout the fluid core and a component that is applied directly to the solid core; the forcing that we use is the same as that used in one of our previous studies [24]. The amplitude of the forcing is held constant over the range of frequencies considered (0–10 cycles per year); to reproduce the amplitude of the observed LOD oscillation requires a forcing that results in a torque on the inner core of the order 10^{17} N m.

The LOD response derived from the model is processed in the same manner as the data so that the spectra obtained are directly comparable. The sampling frequency of the data is greater than that of the model due to computational considerations; however, the model sampling is sufficiently high to accurately capture the oscillation of interest. In this study, the free parameters of the model are the strength of the gravitational coupling between the mantle and inner core (Γ_g) and the relaxation time of the inner core (τ). (For other relevant physical parameters see Table 1.) The amplitude of the observed spectral peak can always be obtained by adjusting the amplitude of the applied forcing; for simplicity, we eliminate this degree of freedom from the model by comparing the spectra after normalising with respect to the maximum amplitude in the frequency range of interest.

Table 1
Physical parameters used in torsional oscillation models

Parameter	Symbol	Value
Radius of ICB	r_I	1.22×10^6 m
Radius of CMB	r_f	3.48×10^6 m
Fluid core density	ρ_f	1.2×10^4 kg m^{-3}
Inner core moment of inertia	C_i	5.87×10^{34} kg m^2
Mantle moment of inertia	C_m	7.12×10^{37} kg m^2
Radial magnetic field in fluid	B_s	0.2 mT
Magnetic field across ICB	B_i	3.5 mT
Magnetic field across CMB	B_m	0.5 mT
Core conductivity	σ_f	5×10^5 S m^{-1}
Mantle conductance	$\sigma_m\Delta$	10^8 S

3. Results and discussion

Fig. 1 displays the spectrum obtained from the LOD data (solid line). The observed oscillation has a period of 5.84 yr, as determined by a cubic spline fit to the observed spectrum. The uncertainty associated with the numerical fit to the observed spectrum is small; however, due to the relatively short duration of the time series, spectral spreading significantly limits the frequency resolution of the data set. For a time series of duration, t, the frequency resolution of the spectrum is $\Delta f \simeq 1/t$ (e.g. [26]). Based on the frequency resolution of the considered ~42 yr record of LOD we adopt a value of 5.8 ± 0.8 yr as the period of the observed LOD oscillation. This oscillation is likely the result of interaction between the mantle and the core. Fig. 1 also shows the spectra obtained from two models of core–mantle coupling, one which includes gravitational coupling between the mantle and inner core, and one which does not.

Torsional oscillations represent one class of core normal modes that are known to influence LOD. For periods near that of the observed LOD oscillation individual torsional oscillation normal modes are found to have periods separated by about one half year. The frequency spacing between interannual torsional oscillation normal modes is less than the frequency resolution of the time-limited time series; thus, the individual modes are not resolved in the model spectrum. Similarly, interannual torsional oscillations that may be present in the Earth's fluid core would not be individually resolved in the observed spectrum due to the effect of spectral spreading. The observed oscillation cannot, therefore, be attributed to a single, interannual, torsional oscillation normal mode; instead, it must be compared to the excitation produced by forcing a suite of closely spaced, interannual, torsional oscillation normal modes.

However, as seen in Fig. 1 (dotted line), a suite of torsional oscillations are not capable of reproducing the distinct interannual peak when only CMB coupling mechanisms are considered (see also [24]). In contrast, when gravitational coupling is included in the model the MICG normal mode results in a distinct interannual peak in the LOD spectrum (Fig. 1, dashed line). We therefore interpret the observed 5.8 yr oscillation in LOD as the signature of the MICG normal mode. The period of the observed oscillation is matched by tuning

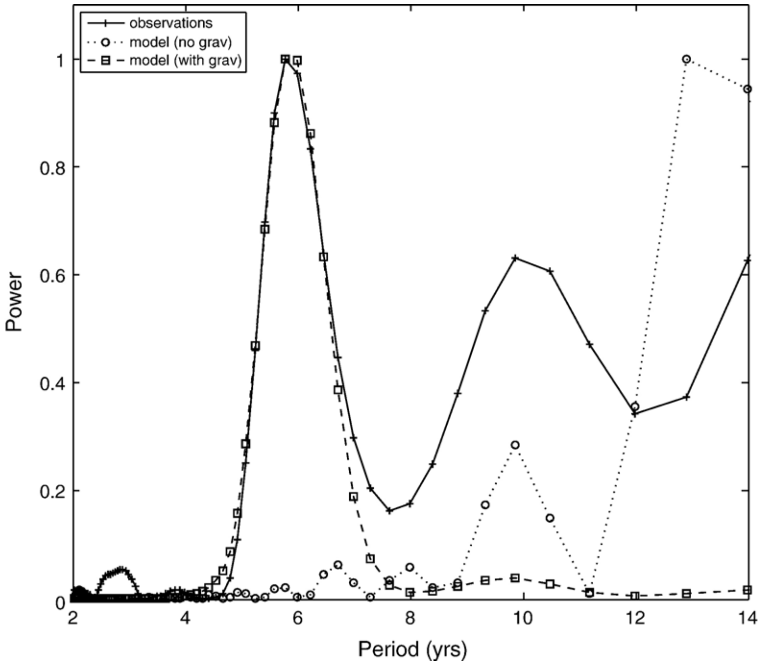

Fig. 1. Spectra of the torque on the mantle versus period for the observations (solid, crosses) and the model with (dashed, squares) and without (dotted, circles) gravitational coupling between the mantle and inner core. For the model with gravitational coupling $\Gamma_g = 2.96 \times 10^{20}$ N m. The inner core is assumed to be rigid ($\tau = \infty$). For other model parameters see Table 1.

of the gravitational coupling constant, Γ_g, to a value of 2.96×10^{20} N m.

Γ_g is a measure of the aspherical (more precisely, the non-zonal) density distribution of the mantle, including flow-induced deformation at the CMB [25]. Deformation of the CMB is of particular importance to determining the strength of the gravitational core–mantle coupling, due to both the large density jump

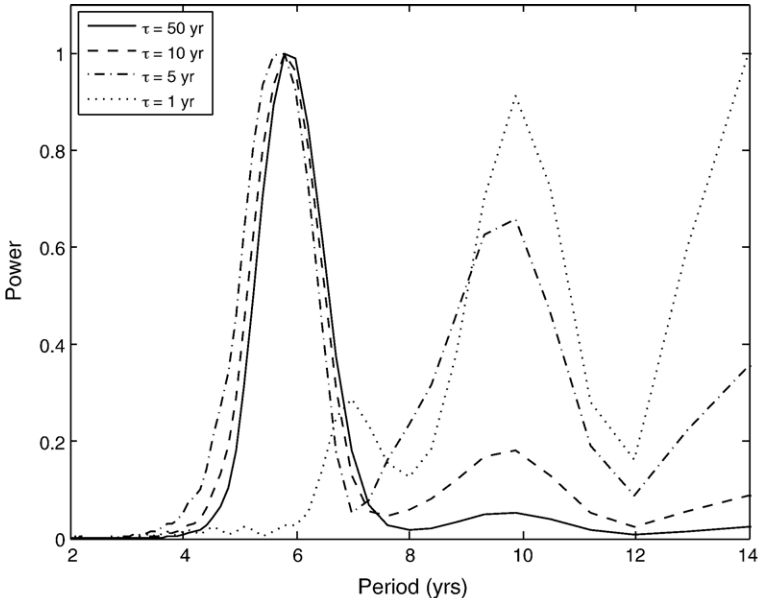

Fig. 2. Mantle torque spectra of models with τ equal to 1 (dotted), 5 (dash-dot), 10 (dashed) or 50 (solid) yr. In all cases $\Gamma_g = 2.96 \times 10^{20}$ N m. For other model parameters see Table 1.

across the CMB and its relative proximity to the inner core. However, the coupling strength depends on density variations throughout the mantle, in part because the shape of the equipotential surfaces inherently depends on the integrated mantle density field, but also because the CMB deformation arises due to mantle flow driven by the overlying density heterogeneities. Lower mantle heterogeneity is dominated by the degree two, order two harmonic (e.g. [27]); therefore, the gravitational coupling constant primarily provides a constraint on the integral of this density component (see [25] for details).

To this point we have assumed a rigid inner core; however, the Earth's inner core is likely to be deformable on the timescale of the MICG oscillation. Work based on mineral physics theory suggests a constraint on inner core viscosity of 10^{11} Pa s, corresponding to an inner core relaxation time, $\tau \simeq 60$ s [28]. Constraints based on the inferred rate of inner core super-rotation allow much higher viscosities; the rate of inner core super-rotation inferred from seismic records leads to an upper bound on inner core viscosity of the order of 10^{17} Pa s [29–31]. The existence of the MICG mode requires an inner core that is not too soft. If the inner core relaxation time is sufficiently short, then the gravitational torque associated with misalignment between the mantle and inner core density fields will be entirely compensated by viscous deformation and the inner core will be free to rotate relative to the mantle.

Fig. 2 shows the effect of inner core relaxation on the model spectrum of the MICG oscillation. The model response for $\tau = 50$ yr (solid line, Fig. 2) is similar to that obtained with a rigid inner core (dashed line, Fig. 1). A relaxation time of 1 yr is sufficiently short that the model cannot match the observed spectrum. As τ is progressively shortened, the spectrum approaches that of the model without gravitational coupling (dotted line, Fig. 1). The inner core relaxation time must be at least several years for the model to match the observed MICG oscillation; if deformation occurs throughout an isoviscous inner core, then this constraint on τ corresponds to a lower bound on inner core viscosity of the order of 10^{17} Pa s [31]. This viscosity value is the same order of magnitude as the upper bound implied by the observationally inferred rate of inner core super-rotation, taken together these results could provide a tight constraint on the allowed value of inner core viscosity.

The value of τ also affects the period of the peak in the model spectra. However, the period of the MICG mode is predominantly determined by the strength of the gravitational coupling between the mantle and inner core. Fig. 3 shows the effect of changing the assumed value of Γ_g for models with $\tau = \infty$; similar results are obtained for a non-rigid inner core. A 15% change in the

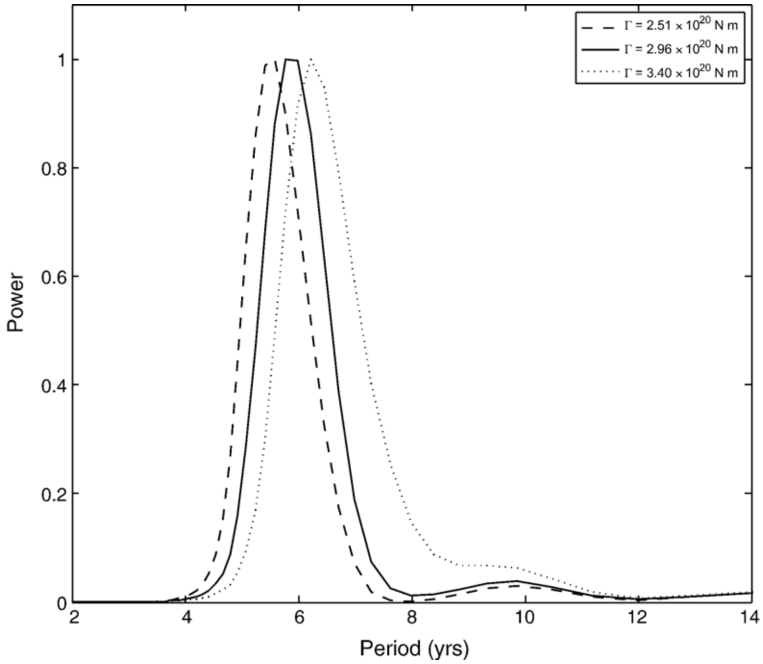

Fig. 3. Mantle torque spectra of models with Γ_g equal to 2.51 (dotted), 2.96 (solid) or 3.40 (dashed) × 10^{20} N m. The inner core is assumed to be rigid ($\tau = \infty$). For other model parameters see Table 1.

assumed value of Γ_g shifts the peak of the model spectrum by ~0.4 yr. (This difference is large enough to be evident in the comparison between model spectra even though it is only about one half of the uncertainty introduced by the spectral spreading.) The MICG normal mode period is shifted more by a 15% change in Γ_g than by order of magnitude changes in τ. The observed period of the MICG mode can thus be used to constrain the strength of the gravitational coupling between the mantle and inner core. The value of $\Gamma_g = 3.0 \times 10^{20}$ N m is close to the value obtained from simulations of mantle flow [32,33]. At present the excess flattening of the CMB (i.e. degree 2, order 0 deformation) determined by VLBI measurements [34] provides the best constraint on the large scale dynamics of the lowermost mantle. The period of the MICG mode depends on the non-zonal structure of the mantle and provides a new constraint for models of mantle flow.

4. Conclusion

A 5.8 yr oscillation in LOD has been found from analysis of a 42 yr time series corrected for atmospheric effects. Models of core–mantle interaction reproduce the observed oscillation only when gravitational coupling between the mantle and inner core is considered. The model fit to the LOD oscillation provides strong observational support for mantle–inner core gravitational coupling and suggests that the observed variation corresponds to a normal mode of oscillation. The period of this normal mode is controlled by the non-zonal density structure of the mantle. Existence of the mantle–inner core gravitational normal mode implies that the inner core responds primarily through solid-body rotation, not viscous deformation, when perturbed from its equilibrium orientation. The characteristic relaxation time of the inner core shape is constrained to be at least several years, corresponding to a lower bound of the order of 10^{17} Pa s for the viscosity of the inner core.

Acknowledgments

This work was supported by the National Science Foundation EAR-0309531.

References

[1] IERS, International Earth Rotation Service, Annual Report, Observatoire de Paris, 1998.

[2] J. Wahr, The Earth's rotation, Annu. Rev. Earth Planet. Sci. 16 (1988) 231–249.

[3] R. Hide, J. Dickey, Earth's variable rotation, Science 253 (1991) 629–637.

[4] R. Rosen, The axial momentum balance of the earth and its fluid envelope, Surv. Geophys. 14 (1993) 1–29.

[5] T. Eubanks, Variations in the orientation of the earth, in: D. Smith, D. Turcotte (Eds.), Contributions of Space Geodesy to Geodynamics, vol. 24, American Geophysical Union, Washington, DC, 1993, pp. 1–54.

[6] R. Abarca del Rio, D. Gambis, D. Salstein, Interannual signals in length of day and atmospheric angular momentum, Ann. Geophys. 18 (2000) 347–364.

[7] J. Chen, C. Wilson, B. Tapley, Interannual variability of low-degree gravitational change, 1980–2002, J. Geod. 78 (2005) 535–543, doi:10.1007/s00190-004-0417-y.

[8] J. Dickey, P. Gegout, S. Marcus, Earth-atmosphere angular momentum exchange and ENSO: the rotational signature of the 1997–98 event, Geophys. Res. Lett. 26 (1999) 2477–2480.

[9] J. Dickey, S. Marcus, O. de Viron, Coherent interannual and decadal variations in the atmosphere–ocean system, Geophys. Res. Lett. 30 (2003) 1573–1576, doi:10.1029/2002GL016763.

[10] R. Holme, O. de Viron, Geomagnetic jerks and a high-resolution length-of-day profile for core studies, Geophys. J. Int. 160 (2005) 435–439, doi:10.1111/j.1365-246X.2004.02510.

[11] K. Wolter, M. Timlin, Monitoring ENSO in COADS with a seasonally adjusted principal component index, Proc. of the 17th Climate Diagnostics Workshop, NOAA/N MC/CAC, NSSL, Oklahoma Clim. Survey, CIMMS and the School of Meteor. Univ. of Oklahoma, Norman, OK, USA, 1993, pp. 52–57.

[12] K. Wolter, M. Timlin, Measuring the strength of ENSO—how does 1997/98 rank? Weather 53 (1998) 315–324.

[13] C. Ropelewski, P. Jones, An extension of the Tahiti-Darwin Southern Oscillation Index, Mon. Weather Rev. 115 (1987) 2161–2165.

[14] Japan Meteorological Agency, Marine Department, Climate Charts of Sea Surface Temperatures of the Western North Pacific and the Global Ocean, 1991.

[15] D. Jault, C. Gire, J.-L. Le Mouël, Westward drift, core motions and exchange of angular momentum between core and mantle, Nature 333 (1988) 353–356.

[16] A. Jackson, J. Bloxham, D. Gubbins, Time-dependent flow at the core surface and conservation of angular momentum in the coupled core–mantle system, in: J.-L. Le Mouël, D. Smylie, T. Herring (Eds.), Dynamics of the Earth's Deep Interior and Earth Rotation, Geophys. Monogr. Ser., vol. 72, American Geophysical Union, Washington, DC, 1993, pp. 97–107.

[17] A. Pais, G. Hulot, Length of day decade variations, torsional oscillations and inner core superrotation: evidence from recovered core surface zonal flows, Phys. Earth Planet. Inter. 118 (2000) 291–316.

[18] R. Hide, D. Boggs, J. Dickey, Angular momentum fluctuations within the Earth's liquid core and torsional oscillations of the core–mantle system, Geophys. J. Int. 143 (2000) 777–786.

[19] S. Braginsky, Torsional magnetohydrodynamic vibrations in the Earth's core and variations in length of day, Geomagn. Aeron. (Engl. Transl.) 10 (1970) 1–8.

[20] S. Braginsky, Short-period geomagnetic secular variation, Geophys. Astrophys. Fluid Dyn. 30 (1984) 1–78.

[21] B. Buffett, Free oscillations in the length of day: inferences on physical properties near the core–mantle boundary, Geodynamics 28 (1998) 153–165.

[22] J. Mound, B. Buffet, Interannual oscillations in length of day: implications for the structure of the mantle and core, J. Geophys. Res. 108 (2003) 2334, doi:10.1029/2002JB002054.

[23] B. Buffett, J. Mound, A Green's function for the excitation of torsional oscillations in the Earth's core, J. Geophys. Res. 110 (2005) B08104, doi:10.1029/2004JB003495.

[24] J. Mound, B. Buffett, Mechanisms of core–mantle angular momentum exchange and the observed spectral properties of torsional oscillations, J. Geophys. Res. 110 (2005) B08103, doi:10.1029/2004JB003555.

[25] B. Buffett, Gravitational oscillations in the length of day, Geophys. Res. Lett. 23 (1996) 2279–2282.

[26] B. Lathi, Linear Systems and Signals, Berkeley-Cambridge Press, Carmichael, CA, USA, 1992.

[27] Y. Gu, A. Dziewonski, W. Su, G. Ekström, Models of the mantle shear velocity and discontinuities in the pattern of lateral heterogeneities, J. Geophys. Res. 106 (2001) 11169–11199.

[28] J. Van Orman, On the viscosity and creep mechanism of Earth's inner core, Geophys. Res. Lett. 31 (2004) L20606, doi:10.1029/2004GL021209.

[29] J. Zhang, X. Song, Y. Li, P. Richards, X. Sun, F. Waldhauser, Inner core differential motion confirmed by earthquake waveform doublets, Science 309 (2005) 1357–1360, doi:10.1126/science.1113193.

[30] K. Creager, Inner core rotation rate from small-scale heterogeneity and time-varying travel times, Science 278 (1997) 1284–1288.

[31] B. Buffett, Geodynamic estimates of the viscosity of the Earth's inner core, Nature 388 (1997) 571–573.

[32] A. Forte, A. Dziewonski, R. Woodward, Aspherical structure of the mantle, tectonic plate motions, nonhydrostatic geoid, and topography of the core–mantle-boundary, in: J.-L. Le Mouël, D. Smylie, T. Herring (Eds.), Dynamics of the Earth's Deep Interior and Earth Rotation, Geophys. Monogr. Ser., vol. 72, American Geophysical Union, Washington, DC, 1993, pp. 135–166.

[33] P. Defraigne, V. Dehant, J. Wahr, Internal loading of an inhomogeneous compressible earth with phase boundaries, Geophys. J. Int. 125 (1996) 173–192.

[34] P. Mathews, T. Herring, B. Buffett, Modeling of nutation and precession: New nutation series for nonrigid Earth and insights into the Earth's interior, J. Geophys. Res. 107, doi:10.1029/2001JB000390.