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GEOPHYSICS

A Mechanism of Temperature Inhomogeneity Formation in the Ocean Stratified by Temperature and Salinity

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Seawater is a two-component medium, whose density depends on temperature and concentration of admixture (salinity). Numerous measurements of these variables in the ocean revealed the existence of clearly manifested horizontal and vertical inhomogeneities (see, for example, [1–4] and references therein). In particular, situations with numerous interfaces (steps) on vertical temperature and salinity profiles (fine structure of the ocean) are widespread. It is generally accepted that the main mechanisms of the fine structure formation are internal wave breaking and convection caused by double diffusion. In our work, we consider a mechanism related to the nonstationary wave process of mechanical equilibrium establishment, namely hydrostatic adjustment. It is shown that in stably stratified two-component medium, initial temperature and concentration inhomogeneities do not disappear at the find stage of this process; i.e., the medium conserves the memory about the initial perturbation (this is excluded in usual one-component fluids stratified only by temperature). The peculiarities of the structure of final distributions are studied. The effect of formation of discontinuities from smooth perturbations is described. The mechanism considered here supplements the previously studied mechanisms.

LINEAR THEORY

The equation of state for saline seawater is written to a high accuracy in the following form [1]:

$$\rho = \rho_*(1 - \alpha T + \beta s). \tag{1}$$

Here, ρ is density, ρ_* is the value of ρ at constant mean values of temperature T_* and salinity s_*; T, s are corresponding deviations from the mean values; α is the coefficient of thermal expansion; and β is the coefficient of salinity compression. Taking into account Eq. (1), the adiabatic motions of the medium are described by a closed system of hydrodynamic equations and transport of admixture:

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\nabla p - g\mathbf{k}, \quad \operatorname{div}\mathbf{u} = 0, \tag{2}$$

$$\frac{dT}{dt} = 0, \quad \frac{ds}{dt} = 0, \tag{3}$$

where $\mathbf{u}$ is a velocity vector with components u, v, w along horizontal axes x, y, and upward directed axis z; p is pressure; g is acceleration due to gravity; $\mathbf{k}$ is vertical ort; and $\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{u}, \nabla)$ is the operator of total derivative.

Let us analyze the behavior of perturbations induced in a linearly stratified two-component medium, which was initially at rest. The corresponding initial conditions for (1)–(3) are written in the following form:

$$\mathbf{u}|_{t=0} = \mathbf{0}, \quad T|_{t=0} = \gamma_T z + T_i(\mathbf{x}), \quad s|_{t=0} = \gamma_s z + s_i(\mathbf{x}), \tag{4}$$

where $\mathbf{x} = (x, y, z)$, T_i, s_i are specified initial perturbations. Let us consider that the background equilibrium state is convective stable. This corresponds to a decrease in background density $\bar{\rho}(z) = \rho_*(1 - \gamma z)$ with height. In the stable state $\gamma = \alpha\gamma_T - \beta\gamma_s > 0$.

It is convenient to characterize the relative contribution of temperature and salinity to the background density stratification by a dimensionless parameter

$$\eta = \frac{\beta\gamma_s}{\alpha\gamma_T}. \tag{5}$$

Usually, $\eta = 0$ corresponds to a one-component medium stratified by temperature. It is obvious that $\gamma = \alpha\gamma_T(1 - \eta)$.

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In a stably stratified infinite medium, initial horizontal inhomogeneities of the density field $\rho_i = -\rho_*(\alpha T_i - \beta s_i)$ lead to the appearance of wave motions, which smooth the density inhomogeneities and attenuate after long time periods. The corresponding wave process of hydrostatic adjustment can be most easily investigated in the case of small amplitude perturbations. Denoting the small perturbations of thermodynamic variables by primes and using the Boussinesq approximation, we shall get the following linear system instead of (1)–(3):

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{\rho_*}\nabla p' + g(\alpha T' - \beta s')\mathbf{k}, \quad \mathrm{div}\mathbf{u} = 0, \quad (6)$$

$$\frac{\partial T'}{\partial t} + \gamma_T w = 0, \quad \frac{\partial s'}{\partial t} + \gamma_s w = 0 \quad (7)$$

with initial conditions

$$\mathbf{u}|_{t=0} = \mathbf{0}, \quad T'|_{t=0} = T_i(\mathbf{x}), \quad s'|_{t=0} = s_i(\mathbf{x}).$$

For dimensionless buoyancy

$$\sigma = -\frac{\rho'}{\rho_*} = \alpha T' - \beta s'$$

system (6), (7) is reduced to one equation

$$\frac{\partial^2}{\partial t^2}\Delta_3\sigma + N^2\Delta_2\sigma = 0, \quad (8)$$

where $N = \sqrt{g\gamma}$ is the Brunt–Väisälä frequency, Δ_3 and Δ_2 are 3D and 2D (horizontal) Laplacians. Equation (8) is the main equation of the linear theory of internal gravity waves [5] (this equation is usually formulated for the vertical velocity component). The general solution of the Cauchy problem for this equation is given in [6]. It follows from the corresponding solution describing the process of spatial wave scattering that the density perturbation attenuates in the course of time: $\sigma \to 0$ at $t \to \infty$.

It is clear that, in a one-component medium (whose density depends only on temperature), temperature perturbation T' attenuates together with σ. The behavior of T' in a two-component medium is completely different. Let us denote the final (at $t \to \infty$) perturbations of temperature and salinity as T'_f, s'_f. In order to find these perturbations, we shall use a simple local conservation law, which is obtained by excluding w from equations (7):

$$\frac{\partial r}{\partial t} = 0, \quad r = \gamma_s T' - \gamma_T s'. \quad (9)$$

According to (9), the field of $r(\mathbf{x})$ does not change in time:

$$\gamma_s T'_f - \gamma_T s'_f = \gamma_s T_i - \gamma_T s_i. \quad (10)$$

In the final state, $\sigma = 0$; i.e., $\alpha T'_f = \beta s'_f$. From the latter relation and (10) we get the following relations

$$T'_f = \frac{\alpha\eta T_i - \beta s_i}{\alpha(\eta - 1)}, \quad s'_f = \frac{\alpha\eta T_i - \beta s_i}{\beta(\eta - 1)}, \quad (11)$$

which indicate that perturbations of T' and s' do not disappear in a two-component medium even at the final stage of the hydrostatic adjustment. These perturbations compensate each other in the density field ($\sigma_f = 0$) forming stationary thermohaline trace. In a real two-component medium, the trace would be naturally destroyed during the characteristic time of dissipation, but this characteristic time exceeds significantly the time scale of hydrostatic adjustment.

We note that conservation law (9) is a linear form of freezing equation specific for two-component media $\frac{d\mathbf{R}}{dt} = (\mathbf{R}, \nabla)\mathbf{u}$ for vector field $\mathbf{R} = \nabla T \times \nabla s$, which is tangent to the lines of intersection between surfaces $T =$ const and $s =$ const. This equation follows from Eqs. (3). For the perturbations of small amplitude,

$$\mathbf{R} = (\gamma_T\mathbf{k} + \nabla T') \times (\gamma_s\mathbf{k} + \nabla s') \sim \mathbf{R}' = \nabla r \times \mathbf{k},$$

$$\frac{\partial \mathbf{R}'}{\partial t} = 0.$$

The latter equation is equivalent to (9).

Let us analyze the peculiarities of final distributions in case $s_i = 0$ (initial perturbation of salinity is absent):

$$T'_f = \frac{\eta}{\eta - 1}T_i, \quad s'_f = \frac{\alpha}{\beta}\frac{\eta}{\eta - 1}T_i. \quad (12)$$

At $\eta = 0$ (one-component medium), the trace of the initial perturbation is absent: $T'_f = s'_f = 0$. At $\eta \neq 0$, taking account of limitation $\gamma > 0$, three qualitatively different situations corresponding to different values of parameter $m = \frac{\eta}{\eta - 1}$ are possible.

(1) Background stratifications of temperature and salinity are stable: $\gamma_T > 0$, $\gamma_s < 0$, $\eta < 0$. In this case, $0 < m < 1$; i.e., the final perturbation conserves the sign of the initial perturbation, but its amplitude is smaller.

(2) Temperature stratification is not stable, but the system is stabilized by stable salinity stratification: $\beta\gamma_s < \alpha\gamma_T < 0$, $\eta > 1$. In this case, $m > 1$; i.e., the amplitude of perturbation in the final state is always greater than the initial amplitude, and $\eta \to 1 + 0$ (approximation to neutral density stratification) $T'_f \to \infty$ ($m \to \infty$). The effect of intensification of perturbations in stably stratified two-component medium can be understood from the following considerations. Initial positive perturbation of temperature leads to the appearance of upward motions in the medium, which transport warm water from below at $\gamma_T < 0$. If density stratification is close to neutral, it does not prevent intense development of these motions, which results in the formation of an intense trace.

(3) Salinity stratification is not stable, but the system is stabilized by stable temperature stratification: $\alpha\gamma_T > \beta\gamma_s > 0, 0 < \eta < 1$. In this most interesting case, $m > 0$ and $|m| \to \infty$ at $\eta \to 1 - 0$. Thus, final and initial perturbations have opposite signs; for example, a cold trace is formed in the medium as a response to initial heating. At first glance, this is an unexpected effect of negative heat capacity of stratified two-component media [7, 8], but it has a simple explanation. Intense vertical motions, which transport cold water from below, develop at $\gamma_T > 0$ and density stratification close to neutral.

Let us note one more peculiarity of the final perturbation. It follows from Eqs. (11) that discontinuities (jumps) in the initial distributions T_i, s_i are also conserved at the final stage. If a discontinuity exists at the initial moment only in the field of one variable, then according to (11), it also appears in the field of another variable during the hydrostatic adjustment. As shown below, the universal property of nonlinear dynamics is in the formation of discontinuities from smooth initial perturbations.

NONLINEAR THEORY

Let us now investigate the theory of final perturbations in a nonlinear problem. For simplicity, we shall consider two-dimensional motions occurring on plane (x, z). We also assume that, at the initial time moment, $s_i = 0$, while the temperature perturbation $T_i(x, z)$ is localized in a horizontal plane: $T_i \to 0$ at $|x| \to \infty$. The corresponding initial density distribution is $\rho_0(x, z) = \bar{\rho}(z) + \rho_i(x, z)$, where $\bar{\rho} = \rho_*(1 - \gamma z)$, $\rho_i = -\alpha\rho_* T_i$.

In a nonlinear problem, variables ρ, T, s are Lagrangian invariants, i.e., values conserved in each fluid particle. Using this fact, one can unambiguously find the distribution of these variables in the final state. Let us first show that if mechanical equilibrium is established in the medium, then in this state $\rho_f(z) = \bar{\rho}(z)$; i.e., the final distribution coincides with the background one (perturbation of density disappears). Indeed, it follows from the definition of the Lagrangian invariants that for each $t > 0$ $\rho = \rho_0(x_0, z_0)$, where x_0, z_0 are initial (Lagrangian) coordinates of a fluid particle. From here, we get $\rho_f(z) = \rho_0(x_0, z_0)$; i.e.,

$$\rho_f(z) = \bar{\rho}(z_0) + \rho_i(x_0, z_0). \tag{13}$$

Proceeding to the limit $|x_0| \to \infty$ in (13) and taking into account that in this case $\rho_i \to 0$, $z_0 \to z$, we obtain $\rho_f(z) = \bar{\rho}(z)$.

Let us assume that $\rho_f(z) = \bar{\rho}(z)$ in (13). In this case, Eq. (13) would be written as

$$\rho_*(1 - \gamma z) = \rho_*(1 - \gamma z_0) - \alpha\rho_* T_i(x_0, z_0),$$

From this, we get

$$z = z_0 + \frac{\alpha}{\gamma} T_i(x_0, z_0). \tag{14}$$

The physical sense of Eq. (14) is quite clear. It determines the dependence of the final (Eulerian) vertical coordinate z of a fluid particle on the initial (Lagrangian) coordinates. If this dependence is known, a similar dependence $x = x(x_0, z_0)$ for the horizontal coordinate can be found from the equation of continuity in Lagrangian variables

$$\frac{\partial(x, z)}{\partial(x_0, z_0)} = 1. \tag{15}$$

Seeking the solution of (15) in form $x = x(x_0, z)$ we get $\frac{\partial x}{\partial x_0} = \left(\frac{\partial z}{\partial z_0}\right)^{-1}$. From this, x is found by integration. Thus, Eqs. (14), (15) form a closed system of equations for finding the field of Lagrangian displacements of fluid particles.

Let us find the final distributions of temperature and salinity. The following relations follow from the definition of Lagrangian invariants and initial conditions (4)

$$T_f = \gamma_T z_0 + T_i(x_0, z_0), \quad s_f = \gamma_s z_0, \tag{16}$$

which together with relations $x = x(x_0, z_0)$, $z = z(x_0, z_0)$ give parametric presentation (x_0, z_0 are parameters) of functions $T_f = T_f(x, z)$, $s_f = s_f(x, z)$. Equations (16) can be transformed by taking into account that, according to (14), $z_0 = z - \frac{\alpha}{\gamma} T_i$. Substituting the latter equation into (16), we get

$$\begin{aligned} T_f &= \gamma_T z + \frac{\eta}{\eta - 1} T_i(x_0, z_0), \\ s_f &= \gamma_s z + \frac{\alpha}{\beta}\frac{\eta}{\eta - 1} T_i(x_0, z_0). \end{aligned} \tag{17}$$

Let us compare Eqs. (17) with the results of the linear theory. It is easy to understand that Eqs. (12) follow from (17) for the deviations from the background distributions, with the only difference that the left and right parts of (12) are now expressed in the Eulerian and Lagrangian coordinates, respectively. Within the linear theory, $x_0 \sim x$, $z_0 \sim z$, and Eqs. (12), (17) are equivalent. In the nonlinear problem, relation (15) between the coordinates determined from (14) leads to a complex configuration of the initial distributions. The most clearly manifested peculiarity is related to the formation of discontinuities (jumps) along the vertical.

Let us consider the case of initial distribution specified as $T_i = \Delta T h\left(\frac{x}{L}\right)\tau\left(\frac{z}{H}\right)$, where ΔT, L, H are amplitude and horizontal and vertical scales of T_i, respectively. We assume that function $h(x)$, decreasing in infinity, is even (not negative) and satisfies condition

$h(0) = 1$. For the dependences between dimensionless (normalized by L and H, respectively) Eulerian and Lagrangian coordinates from (14) and (15), we shall get

$$z = z_0 + ah(x_0)\tau(z_0),$$
$$x = x_0 - a\int_0^{x_0} \frac{h(x_0)\tau'(z_0)}{1 + ah(x_0)\tau'(z_0)}dx_0, \quad (18)$$

in which we introduced dimensionless amplitude parameter $a = \frac{\alpha\Delta T}{\gamma H}$ and denoted the derivative by prime. The integrand in (18) is assumed as $z_0 = z_0(x_0, z)$ (relation for z is solved with respect to z_0).

According to (16), (18), the vertical distribution of T_f over the symmetry axis $x_0 = 0$ of the temperature perturbation is described by parametric (z_0 is a dimensionless parameter) expressions

$$z = z_0 + a\tau(z_0), \quad T_f = \gamma_T H[z_0 + a(1-\eta)\tau(z_0)]. \quad (19)$$

Dependence (19) for various values of parameter a in the case of localized distribution $\tau(z_0) > 0$ and $\eta < 0$ is shown in Fig. 1 (vertically infinite medium is considered). As seen in the figure, with the increase of a at certain value $a = a_{cr}$, a discontinuity is formed at point $z = z_{cr}$ on the vertical profile T_f. A similar discontinuity is also formed in the vertical profile of salinity. These discontinuities compensate each other so that the vertical distribution of density remains smooth (linear). It is not difficult to show that the appearance of the discontinuity is associated with the inflection point z_* on the graph of dependence $z = z(z_0)$. This point is found from equation $\tau''(z_0) = 0$ (prime denotes derivatives). Since

$$\frac{\partial T}{\partial z} = \frac{\frac{\partial T}{\partial z_0}}{1 + a\tau'(z_0)},$$

we shall find for the critical parameters:

$$a_{cr} = -\frac{1}{\tau'(z_*)}, \quad z_{cr} = z_* + a_{cr}\tau(z_*).$$

In particular, at $\tau(z_0) = \frac{1}{1+z_0^2}$, elementary calculations yield:

$$a_{cr} = \frac{8\sqrt{3}}{9}, z_{cr} = \sqrt{3}.$$

Using these values, it is possible to estimate the critical value ΔT. Discontinuity appears when this value is exceeded: $\Delta T = \frac{a_{cr}\gamma H}{\alpha}$ (this value linearly depends on the scale of perturbation H). At $H = 10$ m, $\alpha = 2 \cdot 10^{-4}$ K^{-1}, $\gamma = 9 \cdot 10^{-7}$ m^{-1} ($N = 3 \cdot 10^{-3}$ s^{-1}), we get $\Delta T = 7 \cdot 10^{-2}$ K; i.e., initial perturbation with an amplitude of the order of one-tenth of a degree already leads to the formation of discontinuity.

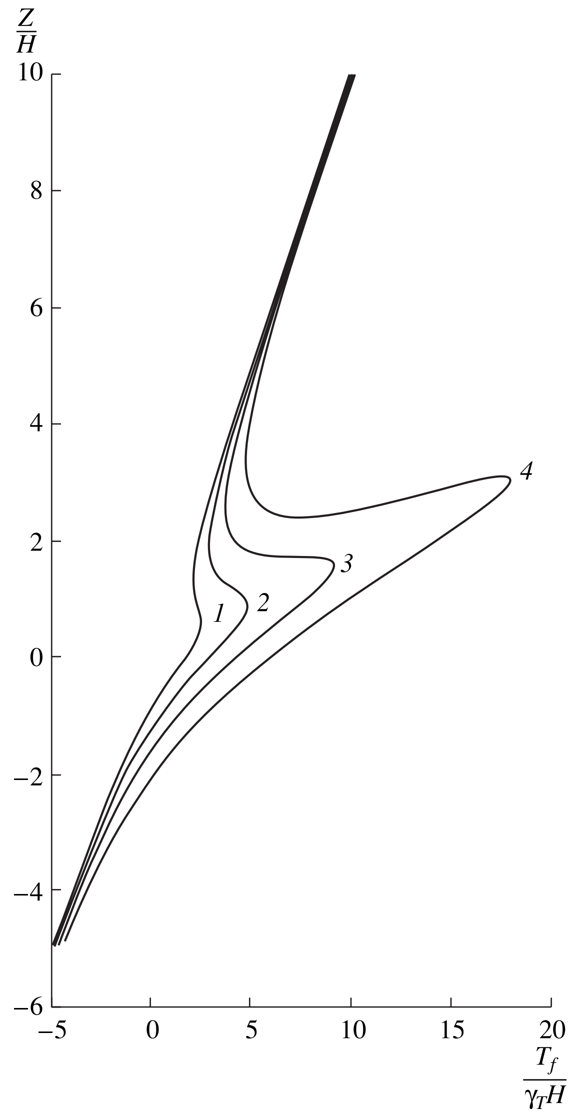

Fig. 1. Vertical temperature distribution along the symmetry axis of the perturbation at $\tau(z_0) = \frac{1}{1+z_0^2}$, $\eta = -5$, and values of parameter $a = 0.4$ (*1*); $a = 0.8$ (*2*); $a = a_{cr} = \frac{8\sqrt{3}}{9} \approx 1.53$ (*3*); $a = 3$ (*4*).

Let us emphasize that the effect of discontinuity formation has a universal character unrelated to the details of the structure $\tau(z_0)$. Its physical interpretation is not difficult. Indeed, according to (14), the vertical displacement of a fluid particle $l = z - z_0$ is determined by intensity of ΔT of the initial perturbation T_i. At $\Delta T > 0$, warmer particles ascend and gain the relatively cold ones, which leads to the formation of a discontinuous profile resembling the known N-wave in gas dynamics.

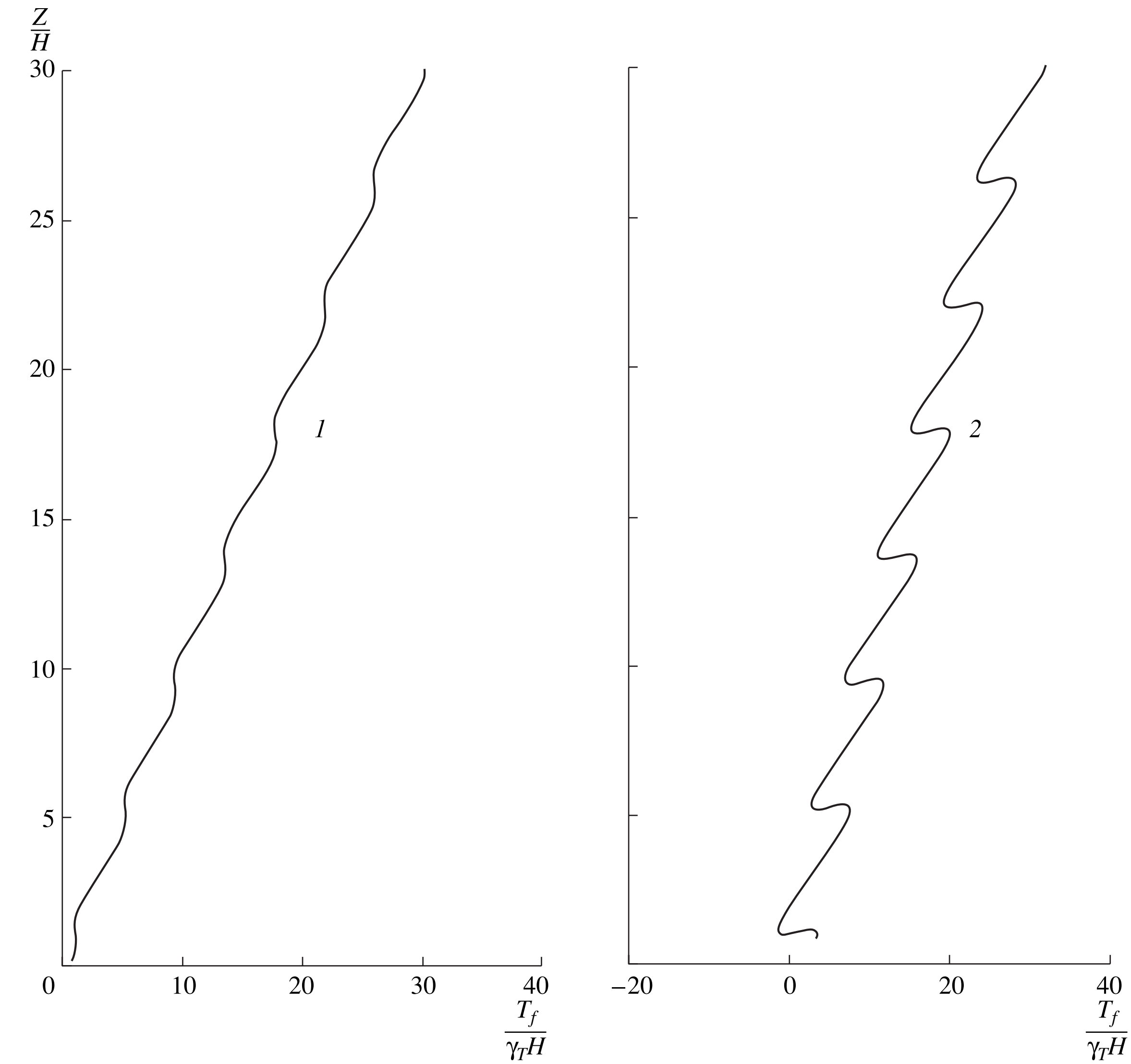

Fig. 2. Vertical temperature distribution along the symmetry axis of the perturbation at $\tau(z_0) = \cos(\lambda z_0)$, $\lambda = 1.5$, $\eta = -2$, and values of parameters $a = 0.2$ (*1*), $a_{cr} \approx 0.95$ (*2*)

Such profiles are recorded sufficiently often. They are characteristic of temperature inversions [1].

Dependence (19) at $\tau(z_0) = \cos(\lambda z_0)$ for two values of parameter a is shown in Fig. 2. This figure demonstrates that in the case of periodical (by vertical) initial perturbation, which is generated, for example, by an internal wave, a stationary sawtooth-shaped temperature profile can be formed, in which layers with constant gradients are separated by sharp interfaces (inversions). The condition of formation of discontinuities for a periodical perturbation is written as $a > a_{cr} = \frac{1}{\lambda}$. The corresponding critical value ΔT at $\lambda = 1$ and the values of parameters given above are equal to $4.5 \cdot 10^{-2}$ K. It is clear that if the initial perturbation is aperiodic (it has positive and negative phases), a profile with irregular distribution of interfaces would be formed. Such profiles are characteristic of the fine structure of the ocean, which, according to the definition given in [1, page 119], is an irregular or systematic alternation (by depth) of the intervals with low and high vertical gradients of a certain property. We note that characteristic values of temperature jumps are a few tenths of a degree for the fine structure. This corresponds to supercritical regimes.

Thus, the scenario of mechanical adjustment of equilibrium in a stratified two-component medium (seawater) is different than in usual fluids stratified only by temperature. Initial horizontal inhomogeneities of temperature and admixture concentration (salinity) in a two-component medium do not disappear even at the final stage of this process and they form a long-living thermohaline trace. The structure of the trace is not trivial. Depending on the relative contribution of these two factors to the background density stratification, the situations are possible when the deviation from the temperature background at the final stage significantly exceeds the amplitude of the initial deviation and has an opposite sign. The universal peculiarity of the structure of the trace is in the formation of vertical discontinuous (stepwise) distributions from initial smooth perturbations. Such distributions are characteristic of the oceanic fine structure.

It should be emphasized that mutual compensation of temperature and admixture concentration takes place in the final states in the density field. In other words, clearly manifested horizontal and vertical inhomogeneities of the temperature and salinity fields can exist in a stratified ocean in the state of mechanical equilibrium and smooth density field. Such peculiarity was repeatedly noted in the analysis of data of oceanographic observations [3, 4]. It is easy to make sure that in the often cited results of Molcard and Williams [1], (Chapter 3, Fig. 3), the steps on vertical profiles of temperature and salinity are compensated in such a manner that their contribution to the density field is equal by the absolute value and opposite by the sign. The authors of [4] mention compensation (although not complete) in the density field. It seems reasonable to make an experimental test of the regularities found in this work.

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