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GEOPHYSICS

Self-Similarity of Energy Structure of Seismicity in the Baikal Region

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In this paper, the energy structure of seismicity (ESS) in the Baikal region is studied within the framework of the self-similarity theory when the distribution of shocks is extrapolated by inverse cascade over the scale of K_P energy classes similar to Kantor dust. It was shown that the Guttenberg–Richter law reflects self-similarity of the ESS, whose Hausdorff dimension D depends on the scale used to estimate the earthquake magnitude. If the slope of the recurrence plot b decreases, the Hausdorff dimension of the ESS asymptotically approaches $D \to 1$. The stages of geophysical medium self-organization in the Baikal region lead to simplification of the ESS, which is caused by the regularity of the seismic process energy under the influence of strong shocks. For the same reason, the ESS is more uniform at flanks of the Baikal region than in the central part of the Baikal rift zone (BRZ).

The modern theoretical and computer models of seismicity are developed as a new foundation for studying spatiotemporal and energy structure of seismicity and predicting strong earthquakes. These models envisage a wider range of the major features of prediction of seismicity by joint analysis of the models and phenomenology: scaling, similarity, self-similarity, and predictability at different scales of averaging. It is known that invariance (with respect to multiplicative scale variations) is provided by self-similarity of spatiotemporal processes [1]. Self-similarity applied to random sets is not a very strict notion. It is sufficient that the parts and the whole in a reduced scale have the same distributions. Despite their fundamental importance, these peculiarities of the prediction of natural seismicity in the investigations of the Baikal ESS were not used practically so far. Currently, the spatiotemporal variations in the energy of the seismic process are traditionally characterized by the variations in the slope of the recurrence plot [2, 3]. Such power laws describe the statistical distribution of the wide spectrum of natural phenomena and "…form the natural family tree, for which the Guttenberg–Richter law is one of its branches" [4, page. 139].

Monitoring of seismicity is a sensitive tool in the study of seismic tectonic processes and the prediction of earthquakes. For example, according to the model of calm seismic periods, the mean rate of earthquake flux in a vast region around an expected strong shock increases when the flux decreases within the earthquake source zone [5]. Analysis of seismicity in the Baikal region showed that the strongest shocks are usually a response to reconstruction and inversion of the stress field during self-organization of the Baikal rift lithosphere [6, 7]. Such investigations point to the fact that strong seismic events are caused by diverse global, regional, and local geodynamic phenomena that induced variations in the stressed-and-strained state or rheological state of the medium. The available data suggest that trends of tectonic impact and different properties of the lithosphere (spatial inhomogeneity, hierarchic block structure, different types of nonlinear rheology, gravity processes, physicochemical and phase transitions, and migration of fluids) are manifested in the properties of earthquake fluxes, whose energy structure is characterized by the recurrence plots.

In experimental seismology, the recurrence plot is written in the form of a correlation equation between the logarithm of the number of earthquakes $\log N$ and magnitude M (or energy class K). In a log-linear coordinate system, the equation has the linear form [2]

$$\log N = A + bM, \tag{1}$$

where coefficient A is seismic activity and coefficient b (or γ) is the slope of the recurrence plot. Since the gaps

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in the records of representative earthquakes are not likely, the coefficients in Eq. (1) depend mainly on the accuracy of determination of magnitudes and Z of scales, as well as on the amount of initial data and methods of their processing. Different versions of the classification of earthquakes by energy are used in Russia, while the value of seismic events is estimated in magnitudes in international seismological practice. At present, we already have a few tens of magnitude scales, and researchers are proposing new scales that characterize the spectral density of earthquake records in a specified frequency interval. This complicated situation can be overcome by coordination of the major dynamic scales and transition to earthquake characteristics that reflect the real physical parameters of the source: seismic moment M_o and total energy E of the earthquake. Due to the existence of numerous magnitude scales and the problem of their coordination, researchers are more frequently using the moment magnitude M_W [8, 9]. A parameter determining the moment magnitude of a strong earthquake appears as a seismic moment—a geophysical property equal to the product of the fault zone area, rigidity of the medium, and mean displacement along the fault. Actually, this is "…potential work, i.e., possible energy needed to overcome the friction forces over the fault surface" [10, p. 12].

The most hazardous part of the Baikal region in terms of seismicity (φ = 48.0°–60.0° N, λ = 96.0°–122.0° E) is the riftogenic series of depressions extending from the northern part of Mongolia along Lake Baikal to southern Yakutia. Intensity of the shocks in the *Catalog of Earthquakes in the Baikal Region* is estimated on the basis of the energy class scale K_P [11]. In the 20th century, the Baikal region witnessed approximately 40 earthquakes of the most hazardous type ($K_P \geq 15$) that can provoke large-scale human and material losses [12]. Recently, scientists have proposed a new method for estimation of the strength of earthquakes in the Baikal region based on seismic moment M_o and moment magnitude M_W. The seismic moment of earthquakes is calculated on the basis of data on amplitudes and periods of maximal displacements in 3D transversal waves for approximately 80 000 shocks with $K_P \geq 6$, which makes up 95% of the total number of seismic events recorded in the region from 1968 to 1994. Seismic moments of earthquakes in the Baikal region were determined for the Brune fracture model [13] using the following relation:

$$M_o = \frac{4\pi\rho r V^3 \Phi_o}{\Psi_{\theta\varphi}}, \qquad (2)$$

where M_o is the seismic moment, din cm; $\rho = 2.7$ g/cm^3 is the density of the medium; $V = 3.58$ km/s is velocity of 3D transversal wave propagation; r is hypocentral distance, km; $\Psi_{\theta\varphi} = 0.6$ is the value of the directional function of radiation from the source [10]; and Φ_o is the level of amplitude spectrum of displacement momentum [14]. Momentum magnitude M_W is related to the seismic moment M_o (H m) of the earthquake by means of relation [8]

$$M_W = \frac{1}{1.5}(\log M_o - 9.1). \qquad (3)$$

The relations of the earthquake recurrence plots for scales K_P (4) and M_W (5) were obtained on the basis of sampling from 29 040 shocks with $8 \leq K_P \leq 14$ and $3 \leq M_W \leq 6$ in the Baikal region from 1968 to 1994:

$$\log N = (8.20 \pm 0.10) + (-0.49 \pm 0.01)K_P, \quad \rho = -0.99 \pm 0.01, \quad F = 29.4, \qquad (4)$$

$$\log N = (7.76 \pm 0.41) + (-1.06 \pm 0.08)M_W, \quad \rho = -0.96 \pm 0.03, \quad F = 8.9, \qquad (5)$$

where ρ is the correlation coefficient and F is the Fischer criterion, which indicates that both equations describe the data at 1% significance level. The slope of the recurrence plot $\gamma = -0.49 \pm 0.01$ (4) for this sampling of earthquakes corresponds within the standard deviation to the slope of the plot $\gamma = -0.50$ obtained for representative earthquakes with $K_P \geq 8$ recorded in the Baikal region from 1964 to 1997 [3]. This indicates the representative character of the data set used here. Equations (4) and (5) show that the same seismic process is characterized by different parameters of the recurrence plot.

The fractal measure can be presented by correlated fractal sets varying according to a power law with different exponents—this idea opens a new field for application of fractal geometry to geophysical systems of seismic genesis. In [6, 7], we suggested that cyclic synergetic processes occurring over millions of years since the terminal Early Cretaceous–Paleocene (origination of the South Baikal depression, the historic core of the Baikal rift system [15]) in the BRZ lithosphere are responsible for natural spatiotemporal and energy-fractal structures. The term 'natural fractal' is applied to denote natural structures that can be applied in the form of a fractal set depending on the objective of presentation [1]. Natural fractals include a random element, and their regularities or irregularities obey statistical laws. Such fractals tend to scale invariance, at which the degree of their irregularity and fragmentation is the same for all scales. In order to demonstrate the similarity of the energy structure of the natural seismicity in the Baikal region and inverse cascade of Kantor dust [1], the sequences of iterations of the ESS in the Baikal region are shown in Fig. 1. On average, at $\gamma \approx -0.3$, one earthquake with K_P (first iteration) corresponds to two shocks with K_P-1 (second iteration), and so on (Fig. 1a). At $\gamma \approx -0.5$, one earthquake with K_P (first iteration) corresponds to three shocks with K_P-1 (second iteration), and so on (Fig. 1b). At $b \approx -1.0$, one earthquake with M_W (first iteration) corresponds to ten shocks with M_W-1 (second iteration), and so on (Fig. 1c). By analogy with the Kantor dust [1], the Hausdorff dimension of a discrete seismic process plot (with $\gamma \approx -0.3$ for the K_P scale) can be estimated by the relation

$$D \approx \frac{\ln 10^{|-0.3|}}{\ln(2 \cdot 10^{|-0.3|} - 1)} \approx \frac{\ln 2}{\ln 3} \approx 0.63,$$

which characterizes the degree of filling of the accommodating space. If plot $\gamma \approx -0.5$, the Housdorff dimension is equal to $D \approx \frac{\ln 3}{\ln 5} \approx 0.68$. For the M_W scale at $b \approx -1.0$, $D \approx \frac{\ln 10}{\ln 19} \approx 0.78$. In the format of the figure, the computer resolution provides a presentation of five and four iterations for the K_P scale and a presentation of two iterations for the M_W scale. This indicates that the structure of the earthquake distribution by the magnitude scale is more complex. If the slope of the recurrence plot decreases, the Hausdorff dimension of the ESS would asymptotically approach to

$$D \to \frac{\ln C}{\ln(2C-1)} \to 1, \tag{6}$$

where $C = 10^{|b|}$. One can suppose that, within the theory of self-similarity, the formation of the family tree [4] is described by Eq. (6), whose spectrum of solutions is shown in Fig. 2.

Figure 3 shows a plot of time variation of the Hausdorff dimension of the energy structure of annual D_Y and total accumulated (with a 1-yr-step) D_S samplings of earthquakes with $K_P \geq 8$ recorded in the Baikal region from 1964 to 2002. One should note the minimal values of $D_Y \approx 0.63$ and $D_S \approx 0.66$ recorded after geodynamic activation of the lithosphere and the great earthquakes of 1967. The D_Y value decreased in 1981 and early 1990 after the inversion of the stress field during self-organization of the BRZ lithosphere and a series of five shocks with $K_P = 14$ in 1981 and strong earthquakes in the late 1980s–initial 1990s [6, 7]. It is likely that such simplifications of the ESS are caused by self-organization of the geophysical medium and ordering of the energy state of the seismic process under the influence of strong shocks. From the beginning of the 1970s, the D_S value increased monotonously, and its variations do not exceed the standard deviation even after sufficiently strong shocks (Fig. 3). Hence, the influence of great earthquakes on the ESS and the long-term memory about the rearrangement of the energy of the seismic process in the Baikal region is extraordinarily strong.

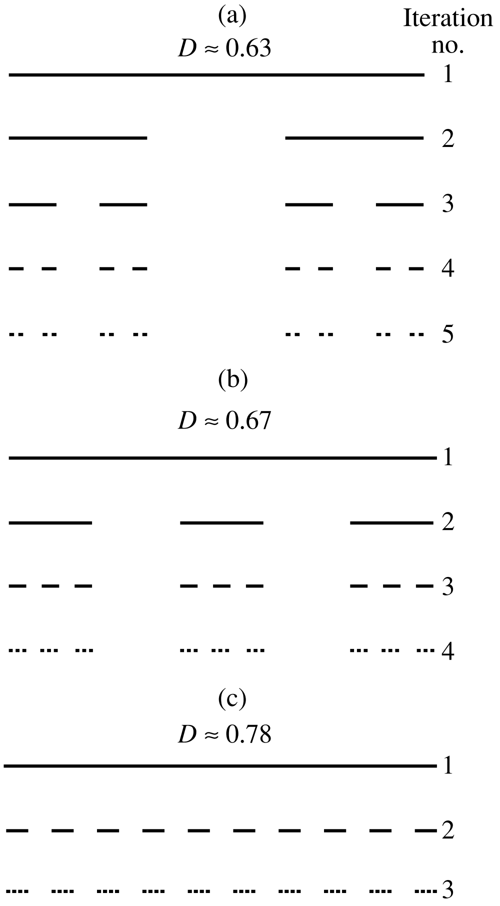

Fig. 1. Graphic presentation of the iteration's sequence of the energy structure of natural seismicity in the Baikal region. (a, b) Scale of energy classes K_P ($\gamma \approx -0.3$, $\gamma \approx -0.5$); (c) scale of momentum amplitudes M_W ($b \approx -1.0$).

Figure 4 shows a chart of the ESS in the Baikal region for the scale of energy classes K_P plotted on the basis of D contour lines determined for the samplings of $n > 100$ earthquakes with $K_P \geq 8$ recorded from 1964 to 2002 within a grid of 2.0° × 2.0°. The data was averaged using a step of 1° by latitude and longitude, and the D values were attributed to the grid centers. The chart shows that the energy property of seismicity is more uniform and structured at the flanks of the Baikal region than in the central part of the BRZ. It is obvious that the observed pattern reflects the fact that strong earthquakes with $M \geq 6$ occur more frequently at the flanks (Fig. 4), while the central part of the BRZ is dominated by weak shocks including many swarms of seismic events. The Baikalian ESS was characterized by statistical multiplexing owing to the superposition of the aftershocks, swarms, and background shocks that triggered the joint self-similar process. Since the nature of these seismic events is different, one can suppose that the ESS of individual fluxes of aftershocks, swarms, and background shocks would be different.

It is noteworthy that individual properties of the spatiotemporal ESS of the Baikal region can change depending on the accepted earthquake scale. However, the main features and trends would be retained. The

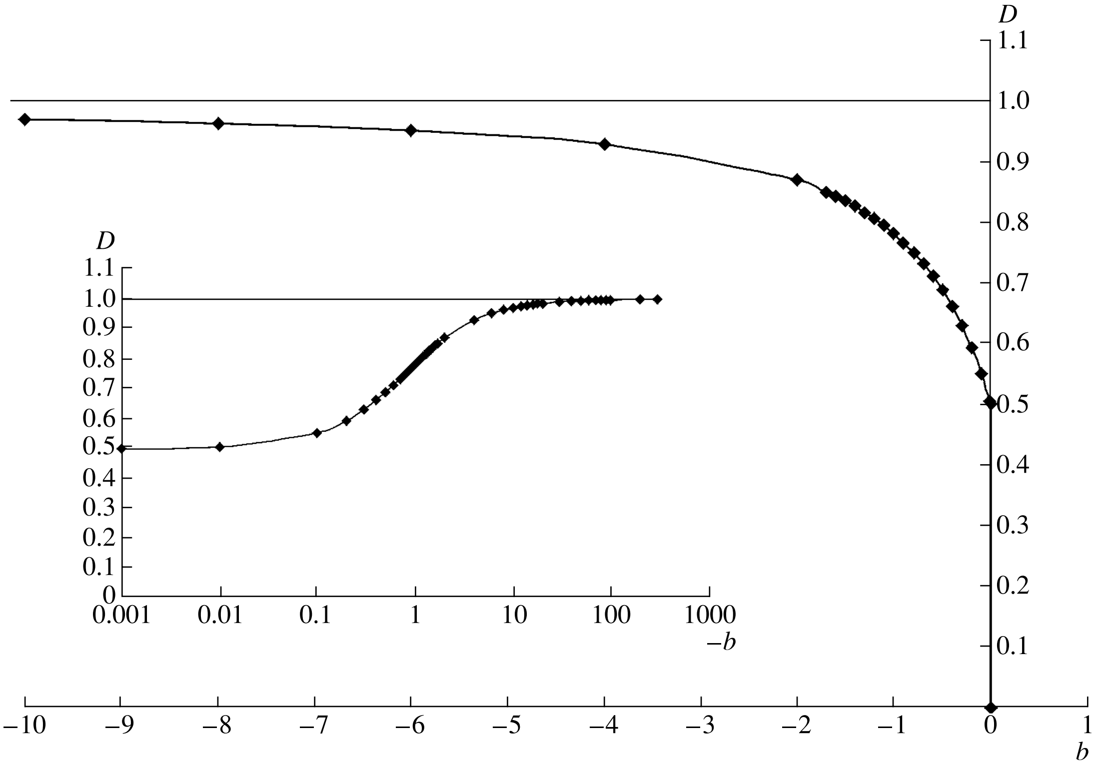

Fig. 2. Spectrum of Hausdorff dimensions D for the ESS as a function of the slope of the recurrence plot b. Asymptotic approximation $D \rightarrow 1$ is shown in the inset.

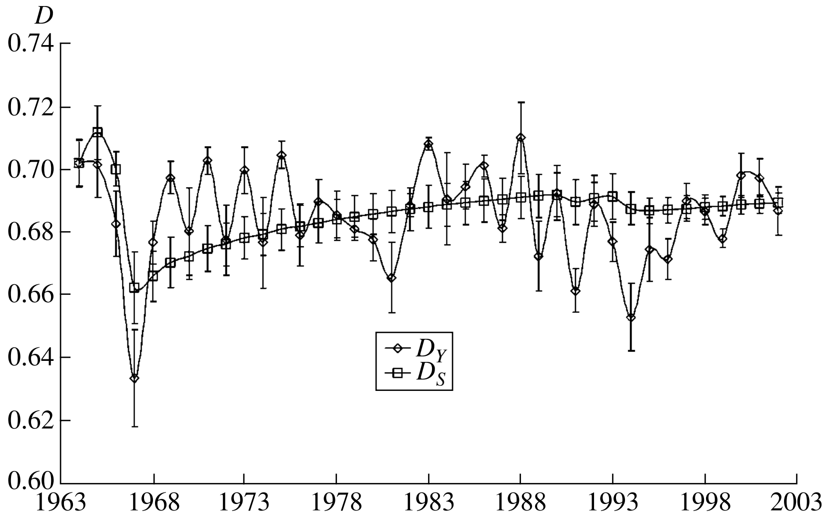

Fig. 3. Time variations in the Hausdorff dimension of the energy structure of annual D_Y and total accumulated D_S of earthquake samplings in the Baikal region.

results of our work suggest that real self-similarity of the ESS can be estimated with the Hausdorff dimension D using a physically substantiated absolute scale for estimating the strength of the earthquake. Investigations in this field should decipher (i) new statistical trends in variations of self-similarity parameters of the spatial-energy structure and dynamics of seismicity; (ii) their relation to the structure of inhomogeneities in the active geophysical medium; and (iii) major trends in the evolution of self-organization during the transitional seismic regime related to inversion of the stress field in the BRZ lithosphere.

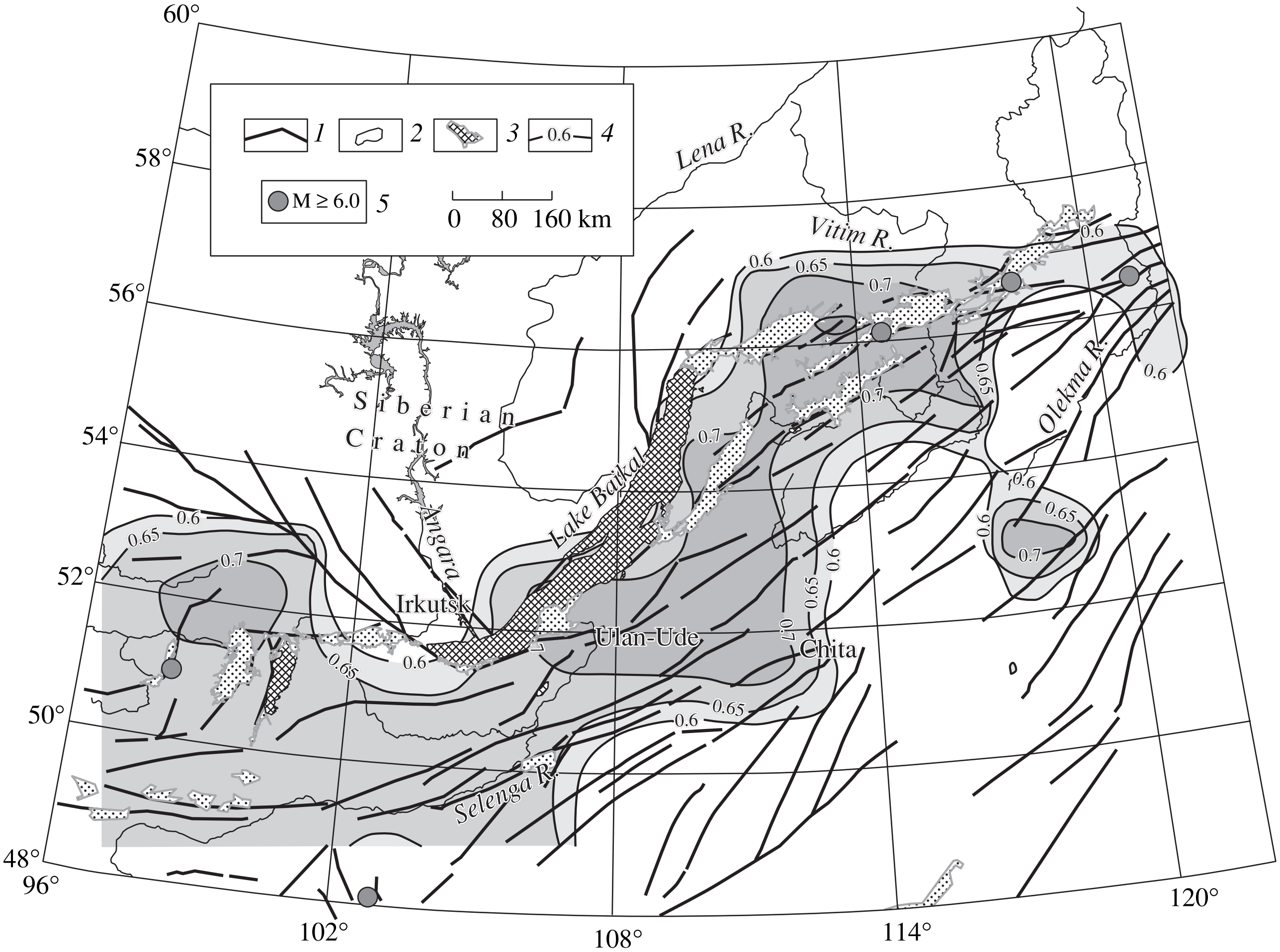

Fig. 4. Chart of the ESS in the Baikal region. (*1*) Major fractures; (*2*) depressions; (*3*) lakes, (*4*) contour lines of the Hausdorff dimension *D*, (*5*) instrumentally recorded earthquakes with $M \geq 6.0$ (5).

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