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GEOCHEMISTRY

Fractal Features and Morphological Differences between Periodic Infiltration and Diffusion Metasomatic Zonation

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According to the theory based on the principle of local chemical equilibrium in the solution–rock system [1], the metasomatic zonation is divided into the infiltration and diffusion dynamic types. The formation of zonality during highly nonequilibrium metasomatic processes can produce rhythmic-banded structures that complicate the vectorial (unidirectional) zonation [2]. To interpret such structures, the theory of zonation has been supplemented by a dynamic–kinetic model of the periodic (rhythmic-banded) metasomatic zonation [3, 4]. The model of formation of rhythmic-banded metasomatic structures considers such an infiltrational process, when the solvent filtrates through a uniformly porous rock together with dissolved components. Another type of zonation is related to diffusion of components in an immobile solvent (pore solution of rock). The formation mechanism of rhythmic-banded structures in this case is principally the same as the mechanism of infiltration metasomatism, but morphology of the formed structures differs substantially.

As has been established in [3], a certain relationship between rates of fast reactions (oxidation and reduction of Fe ions in solution) and slow reactions (dissolution and precipitation of minerals) controls the development of oscillations and waves in solution during the formation of banded skarns. The limit cycle, which suggests formation of a dissipative structure in solution and periodic zonation in rock, represents an attractor of the dynamic system. However, morphology of real dissipative structures of metasomatic zonation corresponds to more complex models. The analysis of morphology of real structures that requires application of modern nonlinear methods yields important information for the development of a complex model. The aim of this paper is to apply wavelet analysis to the parameterization of real rhythmic-banded structures formed as a result of different metasomatic mechanisms.

The wavelet analysis is widely applied for processing genetically different fractal signals and images, e.g., seismic signals, satellite images of clouds, turbulent fields, photographs of minerals, metallographic images of deformed metals, and so on [5, 6].

During parameterization of fractal morphological features of the metasomatites, an image of thin section is digitized and profiles perpendicular to the direction of banding in the image are analyzed. Profiles of $f(x)$ represent the distribution of sample color intensity along a chosen direction and reflect phase distribution in the sample. The identification and classification of profile features (phase boundaries) is an objective of parameterization. The wavelet transform of profile 1D consists in its expansion along the base composed of a soliton-type function ψ (wavelet) by stretching and shearing:

$$[W^{\psi}f](a, b) = \frac{1}{\sqrt{|a|}}\int_{-\infty}^{\infty} f(x)\psi^*\left(\frac{x-b}{a}\right)dx, \qquad (1)$$

where a is the extension/compression parameter, and b is the shear parameter. Expansion (1) distributes signal $f(x)$ by scales a, thereby retaining information on spatial localization of features with aid of parameter b (Fig. 2). This approach makes it possible to identify the hierarchy of characteristic zonation scales in the sample and to establish interrelations between the scales.

We investigated samples of the infiltrational-banded wollastonite–hedenbergite skarn (Dal'negorsk boron deposit, Primorye) and the complex-banded hydrother-

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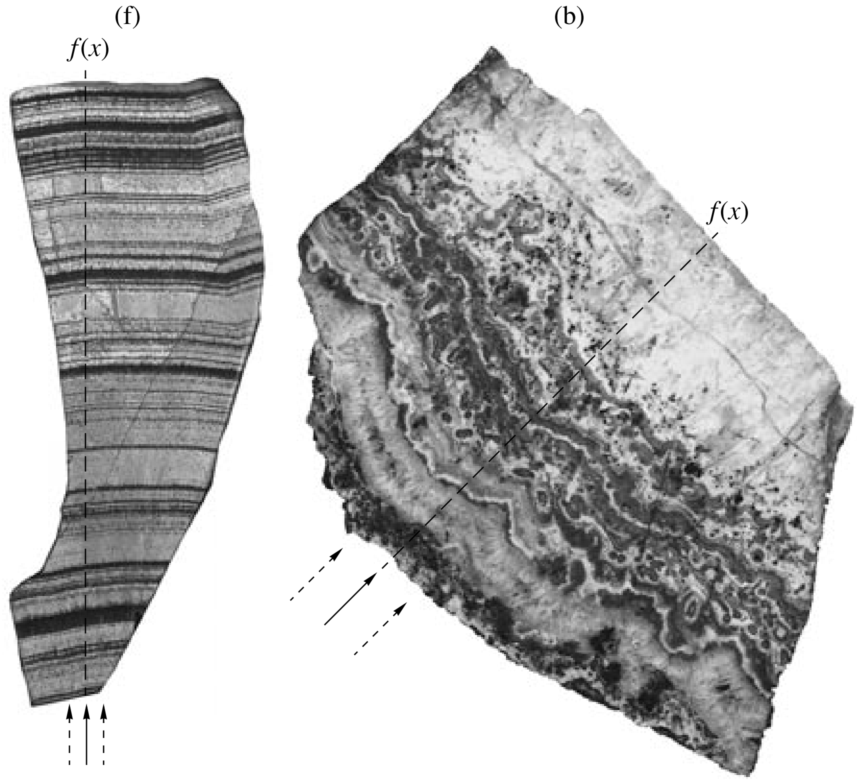

Fig. 1. Rhythmic-banded structures of the studied metasomatites. (a) Banded infiltration wollastonite–hedenbergite skarn (Dal'negorsk boron deposit, Primorye); (b) complexly banded hydrothermal gold–silver vein (Dukat deposit, Russian Northeast). Arrows indicate the direction of the analyzed phase profiles $f(x)$.

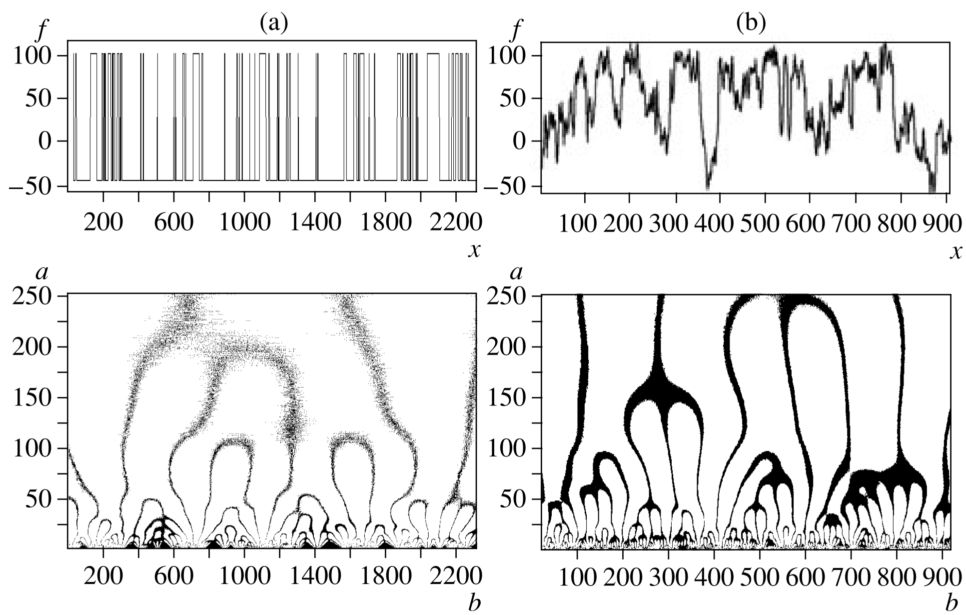

Fig. 2. Wavelet spectrograms for typical phase profiles of (a) infiltration and (b) diffusion metasomatites that control hierarchy of phase dimensions. Axes b and a of the spectrogram correspond to the spatial coordinate and dimension of the phase, respectively.

mal quartz–gold–silver veins (Dukat deposit, Russian Northeast).

The rhythmic-banded skarn sample (Fig. 1a) consists of the prevalent wollastonite (light), thin hedenbergite bands, and an admixture of datolite. Wollastonite forms fibrous aggregates of dendritic crystals elongated perpendicular to the banding. Other minerals are represented by isometric grains. The dark bands vary in thickness and show a hierarchy of scales. Bands of the lowest level have a thickness of one hedenbergite grain. The boundaries of the bands are the fronts of the replacement. In our case, they are represented by smooth spherical surfaces without marked curvatures.

In the quartz–chlorite–adularia sample from the gold–silver vein (Dukat deposit), the rhythmic banding is observed in segments composed of Fe-berthierite, adularia, and quartz with magnetite dissemination. The banding is usually conformable with vein contacts. The quartz–adularia–chlorite vein cuts and metasomatically replaces rhyolite (quartz + orthoclase + plagioclase). The severely altered silicified relicts of rhyolite are incorporated into the vein selvages. The diffusion interaction of the solution from the vein with the pore solution in rhyolite produced the following metasomatic zonation (from wall rock into the vein): (0) rhyolite, (1) zone of large orthoclase crystals, (2) orthoclase + quartz + chlorite, (3) chlorite + orthoclase ± quartz, and (4) veined quartz. Zone 1 is composed of a large-columnar aggregate of orthoclase crystals with the vertex oriented toward the vein axis in compliance with the direction of their growth. Zone 2 is a rhythmic intercalation of the curved bands of dark gray Fe-chlorite and the light (orthoclase + quartz) bands. Zone 2 is separated from zone 3 by a thin quartz band that hosts a chain of concentrically zoned (orthoclase + chlorite) spheres. Zone 3 is mainly composed of chlorite with fragments (patches) of chlorite–orthoclase rhythmic bands. The fragments are dispersed chaotically within this zone. When the patches approach zone 2, they line up conformably with banding in zone 2.

Samples of infiltration and diffusion metasomatites are characterized by rhythmic banding of principally distinct morphologies. Therefore, we should use different parameterization methods: the monofractal approach for the infiltration metasomatites and the multifractal approach for the diffusion metasomatites.

(1) The morphology of infiltration metasomatites is mainly composed of two components represented by light and dark bands. Therefore, we used the two-color spectrum for their digitization (Fig. 2a shows a typical profile). The signal is formed by isolated features of one type (δ-shaped boundaries of zones). A set of the present scales and the fractal dimension of a properties-carrier are possible parameters.

(2) The diffusion metasomatites are characterized by "turbulent" morphology, and the size of diffuse boundaries between zones is comparable with the size of the zones. Therefore, the sample was digitized in a gray scale (Fig. 2b shows a typical profile). The signal is formed by a continuous totality of features of various types owing to a significant role of diffuse boundaries between the zones. In this case, the set of the present scales and the multifractal spectrum of the features are parameters of the signal.

The analyzed profiles represent functions correlated with chemical compositions of samples. Thus, the results of analysis may be applied to develop a complicated dynamic model of metasomatic zonation.

The aforementioned parameters were quantitatively assessed based on the following procedures of wavelet analysis.

The dimension of feature-carrier D was determined from the behavior of the number of local maximums $N(a)$ of wavelet expansion (1) depending on scale a. In the case of a self-similar signal, this relationship bears the following exponential character:

$$N(a) \sim a^D, \tag{2}$$

where the local maximums are lined up along parameter b at constant a. For 1D profiles of $f(x)$, dimension D differs from 1 only when the signal has specific features in a limited number of points and represents a smooth function in other points. Such signals can be exemplified by a devil's ladder, where the features are distributed in compliance with Cantor discontinuum.

The concept of Lipschitz–Hölder smoothness is used to classify specific features of a profile. Profile $f(x)$ at point x_p has a feature with Lipschitz–Hölder exponent α, if the following condition is fulfilled in the vicinity of this point:

$$|f(x_p + \Delta x) - f(x_p)| \le C|\Delta x|^{\alpha}. \tag{3}$$

In the case of nonisolated features, one constructs multifractal spectrum $D(\alpha)$, which determines the set of features of the signal and the dimension of the respective carriers. The multifractal spectrum may be constructed by wavelet expansion in the following way [7]. The partition function

$$Z(q, a) = \sum_{W\max} |W(a, b)|^q;$$

is constructed at local maximums of wavelet expansion. Then, one finds the function of scale exponents $\tau(q)$ that characterizes the exponential behavior of partition function Z depending on scale a

$$Z(q, a) \sim a^{\tau(q)}.$$

The sought multifractal spectrum $D(\alpha)$ is related to $\tau(q)$ by Legendre transform

$$D(\alpha) = \min\left[q \cdot \left(\alpha + \frac{1}{2}\right) - \tau(q)\right].$$

The chosen scales can be efficiently identified by a smoothed analog of Fourier power spectrum (scalogram)

$$E_W(a) = \langle |W(a, b)|^2 \rangle_b,$$

which represents wavelet coefficients averaged by parameter b. The maximums of scalogram correspond to the scales with the periodic component. Moreover, the real scale and parameter a are linearly interrelated. While analyzing the images, it is convenient to use a

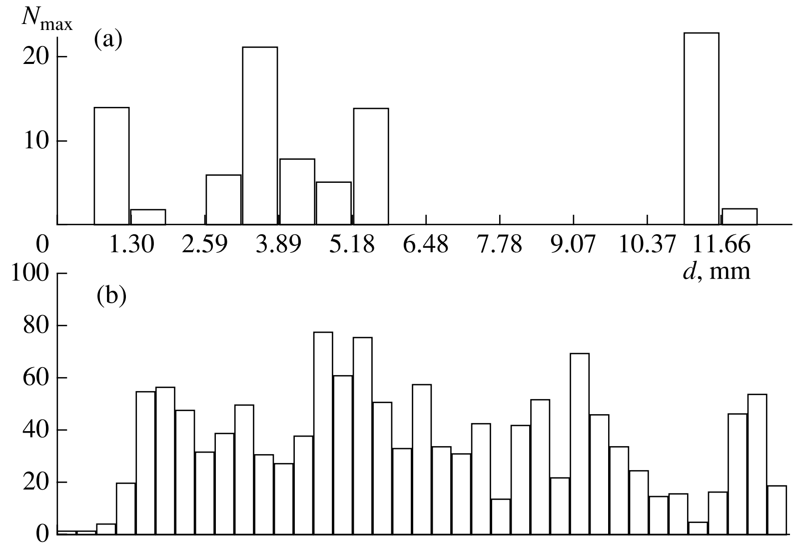

Fig. 3. Frequency histograms of the appearance of characteristic phase dimensions for (a) infiltration and (b) diffusion metasomatites.

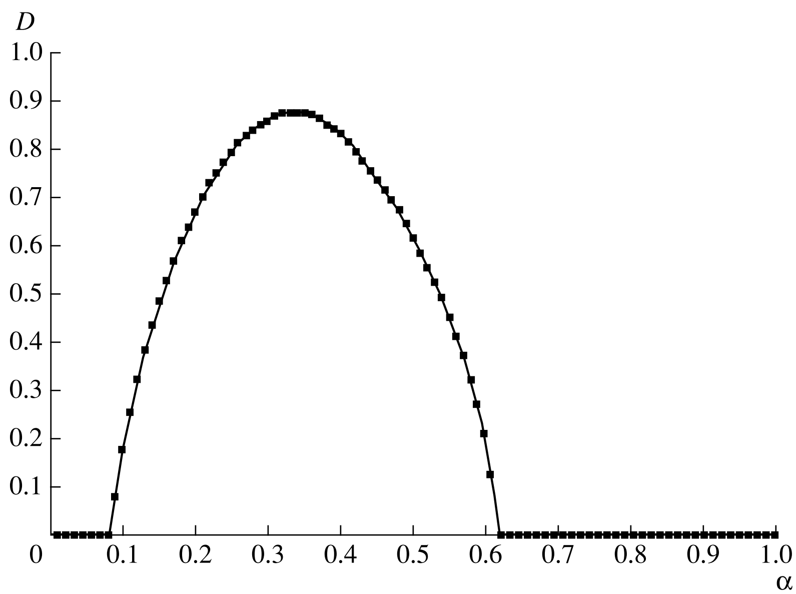

Fig. 4. Multifractal spectrum of a phase profile of the diffusion metasomatite. Axis x determines the feature type (diffuse character) of phase boundaries. Axis y is the fractal dimension.

histogram that displays frequency of appearance of various scales for the entire totality of image profiles.

The analysis of morphological profiles of the banded skarn with a method of local maximums (2) has revealed its self-similarity with fractal dimension $D \sim 0.7$. The structure has a finite set of strictly chosen scales: ~1, 3, 5, and 11 mm (Fig. 3a). The stochastic analog of Cantor discontinuum may be suggested as a model of such morphology.

The diffusion metasomatite profiles include a continuous spectrum of scales, which complicates parameterization with the aid of the chosen scales (Fig. 3b). At the same time, the multifractal spectrum $D(\alpha)$ of the diffusion metasomatite sample suggests the presence of features ranging from $\alpha_{min} \sim 0.15$ to $\alpha_{max} \sim 0.6$ with maximum at $\alpha \sim 0.35$(Fig. 4). The spectrum with such parameters is similar to the multifractal spectra of turbulent velocity fields [8].

The morphological differences of the infiltration and diffusion zonation are related to dynamic causes, namely, instability of the flat front of mineral replacement during diffusion and its stability during infiltration. In this paper, we proposed an approach for parameterization of macrostructure of rhythmic-banded metasomatites of both infiltration and diffusion types. We have shown that the infiltration structures may be regarded as a stochastic fractal similar to Cantor discontinuum. We propose to use the fractal dimension and relationship of characteristic scales for their parameterization. The diffusion-related metasomatic samples represent stochastic multifractals, and the multifractal spectra are necessary for their parameterization. We can use results of the parameterization as fitting parameters for the construction of a complicated kinetic equation of dynamic model of metasomatic zonation.

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