Models, data and mechanisms: quantifying wildfire regimes

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Abstract: The quantification of wildfire regimes, especially the relationship between the frequency with which events occur and their size, is of particular interest to both ecologists and wildfire managers. Recent studies in cellular automata (CA) and the fractal nature of the frequency–area relationship they produce has led some authors to ask whether the power-law frequency–area statistics seen in the CA might also be present in empirical wildfire data. Here, we outline the history of the debate regarding the statistical wildfire frequency–area models suggested by the CA and their confrontation with empirical data. In particular, the extent to which the utility of these approaches is dependent on being placed in the context of self-organized criticality (SOC) is examined. We also consider some of the other heavy-tailed statistical distributions used to describe these data. Taking a broadly ecological perspective we suggest that this debate needs to take more interest in the mechanisms underlying the observed power-law (or other) statistics. From this perspective, future studies utilizing the techniques associated with CA and statistical physics will be better able to contribute to the understanding of ecological processes and systems.

In many regions of the world, wildfires are common and are considered an integral component of ecosystem functioning. However, wildfires also pose a threat to humans, their activities and livelihoods, and repeated fires can negatively affect ecosystem functioning (Bond & van Wilgen 1996). Thus, understanding and managing the relationships between wildfires, ecological systems and human activity is important. The combination of the timing, frequency and magnitude of all disturbances occurring in a given region is known as the 'disturbance regime'. Recently, much research has considered one particular aspect of the disturbance regime: the frequency–area distribution of wildfires in a given area. Here, we will focus on disturbance by wildfires.

Examination of these statistics in the context of wildfire activity is not new (e.g. Minnich 1983; Baker 1989; Strauss *et al.* 1989, among many others), but recently there has been considerable debate regarding the 'heavy-tailed' (i.e. the tail decreases at a relatively slow rate) nature of these frequency–area distributions. One specific class of heavy-tailed distribution is a power-law (fractal) where the frequency–area distribution has no inherent scale (it is scale-invariant). The presence of such scaling relationships has been noted widely in many features of biological and ecological systems (e.g. Brown *et al.* 2002). In the wildfire literature, discussion has particularly addressed whether these heavy-tailed frequency–area distributions are power-law in nature, and what the implications of such a power-law distribution might be.

Much of the present debate on the heavy-tailed nature of 'real' wildfire areas is the result of research in the early 1990s, where simple 'forest-fire' cellular-automata (CA) models were found to produce power-law size frequency distribution – a characteristic linked with self-organized criticality (SOC) (Bak *et al.* 1990; Drossel & Schwabl 1992; Clar *et al.* 1996). Malamud *et al.* (1998) then produced the first detailed research showing that both the forest-fire CA model and 'real-world' wildfires exhibit robust power-law frequency–area distributions. Since then, other authors have presented data and analyses with the aim of variously confirming or refuting the assertion that real-world wildfire frequency–area distributions follow a power-law distribution (e.g. Ricotta *et al.* 1999, 2001; Cumming 2001; Ward *et al.* 2001; Reed & McKelvey 2002; Schoenberg *et al.* 2003).

In this paper, we will examine the history and nature of this discussion, before suggesting what direction it might take, or be most useful to take in the future. We approach this topic from a broadly ecological perspective, emphasizing the need for consideration of the ecological (or otherwise) mechanisms driving observed wildfire frequency–area distributions. We will begin by

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examining the most recent papers in this area, before establishing the state of current research in this area and suggesting what avenues of future research on this topic might prove fruitful.

Ecological examination of wildfire frequency–area distributions

Consideration of wildfire and other disturbance regimes (the spatio-temporal dynamics of recurrent disturbance events) has a long history in ecology. The dynamics of *succession–disturbance* in ecology is important when considering wildfires. *Succession* is the change in ecological community composition (in essence the relative abundance of the different species in the community) and function (the ways in which the abiotic and biotic components of the community are linked) through time.

Disturbance is the disruption of an ecosystem, community, or species' populations by any relatively discrete event in space and time, with a resultant change in the physical environment (White & Pickett 1985). Until the 1950s and 1960s, ecologists' views on succession–disturbance dynamics were dominated by the perspective of Frederick Clements (1916, 1928, 1936). Clements' conceptualization of the community emphasized equilibrium and stability, as encapsulated by the 'balance of nature paradigm'; in this view disturbance events were seen as unnatural, as they moved the system away from its 'natural' equilibria (the so-called 'climax' condition). Ecosystem management conducted from this perspective, therefore, aimed to minimize disturbance events and their impacts, resulting in policies such as fire suppression.

More recently, ecologists have accepted the fundamental importance of apparently random events, such as disturbance, in structuring ecosystems, and have adopted a more scale-sensitive, disequilibrial view (Wu & Loucks 1995; Perry 2002). With this shift has come increasing interest in characterizing the three key dimensions of the disturbance regime: size, frequency, and intensity. Recently, there has been some attention focused on determining whether large, infrequent disturbance have a qualitatively different effect than small, frequent ones (Romme *et al.* 1998; Turner *et al.* 1998). It is with this historical perspective in mind that we need to consider ecological approaches to quantifying wildfire regimes, in contrast to the model-based approaches we discuss later.

The frequency of disturbance events is very important in terms of the evolution of the reproductive strategies that different species adopt (e.g. the number and size of offspring produced, the energy invested per reproductive event, and so on); these reproductive strategies are sometimes also known as life history traits (Bond & Keeley 2005). For example, the optimal time after disturbance for a species to maximize seed storage (in either the crown or soil seedbank) will be influenced by the average time between wildfire events (Enright *et al.* 1998*a, b*). What constitutes a 'frequent' wildfire will vary from ecosystem to ecosystem, depending on factors such as rates of biomass production, the nature of other disturbance agents operating alongside fire (e.g. wind-throw) and regeneration rates. An intriguing body of ecological theory suggests, however, that intermediate disturbance frequencies will promote the highest levels of biodiversity (the 'intermediate disturbance hypothesis', Connell 1978). Early quantitative studies of the wildfire regime, conducted in the 1950s and 1960s, emphasized frequency – in essence an estimate of the probability distribution of survival or mortality from wildfire(s) (Johnson & Gutsell 1994). Early efforts (e.g. Spurr 1954) were often somewhat *ad hoc* studies of wildfire occurrence, and are perhaps better seen as wildfire 'history' studies. However, Heinselman (1973), in a seminal study, mapped the time-since-wildfire-year, on the basis of stand ages, in the Boundary Waters Canoe Area in Minesotta (USA). On the basis of this map Heinselmann estimated survivorship from wildfires in the landscape.

Since the late 1970s, a number of statistical methods and distributions that might be suitable for describing wildfire frequency have been developed and applied, with much emphasis on the Weibull and negative exponential distributions (see Johnson & van Wagner 1985). These statistical models allow empirical assessment of relationships between spatio-temporal variation in wildfire frequency and other environmental factors (Johnson & Gutsell 1994). Considerable debate remains over the drivers of spatio-temporal variability in wildfire frequency (in particular the relative roles of weather v. fuels), and unravelling these patterns is a focus of current work (e.g. Bessie & Johnson 1995). Although sophisticated statistical tools are available for modelling fire frequency (e.g. Presiler *et al.* 2004; Reed & Johnson 2004), the stumbling-block is often collecting adequate empirical data to represent the processes and designing adequate sampling strategies for this data collection (Johnson & Gutsell 1994).

Although much research effort has focused on the frequency component of the wildfire regime, other ecologists have considered the size (i.e. burned area, often equated with severity) component of the wildfire regime. As different ecosystems respond differently to wildfires, what constitutes a severe event will also vary (Moritz 1997). The diverse effects of wildfire suppression efforts have received considerable attention in this context.

Minnich (1983) compared the frequency–area distribution for regions in Southern California that had been subject to wildfire suppression, with that of regions in northern Baja California that had not. He found that in regions subject to suppression, large intense wildfires occurred (possibly larger than had occurred pre-suppression), and that total burned area was the same as in regions where wildfires were unsuppressed. Although subsequently there has been much debate concerning the existence and significance of this difference (e.g. Strauss *et al.* 1989; Chou *et al.* 1993; Keeley & Fotheringham 2001; Minnich 2001), this research stimulated interest and debate in the appropriate methods for characterizing and comparing empirical wildfire frequency–area distributions (see also Miyanishi & Johnson 2001; Ward *et al.* 2001; Bridge *et al.* 2005, for discussion regarding Ontario). The consensus appears to be that the majority of empirically observed wildfire size distributions are heavy-tailed (e.g. Malamud *et al.* 1998, 2005).

The forest-fire cellular automata model

The forest-fire cellular automata model rose to prominence amidst the suite of models used by Per Bak and others to examine and propound the theory of self-organized criticality (SOC) in dynamical systems (Bak *et al.* 1987, 1988). Self-organized criticality was first presented by Bak *et al.* (1987) as the concept that dynamical systems order themselves naturally to a critical state regardless of initial conditions and independent of any exogenous driving force. Although the exact definition of SOC is often unclear, Turcotte (1999, p. 1380) suggests a working definition for SOC 'is that a system is in a state of self-organized criticality if a measure of the system fluctuates about a state of marginal stability'. Bak (1996) suggested that, at the critical state, small inputs to a system can cause events of any magnitude in intermittent periods of activity and that prediction of the size of a specific future event is impossible. The frequency–area distribution of events in this type of system will exhibit power-law (i.e. critical) behaviour (Bak & Tang 1989).

Bak *et al.* (1987) first presented the concept of SOC using models of coupled-pendulums and, more famously, sandpiles (for further discussion on the sandpile model, see paper in this volume, Malamud & Turcotte 2006). Later, Bak *et al.* (1990) used a cellular automaton (CA) to model forest fires. Although there are many variations on this forest-fire model, details of the rules that define the mechanics of the simplest of these models can again be found in Malamud & Turcotte (2006); an example of the progression of this simple forest-fire model is given in Figure 1. Using the simplest version of the forest-fire CA model, Bak *et al.* (1990) and Drossel & Schwabl (1992), amongst others, found that uniformly injected energy ('trees') is dissipated ('burning trees') in a spatially self-similar (fractal) manner. The forest-fire CA model was shown to self-organize itself to a state where fire sizes (patches of contiguous cells that burn in a single burning 'event') exhibited power-law frequency–area statistics.

Similar models had previously been considered in the context of percolation theory (Albinet *et al.* 1986; Beer & Enting 1990, 1991). Site-percolation models are specified by a parameter, p, which defines the probability of a site in a lattice being occupied. At a critical point ($p_c \cong 0.59275$) clusters traversing the entire lattice (spanning clusters) form. Although these models are not self-organizing through time, they do show critical behaviour in the sense that the model shows quite different regimes of behaviour depending on the value of the tuning parameter. These percolation-based models have received less attention than the forest-fire CA model in the context of SOC, which rapidly attracted interest because of its apparent potential to contribute to the understanding of natural dynamical systems (they are, however, the basis of much research in statistical physics). Hergarten (2002) makes this same point with reference to a CA model, essentially the same as that used by Drossel & Schwabl (1992), proposed by Henley (1993). Turcotte (1999) provides a good comparison of the CA forest-fire model and site-percolation models.

A non-cumulative frequency–area distribution is considered power-law if

$$f(A) \sim A^{-\beta} \quad (1)$$

where $f(A)$ is the frequency density, that is, the number of wildfires with burned area A (properly normalized to 'unit' bins), and β is a constant.

Plotting the frequency densities against area in log–log space produces a straight line with slope $-\beta$. Power-law frequency–area behaviour has often been interpreted as a sign that a system is in an SOC state, as it suggests emergent global properties that have risen from simple local interactions (Solow 2005). As a result, the presence of power-law frequency–area statistics in the forest-fire CA *model* are suggestive of SOC behaviour, particularly because the properties are robust to the values of various forcing parameters and the values of the critical exponents of the model appear 'universal' (i.e. the frequency–area power-law exponent

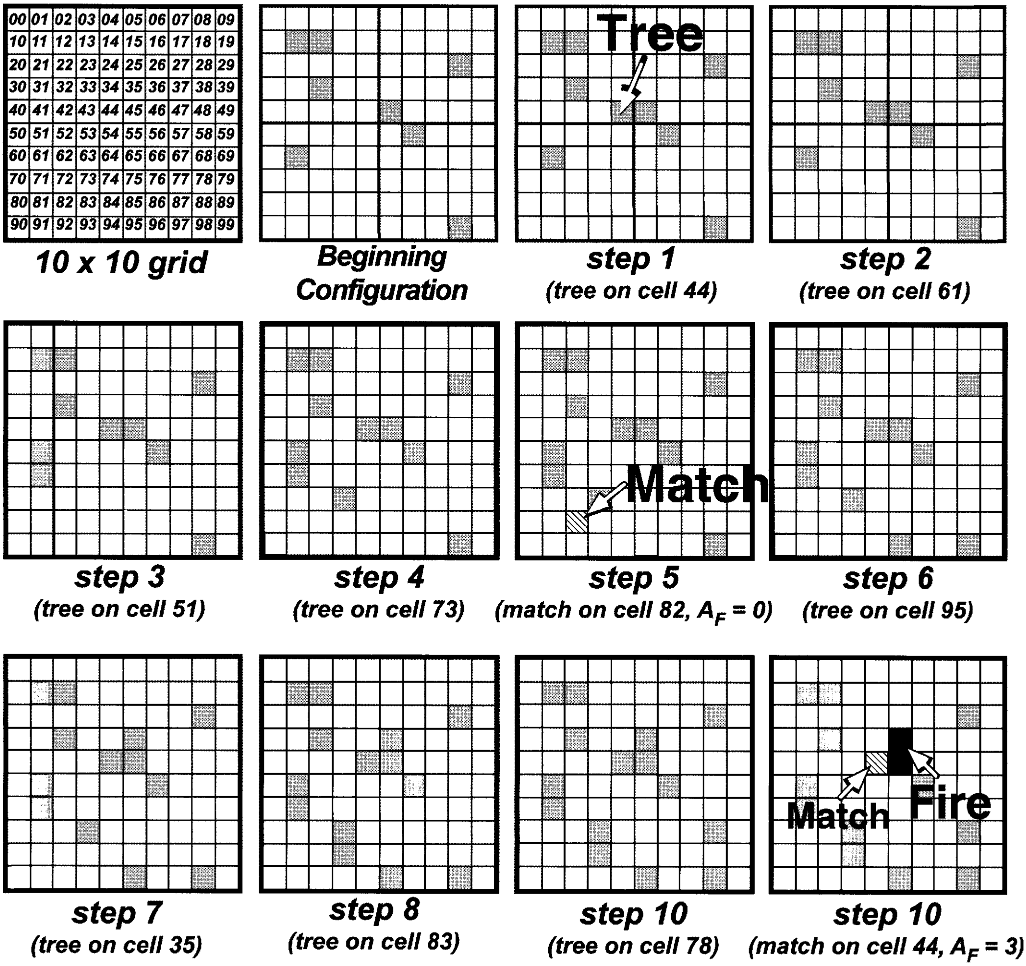

Fig. 1. Illustrative sequence of the forest-fire cellular automata (CA) model. This example illustrates ten time steps of a 10×10 model grid. Grid cells may be unoccupied (depicted by white) or occupied by trees (depicted by grey). Trees are randomly dropped on the grid at each time step, and if a cell is unoccupied, then a tree is 'planted', as in Step 1. If a cell is already occupied, then nothing happens when a tree is dropped on it (e.g. Step 2). At every $1/f$ time step a 'match' (depicted here by a hatched cell) is randomly dropped on the grid, where f is the sparking frequency. In this example, $1/f = 5$. If the match falls on an unoccupied grid cell, a wildfire is *not* ignited (e.g. Step 5) and the model moves to the next time step where tree drops are again attempted. However, if a match falls on an occupied site the tree burns and the model fire spreads to all neighbouring non-diagonal occupied sites (e.g. step 10, with a model fire size of $A_F = 3$ cells).

remains independent of any model parameters; Clar *et al.* 1994).

Based on the dynamics of these simple models, proponents of SOC suggested that it 'might be *the* underlying concept for temporal and spatial scaling' in dynamical systems (Bak *et al.* 1987, p. 384) and even that it might define *How Nature Works* (Bak 1996). With hindsight these claims seem overstated, particularly considering the way in which this modelling process proceeded. As Hergarten (2002) points out, modelling in earth and environmental sciences often starts by observing a set of phenomena and then proceeds to attempt to represent these phenomena as accurately as possible in a modelling framework. However, these much more abstract CA models were developed by statistical physicists with little regard to the actual processes they were representing, with the 'forest fire' label originally intended more as a metaphor rather than a claim of representation. As generalized models of abstract systems these experiments are interesting, but whether their behaviour is present in the real world has been questioned (e.g. Reed & McKelvey 2002). There are of course many questions that remain, but two of interest to earth scientists and, in particular, ecologists, include (1) whether the patterns and behaviours observed in the CA models are found in nature? and (2) whether 'real-world' forests and their disturbance regimes are/show SOC?

Confronting models with data

Observed frequency–area distributions in nature

Malamud *et al.* (1998) were the first to examine whether the power-law frequency–area distributions of model fires in the forest-fire CA model were also characteristic of 'real-world' wildfire regimes. Examining data sets from four study areas they found that wildfire frequency–area distributions followed a power law (see Table 1). In these four regions, spread around the globe and with widely varying environmental conditions, frequency–area power-law behaviour was observed over up to six orders of magnitude, with the power-law exponent $\beta = 1.3–1.5$. Malamud *et al.* (1998) also attempt to interpret some of the parameters in the forest-fire CA model in the context of observed fire dynamics and regimes. The 'sparking frequency' parameter (the frequency with which model fires are given the potential to start by a match) was directly compared to management strategies practised in Yellowstone National Park, and the implications of changes in the parameter values discussed with reference to real-world events. Although the exponents in the frequency–area power-law relationship differ between the real world and model data (with the model data exhibiting consistently smaller β values), the suggestion by Malamud *et al.* (1998) was that wildfires could be quantified in nature by using the same frequency–area scaling relationship found in the forest-fire CA model (see also Turcotte 1999). The implication was that the ecological systems in which real wildfire regimes exist may potentially exhibit SOC behaviour in the same way as the CA models appear to.

Malamud *et al.* (1998) were not the first to find power-law scaling in wildfire frequency–area statistics (e.g. Minnich 1983, as discussed above in the section on ecological studies). However, they were the first to compare wildfire regimes in nature and in the forest-fire CA model. Furthermore, the prominent location of this publication, allied with enthusiasm in much of the scientific community at the time for SOC, meant that a flurry of similar studies making similar analyses of real wildfire regimes soon followed (see Table 1). The majority of these studies (including the one by Malamud *et al.* 1998) were hampered by low spatial and/or temporal resolution, with relatively few wildfire records.

Recently, however, Malamud *et al.* (2005) have examined a much larger, high-resolution (spatial) data set detailing the burnt area, location and cause of ignition of 88,916 wildfires on United States Forest Service land across the conterminous USA for the period 1970–2000 (this is also discussed in Malamud & Turcotte 2006, in this volume). The large amount of data allowed Malamud *et al.* to examine different subregions of the conterminous USA, and compare them with each other. For each of the 18 regions examined, excellent power-law relationships were found between the non-cumulative number of wildfires and burned area, with $\beta = 1.30–1.81$. Two examples are given in Figure 2, showing the two extremes of values of β obtained by the authors. In Figure 2a are presented the frequency–area statistics for 16,423 wildfires in the *Subtropical* ecoregion division (within the southeastern part of the USA) and in Figure 2b, 475 wildfires in the *Mediterranean* ecoregion division (within California, USA). In both cases, excellent correlations are obtained with the power-law relationship (1), with $\beta = 1.81 \pm 0.07$ (± 2 s.d.) for the *Subtropical* ecoregion and $\beta = 1.30 \pm 0.05$ (± 2 s.d.) for the *Mediterranean* ecoregion.

The values of β for model fires in the forest-fire CA model are consistently lower than those of real-world wildfires. This indicates a reduced contribution to the wildfire regime of small wildfires, and a corresponding increased contribution of large wildfires, when comparing the real world with CA models (as observed by Malamud *et al.* 1998). Why there is such a consistent disparity is a question that needs to be addressed if links between the forest-fire CA model and real wildfire regimes are to be made.

Heavy tailed, but what flavour?

The large majority of studies have found heavy-tailed wildfire frequency–area distributions (Table 2), with the implication being that extreme events are perhaps not as extreme (or surprising) as they are often perceived to be (Katz *et al.* 2005). There is, however, considerable debate as to what type of probability distribution best describes these data. Empirically driven studies have used a range of heavy-tailed distributions, including the Weibull and the generalized Pareto among others, whereas studies arising from

Table 1. *Selected wildfire frequency–area studies presenting power-law statistics*

Author	Study area (time period) [no. of wildfires examined]	Non-cumulative power-law exponent (β)	Orders of magnitude over which power-law behaviour found	Comments
Malamud *et al.* (1998)	(a) USA Fish & Wildlife Service lands (1986–1995) [4284]	(a) 1.31	(a) 5.0	
	(b) Western USA (1150–1960) [120]	(b) 1.34	(b) 2.5	
	(c) Alaska, USA (1990–1991) [164]	(c) 1.43	(c) 2.0	
	(d) Australian Capital Territory (1926–1991) [298]	(d) 1.49	(d) 3.0	
Malamud *et al.* (2005)	(a) Conterminous USA (1970–2000) [88,916 wildfire fires over 18 different study areas (ecoregions)]	(a) 1.30–1.81	(a) 4.5–5.5	Error bars on β mostly in range ± 0.05–0.09 (± 2 s.d.)
McCarthy & Gill (1997)	(a) Australia: Victorian Forests (?) [?]	(a) $\sim$1.4	(a) 4.0	β read manually off the authors' graphs of non-cumulative frequency–area
	(b) Australia: Ku-ring-gai Chase National Park (?) [?]	(b) $\sim$1.2	(b) 4.0	
Niklasson & Granstrom (2000)	(a) Northern Sweden: (1499–1650) [28]	(a) $\sim$1.4*	(a) 3.0	
	(b) Northern Sweden: (1650–1880) [126]	(b) $\sim$1.9*	(b) 3.0	
Ricotta *et al.* (1999)	(a) Italy: Liguria (1986–1993) [9164]	(a) 1.72*	(a) 2.0	Outside the two orders of magnitude over which authors found power-law behaviour, the frequency–area distributions 'deviated' significantly
Ricotta *et al.* (2001)	(a) Italy: (i) Molise (1986–1993) [1236]; (ii) Cilento (1975–1996) [508]; (iii) Simbruini Mountains (1970–1997) [215]	(a) 1.2–1.3 & 1.9–2.1	(a) to (d) 1.0 to 3.5	For each of the eight regions studied, the authors applied power-law fits to two (Italy, France, Greece) or three (Spain) ranges of areas
	(b) Spain: (i) Sierra de Gredos (1974–1999) [2105]; (ii) Alicante (1973–1996) [132]; (iii) Ribera d'Ebre (1983–1997) [204]	(b) 1.1–1.3, 1.3–1.5 & 2.2		
	(c) France: Venaco Region (1977–1997) [335]	(c) 1.2 & 1.8		
	(d) Greece: Mount Penteli (1954–1996) [336]	(d) 1.2 & 1.5		
Song *et al.* (2001)	(a) China: Nei-Meng-Gu	(a) 1.23	(a) 2.5	
	(b) China: Hei-Long-Jiang	(b) 1.28	(b) 2.5	
	(c) China: Guang-Xi	(c) 1.63	(c) 1.2	
	(d) China: Yun-Nan (China: all 1950–1989) [number of wildfire records not given]	(d) 1.76	(d) 1.2	
Turcotte & Malamud (2004)	(a) Ontario, Canada (1976–1996) [15,308]	(a) 1.38	(a) 5.5	
Ward *et al.* (2001)	(a) Ontario, Canada (1976–2000) [484–3279]	(a) 1.13–1.25	(a) 5.0–6.0	
Zhang *et al.* (2003)	(a) Russia (2001) [2764]	(a) 2.03	(a) 3.0	Smallest fire observed was 2 km^2 due to nature of satellite data

*Authors originally did cumulative frequency–area distributions. So that all results are presented as equivalent non-cumulative frequency–area, we have added 1.0 to the power-law exponents authors found using cumulative frequency–area distributions, to arrive at equivalent non-cumulative frequency–area distribution β. Note that any results obtained using cumulative frequency–area distributions must be treated with caution, as cumulative frequencies can obscure underlying trends in finite data sets (Main 2000).

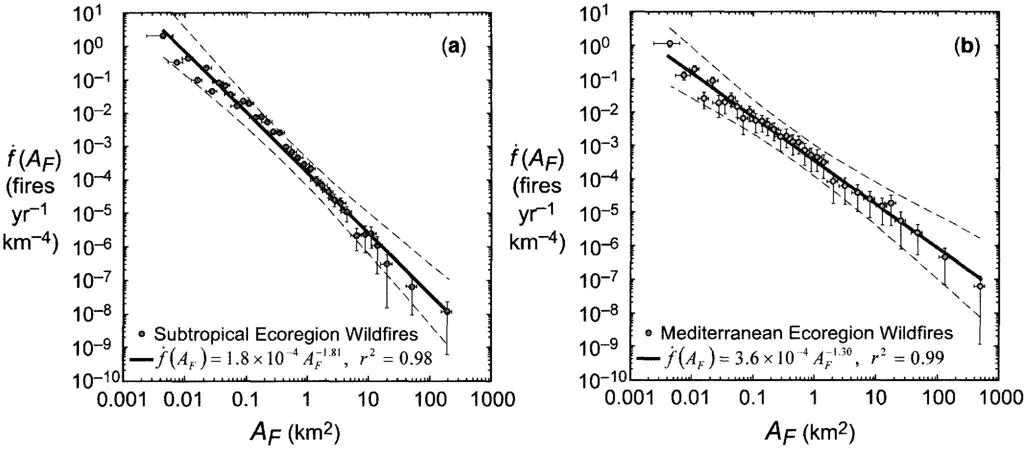

Fig. 2. Normalized frequency–area wildfire statistics for (**a**) *Mediterranean* and (**b**) *Subtropical* ecoregion divisions, for the period 1970–2000 (figure after Malamud *et al.* 2005). Shown (circles) are normalized frequency densities $\dot{f}(A_F)$ (number of wildfires per 'unit bin' of 1 km^2, normalized by database length in years and USFS area within the ecoregion) plotted as a function of wildfire area A_F. Also shown for both ecoregions is a solid line, the best least-squares fit to Eq. (1), with coefficient of determination r^2. Dashed lines represent lower/upper 95% confidence intervals, calculated from the standard error. Horizontal error bars are due to measurement and size binning of individual wildfires. Vertical error bars represent two standard deviations (± 2 s.d.) of the normalized frequency densities $\dot{f}(A_F)$.

exploration of the forest-fire CA model have emphasized power-law distributions (see Table 1). Some have queried whether 'true' power-laws would be expected in nature (e.g. Bolliger *et al.* 2003), and others whether observed wildfire regimes do actually follow a power-law (e.g. Reed & McKelvey 2002). Malamud *et al.* (2005) acknowledged that there will always be upper and lower cutoffs in nature for any power-law behaviour and that a true mathematical power-law (fractal) would be impossible in nature (see also Brown *et al.* 2002). There will always be a lower limit to what can be described as a wildfire, and measurement accuracy of the smallest wildfires is problematic. At the upper bound both Reed & McKelvey (2002) and Malamud *et al.* (2005) cite topographic influences restricting wildfire spread and therefore putting a constraint on the largest possible wildfire. Such effects are analogous to the finite-grid size effect observed in the forest-fire CA model. Schenk *et al.* (2000) showed the finite-grid effect results in a collapse of the frequency–area power-law scaling relationship when the correlation length (a measure of the radius of the largest tree cluster) becomes large relative to the size of the system (i.e. the size of the grid).

Table 2. *Examples of heavy-tailed wildfire frequency–area distributions suggested by recent studies**

Nature of distribution	Author	Study area (time period) [no. of wildfires examined]
Power law	See Table 1	
Negative exponential	Baker (1989)	Minnesota (1727–1868) [not stated]
Weibull distribution	Reed & McKelvey (2002)	(a) Sierra Nevada, California (1908–1992) [2536]
		(b) Nez Perce NF, Idaho (1870–1994) [1795]
		(c) Clearwater NF, Idaho (1910–1999) [884]
		(d) Yosemite NP, California (1930–1999) [3190]
		(e) N.E. Alberta (1961–1998) [5478]
		(f) Northwest Territories (1992–1999) [2544]
Truncated power law	(a) Burroughs & Tebbens (2001)	(a) Australian Capital Territory (1926–1991) [298]
	(b) Cumming (2001)	(b) N.E. Alberta (1980–1993) [2898]
	(c) Schoenberg *et al.* (2003)	(c) LA County, California (1950–2000) [548]

*These studies are based on empirically observed, rather than model-derived, data and analyses. Although a variety of different distributions are suggested, all are heavy-tailed in nature and many suggest power-law behaviour over a limited range of magnitudes.
NF: National Forest; NP: National Park; LA: Los Angeles.

As noted above (section on ecological studies), studies in the 1970s to the 1990s examined wildfire frequency–area relationships and suggested that they follow distributions other than the power law (e.g. Baker 1989). More recent studies have also suggested that wildfire distributions are not truly power-law, in the sense that they are not power-law across all event magnitudes (see Table 2). For example, Burroughs and Tebbens (2001) examined the same Australian Capital Territory (ACT) data used by Malamud *et al.* (1998) in a cumulative plot and proposed that a truncated power law offered a better fit. This then, is one criticism of the contention that real-world wildfire regimes show similar behaviour to those of the forest-fire model – the fact that real-world wildfires may not actually exhibit power-law (i.e. scale-invariant) behaviour in their frequency–area statistics. Reed & McKelvey (2002) specifically considered the question of whether power-law fire size distributions existed in real-world data as suggested by the presence of SOC behaviour. Examining six regions in North America (see Table 2), the authors suggested that the presence of power-law behaviour in wildfire size distributions was exaggerated, and that variations on the Weibull distribution provided the best fit to data. Reed & McKelvey (2002) did concede that power-law behaviour was in evidence over a *limited* range of wildfire sizes for some regions.

The examples cited here (Table 2) consider distributions other than the power law provided better fits to empirical data. However, for all the regions studied, these alternative distributions are 'heavy-tailed', with many of them very closely related to the two-parameter power-law distribution, but with additional parameters. A number of these distributions still exhibit scale-invariance over some part of the range of wildfire sizes they examine. Discrepancies between these distributions are most prominent in the tails where infrequent, extreme events cause distortion. More generally, there are other distributions that may explain the power-law distributions observed; for example, the log-normal and exponential distributions look quite linear in log–log space, especially when the distribution extremes are 'veiled' (May 1975; Brown *et al.* 2002). The infrequent, but very large events, in the heavy tail of the wildfire size distributions are of most concern for fire and forestry managers, but it is also the area where the most uncertainty lies. Wildfire size distributions that inaccurately represent the upper tail are problematic for managers. However, it is often the case when dealing with probabilistic hazard forecasting that the uncertainty for the recurrence intervals for the largest events (e.g. earthquakes and floods) is very large, and managers 'make do' with what is available. Thus, we believe it is important that any attempt to fit a specific heavy-tailed distribution to data is accompanied by error bars and a measure of the confidence of those fits (Fig. 2) so that the uncertainty by the hazard managers can be fairly addressed. In many studies error terms and measures of confidence are not provided. The debate regarding whether power-law frequency–area distributions and associated SOC-type behaviour, as found in the forest-fire CA model, are also found in nature remains open; in the next section, we discuss possible new directions for this effort.

Whichever flavour, what does this all mean for future research?

Perspectives on SOC

The current state of research contests the presence of power-law behaviour in real wildfire regime frequency–area distributions as evidence of SOC-type mechanisms (Gisiger 2001; Frigg 2003; Solow 2005). It should be remembered that the FFCA, and other models of its type, is a metaphor for SOC behaviour rather an explicit representation of a specific system and its associated suite of processes. The spatially random recovery (re-growth) of trees is a weak assumption as seed dispersal processes will determine tree regeneration patterns (this type of recovery was simulated in a recent model of mussel-bed disturbance, Guichard *et al.* 2003).

There are many other mechanisms by which power-law behaviour can be generated in nature – using the presence of power laws to determine whether a system is SOC suffers from the problem of under-determination (e.g. Oreskes *et al.* 1994; Frigg 2003). For example, another mechanism that has been proposed to explain the presence of power-law size–frequency distribution in many systems is Highly Optimized Tolerance (HOT; Carlson and Doyle 1999, 2002; Doyle and Carlson 2000). HOT takes a rather different view of complex systems than SOC, focusing on 'designed' systems (whether engineered or subject to natural selection) that are optimized to be robust in the face of environmental uncertainty. In itself the simple presence of a power law is a very weak test of the presence of any specific generating mechanism, and the way investigation is currently being pursued does not improve on this. After all, how good does a power-law relationship have to be to show SOC behaviour? However, irrespective of the origins of power-law behaviour, and despite the criticisms levelled by some authors advocating alternative distributions with better fits to the data, the power law is currently the most parsimonious model available to describe wildfire frequency–area distributions.

The implication that SOC behaviour demands true power-law behaviour over all magnitudes of events is, of course, in reality impossible. Only in an infinite system is this possible, but the world is finite. Therefore, not only is it no surprise that we do not find 'true' power-law behaviour in nature, but upper and lower cutoffs in any observed power-law behaviour are inevitable. However, possibly the biggest problem regarding the credibility of studies examining wildfire (or other 'natural') distributions and finding, or advocating, power laws, is the feeling that the researcher is simply fitting a line through their data with little regard for what this means or how this new finding might be used. In terms of interpreting observed distributions as the fingerprints of underlying process, differences between distributions (no matter how subtle) imply different generating mechanisms. As Brown *et al.* (2002, p. 622) comment 'In current applications of statistics to biological or ecological data, there is often an unfortunate tendency to be satisfied with the "model" or equation that gives a good fit. It is important, however, to consider the implications of the particular mathematical form of the equation'. Examining abstract systems (models) is a valid scientific pursuit in itself (as the forest-fire CA model, and so on, were initially used), but once we enter the realm of the actual, questions of 'Why?' and 'What use?' become of greater importance. Specifically, here we propose the questions 'Why is this system exhibiting power-law behaviour?' and 'To what use can this power-law nature be put?' should drive future research in this field.

Mechanisms and causal processes

Recently, some authors have begun to examine why power laws are observed in both models and nature. For example, Reed & Hughes (2002) suggest that if stochastic processes growing in an exponential manner are 'killed' at random (the burning of trees in the forest-fire CA model), the distribution of this killed state will follow a power law in one or both tails. Yang (2004) suggests power-law behaviour in SOC systems is the result of a balance between competitive trends. Specifically, power-law behaviour occurs when the probability of a site being in an 'active' state (rather than 'inactive') at the next time step is close to 0.5 (i.e. in the forest-fire CA model, the probability of being a tree versus being empty due to burning). Attempts to link the theory and patterns observed in SOC-type models to observed ecological patterns and processes are also on the horizon. Pascual & Guichard (2005) highlight the differences between three types of criticality ('classical', 'self-organized' and 'robust') and their relevance to disturbance patterns observed in ecological systems. These authors suggest that systems with subtle variation in their relationships between disturbance and recovery show different types of critical behaviour. Greater consideration of the processes driving SOC-type, power-law behaviour is required and links to observed processes and pattern, such as that demonstrated by Pascual & Guichard (2005) should be welcomed.

Despite the issues regarding wildfire frequency–area distributions, and the spatial restrictions on power-law behaviour, Malamud *et al.* (2005) emphasize the usefulness of power-law distributions for describing and studying the drivers of wildfire regimes at regional and continental scales. By spatially disaggregating data into Bailey's (1995) ecoregions (geographic areas of similar climate, vegetation and soil), and normalizing frequency–area statistics by ecoregion area and number of years in each data set, differences between wildfire regimes could be compared and contrasted with reference to putative broad-scale environmental drivers. Results showed an apparent east–west gradient in the power-law exponent (β values) across the conterminous USA, over 18 different ecoregions, potentially due to forest fragmentation and/or human population densities. Holmes *et al.* (2004) used a similar methodology to examine differences in Florida wildfire regimes according to vegetation type ('flatwoods' v. 'swamp'). Malamud *et al.* (2005) suggested that examining relationships between past and current climates could aid understanding as to how wildfire frequency–area scaling might change under future modified climate conditions. This type of approach is advocated by Brown *et al.* (2002), who believe that improved analysis of empirical patterns is required (in particular they advocate looking for systematic deviations from self-similarity).

Forecasting and prediction v. explanation

Malamud *et al.* (2005) used their power-law frequency–area statistics to do probabilistic hazard analysis, where they calculated wildfire recurrence intervals – the average time between events of a given area or larger – for spatial 'areas' of 1000 km^2 in each of the ecoregions. A map of expected recurrence intervals for wildfire areas of 10 km^2 or greater was presented for the 18 ecoregions of the conterminous USA (Fig. 3). Examples of recurrence intervals found included 2 ± 1 years (± 2 s.d.) in the *Mediterranean* ecoregion and 203 ± 99 years (± 2 s.d.) in the *Warm Continental* ecoregion. In other words, for the Mediterranean ecoregion, in any 1000 km^2 'area' in this ecoregion, the analyses of Malamud *et al.* (2005) would indicate on average one wildfire with burned area greater than 10 km^2 every 1–3 years,

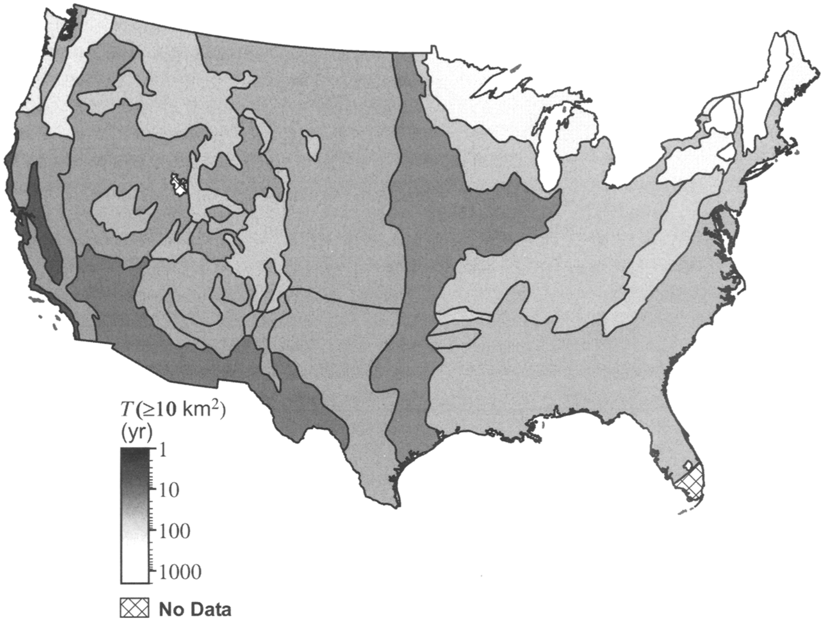

Fig. 3. Spatial mapping of wildfire recurrence intervals for the conterminous USA by ecoregion division (figure after Malamud *et al.* 2005). Based on a power-law frequency–area relationship from empirical data for the period 1970–2000 (see Fig. 2), recurrence intervals show how many years on average a wildfire of 10 km^2 or larger would be expected in spatial areas of 1000 km^2 within each ecoregion division. The legend colours go from black (small recurrence intervals) to white (large recurrence intervals), representing 'high' to 'low' hazard, with the legend scale in years increasing logarithmically.

or 33–100% probability of occurring in any given year. This is compared to the *Warm Continental* ecoregion, where there is a 0.3–1.0% probability for the same size wildfire (10 km^2 or greater) occurring in any given year.

Even with the large error bars, these results are useful for broad generalities of wildfire risk in the conterminous USA. The question is whether they are useful for accurate probabilistic hazard forecasting on a finer geographic scale compared with more complex models that might provide more accurate assessment of the assessment at the cost of needing greater parameterization (e.g. Presiler *et al.* 2004). However, not all scientific models must be used for prediction, and Malamud *et al.* (2005) argue that the two-parameter 'parsimonious' power-law model is a useful tool to learn about wildfire regimes at broad, regional scales as other authors have recently emphasized (Cleland *et al.* 2004; Schoennagel *et al.* 2004).

These studies do not lend themselves easily to applied wildfire management in terms of specific probabilistic hazard estimation or wildfire prediction (i.e. when and where a fire will occur) at specific points in the landscape. Rather these studies are useful to examine the behaviour of wildfire regimes at broader scales – for broader management issues over longer time-scales and larger spatial extents. However, the examination of the driving forces and most important processes between regions increases understanding of these systems and will become increasingly useful for management. Describing wildfire regimes through the simplified assumption of power-law behaviour will be one aspect of this examination of the driving forces behind wildfire regimes.

Finally, there are possible links between other more complicated statistical tools and the simple power-law approach outlined here. Katz *et al.* (2005, p. 1133) comment that 'an apparently unappreciated connection between the existence of power laws in ecology and statistical extreme event theory has been identified'. Although the usual application of statistics emphasizes the mean and variance as a probability distribution's parameters of interest, extreme event statistics (and extreme value theory) focus on a variable's extremal values (Gaines & Denny 1993; Katz *et al.* 2005). Moritz (1997) compares the 'extremal fire regime' (i.e. the distribution of the largest wildfires in each year) in two regions of the Los Padres National Forest (California, USA) in relation to wildfire suppression and climatic forcing events. Although seemingly comparatively infrequently used in fire regime studies, the work of Moritz (1997) suggests that the approach has potential utility for comparison of wildfire regimes over broad space–time scales, and also for exploring the relative importance of different forcing mechanisms (by including other environmental factors as covariates in models of the extremal wildfire regime). A similar approach has been taken by Alvarado *et al.* (1998). Gaines & Denny (1993) also comment on the observed spatial consistency of parameter estimates of extreme value distribution, and go so far as to consider that this may be indicative of the 'existence of underlying principles governing these phenomena' (p. 1677). The question of what drives and/or constrains spatio-temporal variability in model coefficients and observed probability distributions remains to be adequately addressed.

Conclusion

Quantitative description of disturbance regimes is of considerable interest to ecologists and others, from both theoretical and applied standpoints. Wildfire frequency–area distributions have received ongoing attention. On the one hand ecologists are interested in unravelling the importance of 'extreme' events for ecosystem composition and function, on the other there has been considerable interest in the idea that power-law frequency–area wildfire distributions indicate the presence of 'self-organized criticality'. From an applied perspective there is an obvious interest in being able to predict the likelihood of extreme events for hazard management and mitigation. Although heavy-tailed distributions typify observed wildfire frequency–area distributions, there is considerable debate over the exact nature of the probability distribution(s) that best describe these data; some authors have strongly advocated power laws as best descriptors while others have not.

This debate has been muddied by interpretations of systems being self-organized critical depending on what type of distribution is observed. Although power laws provide a simple and parsimonious description of many observed frequency–area distributions, more caution needs to be taken in ascribing the presence of power laws to a system being in the self-organized critical state than has often been the case. Studies examining the 'Why?' and 'What use?' of power laws in nature should be extended in the future. Irrespective of theoretical debates regarding the complexity theory underlying wildfire distributions, or regarding the distributions themselves, future studies should utilize techniques associated with self-organized criticality, cellular automata modelling and statistical physics to build bridges toward the ecological community. In turn, this will allow them to become of greater value as tools for examining ecological processes and systems, while attempting to improve understanding of fundamental underlying natural laws.

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