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Journal of volcanology and geothermal research

Journal of Volcanology and Geothermal Research 150 (2006) 132-145

www.elsevier.com/locate/jvolgeores

# Unrest at the Campi Flegrei caldera (Italy): A critical evaluation of source parameters from geodetic data inversion

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> Received 24 May 2004; received in revised form 27 September 2004 Available online 8 August 2005

#### Abstract

We have analysed the deformation documented during unrest at the Campi Flegrei caldera, Italy, between 1981 and 2001. Via inverse modelling, we constrain the location, geometry and size of the source responsible for the continuing period of surface deformation. We present a critical re-evaluation of results from previously published models and for the first time invert post-1994 data to infer source parameters. Our evaluation is based on constraints from additional horizontal displacement data, mechanical properties of the country rocks, effects of volcanic surface loading and on other geophysical and geochemical observations.

We invert leveling and tide-gauge data for a spherical point (Mogi-model) source, a penny-shaped crack and finally a prolate spheroid. Despite the good qualities of fit of both the Mogi-model and the penny-shaped source to the vertical displacement data, our critical evaluation of the implied source properties forces us to reject these models. We propose instead a vertical prolate spheroid located about 800 m East of Pozzuoli at a depth of 2.9 km (95% confidence bound 2.0 to 4.2 km) with an aspect ratio of 0.51 (95% bounds 0.37–0.69) as a more appropriate source model. This model best accounts for the criteria employed and the observed deformation between 1981 and 2001. Combined with results from the inversion of gravity change data (1982–1984) for the spheroidal source, we infer a hybrid nature of the source including both magmatic and hydrothermal components. © 2005 Elsevier B.V. All rights reserved.

Keywords: caldera unrest; geodesy; data inversion; magma; hydrothermal system; Campi Flegrei

# 1. Introduction

# 1.1. Geological setting and history of unrest

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The Campi Flegrei caldera (CFc, Fig. 1) is a morphological structure including submerged and continental parts at the western edge of the Bay of Naples, Italy. The caldera was formed from two

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Fig. 1. Digital elevation model of Campi Flegrei area. (a) The circumference of the area affected by ground movements since 1972 with the town of Pozzuoli located in its centre is highlighted with a broken line. This area coincides with the morphological expression of the Campi Flegrei caldera (CFc). (b) Close-up of the CFc indicating the location of the 16 benchmarks where deformation data analysed in this paper have been obtained between 1981 and 2001 (after Gottsmann et al., 2003). 2—Posillipo, 3—Nisida, 4—Bagnoli, 5—La Pietra, 6—Gerolomini, 7—Via Napoli, 8—Serapeo, 9—Arco Felice, 10—Baia, 11—Miseno, 12—Astroni, 13—Via Campana, 14—Monte Ruscello, 15—Solfatara, 16—Quarto, 17—Piazza Esedra.

main collapse events at 37 and 12 ka (Rosi et al., 1983; Orsi et al., 1992). Post-caldera volcanic activity has been mainly explosive, the latest eruption occurred in 1538 (Di Vito et al., 1987). The caldera appears to have undergone resurgence since 10 ka which led to the dissection of the caldera floor into individual blocks (Orsi et al., 1996). Results from drilling (Chelini and Sbrana, 1987) and geophysical investigations (Cassano and La Torre, 1987; Vanorio et al., 2004) suggest that a shallow magma reservoir resides beneath the CFc with its top at 4–5 km depth. Based on numerical simulations, Wohletz et al. (1999) propose the existence of long-lived magma reservoirs beneath the CFc totalling a minimum volume of  $450 \text{ km}^3$ .

Campi Flegrei is renowned as a site of continual slow vertical movements (bradyseisms). Our current understanding is that the caldera has been deflating for at least 2000 yr (Parascondola, 1947). Before 1968, the rate of deflation was measured at 15 mm/year at "Serapeo", an ancient Roman market near the harbour of Pozzuoli (Fig. 1). Despite the overall deflation, periods of uplift occurred in the early 1500s (Di Vito et al., 1987) and since 1969 (Di Vito et al., 1999). The eruption in 1538 appears to have been preceded by decades of uplift cumulating in 8 m of inflation, shortly before the eruption (Di Vito et al., 1987). The latest periods of tumescence (1970–72 and 1982–84) have resulted in a net uplift of 3.5 m. Both periods are characterised by episodes of rapid uplift on a background trend of slow deflation.

# 1.2. Dynamics of deformation since 1970 and previous work

The CFc has been similarly affected by uplift and deflation since 1970. Deformation is centred around Pozzuoli and decreases smoothly away from Pozzuoli becoming insignificant about 6–7 km from the town centre, just beyond the caldera rim (Fig. 1;Corrado et al., 1977; Berrino et al., 1984; Berrino, 1998).

The deformation pattern associated with uplift and deflation at the CFc has been modelled and interpreted by various authors (Table 1) who suggest a source of volume change located at depths between 2.7 and 5.4 km. Results from these studies are not necessarily directly comparable, as the modelling parameters differ significantly. For example, Avallone et al. (1999) and Lundgren et al. (2001) modelled the background deflation inverting data from radar measurements but did not consider the previous source pressure increase during inflation. We suggest that in order to understand the mechanisms of caldera unrest at Campi Flegrei, the concurrent inflation and deflation must be modelled together. Failure to account for the net combined changes may result in inferring unrealistic physical properties for the source. Regarding the nature of the uplift and rapid deflation source, some authors (Corrado et al., 1977; Berrino et al., 1984; Dvorak and Berrino, 1991) infer a shallow magma reservoir beneath the caldera, while others (Bonafede and Mazzanti, 1998) identify the hydrothermal system at Campi Flegrei as a more likely source. The currently ongoing slow deflation is generally interpreted to result from an ongoing, long-term pressure decrease in the source. Despite this general agreement, there is substantial controversy as to the nature of the source and pressure drop. Potential sources are i) magma removal; ii) ground water removal from water saturated deposits that fill the caldera (Berrino, 1994; Dvorak and Berrino, 1991); iii) exhaustion of overpressure in the hydrothermal system (Bonafede and Mazzanti, 1998; Lundgren et al., 2001) or some combination of each.

In this paper, we present a new analysis of geodetic data recorded over two decades (1981–2001) at Campi Flegrei. Reassessment of the period of inflation (1982–84) illustrates that despite the good agreement between measured and modelled data, employing 'simple' spherical source models at the CFc requires unrealistic assumptions of physical and mechanical properties for the source and encasing rocks. We present results from a joint and comprehensive inversion of deformation data and use plausible boundary conditions to test the validity of inferred source parameter values.

#### 2. Deformation monitoring

Vertical deformation at Campi Flegrei has been recorded in the region since 1970 (Corrado et al., 1977; Berrino et al., 1998). Since 1981, deformation measurements have been supplemented by microgravity surveys (Berrino et al., 1984; Berrino, 1994). The deformation benchmarks shown in Fig. 1 coincide with the locations of gravity stations. The entire set of deformation data obtained in conjunction with gravity data at the CFc between 1981 and 2001 was

Table 1

Compilation of source depths (H) for caldera unrest at Campi Flegrei inferred from various investigations of surface deformation

H (km)	Period	Data; best-fitting source model	Reference
2.8	Inflation (1982-84) and deflation (1985-93)	Levelling; point	Berrino (1994)
3.0	Inflation (1970-72, 1982-84)	Levelling; rectangular sheet	Dvorak and Berrino (1991)
5.4	Inflation (1970–72, 1982–84)	Levelling; oblate spheroid	Bianchi et al. (1987)
3.0	Inflation (1970–72, 1982–84)	Levelling; rectangular sheet	Bonasia et al. (1984)
2.7	Deflation (1994–1997)	SAR; point	Avallone et al. (1999)
3.0	Inflation (1982–84)	Levelling; point	Fernandez et al. (2001)
2.9	Deflation (1993–98)	SAR; rectangular sheet	Lundgren et al. (2001)

Note, this list is not exhaustive.

published recently in Gottsmann et al. (2003) who also provide details on the network of 18 stations. In this paper we employ the vertical deformation data reported in Gottsmann et al. (2003) for inversion. Data obtained since 1994 have not previously been inverted in order to constrain source parameters.

The occupation protocol and measurement techniques are reported in detail in Gottsmann et al. (2003) and are only briefly recapitulated here. Deformation has been measured using tide gauges and precise levelling. For each survey, several elevation measurements were performed for each station. The uncertainty in height changes at each benchmark is typically about 4 cm. Following the approach by Gottsmann et al. (2003), the data were grouped into three different episodes during the recent period of caldera unrest: i) inflation (1982-1984), rapid deflation (1985-1987) and slow deflation (1988-2001). Data from each episode are inverted separately in order to discriminate between variations in source parameters such as position, depth and geometry as well as pressure and volume changes.

#### 3. Computational modelling

All geodetic modelling is performed by inverting vertical deformation data for an axi-symmetric source geometry in an elastic, homogeneous, isotropic halfspace. Variables and constants employed are listed in Table 2. One of the most critical model parameters is crustal rigidity, i.e., shear modulus,  $\mu$ , and source pressure change,  $\Delta P$ , are inversely correlated. Values of the rigidity of the lithospheric crust reported in previous studies on the CFc vary between 1.6 (Bianchi et al., 1987) and 30 GPa (Lundgren et al., 2001; Dvorak and Gasparini, 1991) and pressure changes deduced from geodetic modelling varied accordingly over some orders of magnitude. Studies on seismic wave speeds in altered volcanic rocks have shown that the shear modulus may be smaller than 5 GPa (Vanorio et al., 2003). We are aware that the selection of a uniform elasticity is an oversimplification as the crust is not a homogeneous half-space. In order to account for the existence of a long-lived magmatic system, high heat flow and large geothermal gradient (Chelini and Sbrana, 1987) at Campi Flegrei, we chose a shear modulus of 10 GPa.

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$\mu \qquad Shear modulus of elastic \qquad 1.00E+10  Pa \\ half-space \\ \lambda \qquad Lame constant \qquad 1.00E+10 \\ \nu \qquad Poisson ratio \qquad 0.25$	Т	Tensile strength		Pa
$\begin{array}{c} \mu & \text{Bindar includes of cluster} & 1.001 \times 10^{-1} \text{ f a} \\ \text{half-space} \\ \lambda & \text{Lame constant} & 1.00E+10 \\ \nu & \text{Poisson ratio} & 0.25 \end{array}$	1 s 11	Shear modulus of elastic	1.00E + 10	Pa
λLame constant $1.00E+10$ νPoisson ratio $0.25$	μ	half-space	1.002 10	14
v Poisson ratio 0.25	λ	Lame constant	$1.00E \pm 10$	
	D D	Poisson ratio	0.25	
a Semi-major axis of prolate m	a	Semi-major axis of prolate	0.23	m
spheroid	.,	spheroid		
<i>h</i> Semi-minor axes of prolate m	h	Semi-minor axes of prolate		m
spheroid	~	spheroid		
$\gamma^2$ , $\tilde{\gamma}^2$ Chi-square, reduced chi-square	$\chi^2, \tilde{\chi}^2$	Chi-square, reduced chi-square		

We inverted the data for each of the three episodes (inflation, rapid deflation and slow deflation) for the following finite source geometries (Fig. 2): i) a Mogi-type point source (Mogi, 1958), ii) a uniformly pressurised spherical cavity (McTigue, 1987), iii) a penny-shaped crack (Fialko et al., 2001a) and iv) a prolate spheroid (Yang et al., 1988). The spherical



Fig. 2. Sketch of source geometries employed for data inversion. From left: Mogi-type point (Mogi, 1958), finite sphere (McTigue, 1987), penny-shaped crack (Fialko et al., 2001a) and prolate spheroid source (Yang et al., 1988). See Table 2 for notations.

cavity and the Mogi-type point source are essentially the same. However, we present results from both inversions, as the solution for a finite spherical cavity allows us to quantify the radius vs. pressure change relationship as outlined below. In all models the source is located at point  $x_0$  and  $y_0$  and depth *H* from the surface. The relative topography of the area is only a few hundred meters and was not considered for inversion.

The vertical surface deformation  $(U_{zs})$  produced by a volume change,  $\Delta V$ , within a spherical Mogi source (Mogi, 1958; McTigue, 1987) is

$$U_{\rm zs} = (1 - v) \frac{\Delta V}{\pi} \frac{H}{R^3}.$$
 (1)

The spherical body will expand or contract under a pressure change  $\Delta P$  by

$$\Delta \alpha = \frac{\alpha \Delta P}{4\mu} \quad (\text{Hagiwara, 1977}). \tag{2}$$

A spherical body whose radius is far smaller than its depth may be represented by the mathematical concept of a point source (Mogi, 1958). The volume change of the source can be related to a pressure change via

$$\frac{\Delta V}{\pi} = \frac{\alpha^3}{\mu} \Delta P \quad (\text{McTigue}, \ 1987). \tag{3}$$

Vertical surface displacements for a pressurised horizontal circular (penny-shaped) crack were computed using algorithms presented in Fialko et al. (2001a). This finite oblate cavity essentially represents a body with a sill-like geometry of radius  $\alpha$  at depth *H* from the surface. Here, we present the results for the best-fit model as the ratio of crack depth to crack radius

$$h = \frac{H}{\alpha}$$
 (Fialko et al., 2001a). (4)

For  $h \ge 1$ , the deformation source is essentially a "point crack" (Fialko et al., 2001b). The maximum amplitude of surface uplift  $U_{zc}^{max}$  for a point crack is given by

$$U_{\rm zc}^{\rm max} = \frac{4}{\pi} \frac{1-\nu}{\mu} \Delta P \frac{\alpha^3}{H^2}$$
 (Fialko et al., 2001b). (5)

We have also inverted the deformation data for a pressure change  $\Delta P$  within a prolate spheroidal source employing the analytical solutions presented in Yang et al. (1988). The spheroid has a semi-major axis, *a*, and two semi-minor axes of equal value, *b*. The model also allows for a non-vertical geometry (dipping) of the spheroid with dip angle  $\theta$  and dip direction  $\phi$ .

Tiampo et al. (2000) have shown that the volume change induced by a pressure change in an spheroidal cavity at depth may be quantified via

$$\Delta V = \frac{\Delta P}{\mu} \pi a b^2. \tag{6}$$

We have applied an inverse-modelling procedure in order to obtain vertical displacements from each source model based on defined modelling parameters. First, a grid search algorithm was applied to constrain the location and depth of the source via a least squares criterion ( $\chi^2$ ; Bevington and Robinson, 1992) using a unit pressure change at each grid data point via

$$\chi^{2} = \sum_{i=1}^{N} \frac{(C_{i} - E_{i})^{2}}{\sigma_{i}^{2}}$$
(7)

where  $C_i$  are the calculated or modelled displacements,  $E_i$  are the expected or measured displacements, N the total number of the population and  $\sigma$  is the standard deviation of the displacements, which in our case is 4 cm.

Once a region with similar and small  $\chi^2$  results was identified, a grid refining technique was applied by reducing the grid node distances from 100 to 10 m and the model parameters were then varied independently. Finally, the solution giving the smallest  $\chi^2$  was selected for each of the four source models. In order to compare quality of fits obtained from the individual models we use a reduced  $\chi^2$  criterion  $\tilde{\chi}^2$ , where

 $\tilde{\chi}^2 = \chi^2 / (n - m)$ ; *n* is the number of observations and *m* is the number of free parameters of the models. The difference n - m is the number of degrees of freedom for each model.

## 4. Constraining the source: results and discussion

#### 4.1. Source location

Table 3 presents the best-fit parameters from data inversion for each source model and each deformation period. All four models give the same location (within error) of the centre of the pressure source responsible for caldera unrest between 1981 and 2001. The inferred surface projection of the source is located approximately 0.8 km east of the town centre of Pozzuoli for all deformation periods. This suggests that the source responsible for uplift between 1982 and 1984 is also responsible for both the fast and slow deflation since the beginning of 1985. Our location of the pressure source matches, within error, the location of the sources proposed by Dvorak and Berrino (1991), Berrino et al. (1984) and Berrino (1994).

There appears however to be a significant discrepancy between our source location (ca. 800 m east of Pozzuoli) and that presented in Avallone et al. (1999) who propose the existence of a Mogi-type source of pressure change located off-shore, ca. 800 m southwest of Pozzuoli.

Table 3

Modelling parameter values obtained from data inversion of vertical displacement data

Model	x <sub>o</sub>	Уo	Depth (H) km	Radius (α) km	a km	b km	$\Delta P$ MPa	$\frac{\Delta V}{*10^6 \text{ m}^3}$	$\tilde{\chi}^2$
Inflation (1982–1984)									
Sphere	426,550	4,519,250	2.7	0.7			530	64	12.7
Penny-shaped crack	426,500	4,519,200	3.0	1.0			172	87	4.5
Prolate spheroid	426,550	4,519,240	2.9		2.2	1.4	52	71	16.1
Rapid deflation (1985-	-1987)								
Sphere	426,550	4,519,250	2.7	0.7			-130	-15	2.0
Penny-shaped crack	426,500	4,519,200	3.0	1.0			-42	-21	1.1
Prolate spheroid	426,550	4,519,220	2.9		2.2	1.4	-14	-19	4.8
Slow deflation (1988–2	2001)								
Sphere	426,580	4,519,320	2.7	0.7			-133	-15	1.1
Penny-shaped crack	426,580	4,519,320	3.0	1.0			-42	-21	1.9
Prolate spheroid	426,550	4,519,250	2.9		2.2	1.4	-14	- 19	2.2

Despite significant differences in the values of source parameters, results from all the models allow us to suggest that volume changes during the fast deflation episode between 1985 and 1987 are on the same scale as the volume decrease during slow deflation between 1988 and 2001. The total volume decrease in the source between 1985 and 2001 is approximately 50% of the volume increase during inflation between 1982 and 1984 (Table 3).

# 4.2. Parameter constraints for spherical source

#### 4.2.1. Depth, volume and pressure changes

The best-fit solution for a Mogi-type point source model implies a source depth of 2.7 km for all periods of deformation. This depth is in very good agreement with results presented by other authors employing the Mogi-model (Berrino et al., 1984; Berrino, 1994). Note also, that despite the disagreement on the location of the source between our data and that in Avallone et al. (1999), our source depth matches the depth constrained by these authors. However, previous authors only consider a very small period of deformation and not the longer sequence modelled here. Our results from inverting the data for a Mogi-type source undergoing pressure changes  $\Delta P$  during the three deformation periods are shown in Fig. 3. Volume changes calculated via Eq. (1) for each of the periods are presented in Table 3. Following Eq. (3) pressure changes within a Mogi-type source cannot be solved independently of the source dimensions (Eq. (3); McTigue, 1987). Thus, geodetic data inversion cannot provide an independent simultaneous solution for both pressure changes and source dimensions. This ambiguity between pressure change and source dimension requires a number of assumptions on source parameters or parameter boundary values in order to obtain valuable information. For example, for the period of inflation the best fitting Mogi model suggests a source at 2.7 km depth undergoing a volume change of  $64 * 10^6$  m<sup>3</sup> (Table 3). The associated pressure change vs. sphere radius relationship for such a model is displayed in Fig. 4. For a radius of 0.7 km a pressure increase of more than 500 MPa (Eqs. (1),(2); Fig. 4) would be required. For the same amplitude of vertical surface deformation and the same quality of fit, a smaller source radius would require source overpressures to increase even more drastically (Fig. 4).



Fig. 3. Normalised vertical displacements  $(U_z/U_z^{max})$  as a function of distance *r* from the inferred surface projection of the source from data inversion for a Mogi source (Mogi, 1958; point and sphere) and a penny-shaped crack (Fialko et al., 2001a). The maximum vertical displacements obtained from the two source models match to within 1 cm.  $U_z^{max}$  is taken as the average of the two values. Results are shown for three different periods of deformation observed at Campi Flegrei between 1981 and 2001: inflation, rapid deflation and slow deflation (after Gottsmann et al., 2003). Results for the Mogi source are obtained from inversion for a source situated at 2.7 km depth. Results for the penny-shaped crack are obtained for a source with a depth vs. radius ratio h of 3.0. The solutions for the models were obtained by employing parameters as specified in Table 2.

### 4.2.2. Pressure boundary values

It is reasonable to relate the overpressure in the source to a fracture criterion of encasing rocks, i.e., the tensile strengths. Dry, cold competent rocks have maximum tensile strengths of 40–50 MPa. The tensile strength of encasing rocks in a longstanding volcanic environment such as Campi Flegrei is expected to be significantly less; 20 MPa or less would be expected for such an environment (Pinel and Jaupart, 2003; Folch and Marti, 1998). In the absence of a substantial volcanic edifice at Campi Flegrei, which could

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Fig. 4. The pressure change vs. source radius relationship for the best-fitting spherical source (Mogi, 1958) for a depth of 2.7 km and a volume change of  $64 * 10^6$  m<sup>3</sup> during inflation between 1982 and 1984 (Table 3; Eq. (2)). Also shown is the expected tensile strength ( $T_s$ ) of encasing rocks as a criterion to initiate fracturing during source pressurisation.

increase the required source overpressure to initiate wall rock fracturing (Pinel and Jaupart, 2003), a pressure increase of more than 50 MPa would have certainly fractured the country rock, for example if pressure changes within a hydrothermal system are considered to be the source for the observed deformation. From gravity inversion, Berrino et al. (1984) deduced a density of 2500 kg/m<sup>3</sup> for a Mogi source at 2.7 km depth, which suggests a predominantly magmatic nature. In that case, a pressure increase due to a magmatic intrusion could be dissipated by viscous deformation of the surrounding rocks (Dragoni and Magnanensi, 1989), a scenario that is not addressed in the framework of purely elastic behaviour considered here. Newman et al. (2001), for example, showed for the case of inflation of the Long Valley caldera that correcting for a viscoelastic halo around a point-source intrusion at depth might reduce the associated pressure increase by about one order of magnitude. Following the same rationale and allowing for a thick viscoelastic shell would similarly reduce the required pressure increase to perhaps around 50 MPa. Applying a viscoelastic layered Earth model, with an equivalent viscosity, Fernandez et al. (2001) deduced pressure changes of less than 10 MPa within a magmatic point source at 2.8 km depth for the period of rapid inflation.

#### 4.2.3. Constraints from other observations

A magmatic intrusion at such shallow depth is however difficult to reconcile with the lack of significant temperature anomalies in boreholes up to 2.5 km depth around the centre of the CFc, the observed geothermal gradient (Chelini and Sbrana, 1987), the lack of increased magmatic components in hydrothermal fluids (Martini, 1986), and constraints from seismological investigations (Vanorio et al., 2005) that propose the existence of the top of a magma reservoir at about 4–5 km depth.

If the spherical source is positioned at a depth of 5.4 km with a radius of 2.5 km, the pressure change required during inflation falls to less than 20 MPa, and this would also provide a reasonable explanation for the observed earthquake pattern. (Aster and Meyer, 1988). However, the resultant  $\chi^2$  of >5000 reflects an unacceptable quality of fit.

Pressure changes in a spherical source computed for the ongoing period of slow deflation are of the order of 200 MPa, if reasonable source dimensions are assumed (Table 3). Such large-scale depressurisation of the sub-surface system appears unrealistic for two reasons: i) the preceding pressure increase is most likely less than the tensile strength of surrounding rocks as explained above and thus marks a datum against which the subsequent pressure decrease can be assessed and ii) hydrostatic pressure compensation would ultimately result in the brittle failure of roof rocks into the depressurised reservoir, which has not been observed at the CFc. We therefore conclude that it is not satisfactory to use a shallow Mogi-type source geometry to account for the recent deformation pattern at Campi Flegrei, despite the good fit to the vertical deformation data of the Mogi-model for a point source located at 2.7 km depth.

#### 4.3. Parameter constraints for sill-like source

Results from inverting the data for the pennyshaped crack are shown in Fig. 3. The best fit to the data was obtained by inverting a crack with a depth/ radius ratio of 3.0. The crack can hence be regarded mathematically as a "point crack" (cf. Eq. (5)). A pressurised crack fits the vertical deformation data better than the Mogi source (Table 3). However, the same ambiguity between source size and pressure changes applies for inverting the deformation data for a penny-shaped crack.

The modelling parameters for the data inversion for all three periods of deformation using this source geometry are given in Table 3. Note, that despite the excellent fit of the model to the data, the pressure changes required to explain the observed uplift between 1982 and 1984 are in excess of 200 MPa if the source is assumed to be at a depth of 3 km. This depth was proposed by Dvorak and Berrino (1991) for a sill-like source. Our calculated volume change of  $87 \times 10^6$  m<sup>3</sup> within the penny shaped crack matches almost exactly the volume change inferred for a rectangular sheet by Dvorak and Berrino (1991). Following the line of argument presented for the case of the Mogi source, a depth of 3 km for a sill-like source has to be rejected on the basis of the required overpressures if the source of deformation is regarded as nonmagmatic, i.e., hydrothermal. A source depth of 5.8 km is required in order for overpressures to be less than 40 MPa. However, a pressure increase less than 20 MPa (a realistic tensile strength of the country rock), can only be derived from this model by a crack located at a depth of at least 10 km with a radius of 3.3 km. Seismicity during the 1982-1984 uplift was restricted to depths between 1 and 5.5 km with a concentration at 2-4 km beneath the centre of the caldera (Aster and Meyer, 1988). Pressurising a sill-like hydrothermal source at a depth of 10 km does neither seem consistent with the observed earthquake hypocentre pattern nor does it appear to be a reasonable geological scenario. If the source of inflation is considered to be a sill-like magmatic intrusion, the same thermal, geochemical and seismological reservations for its validity apply as for the case of the spherical source. Nevertheless, basing our evaluation solely on the observed vertical deformation we cannot rule out a sill-like injection at 3 km depth where the pressure change is dissipated by inelastic behaviour of a surrounding halo. We shall see below however that there is an additional critical constraint that limits the application of a sill-like source to explain the observed deformation pattern during inflation.

Taken in isolation, a pressure decrease of 24 MPa in a sill-like source located at 5.6 km depth appears a

realistic value for the ongoing episode of slow deflation. From geodetic data inversion alone we can neither unambiguously propose nor exclude a silllike body at a depth of 5.6 km as a possible source for the present episode of slow deflation and other observations and results need to be taken into account. For example, "miniuplifts" during the overall episode of slow deflation have recently been attributed to pressure changes in a shallow hydrothermal system beneath the CFc (Gaeta et al., 2003). These results are inconsistent with a deeper-seated source (>5 km) for the ongoing slow deformation. We therefore regard a sill-like source at a depth of around 5 km to be unsatisfactory to explain the observed surface subsidence between 1988 and 2001.

## 4.4. Parameter constraints for spheroidal source

#### 4.4.1. Depth and volume changes

Inverting the deformation data for a prolate spheroid (Yang et al., 1988), also includes the ambiguity between pressure increase and source dimension. The best solution from the inversion suggests the centre of the spheroid located about 800 m east of the town of Pozzuoli at 2.9 km depth with an aspect ratio b/a of 0.51 for all periods of deformation (Fig. 5). Although we have inverted for a vertical prolate spheroid, the solution for a source with a sub-vertical (84°) northward dip of its major axis provides an equally good fit. Due to the lack of data immediately to the south of the inferred source location we are unable to assess the statistical significance of this inclination.

The volume increase deduced for the period of inflation (1982–84) is  $71 * 10^6$  km<sup>3</sup>. We infer an annual volume decrease of about  $9 * 10^6$  m<sup>3</sup> during rapid deflation between 1985 and 1987 which compares with a rate of ~1.3  $10^6$  m<sup>3</sup>/year between 1988 and 2001. Almost one order of magnitude decrease in the rate of annual volume loss from 1988 onwards is necessary for the spheroidal source. The quality of fit to the vertical displacement data from inversion for the prolate spheroid is poorer compared with those obtained for the other two source models, in particular for periods of inflation and rapid deflation (Table 3).

However, there are some constraints obtained from the inversion that are worth addressing: i) the pressure changes required to fit the data are the smallest of the three models (see next section), ii) the source extends



Fig. 5. Normalised vertical displacements  $(U_z/U_z^{\text{max}})$  as a function of distance from the inferred surface projection of the source from data inversion for the vertical prolate spheroid situated at a depth of 2.9 km with semi-major axis, *a*, and semi-minor axes, *b*, of 2.2 and 1.4 km, respectively. Pressure and volume changes for (a) inflation 1982–1984; (b) rapid deflation 1985–1987 and (c) slow deflation 1988–2001 are shown in Table 3.

from a depth of about 6 km to within one km of the surface covering the area of prominent seismicity during the period of inflation (Aster and Meyer, 1988) and iii) the source includes areas of anomalous P- and S-wave ratios obtained from seismic tomography (Vanorio et al., 2004).

#### 4.4.2. Pressure changes

For the prolate spheroid source model, the pressure changes required to produce the observed deformation between 1982 and 2001 are 52 MPa for inflation and

14 MPa for rapid and slow deflation, respectively. The absence of significant surface fracturing and the lack of magmatic gases emitted at Campi Flegrei during the period of inflation (Martini, 1986; Martini et al., 1991; Tedesco et al., 1988) suggest that the pressurisation of the source was insufficient to promote substantial fracture propagation to the surface. Of all the models employed in this study, the pressure increase deduced for the prolate spheroid source provides the most satisfactory explanation for the observed phenomena during inflation. It is worth noting that reducing the rigidity to 5 GPa, similar to values employed by Bianchi et al. (1987) and consistent with elastic properties for altered caldera-fill successions (Vanorio et al., 2003), would result in an overpressure of less than 10 MPa for the period of inflation, well within the limits of the assumed maximum tensile strength of encasing rocks and of the same order of magnitude as those reported in Fernandez et al. (2001). In the case of the point and sill-like sources, the overpressures would reduce to 250 and 100 MPa, respectively, but are still significantly larger than the tensile strength of the encasing rocks.

# 4.4.3. Horizontal displacements

It is well known that pressure changes within different source geometries may result in similar vertical surface deformation patterns (e.g., Dietrich and Decker, 1975; McTigue, 1987). A solution to the source geometry problem could be obtained by evaluating horizontal displacements; their dependence on source geometry is more sensitive than that on vertical displacements (Dietrich and Decker, 1975; Battaglia et al., 2003). It is only when we compare horizontal displacements reported in Bianchi et al. (1987) with results of the horizontal displacements obtained from the model inversions (Fig. 6) for the surface uplift between 1982 and 83, that we realise that the data inversion for the prolate spheroid gives the best quality of fit to the horizontal displacements. The pennyshaped crack model fails to provide an acceptable quality of fit, while the Mogi model fits the near-field data reasonably well but at distances larger than 2 km from the source the model only provides a fair fit (Fig. 6). This provides further reason for the rejection of a sill-like source at 3 km depth and supports a prolate spheroid source model to explain the observed deformation at least for the period of inflation. In the absence



Fig. 6. Normalised horizontal displacements  $(U_r/U_z^{max})$  observed between 1982 and 1983 and model fits to the data. The maximum vertical displacements obtained from the individual source models match to within 1 cm.  $U_z^{max}$  is taken as the average of the three values.  $\tilde{\chi}^2$  values for the model solutions are 10.7 for the sill model (broken line), 2.44 for the Mogi-model and 1.3 for the prolate spheroid model (solid line). Observed horizontal displacement data from Bianchi et al. (1987).

of horizontal displacement data for the deflation episodes, we present our preferred model on the basis of country rock mechanics and observed geophysical and geochemical phenomena consistent with the vertical deformation.

4.4.4. Confidence bounds and the nature of the source In order to obtain confidence bounds on the spheroidal model parameters we employed a bootstrap percentile method (Efron and Tibshirani, 1986). We have obtained a 95% confidence bound on depth of 2.0 to 4.2 km, aspect ratio b/a 0.37–0.69, volume of 0.049 to 0.088 km<sup>3</sup> and pressure of 28–68 MPa for inflation and 7–23 MPa for rapid and slow deflation, respectively. The volume changes deduced from the spheroidal source for all periods of deformation match within error those obtained by inverting for both the Mogitype and sill-like sources (cf. Table 3).

It is obvious that the results obtained from the inversion of a prolate spheroid require the source to extend to shallow depths, which disqualifies a purely magmatic origin as the source of deformation based on the constraints obtained from other observations as outlined above. The prolate spheroid source would extend into the known hydrothermal system beneath the CFc probed by various deep wells around the area (Chelini and Sbrana, 1987). This would require a contribution from pressure changes within the hydrothermal system to the observed pattern of deformation. In fact, results from gravity inversion for the prolate spheroid for the period of inflation (Gottsmann et al., 2004) give a source density of about 1600 kg/m<sup>3</sup>, which suggests a hybrid nature for the source (both hydrothermal and magmatic components). The fit of the inversion to the gravity residuals presented in Berrino (1994) and Gottsmann et al. (2003) for the period of inflation is shown in Fig. 7. A possible scenario that accounts for the observed inflation may be the combination of deep-seated magmatic processes with mass/ density changes within the shallow-seated hydrothermal system. One could speculate about a coupling between the two sources where the cumulative pressure changes within the two source systems do not exceed the tensile strength of the encasing rocks.

Of all models explored in this study, the prolate spheroid provides the best account for geophysical, geochemical and mechanical constraints and observations over the period of unrest. Homogenous isotropic half-space models for data inversion only provide limited insights into subsurface dynamics at areas where intense faulting may "blur" the relationship between near-field and far field solutions of the inversions. Here, the CFc provides an important case study to investigate these effects (e.g., De Natale et al., 1997; Beauducel et al., 2004), as caldera resurgence since 10 ka has led to a dissection of the area into individual blocks each with different long-term vertical displacements (Orsi et al., 1996). A further limitation of the best-fitting model is the assumption of uniform elasticity. Accounting for a viscoelastic halo around the deep-seated magmatic component of the hybrid source would significantly reduce the required pressure changes to explain the observed deformation. As a



Fig. 7. Fit to gravity residuals for the period 1982–84 from gravity inversion for the prolate spheroid source. The inferred density of the source is 1630 kg/m<sup>3</sup> based on a mass change of  $1.16*10^{11}$  kg (Gottsmann et al., 2004). The residual values are taken from (Gottsmann et al., 2003).

consequence, the pressure changes computed for the prolate spheroid source must be regarded as providing maximum estimates.

# 5. Conclusions

We present results of inversion of vertical deformation data obtained at the restless Campi Flegrei caldera between 1981 and 2001 for spherical, silllike and prolate spheroid source models (Fig. 8). Best-fit solutions of the data inversion for the three source geometries give a source depth of around 3 km about 800 m east of the town of Pozzuoli.

Despite significant differences in the values of the source parameters, results from all the models allow us to suggest that the total volume decrease in the source between 1985 and 2001 is approximately 50% of the volume increase during inflation between 1982 and 1984. Despite the good fits of both the Mogi-model and the penny-shaped source to the vertical displacement data, critical evaluation of the source parameters deduced from the data inversion forces us to reject the



Fig. 8. Schematic South–North cross-section through the central part of the CFc, indicating the location of the inferred prolate spheroid source (to scale). The spheroid may depict an envelope around a hybrid of both hydrothermal and magmatic sources. Also shown is the confidence bound of the source depth. The source shown is consistent with geometric values reported in Table 3. See text for details.

models to explain the observed deformation. Our rejection is based on the evaluation of horizontal displacements, the evaluation of the mechanical properties of the country rocks and constraints from other geophysical and geochemical observations.

We propose a pressurized vertically elongated prolate spheroid, whose centre is located at a depth of 2.9 km (aspect ratio ~0.6) as the most appropriate source model to explain the observed pattern of deformation. A purely magmatic source for the period of inflation appears unlikely based on constraints from other geological and geophysical observations. A pressure increase within a hybrid of both magmatic and hydrothermal sources appears to be a more appropriate model.

For the episode of slow deflation (average ~15 mm/year since 1988) similar to the trend observed before 1968 (see Dvorak and Berrino, 1991 for a compilation of early deformation records) simple thermal relaxation of a magmatic system cannot account for the observed gravity and deformation data (Gottsmann et al., 2003). A combination of deep-seated magmatic effects coupled with processes in the shallow hydrothermal system appears to be a more likely scenario. Discrete "miniuplifts" during the period of overall slow deflation may then be accounted for by small and rapid pressure changes within the (shallow) hydrothermal system beneath the central CFc (Gaeta et al., 2003; Gottsmann et al., 2003).

We conclude with a caution; goodness of fit between modelled and observed vertical displacement data is not the only criterion that must be satisfied to explain observed deformation patterns at restless volcanoes. In addition to horizontal displacements, the validity of physical and mechanical characteristics inferred from these models, needs to be critically assessed via the realistic definition of parameter boundary values, e.g., mechanical failure criteria. Furthermore, data obtained from other monitoring techniques also need to be taken into account. Only then can information from deformation data inversion be confidently incorporated into the evaluation of sub-surface source dynamics for hazard assessment at active volcanic areas.

#### Acknowledgements

We thank Y. Fialko for the generous provision of his computer codes. This work was supported by the EC Research and Training Network "Volcano Dynamics" (HPRN-CT-2000-00060). JG also acknowledges support from an EC Marie Curie Individual Fellowship (HPMF-CT-2002-01969) and a Spanish Ministry of Science grant (Ramón y Cajal program). J. Fernandez and an anonymous referee provided helpful comments. We also thank A. Newman and M. Poland for the editorial handling.

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