

Damage and localization: two key concepts in rock deformation studies

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Abstract: In upper crustal conditions (the brittle field), rocks subjected to a load can experience a deterioration of physical and mechanical properties. This paper treats two conceptual approaches to model the deterioration that results from a crack distribution, whether the distribution is more or less diffuse or is strongly localized. The first part deals with possible ways to characterize diffuse microcracking (*damage*) to establish how the elastic response reflects the microcracking. This is of direct geophysical interest, as elastic wave velocities carry quantitative information on crack content. The second part summarizes two kinds of theory to predict the failure by crack propagation or strain localization, using either fracture mechanics or bifurcation theory. This is of direct geomechanical interest, as what is looked for is an interpretation of localized structures. The third part presents the complementarities between both concepts, damage and localization, to predict failure in rocks.

The birth of faults (i.e. first localization of deformation) is an event that is rarely observed at the field scale. However, it can easily be observed at the laboratory scale when intact rock samples are submitted to deviatoric stresses. Before such an event occurs, one intuitively expects the rock bulk properties to be modified by damage. This is exactly what is observed from various datasets: acoustic emission records, volumetric strain measurements, or elastic wave velocity variations (Ashby & Sammis 1990; Chelidze 1993; Main *et al.* 1993; Zang *et al.* 1996). At low temperature, all of these effects result from crack propagation and opening. We restrict ourselves to the case of low-temperature, upper crustal conditions, where brittle faulting takes place. Obviously this is appropriate close to the Earth's surface, but becomes less valid at greater depths. Ductile deformation (and the related strain-softening or strain-hardening behaviours) and faulting in the lower crust is beyond the scope of this paper.

A similar behaviour is expected at the field scale where rock properties are likely to be modified before an earthquake, although such modifications are likely to be restricted to an area close to the fault (Rudnicki 1988). Again, many observations point to such a behaviour: elastic properties can vary, as indicated by elastic wave velocity records (Scholz 1990; Le Ravalec *et al.* 1996; Crampin & Zatsepin 1997); transport properties such as permeability and electrical conductivity are also modified (Roeloffs 1988; Park *et al.* 1993). Fault and earthquake

mechanics is thus a research field where damage and localization concepts are extremely relevant. Other important areas in geology where the same concepts are very useful are those concerned with reservoir rocks and underground storage.

Damage and localization are intuitively considered above as referring to a spatial strain organization: homogeneous for damage and inhomogeneous for localization. This intuitive approach attempts to capture the rock structure evolution. From this point of view, damage refers here to a more or less diffuse distribution of cracks and localization refers to a macroscopic strain concentration. There is, however, a clear distinction to be made between these intuitive views and the theoretically based concepts of damage and localization. A considerable amount of work has been conducted on both damage and localization during recent decades. There is more than 30 years between the early work of Kachanov (1958), which introduced the damage parameter D in macroscopic continuum mechanics, for ductile deformation, and the more recent work of Kachanov (1993), which is more appropriate to the brittle field and has established on a clear basis how elastic properties of cracked rocks can be derived using crack distribution at the microscopic scale. There is a shorter time period between the early work of Hill (1962), which gives the theoretical framework for localization, and the fundamental results of Rudnicki & Rice (1975), which provide the conditions for localization in dilatant rocks,

but these results have been recently revisited and extended to compacting porous rocks (Olsson 1999; Issen & Rudnicki 2000; Bésuelle 2001).

Although most of the early work on both damage and localization has been developed in fundamental continuum mechanics and not in geology, it has been clear for some time that these concepts are very relevant to understand and quantify rock deformation and rock properties evolution in the upper crust.

It is the purpose of this synthesis paper to recall these basic concepts and show why and how they can be of great interest in upper crustal geology and geophysics. The present work focuses first on damage (in upper crustal conditions where micro-cracking is the main process) at the micro and macro scale, and then on localization. As explained above, we consider the brittle deformation regime so that strains are small ones. The rock is considered to be elastic at low stresses. Increasing stress results in a progressive degradation of elastic moduli as a result of cracking. Beyond a certain stress level, localization takes place. Ductile behaviour and large strains are not considered in this paper.

Damage

The choice of a damage parameter is not straightforward. It should as simple as possible and have a clear mechanical sense. The situation will be very different for ductile failure, as originally discussed by Kachanov (1958), or for the brittle failure of rocks that we examine below. From a rock physics and mechanics point of view, the idea of damage is related to the fact that any rock subjected to loading may experience a progressive deterioration of its physical and mechanical properties. Experimentally, such a deterioration can be demonstrated by the measurement of various properties. For instance, if the rock has an elastic behaviour, its elastic moduli decrease progressively with increasing damage. Although it is intuitive that this deterioration results from a more or less diffuse distribution of cracks, the question is: how can damage be quantified? To answer this question, we present first the theoretical background, which has been established at both the macro and micro scales. We then show through a few examples that it provides an appropriate framework for interpreting the experimental results (see also Dresen & Guéguen 2004).

Damage variables: macroscopic approach

How should we define damage? In the simplest isotropic model, the damage parameter D is a scalar quantity defined as

$$D = \delta S_D / \delta S \quad (1)$$

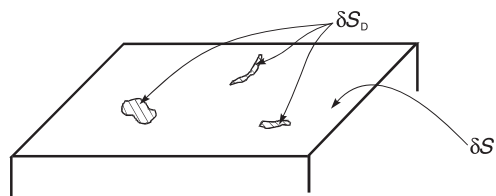


Fig. 1. Geometric definition of D in the isotropic case: $D = \delta S_D / \delta S$.

where δS is the area of intersection of a plane with a representative elementary volume (REV) of the rock and δS_D is the area of the intersections of the same plane with all microcracks and microcavities within δS (Fig. 1). D is a continuous variable defined at the scale of the REV, which varies between zero (undamaged material) and unity (fully damaged material). The above definition allows definition of the effective stress σ_{eff} as the ratio of the load to the surface area that effectively resists the load, i.e. $(S - S_D) = (1 - D) S$:

$$\sigma_{\text{eff}} = \sigma / (1 - D). \quad (2)$$

Recalling that, for a uniaxial compression, Young's modulus E is defined by $\sigma_{\text{eff}} = E\varepsilon$, it results from equations (1) and (2) that in a damaged rock

$$E_D = E(1 - D). \quad (3)$$

E_D can be considered as the effective elasticity modulus of the damaged rock. The introduction of D as explained above is attractive because of its simplicity. It is straightforward to quantify damage through, for instance, elastic wave velocity measurements, which provide directly elastic moduli values and hence D values from equation (3). Such a quantification of damage may be useful for practical reasons but it does not carry any fundamental information on the micromechanics. Moreover, equations (1) and (2) hold only for an isotropic damage, which is not the general case. How can we deal with the 3D situation and anisotropy? An extension of the surface definition of damage can be obtained in two ways. The extension should account for anisotropy and replaces the scalar D by a tensor.

The first way to do this is purely geometrical. A second-rank damage tensor D_{ij} is introduced as follows. Let the area δS , oriented by its normal unit vector with components n_i , become after damage an area $\delta S'$ with normal n'_i . If I_{ij} is the unit second-rank tensor, then D_{ij} is given by

$$(I_{ij} - D_{ij})n_j \delta S = n'_i \delta S'. \quad (4)$$

Does this second-rank tensor account for the variations of the various physical properties of the rock? There is no fundamental reason why this should be so, as again, the micromechanical processes have not been considered.

The second way to generalize the scalar D into a tensor is of greater interest, as it is specific to a given mechanical behaviour and to well-identified micromechanical processes. These processes are, for rocks in upper crustal conditions, those of the micromechanics of cracking. More specifically, let us consider the important case of the elastic properties. Using the above assumption makes it easy to define a fourth-rank damage tensor to quantify the deterioration of the elastic moduli. In the 3D situation, equation (3) is generalized as follows. In the damaged rock, $(\sigma_{ij})_{\text{eff}} = C_{ijkl}\epsilon_{kl}$, where C_{ijkl} is the elastic constants tensor of the intact rock. Its inverse is the elastic compliances tensor S_{ijkl} . In the damaged rock, these tensors become $(C_{ijkl})_D$ and $(S_{ijkl})_D$. Strain can be obtained from $\epsilon_{kl} = (S_{klmn})_D \sigma_{mn}$. Reporting this relation in the expression for $(\sigma_{ij})_{\text{eff}}$ we obtain $(\sigma_{ij})_{\text{eff}} = C_{ijkl} (S_{klmn})_D \sigma_{mn}$. Generalizing equation (2) to the 3D case, this provides the relation

$$(I_{ijmn} - D_{ijmn})^{-1} = C_{ijkl} (S_{klmn})_D \quad (5)$$

where D_{ijmn} are the components of the fourth-rank damage tensor. It is a natural consequence of anisotropic elasticity that a fourth-rank tensor emerges, as the elastic constants tensor is a fourth-rank one. Inverting equation (5) and introducing $\Delta C_{ijkl} = C_{ijkl} - (C_{ijkl})_D$, the components of the damage tensor are obtained in terms of the elastic constants:

$$D_{ijmn} = \Delta C_{ijkl} S_{klmn}. \quad (6)$$

As for equation (3), this allows determination of the damage from measurements of elastic properties. The above model is valid to describe the mechanical behaviour of a rock that is elastic up to a limit given by a plasticity criterion (yield surface). Above that limit, the mechanical behaviour follows some flow rule during loading and is elastic during unloading, as shown in Figure 2. The flow rule provides the incremental plastic strain and is discussed in the section below.

Damage variables: microscopic theories

The above macroscopic approach is phenomenological. It can provide, using equation (5) or (6), the values of the D_{ijkl} components from measurements of elastic constants in both the intact and the damaged rock. However, it does not give any theoretical prediction of the effective elastic constants

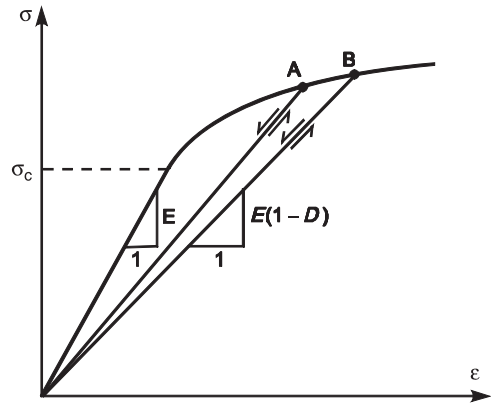


Fig. 2. Idealized stress-strain plot for a brittle rock where inelastic deformation is assumed to be caused by microcracking. The mechanical behaviour is elastic, both in loading and unloading conditions, up to a limit σ_c , which is the elastic limit. The elastic limit defines a yield surface in the stress space. The effective elastic modulus at point A corresponds to unloading. The inelastic increment of strain between A and B corresponds to loading and is due entirely to the incremental increase in compliance as a result of crack extension.

of the damaged rock, which have to be experimentally measured. An independent microscopic approach is required to obtain a theoretical prediction. Such an approach is derived from effective media theories (EMTs) of physics. It is impossible to perform exact theoretical calculations because crack interactions should be taken into account. Crack interactions depend on their separation distance. The ratio crack length/crack separation distance is the parameter that controls crack density. Obviously, crack density is smaller than unity and approaches that value only extremely close to fracture. Two main approximations can be used in EMT calculations. One is the approximation of an effective matrix, and the other is the approximation of an effective field (Kachanov 1993). Effective matrix calculations assume that crack interactions can be obtained by considering that each crack is isolated in a medium that is not the matrix but an effective matrix. A possible way to do this is to derive a set of implicit equations by giving *a priori* to the matrix the (unknown) properties that one wants to calculate (self-consistent method). A simple limiting case is to ignore crack interactions and keep the original matrix as the effective one. This is called the non-interacting crack model and is the simplest model (expected to be valid only at crack densities $\ll 1$). The effective field approximation averages out crack interactions by assuming that each crack sees an

average stress field that is the externally applied field modified by the existence of the cracks themselves. Although various choices are possible, a simple one is to consider that the average stress field is the volumetric average.

In the case of upper crustal conditions, the damage is called brittle because the ratio of plastic strain to elastic strain is very small and we assume here that the damage at the micro-scale is mainly due to microcracking in rocks so that the idealized stress–strain curve is as represented in Figure 2. This means that if the rock is stressed beyond a critical threshold, its original elastic modulus decreases. Beyond the threshold, the crack microstructure is continuously modified. The inelastic increment of strain corresponding to loading from A to B in Figure 2 is entirely due to the incremental increase in compliance, as a result of microcrack extension. The specific values of the elastic moduli for a given crack microstructure (at any point on the stress–strain plot) can be obtained from an EMT model with the appropriate microcrack distribution. In this calculation, cracks are assumed to be frozen in (i.e. prevented from propagating and nucleating). To keep the theory simple, an important assumption has been introduced above. We assume that at a given point in the material, the stress–strain relation is local; that is, stress σ at point M depends on strain ϵ at the same point and not on strain at any other point in the rock. This assumption, which was called the strain equivalence principle by Lemaitre (1996), is identical to the well-known classical approximation in EMT (Beran & McCoy 1970). This is satisfactory as long as cracks are frozen in. It is worth pointing out, however, that a non-local damage theory has been developed by Bazant (1994) to address the problem of a non-uniform system of interacting and growing microcracks. This is of particular importance to handle localization from crack interactions. Such non-local theories provide a way to bridge the gap between damage and localization (see below). This is, however, beyond the scope of this introductory paper.

A detailed overview of the known results on effective elastic properties of cracked solids has been given by Kachanov (1993). Of special interest is the model of *non-interacting cracks*, because, as shown by Kachanov (1993), it remains accurate at high crack densities (up to 0.5; Schubnel & Guéguen 2003), provided the *locations of cracks centres are random*. This model is an effective matrix model; in fact, it is the simplest one, as the matrix is assumed to be the intact rock as discussed above. We give below the results of this model because of their wide range of validity and their broad field of application. That this model is valid in a broad range of crack densities appears to be a

paradox. There are two independent reasons why this is so. The first one is that at a microscopic level, crack interactions exist but approximately compensate. This means that shielding and amplification effects cancel each other, on average. The second reason is that the simplest effective field method (volumetric average) predicts exactly the same results. The volumetric averaged stress field is known to be unmodified by the presence of cracks so that it predicts exactly the same result as the non-interacting crack model. This reflects again the above-mentioned compensating effects.

The calculation of effective elastic moduli in the approximation of non-interacting, randomly oriented cracks in an isotropic matrix was first solved by Bristow (1960). The general method is to calculate the elastic potential energy $f(\sigma_{ij})$ per unit volume for the cracked rock:

$$f = f_0 + \Delta f \quad (7)$$

where f_0 is the potential energy of the undamaged rock and Δf is the additional term related to cracks. By definition of the effective compliances $(S_{ijkl})_D = (C_{ijkl})_D^{-1}$,

$$f = (1/2)\sigma_{ij}\sigma_{kl}(S_{ijkl})_D. \quad (8)$$

Equations (7) and (8) directly provide the effective elastic constants (or alternatively compliances) of the damaged rock if Δf is known. The key point in this approach is the derivation of Δf from micromechanics. As shown by Kachanov (1993), this additional potential energy related to cracks can be expressed as a function of the crack density tensor α_{ij} . An approximate expression of Δf in the case of non interacting cracks in tensile conditions is

$$\Delta f = (1/E_0)[8(1 - \nu_0^2)/3(1 - \nu_0/2)]\sigma_{ij}\sigma_{jk}\alpha_{ik} \quad (9)$$

where E_0 and ν_0 are Young's modulus and Poisson's ratio of the intact isotropic matrix, respectively. The above result is not exact but it is valid with an approximation of less than 10%. The introduction of the crack density tensor α_{ij} results from the micromechanical calculation of the elastic potential. For non-random crack orientations in the case of circular cracks,

$$\alpha_{ij} = (1/V)\sum (c^3 n_i n_j)^k \quad (10)$$

where c is the crack radius and V is the rock volume. The elastic compliances of the cracked rock are found to be modified with respect to the intact

rock by an amount ΔS_{ijkl} . Interestingly, a consequence of the previous approximation is that the effective elastic properties of the cracked rock are orthotropic if the undamaged rock is isotropic in dry conditions. The orthotropy is, in this case, of a simplified type so that only four independent elastic constants exist, corresponding to the four independent components of the $n_i n_j$ distributions (Kachanov 1993). The above approximation introduces an error of less than 10% on the values of the ΔS_{ijkl} components so that it may be considered as acceptable. Within this approximation, the damage is characterized by the crack density tensor α_{ij} , which is a second-rank tensor. Although the elastic constants are anisotropic and described by a fourth-rank tensor, it turns out that a second-order tensor is sufficient to quantify damage within the previously specified conditions and approximations. A complete calculation introduces, in addition to the tensor α , a fourth-rank tensor β , which is small for dry cracks (Sayers & Kachanov 1995). If β is small, an additional assumption to be considered is that of an isotropic random orientation distribution of cracks. This is the simplest situation. The commonly used crack density parameter ρ (Bristow 1960) is sufficient in that case:

$$\alpha_{ij} = (\rho/3)\delta_{ij} \quad (11)$$

where $\delta_{ij} = 0$ if $i \neq j$ and $\delta_{ij} = 1$ if $i = j$. In that case, and only in that case, the damage parameter is a scalar quantity ρ . For an isotropic random orientation distribution of identical circular cracks

$$\rho = (N/V)c^3 \quad (12)$$

where N/V is the number of cracks per unit volume. Then orthotropy does not hold if cracks are allowed to slide. Moreover, if friction is present, deformation is stress path dependent and stress-induced anisotropy develops.

Another important case, which may be important in the crust, is that of fluid-saturated cracks. Then the tensor β is not negligible and it provides direct information on fluid nature and saturation (Schubnel & Guéguen 2003). From a geophysical point of view, the main interest of this approach is that quantitative information on the rock crack and fluid content can be extracted from elastic wave velocities.

Localized deformation

The strain distribution in natural materials may generally follow several types of organization. Depending on the conditions of pressure, temperature, strain rate, strain history and other physical and chemical processes, strain may be more or less distributed through a large domain of the material, as described above. However, in the brittle regime, close to a critical loading (in a broad sense), the displacement field becomes concentrated in localized structures, which absorb the main part of the displacement within the material, for a given set of boundary conditions. The different observations of the localization phenomena at a macroscopic scale can be classified into two main groups, as follows.

(1) *Fractures*. These form a discontinuity of the material and correspond to a discontinuity of the displacement field (Fig. 3a). This discontinuity is quantified in term of displacement jump across the fracture surfaces and the notion of strain is locally irrelevant. Fracture mechanics provides a

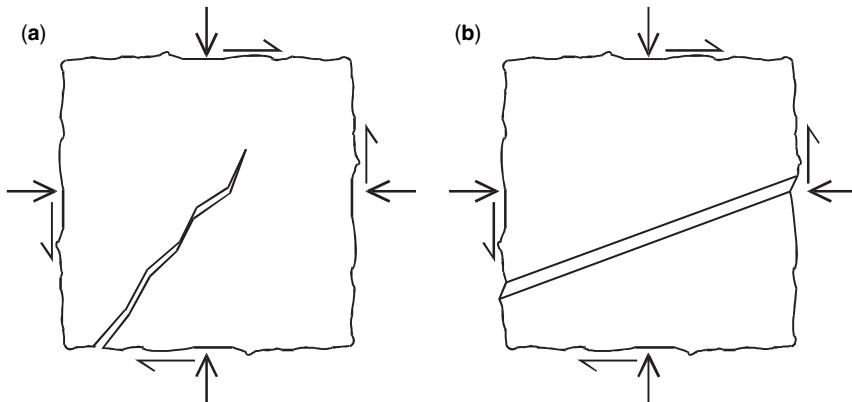


Fig. 3. Schematic description of the two types of localization structures. (a) Crack discontinuity; (b) Strain localization band through a solid.

theoretical approach to analyse the condition of fracture propagation. Fracture mechanics can be applied if initial flaws or cracks exist within the material. This approach is appropriate at several scales, the micro-scale for intragranular cracks or the macro-scale for large fractures. The basic principles of fracture mechanics are outlined below.

(2) *Localization bands*. They have a finite thickness so that the concept of strain within the band is meaningful and can be used. This strain generally has a dominant shear component and the bands are called ‘shear bands’ (Fig. 3b), although compaction bands also exist (e.g. Aydin *et al.* 2006). At a microscopic scale, these bands are generated by a progressive concentration of micro-events: grain sliding and grain rotation for soils; microcracking, grain crushing and cataclastic deformation for rocks (Wong 1982; Menéndez *et al.* 1996; Bésuelle *et al.* 2000; Mair *et al.* 2002; Baud *et al.* 2004; Paterson & Wong 2005). Bulk properties such as permeability or wave speeds are modified in the localization band. Bifurcation theory provides a way to analyse the localization band initiation. This theory considers the localization as an instability, and is presented below. An overview of some experimental results on strain localization in soils and rocks is addressed in more detail by Desrues *et al.* (2007).

Although the two definitions above are clearly different, fractures and shear bands structures are linked in reality. Shear bands, which are initiated by microcrack concentration, can also transform into a macroscopic fracture as a result of the coalescence of cracks (see, e.g. references given by Desrues *et al.* 2007). In this case, the fracture can be considered as the result of a strong deformation inside the band. Reciprocally, the process zone of propagation at the tip of a fracture has a non-zero width, which implies that the material close to the fracture boundary can be damaged within a zone that has a finite thickness. We may note also that localization structures such as *en echelon* crack arrays seem to be intermediate between fractures and shear bands.

Fracture mechanics

It is well known that the existence of a cavity, in a broad sense including a hole or a fracture, in a bulk solid plays the role of a stress concentrator. Pioneer studies by Kolosov (1909) and Inglis (1913) derived the expression of the stress field around an elliptical cavity. Major contributions that initiated fracture mechanics are the Griffith *energy theory* (Griffith 1921), which expresses a condition for crack propagation based on the energy required to separate surfaces in a solid, and the notion of a *stress intensity factor*, proposed by Irwin (1948),

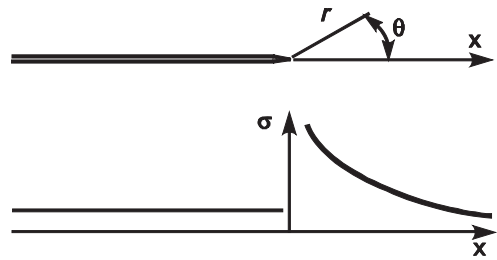


Fig. 4. Schematic illustration of the stress concentration ahead of the crack tip, proportional to $1/\sqrt{r}$.

to characterize the singularity of the stress field near a crack tip, in the linear elastic solution.

Stress concentration in elastic fracture mechanics. To estimate the stress concentration near the crack tip it is helpful to consider a body in which the material has a linear elastic and isotropic behaviour with homogeneous properties. It is convenient to distinguish three types of crack propagation, depending on the loading conditions. Mode I corresponds to a tensile opening of the crack surfaces, mode II to a relative shear sliding of the faces in the direction of the crack propagation (in-plane shear) and mode III to a shear sliding perpendicular to the direction of crack propagation (anti-plane shear). The stress field near the crack tip (Fig. 4) can be approximated by (e.g. Lawn 1993)

$$\sigma_{ij} = K_{\alpha} (2\pi r)^{-1/2} f_{ij}^{\alpha}(\theta). \quad (13)$$

Additional terms that depend on the boundary conditions on the body are negligible near the crack tip. r is the distance from the crack tip, θ is a polar angle defined from the crack plane, K_{α} with $\alpha = \text{I, II, III}$ are stress intensity factors, and f_{ij}^{α} are stress field distribution functions. It is remarkable that, for a given loading mode, the stress field near the crack tip depends on loading applied to the body only through the stress intensity factors.

The crack tip is a singular point of the stress field; from equation (13), it results that the maximum principal stress should be infinite at that point. The linear elastic behaviour assumption leads to overestimation of the stress at the tip. In reality, a material has an inelastic, irreversible mechanical behaviour when stress exceeds a certain value. More realistic models can be used for stress intensity analysis by considering energy dissipative processes in a small zone close to the crack tip (e.g. Irwin 1957; Rice 1968). However, within the elastic approximation, the strain energy in any finite volume around the crack tip remains finite. An alternative approach is to consider the existence of cohesive forces between the crack

faces, near the tip. Such forces allow elimination of the singularity in the stress field (e.g. Dugdale 1960; Barenblatt 1962).

Energy considerations. A complementary approach for crack propagation is based on the energy balance of the solid including the crack. During an infinitesimal crack propagation, the external loads work exceeds the increase of strain energy in the body. The difference, per unit area of new crack surface, is defined as the energy release G (Griffith 1921). The energy release corresponds physically to the work 'needed' to create the new free crack surfaces. Mathematically, this energy corresponds to the displacement of the stress intensity field around the crack tip. For elastic reversible propagation, it is then possible to express G as a linear combination of the square of the stress intensity factors K_α . If irreversible processes are important, the energy release is accounted for using the mathematical device developed by Rice (1968), the so-called path-independent J integral.

Finally, the propagation criterion is expressed when energy release reaches a critical value

$$G = G_c \quad (14)$$

or when stress intensity factors reach critical values K_{ac} . The K_{ac} values can be considered as material parameters and have been intensively studied for various materials in laboratory experiments. Estimations of critical values have also been proposed at large scale for natural crustal faults (e.g. Rudnicki 1980).

Bifurcation theory

In the spirit of the previous approach, a macroscopic fault can be considered as a structure similar to the propagation of a crack or the coalescence of several cracks, which can be nucleated as a result of the existence of initial flaws in the material. Bifurcation theory provides a different way of looking at the localization process. The strain localization area is now considered to be a structure that appears spontaneously within a homogeneous material. At the onset of bifurcation, the initially homogeneous strain field shows a strain concentration in a small area. What are the necessary conditions for such an instability? The theoretical answer goes back to the initial work of Hadamard (1903) on dynamical aspects of wave propagation, and to studies by Thomas (1961), Hill (1962) and Mandel (1964). Rice (1976) reviewed these previous studies for quasi-static conditions and Rudnicki & Rice (1975) identified the strain localization conditions for pressure-sensitive dilatant rocks. The condition

of localization for shear banding as developed by Rice and coworkers is reviewed below. A comprehensive review has been given by Bésuelle & Rudnicki (2004).

An infinite solid is considered initially to be uniformly deformed. The strain is defined as the symmetrical part of the displacement gradient:

$$2\varepsilon_{kl} = \partial u_k / \partial x_l + \partial u_l / \partial x_k$$

where u_k is the displacement vector. The bifurcation analysis determines the conditions for which a non-homogeneous strain field can develop within an infinite planar band. The strain rate inside the shear band $\dot{\varepsilon}^{sb}$ is assumed to be equal to the strain rate outside $\dot{\varepsilon}^o$ plus an additional term, which depends only on the normal direction to the incipient shear band. This additional strain corresponds to the superimposition of a simple shear and a uniaxial deformation and is expressed by n_i , the unit normal vector to the band (Fig. 5), and g_i , which is, *a priori*, an arbitrary vector:

$$\dot{\varepsilon}_{ij}^{sb} = \dot{\varepsilon}_{ij}^o + 1/2(g_i n_j + n_i g_j). \quad (15)$$

The equilibrium condition in the solid can be expressed on the boundaries of the band by the equality of the product of the stress tensor and the unit normal vector (this product has the dimension of a force acting on a unit surface of the boundaries) on each side of the boundaries (Rudnicki & Rice 1975):

$$\dot{\sigma}_{ij}^{sb} n_i = \dot{\sigma}_{ij}^o n_i \quad (16)$$

where $\dot{\sigma}_{ij}^{sb}$ and $\dot{\sigma}_{ij}^o$ are the stress tensor's time derivative inside and outside the shear band. The

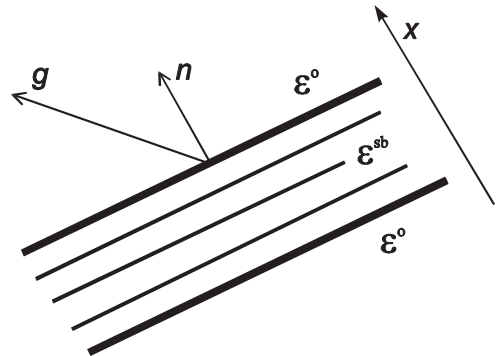


Fig. 5. Schematic illustration of the shear band. n is the unit normal vector to the band and g is an arbitrary vector that depends on the x -direction.

bifurcation condition occurs when these conditions can be met for a non-trivial solution (i.e. a nonzero vector g_i).

The stress time derivative and the strain rate inside and outside the band are assumed to be related by a linear relation using a fourth-rank symmetrical tensor ($L_{ijkl} = L_{jikl} = L_{ijlk}$). The rheological response inside and outside the shear zone can be different. Let us consider here, for example, an elastoplastic law for which the material can load with plastic deformation or unload elastically. However, other categories of constitutive laws can be considered as damage laws. With an elastoplastic law, the material inside the shear band can deform plastically while the material outside the shear zone is unloading elastically:

$$\dot{\epsilon}_{ij}^{sb} = L_{ijkl}^{sb} \dot{\epsilon}_{kl}^{sb} \quad \text{and} \quad \dot{\epsilon}_{ij}^o = L_{ijkl}^o \dot{\epsilon}_{kl}^o \quad (17)$$

Using equations (15)–(17), we obtain

$$(L_{ijkl}^{sb} - L_{ijkl}^o) n_i \dot{\epsilon}_{kl}^o = L_{ijkl}^{sb} n_i n_j g_k. \quad (18)$$

Rice & Rudnicki (1980) have shown that to study the condition of first localization it was justified to consider an elastoplastic behaviour inside and outside the incipient shear band at the onset of bifurcation ($L^{sb} = L^o$). This condition can be viewed as a condition of neutral loading on the boundaries of the band; that is, the material at the boundaries is in a state between elastic unloading and plastic loading. The equilibrium condition (18) is then expressed by

$$L_{ijkl} n_i n_j g_k = 0. \quad (19)$$

A nonzero solution for g_i is then possible when a deformation is reached for which

$$\det(n_i L_{ijkl} n_j) = 0. \quad (20)$$

For small load and for any possible orientation n_i , the determinant is generally positive and decreases when strain increases. Localization becomes possible when it becomes zero for a particular orientation n_i . The vector g_i is an eigenvector of the tensor $n L n$ and if g and n are orthogonal, the additional strain inside the incipient band is pure shear. This strain can have also a normal component, dilating or compacting, which corresponds to a lengthening or shortening in the direction of the normal to the band, if g and n are not orthogonal. That situation corresponds to an increase or a decrease of the porosity inside the band (Bésuelle 2001). The band orientations corresponding to a pure extension band, a dilating and compacting shear band, and a pure compaction band are represented in Figure 6 with respect to the principal stress directions in an axisymmetric compression stress state.

The bifurcation analysis determines an onset of localization, the angle of the band with respect to the principal stress directions and the volumetric/shear strain ratio in the band. The theoretical onset is strongly dependent on the constitutive model so that the material behaviour has to be well known. Intensive investigations of various models have been developed recently, for elastoplastic models with several shapes of yield and potential surfaces (e.g. Molenkamp 1985; Vardoulakis & Sulem 1995) or multilinear models (e.g. Chambon *et al.* 2000). Precise descriptions

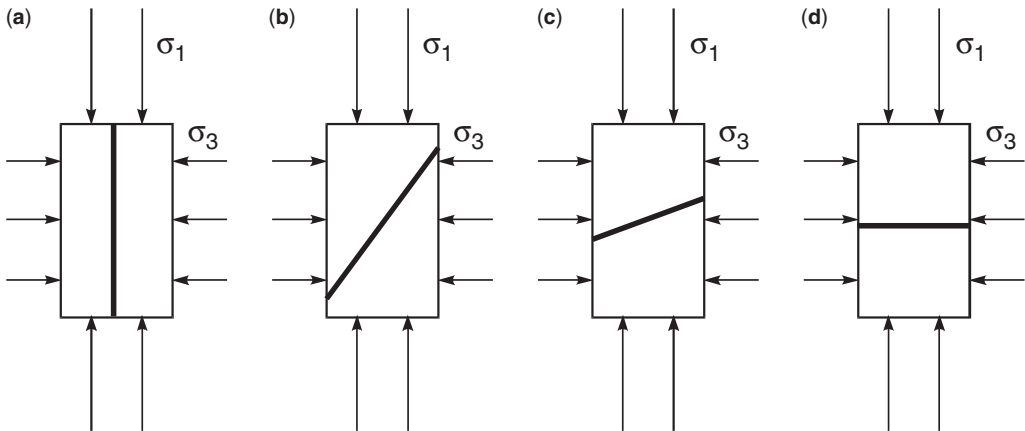


Fig. 6. Schematic illustration of the shear band orientation for an axisymmetric compression stress state with respect to the major (σ_1) and minor (σ_3) principal stress directions, for (a) an extension band, (b) a dilating shear band, (c) a compacting shear band, (d) a compaction band.

of localization (onset, orientation) in laboratory experiments can also help to calibrate the constitutive model in the spirit of a compliance analysis (Vermeer 1982; Desrues & Chambon 2002). In such an approach, some parameters of the law are determined using a comparison between experimental results on strain localization and bifurcation predictions. From a geomechanical point of view, the analysis above allows an interpretation of shear band development as well as compaction band development.

Diffused and localized damage

Damage models, as treated above, link the stress and strain locally in any part of a solid. They are convenient for dealing with rocks that are typically in upper crustal conditions (i.e. in the brittle failure deformation regime). In this case, rock deterioration during loading is induced by a more or less diffuse crack distribution. If we consider a laboratory elementary test, such a law can describe the homogeneous response of the specimen (i.e. before strain localization). In such conditions, rock behaviour is elastic up to a limit where microcracking takes place and degrades the elastic moduli. Strains are small. In counterpart, when strain localization occurs in the specimen, the global response of the specimen becomes a structural response. However, this does not exclude consideration that the damage law is valid in every part of the specimen; that is, inside and outside the incipient deformation band.

As shown above, strain localization can be considered as a problem of loss of uniqueness; that is, for the same initial mathematical problem several solutions become possible, without or with a deformation band. The criteria express when a localization band becomes possible. The bifurcation prediction is strongly dependent on the constitutive law and can be applied to different types of model. Above, we considered the case of an elastoplastic law, but, for example, bifurcation prediction can also be applied to damage models (e.g. Dragon *et al.* 1994). In such a case, strain localization is associated with a damage localization; that is, a concentration of cracks inside a band of finite thickness.

As far as the band thickness is concerned, damage models, and more generally all classical continuum models, are unable to predict it. Only constitutive models with internal lengths can do this. This means that if prediction of incipient localization is possible, prediction of the response after localization (i.e. the post-localization regime) is not possible with classical models. Models that take into account higher displacement gradients are useful. The micropolar models proposed by

Cosserat (1909) with micromechanical considerations for granular material or the second-gradient models developed by Mindlin (1964) and extended to geomaterials (Chambon *et al.* 2001; Bésuelle *et al.* 2006) allow predictions of shear band thickness (Vardoulakis & Sulem 1995). Such formalisms can be extrapolated to damage models (e.g. Kotronis *et al.* 2007). Another approach is the non-local damage models, which introduce also an internal length (Bazant 1994) and allow prediction of band thickness (Pijaudier-Cabot & Benallal 1993).

Conclusion

Damage and localization are two distinct concepts, which are of major importance in geology and geophysics. Considering upper crustal conditions (brittle failure regime), a precise definition of damage parameters is obtained from microcracking processes. Using such parameters allows us to quantify how much a given rock is damaged (i.e. how far it is beyond its elastic limit). Crack and fluid content can be extracted from elastic moduli (or from wave velocities). On the other hand, the completely independent approach of bifurcation theory allows us to know how close a rock is to localization (i.e. how close it is to shear band or compaction band nucleation). To bridge the gap between the concepts of damage and localization, non-local damage theories and second gradient models appear to be very promising. At the field scale, these tools can be used to infer the damage state of a highly stressed area (damaged area) from elastic wave velocity measurements, or to interpret shear band orientations in relation to the *in situ* stresses and the volumetric strain type, compactive or dilatant (localization).

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