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Estimation of Areal Frequency and Mean Trace Length of Discontinuities Observed in Non-Planar Surfaces

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Summary

Mauldon (1998) suggested end-point estimators of areal frequency and mean trace length for a planar sampling window which were recently proved to be unbiased maximum likelihood estimators by Lyman (2003). The present paper is to expand the concept and applicability of the end-point estimators to those for a general non-planar sampling window. The generalized end-point estimators are verified and its applicability for variable discontinuity orientation is checked by Monte Carlo simulation. Standard deviation of estimation error and estimation efficiency of areal frequency and mean trace length are also considered.

Keywords: End-point estimator, areal frequency, mean trace length, non-planar sampling window.

1. Introduction

Most problems of rock engineering cannot be addressed without knowledge of discontinuities in rock mass. Although understanding discontinuities must be a basic step to understand rock mass and rock engineering, there still remains much headroom for the complete realization of natural discontinuities due to the complexity of their mechanical and geometrical properties. By taking a geometrical point of view, we can hardly observe or measure the shape and size of discontinuities in full scale, especially when the rock mass has a large dimension. This has made many researchers develop techniques to estimate the geometrical factors of discontinuities such as size, frequency and orientation, from observed discontinuity traces in a limited sampling exposure.

The estimation technique depends on sampling method: linear (scanline survey) and areal (window) sampling methods. Both methods have been described and discussed by many researchers including Fookes and Denness (1969), Baecher and Lanney (1978), Priest and Hudson (1981) and Priest (1993). Compared with the scanline survey, the areal sampling has an advantage of utilizing a relatively large area of rock and thus being affected less by sampling biases (Mauldon et al., 2001).

When we estimate an areal frequency and mean trace length in an infinite plane by using the areal sampling method, we should notice two kinds of sampling biases: (1) Size bias – longer traces have a higher probability to appear in the finite sampling window. (2) Censoring bias – short traces below a cutoff length are intentionally or inevitably not recorded in the sampling, and long traces beyond the maximum observable length imposed on the finite rock exposure are not to be measured. Priest (1993) referred the former case as 'trimming' and the latter 'curtailment' while Einstein et al. (1983) and Kulatilake et al. (1984) called the former and the latter, 'truncation bias' and 'censoring bias', respectively. The effect of censoring bias by short traces can be negligible by making the cutoff length to be small enough compared to an average trace length (Kulatilake and Wu, 1984b). Estimation of geometrical factors of discontinuities can be regarded as a kind of process to remove/correct the sampling biases.

Main researches for the estimation of areal frequency and mean trace length based on areal sampling are as follows.

Kulatilake and Wu (1984a) proposed a distribution-dependent estimator of areal frequency using a pre-defined trace length distribution and the number of traces corresponding to their trace type. They divided traces into three groups, as Pahl (1981) suggested, according to whether end points of traces were observable or not: those with both ends observed, those with one end observed and those with neither end observed. In their work, traces are assumed to be parallel to each other.

Mauldon (1998) developed an estimation technique of areal frequency based on the principle of associated points (Parker and Cowan, 1976; Laslett, 1982), and named it an end-point estimator of trace density. The end-point estimator of areal frequency is for sampling windows with arbitrary convex boundaries, and for arbitrary trace length and trace orientation distributions.

Pahl (1981) developed an estimator of mean trace length of parallel traces in a rectangular sampling window under the assumption that trace midpoints are randomly and homogeneously distributed in the sampling plane. This estimator is independent of the functional form of trace length distribution and it can be easily obtained by just counting the number of end points of traces observed in the sampling window.

Kulatilake and Wu (1984b) extended Pahl's method to consider a variable discontinuity orientation described by a probability distribution function. They derived the equation of mean trace length for a rectangular sampling window as Pahl did.

Mauldon (1998) advanced Pahl's estimator of mean trace length into the cases of sampling windows with arbitrary convex boundaries. He called it an end-point estimator of mean trace length. Mauldon (1998) also derived a stereological estimator of mean trace length for parallel traces in a rectangular sampling window based on stereological principles (Underwood, 1970).

Zhang et al. (1998) derived an estimator of mean trace length for circular windows which was identical to Mauldon's end-point estimator for circular windows. Mauldon

(1998) and Zhang and Einstein (1998) showed that the end-point estimator of mean trace length is available for arbitrary trace orientations whenever circular sampling windows are adopted.

Lyman (2003) showed that the end-point estimators of areal frequency and mean trace length discussed by Pahl (1981), Mauldon (1998), and Zhang and Einstein (1998) are unbiased maximum likelihood estimators. He emphasized that the end-point estimators are the minimum variance estimators and they are clearly the most desirable estimators.

All estimation techniques introduced above are based on a planar sampling window. Although a rock exposure having a relatively small level of irregularity can be regarded as a planar window, the sampling window often becomes non-planar as the sampling area is extended: for examples, a tunnel surface consisting of curved walls and crown, and rock slopes around small mountains. Extending sampling area means increasing the number of sampled traces and thus it means improving the accuracy of estimation.

This paper presents a technique for obtaining estimates of areal frequency and mean trace length for a non-planar sampling exposure. This is done by improving/generalizing the theory of end-point estimators for a planar window. Developed technique is verified by comparing theoretical values with estimated ones using a Monte Carlo simulation for a curved tunnel surface as a non-planar window. Effect of variable trace orientations on the accuracy of estimation is also checked considering the fact that the end-point estimator of mean trace length assumes parallel traces.

2. Mathematical Models

Pahl (1981) classified a discontinuity trace according to whether both, one or neither end of the trace is visible. This idea of classification is adopted by Laslett (1982), Kulatilake (1984a, b), Villaescusa (1991) etc. Priest (1993) named the three kinds of discontinuity traces *contained*, *dissecting* and *transecting traces* which are similar to Pahl (1981)'s terminology. In Fig. 1, we can see 6 contained traces, 9 dissecting traces and 2 transecting traces.

End-point estimators of areal frequency, λ^a, and mean trace length, μ^L for a planar sampling window are defined by (Mauldon, 1998)

$$\lambda^a = \frac{1}{2A}(N - N^T + N^C) \tag{1}$$

and

$$\mu^L = \frac{A}{W(\theta)}\left(\frac{N + N^T - N^C}{N - N^T + N^C}\right), \tag{2}$$

where,

N is the number of all traces intersecting the window,
N^T is the number of transecting traces in the window,
N^C is the number of contained traces in the window,
A is the area of the window, and
$W(\theta)$ is the width of the window measured in normal direction of the traces which are angled θ from a boundary line of the window.

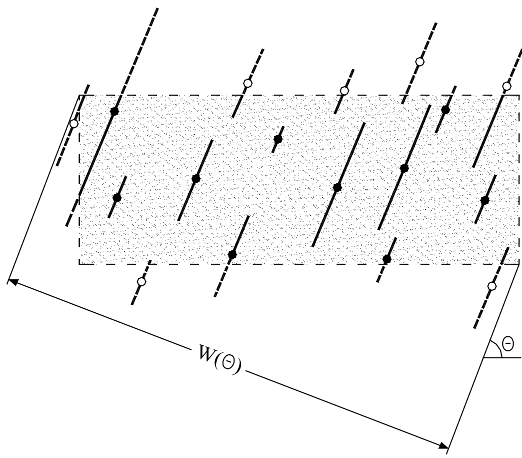

Fig. 1. Parallel traces in a rectangular window. Solid circles indicate the trace centers located in the window while open circles mean the trace centers out of window

For example, λ^a is $10.5/A$, and μ^L is approximately $0.62A/W(\theta)$ applying Eqs. (1) and (2) to traces in Fig. 1. The expression in parentheses of (1) which is same as the denominator in the parentheses of (2) means the number of end-points inside the window, while the numerator in the parentheses of Eq. (2) is equal to the number of end-points outside the window (Mauldon, 1998). The estimated number of trace centers in the window is obtained by dividing the number of end-points inside the window by 2. In Fig. 1, the estimated number is 10.5 which is very near to the real number of 10.

For the shape of discontinuities, polygonal, elliptical or circular models have been suggested and used in numerical modeling. Exact information on discontinuity shape, however, is scant since even 3D exposures like protruding corners in surface outcrops or tunnels do not provide complete information on joint shape (Dershowitz and Einstein, 1988). Circular disc model has been adopted in discontinuity modeling (Baecher et al., 1977, 1978; Barton, 1978; Dienes, 1979; Warburton, 1980; Song and Lee, 2001) due to a mathematical feasibility and some site experiences from which a similar/same trace length distribution was obtained from a few sampling windows in different orientations. There is no evidence, at this stage, to suggest that any other shape model is more realistic than a circle (Priest, 1993). When a discontinuity set consists of randomly located parallel discs, the areal frequency λ^a depends on the orientation of sampling window while mean trace length μ^L does not. Here, the randomly located parallel disc model has been employed in derivation of estimators of areal frequency and mean length of traces observed in non-planar windows.

2.1 Areal Frequency

Main ideas of estimating areal frequency in a non-planar window can be explained by four steps:

1. Division of a non-planar window into sub planar windows,
2. calculation of the areal frequency in each planar window,

3. conversion of the frequency values into normal frequencies, and
4. weighting and summing of the converted values considering a relative area and orientation of each sub window.

The areal frequency, λ^a, of (1) can also be called an apparent areal frequency because it varies according to the orientation relation between discontinuities and a sampling plane. Let λ^A be a 'normal areal frequency' and define it as an areal frequency when discontinuity planes are normal to a sampling plane. If an acute angle between discontinuity planes and the i^{th} sampling plane is ϕ_i, the normal areal frequency, λ^A is obtained by

$$\lambda^A = \frac{\lambda_i^a}{\sin \phi_i}. \tag{3}$$

$1/\sin \phi_i$ of Eq. (3) can be regarded as a weight factor to correct a sampling bias in window sampling similar to Terzaghi's correction factor (Terzaghi, 1965) for a scanline survey. To prevent λ^A of (3) from diverging into an infinite value when discontinuities become parallel to a sampling window, upper limit of $1/\sin \phi_i$ is required. This study followed Priest (1993)'s suggestion of maximum value of 10 for Terzaghi's correction factor, which means that lower values of ϕ_i than 5.7° are forcedly set 5.7°.

Figure 2 shows a non-planar window consisting of three non-parallel sub windows: i, $i+1$ and $i+2$. Although all discontinuities are parallel to each other, an angle between normal vectors of the i^{th} sub window and discontinuities, ϕ_i, and an angle between a discontinuity trace and a boundary line of the i^{th} sub window, θ_i, change as the sub window orientation changes. Here, $\sin \phi_i$ is obtained by a cross product of unit normal vectors of the i^{th} sub window and a discontinuity, $|\hat{n}_i \times \hat{n}_J|$.

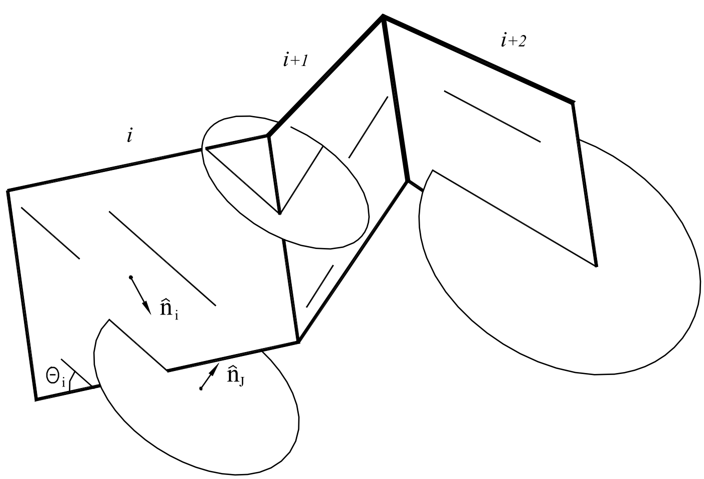

Fig. 2. A non-planar window consisting of three non-parallel rectangular windows and discontinuity discs having a unit normal vector of $\hat{n}_J$

Equation (1) can be expressed for the i^{th} sub window as

$$\lambda_i^a = \frac{N_i - N_i^T + N_i^C}{2A_i}. \quad (4)$$

Then, the normal areal frequency of the i^{th} sub window is obtained from (3) and (4) by

$$\lambda_i^A = \frac{N_i - N_i^T + N_i^C}{2A_i \sin \phi_i}. \quad (5)$$

Now, define an overall normal areal frequency, λ^A, as a weighted sum of the normal areal frequencies obtained in M sub windows:

$$\lambda^A = \sum_{i=1}^{M} w_i \lambda_i^A \quad (6)$$

where, a weight factor, w_i, is defined by a ratio of product of an area and sine function of the i^{th} sub window as

$$w_i = \frac{A_i \sin \phi_i}{\sum_{j=1}^{M} A_j \sin \phi_j}. \quad (7)$$

The weight factor, w_i, is defined as (7) based on the fact that an areal frequency estimated from a larger window or from a window having a bigger angle with discontinuity planes, can be more reliable due to a larger sample size which reduces an estimation error.

By substituting (5) and (7) to (6), the overall normal areal frequency is as follows:

$$\lambda^A = \frac{\sum_{i=1}^{M} (N_i - N_i^T + N_i^C)}{2 \sum_{j=1}^{M} A_j \sin \phi_j}. \quad (8)$$

2.2 Mean Trace Length

As in case of Eq. (4) of areal frequency, Eq. (2) can be re-written for the i^{th} sub window as

$$\mu_i^L = \frac{A_i}{W(\theta_i)} \left(\frac{N_i + N_i^T - N_i^C}{N_i - N_i^T + N_i^C} \right). \quad (9)$$

Although a mean trace length or trace length distribution is not affected by change of orientation of a sampling window in the randomly located parallel disc model, the accuracy of estimation increases as an acute angle between the discs and sampling window increases because, in that case, the discs have more chance to intersect the window. Therefore, we apply the same weight factor, w_i, used in (7) to the mean trace length of the i^{th} sub window, then an overall mean trace length is calculated as

$$\mu^L = \sum_{i=1}^{M} w_i \mu_i^L. \quad (10)$$

The overall mean trace length can be re-written by substituting (7) and (9) to (10) as

$$\mu^L = \sum_{i=1}^{M} \left(\frac{\sin\phi_i A_i^2 (N_i + N_i^T - N_i^C)}{W(\theta_i)(N_i - N_i^T + N_i^C)} \right) \Bigg/ \sum_{j=1}^{M} A_j \sin\phi_j. \tag{11}$$

3. Monte Carlo Simulation

To verify the estimators of areal frequency and mean trace length expressed in (8) and (11), Monte Carlo simulation has been carried out. A curved wall of horseshoe shaped tunnel was adopted as a non-planar sampling window. The tunnel has 22 m of height, 17.5 m of width at maximum and 20 m of length in tunnel axis direction.

Discontinuity discs are generated around the tunnel. Center points of the discs occur in a box zone including the tunnel, and size of the box zone is determined according to the mean diameter of the discs for the simulation result not to be affected by an end effect.

Three kinds of probability functions have been adopted for disc diameter distribution: normal, lognormal and (negative) exponential. Mean diameter of the discs is ranged from 1 m to 8 m, with an increment of 1 m, and volumetric frequency from 0.01 m^{-3} to 0.1 m^{-3} with an increment of 0.03 m^{-3}. Standard deviation of disc diameter is set to be a half of the mean diameter for the normal and lognormal distributions.

Discontinuity orientation has been modeled by Fisher function, where Fisher's constant K, the degree of clustering, is set to be a large number of 105 for verification of Eqs. (8) and (11), and otherwise, it is varied in range of 5 through 35. Dip direction and dip of a discontinuity set is to be 60° and 45°, respectively, in most cases.

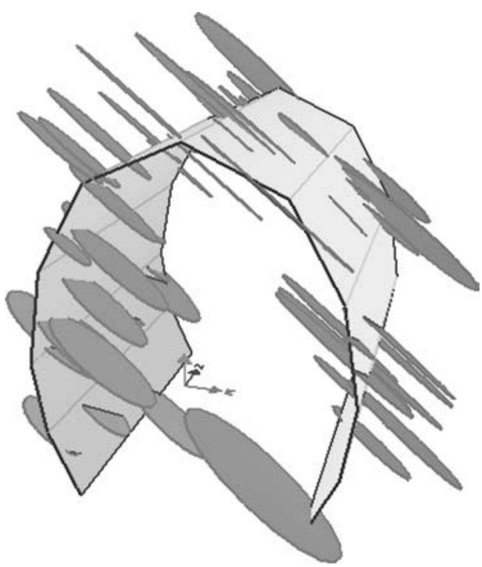

Fig. 3. Circular discontinuities around a tunnel for a verification test

Figure 3 shows an example of the simulated discontinuities and a tunnel, where the mean diameter is 5 m (normal distribution) and volumetric frequency is 0.01 m^{-3}. The tunnel wall consists of 10 rectangular planes used for sub sampling windows.

In each test, mean values of estimated areal frequency and mean trace length were taken after 10 simulations to reduce a variance of the estimated values and thus to easily confirm the trend of error variation at each condition.

3.1 Areal Frequency

3.1.1 Verification

Figure 4 shows an estimation error in percent of the overall normal areal frequency vs. the number of traces in tunnel wall when the discontinuity diameter follows a lognormal distribution. The error decreases in exponential form as the number of sampled traces increases. We can see that the change in volumetric frequency does not directly affect the trend of estimation error. This is also true for the relationship of discontinuity size and estimation error even though the change in diameter is not marked in Fig. 4.

To compare the results from the generalized end-point estimator suggested here with those from the end-point estimator for a planar sampling window, one of the sub windows around the tunnel was taken as a sampling window, and a normal areal frequency was calculated by using Eqs. (1) and (3). Error % of estimated values from a single planar window is plotted in Fig. 4 together with those from the overall

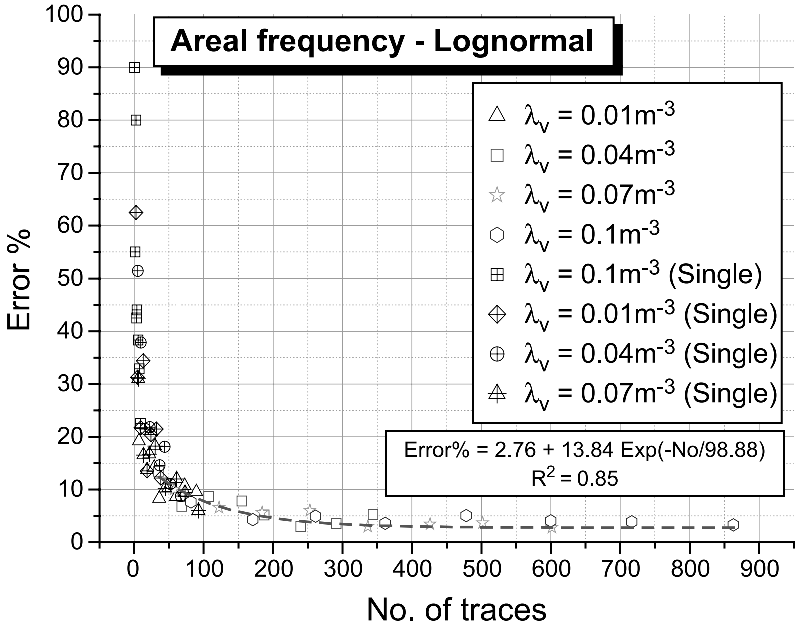

Fig. 4. Error percent of the estimated normal areal frequency from an overall non-planar window and a single planar window for lognormal diameter distribution: λ_V means a volumetric frequency

non-planar window. The former is marked with four kinds of crossed symbols and the latter with empty symbols. Estimation error at the same number of traces is almost same for both cases. A dash-line in Fig. 4 shows a fitting function of the empty symbols indicating the errors from overall non-planar sampling scheme. Its function form is noted in the box at the bottom of the figure. From Fig. 4, we can see that the newly suggested estimator is a generalized or expanded version of the end-point estimator of Mauldon (1998). Actually, Eq. (8) coincides with Eq. (1) when sampling zone is a planar window, except the fact that Eq. (8) deals with a normal areal frequency while Eq. (1) is of an apparent areal frequency.

Estimation error was also checked for cases of normal and exponential diameter distributions but their result has been omitted here because both cases showed similar result with the lognormal case. In case of exponential diameter distribution, for instance, the estimation error % from the overall non-planar window decreases as the sample number (*No*) increases, which is fitted with following exponential function:

$$Error\% = 3.72 + 13.12e^{-No/133.64}(R^2 = 0.75). \tag{12}$$

Equation (12) has similar values of constants and coefficients to those of the fitting function in Fig. 4.

From above simulation result, we can see an important fact besides the validness of the non-planar window sampling method: the estimation error rapidly decreases as the number of traces increases when the number of sampled traces is not high. As the trace number increases further, however, the error is abruptly stabilized past a certain point. In Fig. 4, 100 traces make the error below 10%, but even more 500 traces reduce the error only 5%. When more than 100 traces per discontinuity set is hard to be sampled in a single planar window, the non-planar sampling window technique will be very useful to enhance the sampling size and thus to decrease the estimation error.

3.1.2 Case of Variable Orientation

In verification process above, all of discontinuity discs are set to be parallel. Mauldon's end-point estimator is on an apparent areal frequency and it is originally independent of trace orientation as shown in Eq. (1). The generalized version of Eq. (8), however, includes an orientation variable, ϕ_i, which means an acute angle between a sampling window and a discontinuity plane. This variable is used for converting an apparent frequency to a normal frequency.

To check out a negative effect of variation of disc orientation on the estimation error, the estimation error was evaluated for various Fisher's constant (*K*): 35, 25, 15 and 5. Figure 5 shows the estimation error vs. the number of traces in cases of lognormal diameter distribution. In this figure, the estimation error for a single sampling plane was marked with crossed symbols and added to the result for an overall non-planar window. We can observe that the case of a single window at $K = 5$ shows slightly bigger errors than the other cases. When discontinuity orientation is changed from $60°/45°$ to $0°/45°$, however, the non-planar window method at $K = 5$ showed a

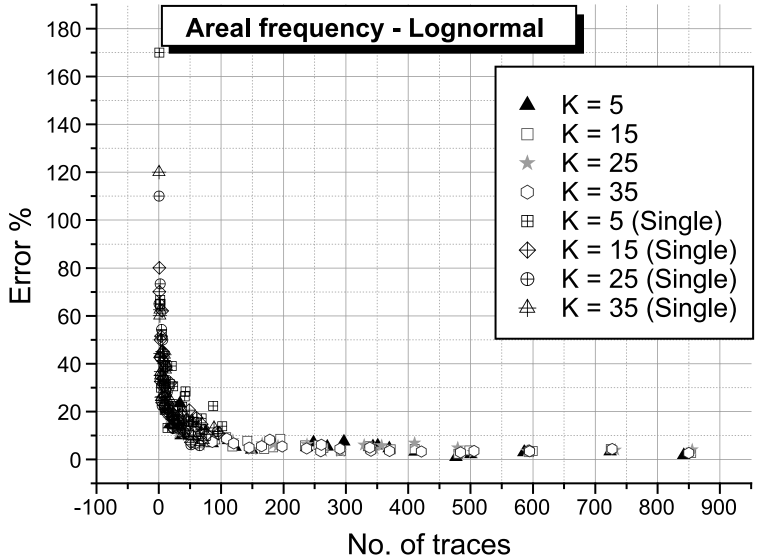

Fig. 5. Error percent of the estimated normal areal frequency for lognormal diameter distribution when discontinuity orientation is variable

bigger error than other cases and the bigger error of a single window at $K = 5$ became negligibly lessened. Considering that most discontinuity sets yield values of K in excess of about 5 (Priest, 1993), the variation of discontinuity orientation does not have a great influence on the normal areal frequency estimation for both of a single plane and a non-planar window sampling methods.

The standard deviations of estimation error for three kinds of diameter distributions showed very similar graphs to those of estimation error %. From these results, it can be said that we can estimate the normal areal frequency with less than 10% of a mean and standard deviation of error by using more than 100 sampled traces.

3.2 Mean Trace Length

3.2.1 Verification

Figure 6 shows an estimation error of the normal areal frequency from a single planar and an overall non-planar window vs. the number of traces in tunnel wall when the discontinuity diameter follows a lognormal distribution. As in the case of areal frequency in Fig. 4, the estimation error exponentially decreases as the number of sampled traces increases, and is not directly dependent on change in volumetric frequency. This exponential decrease is mathematically expressed by the fitting function in the box in Fig. 6. As in the case of areal frequency, the fitting function has similar values of constants and coefficients for the exponential or normal diameter distribution.

Compared with Fig. 4, Fig. 6 shows that the error from an overall non-planar window is a little higher than those from a single window at sample size of 400 or less.

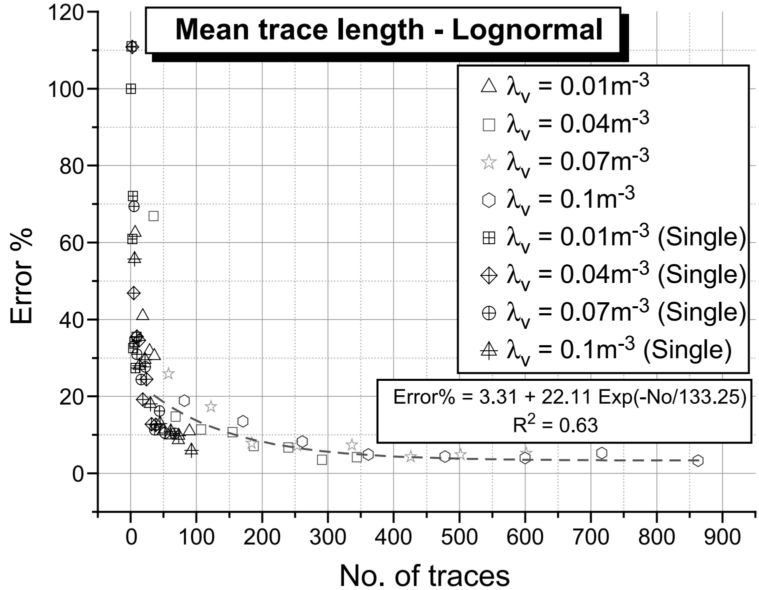

Fig. 6. Error percent of the estimated mean trace length from an overall non-planar window and a single planar window for lognormal diameter distribution

The higher level of estimation error seems to be caused by a structural feature of the end-point estimator: ratios of sampled transecting, dissecting and contained traces are not same between the single planar window and overall non-planar window. Figure 7

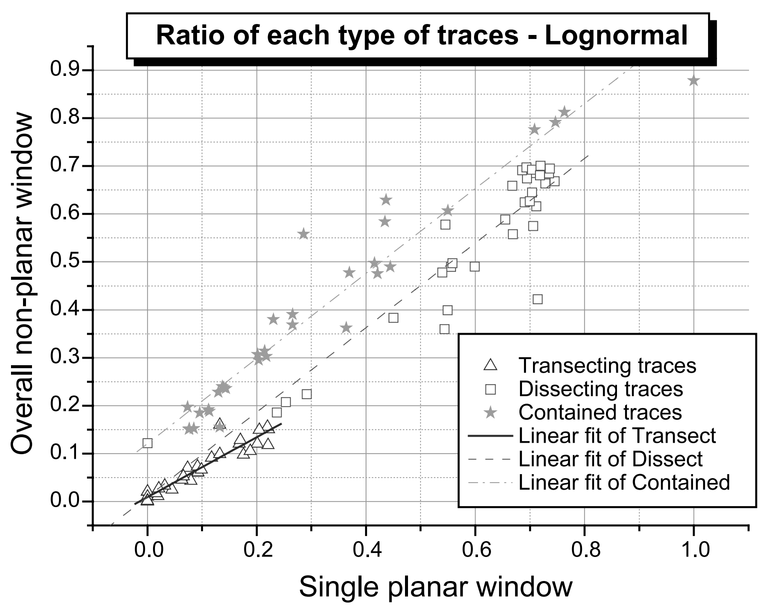

Fig. 7. Relative counts of traces according to their type for an overall non-planar and single planar window

shows the numbers of transecting, dissecting and contained traces divided by a total trace number for each type of sampling window. The number of contained traces (also of dissecting or transecting traces) in the overall non-planar window is obtained by summing the number of contained traces in each sub planar window. When the ratios of dissecting and contained traces in the single planar window increase, those in the overall non-planar window also increase in a very similar rate, while the rate of transecting traces is different from those of dissecting and contained traces. Linear fit functions of dissecting and contained traces in Fig. 7 have slopes of 0.88 and 0.89, respectively. Meanwhile, the slope of linear fit of transecting traces is 0.63. All of the fit functions have a coefficient of determination higher than 0.92.

This aspect does not much affect the areal frequency estimation, because, as shown in Eq. (8), the areal frequency depends only on the numbers of dissecting and contained traces. As for the mean trace length estimation, however, the number of transecting traces is a constituent of Eq. (11). Therefore, the estimation efficiency of the mean trace length using Eq. (11) can be different between the single planar window and the overall non-planar window. More straightforward explanation is as follows: as an area of a sampling window increases, relative amount of dissecting and transecting traces decrease while those of contained traces are enhanced. At this moment, the relative decreased amount of transecting traces is different for a single enlarged planar window and for a non-planar (enlarged) window consisting of small sub windows. This efficiency down of the non-planar window estimation by Eq. (11) is at last due to the inherent geometric limit that it uses small sub windows instead of a large single planar window. In spite of the lower efficiency of the generalized end-point estimator for a non-planar window shown in Fig. 3, its estimation value from 10 sub windows always showed lower error than those from a single planar window in Fig. 6 due to its much higher sample size.

3.2.2 Case of Variable Orientation

As in the case of areal frequency, the orientation of a discontinuity set was modeled by a Fisher function with Fisher's constant K of 35, 25, 15 and 5. Figure 8 shows the estimation error vs. the number of traces in case of lognormal diameter distribution.

In Fig. 8, the mean trace lengths estimated from both of the non-planar window and a single planar window are hardly affected by variation of discontinuity orientation even at $K = 5$, which seems a different aspect from the areal frequency estimation in Fig. 5. When the mean dip direction of discontinuities is changed from 60° to 0°, however, estimation error at $K = 5$ becomes slightly higher than other cases for both sampling methods. Considering a general discontinuity set having K greater than 5, the effect of variable orientation of discontinuities can be ignored as in the case of the areal frequency estimation.

As in the areal frequency estimation, the estimation error and its standard deviation have very similar values at each trace number, regardless of diameter distributions. From this result, it can be said that we can estimate the mean trace length with

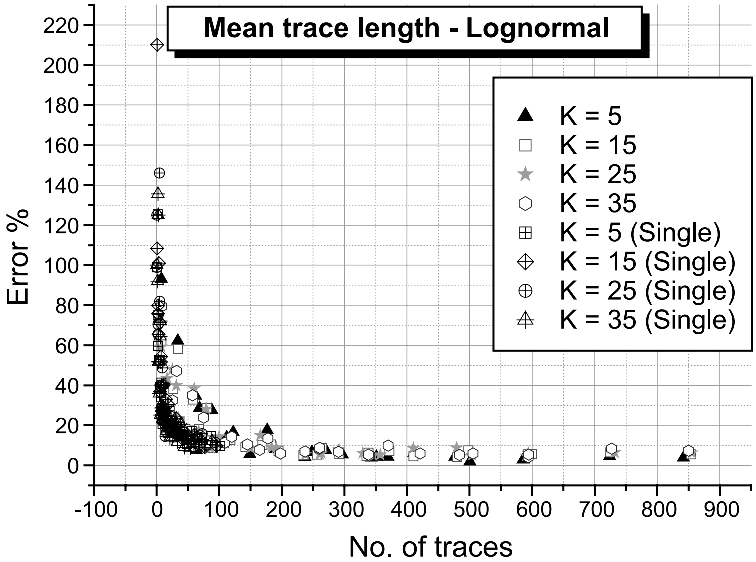

Fig. 8. Error percent of the estimated mean trace length for lognormal diameter distribution when discontinuity orientation is variable

an error whose mean and standard deviation are less than 20% and 10%, respectively, by using more than 100 sampled traces.

4. Example of the Normal Areal Frequency and Mean Trace Length Estimation

For the sake of easy understanding of the suggested methods, the estimation formulas are applied to a simple joint map in Fig. 9. The sampling window is the same as that in Fig. 2 where three rectangular windows serve as sub windows. The sub windows are tagged with A, B and C, respectively. The height of three windows is 10 m and the width is 11 m, 8 m and 9 m, respectively. The acute angle between parallel joints and sub windows is 60° (A), 50° (B) and 30° (C). Each sub window has twenty three (A), thirteen (B) and ten (C) joint traces of which eleven, six and five traces are the contained traces. Only two transecting joints are found in sub window A throughout the sampling region.

Table 1 shows the numbers calculated in the overall estimation procedure. The normal areal frequency (λ_i^A) and weight factor (w_i) at each sub window and the overall normal areal frequency (λ^A) are calculated by using Eq. (5), Eq. (7) and Eq. (6), respectively. The normal areal frequency, λ_A^A and weight factor, w_A at sub window A, for example, are calculated by

$$\lambda_A^A = \frac{23 - 2 + 11}{2 \times 110 \times \sin 60} \approx 0.17 \quad \text{and}$$

$$w_A = \frac{110 \times \sin 60}{110 \times \sin 60 + 80 \times \sin 50 + 90 \times \sin 30} \approx 0.47.$$

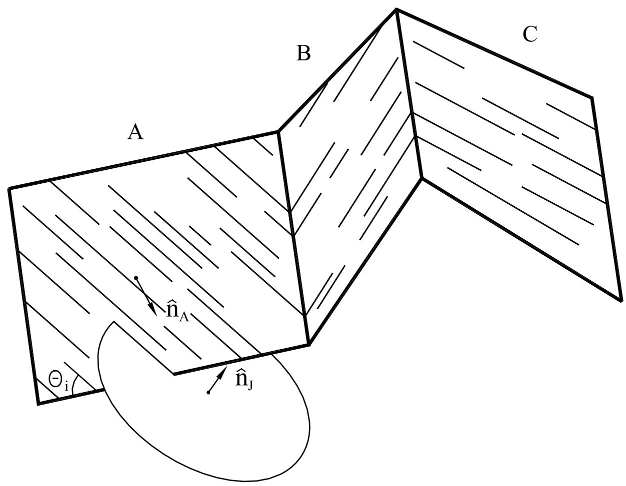

Fig. 9. An example of sampled joint traces for estimating the normal areal frequency and mean trace length

Table 1. Calculation procedure for estimating the normal areal frequency and mean trace length

Sub window	N_i	N_i^T	N_i^C	$A_i(m^2)$	$\phi_i(°)$	$\lambda_i^A(m^{-2})$	w_i	$\lambda^A(m^{-2})$	$\theta_i(°)$	$W(\theta_i)$	$\mu_i^L(m)$	$\mu^L(m)$
A	23	2	11	110	60	0.17	0.47	0.17	55	14.75	3.26	3.04
B	13	0	6	80	50	0.16	0.31		5	10.66	2.76	
C	10	0	5	90	30	0.17	0.22		1	10.16	2.95	

The mean trace length at each sub window (μ_i^L) and the overall mean trace length (μ^L) are calculated by using Eq. (9) and Eq. (10). The span of a rectangular window in direction normal to the joint traces, $W(\theta_i)$ can be expressed by

$$W(\theta_i) = Width \times \sin\theta_i + Height \times \cos\theta_i,$$

where *Width* and *Height* are the horizontal and vertical length of the window, respectively. For example, $W(\theta_B)$ is calculated by

$$W(\theta_B) = 8 \times \sin 5 + 10 \times \cos 5 \approx 10.66.$$

Finally, the overall mean trace length is obtained as 3.04 m as in Table 1.

In real situation, the orientation of joint planes and joint traces is supposed to vary within a certain limit of angle even if the joints belong to the same joint set. In that case, the representative orientation of a joint set should be determined first. Summation of joint normal vectors multiplied by normalized weighting (Priest, 1993) is usually adopted technique for obtaining the representative (mean) orientation, which can be easily implemented with a commercial program such as DIPS. Then, the angular parameters ϕ_i and θ_i in Eq. (5) and Eq. (9) are determined using the mean orientation. Calculation of $\sin\phi_i$ is as explained in Section 2.1 and $\cos\theta_i$ can be easily obtained from a dot product of the orientation vector of a joint trace made by the mean joint orientation and the orientation vector of the horizontal boundary line of the sampling window.

5. Conclusions

Expanding end-point estimators suggested by Mauldon (1998), generalized end-point estimators of areal frequency and mean trace length which are applicable to a non-planar sampling window as well as a single planar window are developed. Main conclusions of this study are as follows.

- Estimation error of areal frequency and mean trace length decreases exponentially as the sample size increases. Therefore, a sample size bigger than a certain value hardly contributes to reduce the estimation error.
- When a discontinuity set is defined by discontinuities having variable orientation of $K > 5$, the normal areal frequency and mean trace length estimated by the generalized end-point estimators are hardly affected by the variable orientation of discontinuities.
- Standard deviation of estimation error in percent by the generalized end-point estimators is almost same with the estimation error itself. From simulation tests, more than 100 sampled traces are needed to estimate the normal areal frequency (mean trace length) by the generalized end-point estimator with 10% (20%) or less estimation error.
- Estimation efficiency of generalized end-point estimator of areal frequency is almost same between a single planar window and a non-planar window, while those of mean trace length are lowered for a non-planar window. This seems to be caused by the fact that transecting traces have a different change rate from contained or dissecting traces for a single sampling window and a non-planar window.

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