

Importance of lateral viscosity variations in the whole mantle for modelling of the dynamic geoid and surface velocities

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Abstract

We investigate the effect of lateral viscosity variations (LVV) in the mantle on dynamic geoid and near-surface mantle velocities. Contrary to the previous studies, we analyse the effect not only of the lithospheric keels and asthenosphere but also of the whole mantle. A 3D global viscosity model is constructed using (a) the S20 seismic tomography model converted to temperature and (b) assumptions about homologous temperature in the mantle Paulson et al. [Paulson, A., Zhong, S., Wahr, J., 2005. Modeling post-glacial rebound with lateral viscosity variations. *Geophys. J. Int.* 163, 357–371]. The conversion parameters provide a depth-dependent viscosity profile, which is generally consistent with the existing studies. Therefore, the horizontal variations are produced self-consistently within this approach. We estimate them as ‘conservative’, thus providing a minimum limit for possible horizontal changes; consequently the obtained results show a low limit for dynamic topography, geoid and surface velocities’ disturbances. To estimate the effect of LVV we have employed the perturbation method of Zhang and Christensen [Zhang, S., Christensen U., 1993. Some effects of lateral viscosity variations on geoid and surface velocities induced by density anomalies in the mantle. *Geophys. J. Int.* 114, 547–551], which is modified to implement mantle compressibility. It has been found that the impacts of the upper and lower mantle are comparable in amplitude. The difference between the initial (only radial viscosity) dynamic geoid and the geoid with implemented LVV reaches –27 to +19 m for the whole mantle LVV, while the effects of the upper and lower mantle are equal to –24 to +16 and –20 to +16 m correspondingly. In general these effects are not correlated. The difference in the geoid response leads to substantially altered velocity-to-density scaling factors (up to 30% compared to the initial ones) obtained in least squares inversion. The effect of the lower mantle LVV on surface velocities is also substantial, in particular with respect to the toroidal component, which does not exist in the initial model.

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1. Introduction

The observed geoid is one of the most important constraints on mantle parameters including the most indefinite value of viscosity variations. However, a determination of the Earth’s mantle structure has an ambiguous solution when using only surface gravity data. A usual way to cope with such a problem is to combine gravity data with other geophysical

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data sets to produce the solution, which fits all the data sets and therefore contains fewer degree of freedom. Seismic tomography models are used for these purposes most often. In global studies this technique is applied starting from the pioneer works by Hager and O'Connell (1981), Ricard et al. (1984), Richards and Hager (1984), Forte and Peltier (1987), King and Masters (1992), Corrieu et al. (1994) and many others.

Despite of the massive efforts, the obtained results are remarkably various and the generalized dynamic model of the Earth does not exist at the moment. There could be several explanations for such an indefinite situation. First of all, there still exists a trade-off between different model parameters and the same fit may be obtained within different model clusters. Secondly, the models analysed are still far from reality. A dynamic response of the Earth surface to internal loading requires a solution of the Stokes' equations together with the Poisson's equation for gravity potential. In the above-mentioned papers the simplified model implies only radial viscosity variations, which provides a possibility to solve the equations separately for each spherical harmonic coefficient. Therefore, it is possible to estimate once the response of the Earth to internal load (density inhomogeneities) placed at different depths, so-called geoid kernels, and then use the kernels in the inversion. Although the method is extremely fast and effective, the effect of lateral viscosity variations (LVV) remains unclear. The main difficulty is that all spherical harmonic coefficients are coupled with LVV, and it diminishes all the advantages of the kernel technique.

Richards and Hager (1989) demonstrated for the 2D convection models that the effect of LVV could be significant for degrees $l > 4$. Koch and Ribe (1989) also investigated some general effects of LVV on the surface observables analysing several simplified models. They found that LVV produce a large (up to 50%) effect, when the load is relatively 'hard' and shallow, but only a small (<15%) one when the load is 'soft' or deep. The geoid anomaly produced by a soft upwelling plume differs only slightly (approximately 13%) from that one generated by an isoviscous plume. By contrast, the viscosity differences associated with subducted slabs could have a larger effect on geoid. Ribe (1992) considered the model consisting of the lithospheric shell with laterally variable thickness and viscosity, which overlies the mantle with depth-dependent viscosity. He demonstrated that LVV and thickness of the lithosphere could produce a large effect on geoid anomalies if the mantle viscosity strongly increases with depth.

Richards et al. (2001) have investigated the effect of LVV in the upper mantle on near-surface mantle velocities. They have demonstrated that combining a pronounced low-viscosity zone and a plastic yield stress to allow localized weakening of the cold thermal boundary layer, results in a distinctly plate tectonic style of convection, with ~30% toroidal surface motion for the 3D case.

Zhang and Christensen (1993) proposed a hybrid finite difference and spherical harmonics' method, which provides a possibility to estimate the effect of realistic LVV within the mantle. The non-linear coupling of the various spherical harmonic modes is calculated by an iterative method. They investigated the effect for long wavelengths ($l = 1-6$) and found that for such wavelengths the effects of LVV on the geoid are lower than those due to changes in radial viscosity structure. However, they can be also significant on the higher modes ($l > 3$) if the viscosity is radially stratified. At the same time, it was found that the misfit between the observed and modelled geoids is not improved by introducing LVV.

Karpychev and Fleitout (2000) calculated the effects on the geoid for the model with LVV in the upper mantle. Beneath the ancient stable continental regions, the viscosity decreases monotonically from its value at the surface to the viscosity at a depth of about 400–600 km. The oceanic lithosphere and the tectonically active continental provinces are underlain by the low-viscosity asthenosphere. The viscosity of the lower mantle is assumed to be constant. Mantle flows are driven by imposed surface velocities and by the density anomalies inferred from tomography models. They found that the geoid differences between the models with and without LVV reach 30%. Contrary to what was proposed in the previous studies, spherical harmonics 2 and 3 are strongly affected by LVV. It is also important that shear stresses at the base of the plates are sensitive to the LVV.

Čadek and Fleitout (1999, 2003) investigated the impact of LVV in the top 300 km of the mantle using the perturbation technique of Zhang and Christensen (1993). In addition to the model of Karpychev and Fleitout (2000) they analysed a possible effect of partial layering of the mantle convection at the 670 km discontinuity. Considering all the effects simultaneously these authors were able to improve remarkably the misfit between the observed and modelled geoid: the partially layered model explains ca. 90% of the observed geoid on the long wavelengths ($l = 2-8$).

Thus, the previous studies showed that the effect of LVV in the upper mantle could be rather significant under certain conditions. At the same time importance of the lower mantle LVV, which could be also significant, remains unclear. Since the effect of viscosity changes is non-linear, it is possible that the inclusion of lower mantle LVV could also modify the impact of the upper mantle. In the present study we investigate the effect of the whole mantle LVV on the dynamic geoid and mantle flow velocities focusing on possible impact of the lower mantle. In contrast to the previous

studies we extend the analysed wavelengths to the 20th spherical harmonic degree, being an approximate limit of the existing seismic tomography models.

2. 3D viscosity model

The 3D viscosity model of the mantle is constructed based on the S-wave tomography model S20a of [Ekstrom and Dziewonski \(1998\)](#). This model is one of the most used in such type of modelling. It provides resolution up to the 20th spherical harmonic degree for isotropic velocity variations. The last aspect is especially important for our study. Despite other models could differ in details from the S20a model, general conclusions about importance of taking into account LVV are valid also for all models with this or higher resolution. We use the procedure proposed by [Paulson et al. \(2005\)](#). The V_s perturbations have been initially converted into density variations. Based on the density variations we have estimated temperature distribution in the mantle (T) and determined a homologous temperature. In the final stage we have calculated viscosity variations (η) applying the following relationship ([Paulson et al., 2005](#)):

$$\eta = A_0 \exp \left(\gamma \frac{T_m}{T} \right), \quad (1)$$

where T_m is a solidus temperature, γ an activation parameter and A_0 is an initial coefficient. The parameters should be chosen separately for different mantle layers depending on P – T conditions and on a phase state of the material, primarily for the upper and lower mantle. So we can also adjust a vertical viscosity profile, which should be close to the results of the previous studies. Thus, LVV are then produced self-consistently within this approach.

There are basically two ways to determine a velocity-to-density scaling factor (VDSF). In the first approach the scaling factor is estimated using joint inversion of seismic tomography data and surface observables (e.g. [Corrieu et al., 1994](#)). Another option is to use mineral physics equations and experimental data (e.g. [Karato, 1993](#)). For the upper mantle we have used the results of joint inversion of the residual mantle anomalies and V_s perturbations in the upper mantle ([Kaban and Schwintzer, 2001](#)). At depths of 70–150 km $\ln(\delta\rho/\rho)/\ln(\delta V_s/V_s) = 0.1$, below 150 km the scaling coefficient increases and reaches 0.25 in the lower part of the upper mantle. It is clear that not all velocity variations presented in the S20a model are induced by temperature variations. Compositional variations could substantially alter velocity–temperature relationship or simply mask the temperature effect. However, we assume that in spite of possible alterations of the computed fields in specific places, general conclusions about importance of incorporating LVV in global dynamic models will be convincing.

As mentioned above, various authors obtained rather different results for VDSF in the transition zone and in the lower mantle (e.g. [Forte and Peltier, 1987](#); [Corrieu et al., 1994](#); [Panasyuk and Hager, 2000](#) and many others). More over, the results depend on the tomography model used in inversion (e.g. [Forte and Perry, 2000](#)). Next, it is not quite clear, how various factors influence velocity, density and temperature in the transition zone and in the lower mantle. Therefore, we have used the value of VDSF at the bottom of the upper mantle (0.25) also for the transition zone and the lower mantle, which is consistent with the mineral physics studies (e.g. [Karato, 1993](#)). It has been decided not to overload this initial model by additional details, which are not well grounded, since it is used to demonstrate principal importance of LVV in the next generation global dynamic models. The density perturbations have been converted into densities applying the depth-dependent thermal expansion coefficient ([Paulson et al., 2005](#)):

$$\alpha = \left(3 - \frac{2h}{h_c} \right) 10^{-5}, \quad (2)$$

where h is the depth and h_c depth to the core–mantle boundary.

To calculate LVV with the (Eq. (1)) we need radial profiles of the horizontally averaged temperature in the mantle and solidus temperature. Since our model is instantaneous we cannot estimate the temperature distribution self-consistently and just take it from literature. The depth-dependent temperature distribution is based on ([Schubert et al., 2001](#)) and the temperature of solidus – on ([Yamazaki and Karato, 2001](#)). Both temperature curves are shown in [Fig. 1](#). The solidus temperature at the bottom of the mantle is slightly increased following ([Schubert et al., 2001](#)).

The only activation parameter γ is required to determine relative variations of viscosity based on temperature anomalies (Eq. (1)). Following ([Paulson et al., 2005](#)) we use $\gamma = 10$ in the upper mantle and $\gamma = 17$ in the lower mantle ([Yamazaki and Karato, 2001](#)). The resulting LVV in the whole mantle are shown in [Fig. 2](#). All details in this figure correspond to the S20a tomography model. The maximum LVV in the lower mantle, being of primary interest for

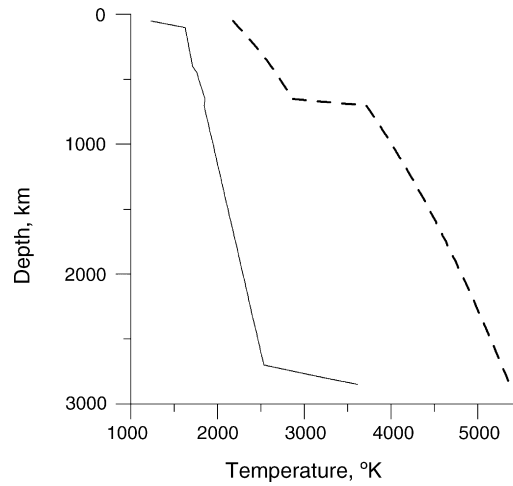


Fig. 1. (a) Radial mantle temperature profile (Schubert et al., 2001). (b) Solidus temperature in the mantle (Yamazaki and Karato, 2001).

this study, are about two orders magnitude and are represented at the whole length scale: from very broad anomalies to small-scale variations. These relative variations are multiplied by the radial viscosity distribution that finally gives the 3D viscosity model. For this study we have used the most general three-layer radial viscosity distribution, which corresponds with most of the previous results. Average viscosity in the upper mantle below 70 km to a depth of 670 km is equal to 10^{21} Pa s. Average viscosity in the lower mantle is 30 times larger as the former one. This model corresponds to the leading coefficients in the Eq. (1): $A_0 = 5 \times 10^9$ Pa s in the upper mantle and $A_0 = 5 \times 10^{13}$ Pa s in the lower mantle. Within the adopted radial temperature distributions (Fig. 1) radial viscosity in the lower mantle is almost constant from 670 down to D'' . The effective viscosity above a depth of 70 km is taken to be 20 times as large as the *average* viscosity in the upper mantle. Low-viscous asthenosphere and high-viscous continental keels appear after introducing the LVV. The upper mantle viscosity variations in the model are likely to be less than in reality, however in the case we are limited by calculation technique (see below). Therefore the effect of LVV in the upper mantle should be considered as a lower limit. Maximum amplitude of the 3D viscosity variations shown in Fig. 2 is about three orders of magnitude.

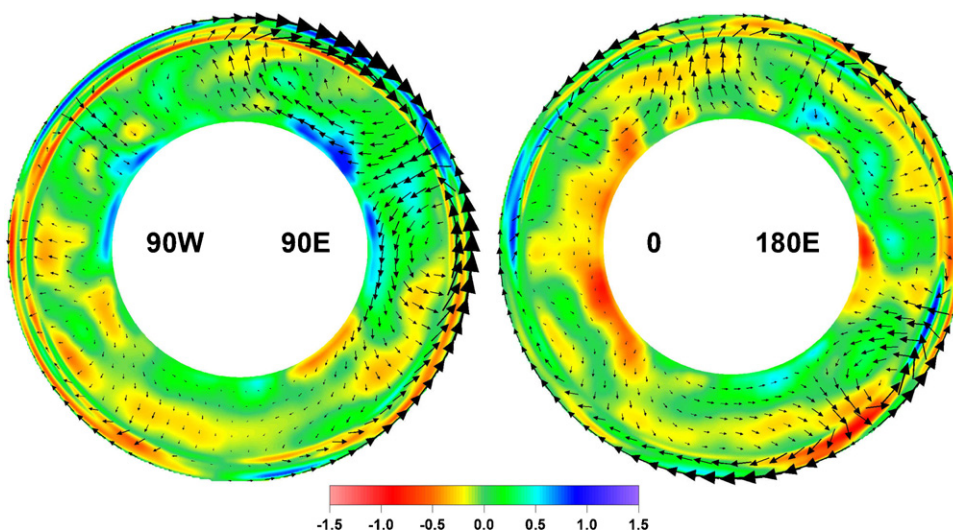


Fig. 2. Cross-sections showing lateral viscosity variations ($\log_{10}$) in the mantle relative to the adopted vertical profile. (Left) 0 – 180° E, (right) 90° W– 90° E. Arrows show mantle velocities (maximal value in the demonstrated cross-sections is equal to 40 mm/year).

3. Numerical technique

Dynamic geoid, surface topography and mantle flow velocities are calculated with the approach suggested by Zhang and Christensen (1993). In the first stage we solve the basic mantle flow equations together with the Poisson's equation for gravity potential using the standard Green functions' method, which has been described in details in many papers starting from the works of Hager and O'Connell (1981), Ricard et al. (1984). The method was modified to implement the latest innovations, e.g. the effects of a compressible mantle and of depth-dependent gravity (Corrieu et al., 1995; Panasyuk et al., 1996). By this we determine geoid perturbations, mantle velocities and stresses induced by any internal load for the model with radially stratified viscosity (Appendix A).

In the second stage the initial solution is modified to account for the effect of LVV. The non-linear coupled terms, appearing in the basic equations due to LVV, are shifted to the right-hand side and the equations are solved iteratively using the standard technique. The additions to right-hand side terms are calculated based from the results of the previous step. When convergence is achieved, the iterative process is stopped (Zhang and Christensen, 1993). For more details and model parameters see Appendix A.

In the next stage the dynamic geoid is calculated for separate mantle layers assuming a constant velocity–density scaling for the given viscosity models. The scaling factors are then rescaled in a least squares adjustment to get a best fit to the observed geoid, i.e. to minimize the function:

$$\chi^2 = \sum_{l,n} \left(N_{\text{obs}}^{l,n} - \sum_j a_j N_j^{l,n} \right)^2 + \sum_j \beta_j (a_j - a_j^0)^2, \quad (3)$$

where N_{obs} is an observed geoid for degree/order l and n ; N_j are geoid variations induced by a layer j ; a_j are unknown scaling factors; a_j^0 are initial scaling coefficients; β_j are the damping factors introduced to stabilize a solution.

We have performed calculations for four different viscosity models. The 1D model with radially dependent viscosity is compared to the models, where LVV are incorporated in the whole mantle. To investigate specific effects of the upper and lower mantle we also consider the models with LVV emplaced separately in the main mantle layers.

4. Results

The calculated geoid for the model with a radially stratified mantle is shown in Fig. 3 in comparison with the observed non-isostatic geoid. In contrast to the previous authors, we have excluded the terms C_{20} and C_{40} from all fields. Origin of these terms in the observed geoid relative to a hydrostatic spheroid is not completely comprehended yet (e.g. Nakiboglu, 1982; Mound et al., 2003). Furthermore, the terms C_{20} and C_{40} dominate in the observed non-hydrostatic geoid, their globally estimated root mean square (rms) is equal to 28.5 m, which is almost identical to the rms of the other terms (31.8 m). Modelling of the terms C_{20} , C_{40} requires precise knowledge of seismic velocity anomalies in the polar areas, which are not sufficiently resolved in the existing global tomography models and their amplitude might be significantly reduced due to damping. Therefore, it has been decided to exclude these terms and consider only those clearly related to the disturbances generated by mantle flows. In addition, the impact of the isostatically compensated lithosphere is also excluded from the observed geoid. The effect has been calculated taking into account available data about crustal structure and including density anomalies in the uppermost mantle (Kaban et al., 2004). This provides a possibility to uncover the effect of deep mantle horizons and of dynamic disturbances of the Earth's surface.

Scaling factors for a radially stratified mantle obtained in a least squares adjustment is shown in Fig. 4. In the lower mantle a conversion coefficient varies insignificantly within the limits of 0.17–0.2, while in the transition zone it exceeds 0.4. These substantial differences will be a subject of our further studies. It turns out that even for a simple three-layer viscosity model it is possible to get a reasonable fit to the observed geoid (Fig. 3). The model explains about 70% of the observed field, being a good result especially taking into account that we have excluded the terms C_{20} and C_{40} accumulating half of the total energy of the long-wavelength non-hydrostatic geoid.

The relative LVV obtained in the previous step (Fig. 2) have been added to the radially stratified model and all the calculations were made for the 3D viscosity models with the method by (Zhang and Christensen, 1993). The geoid for the 3D viscosity model for the whole mantle LVV is shown in Fig. 3 together with the observed field and with

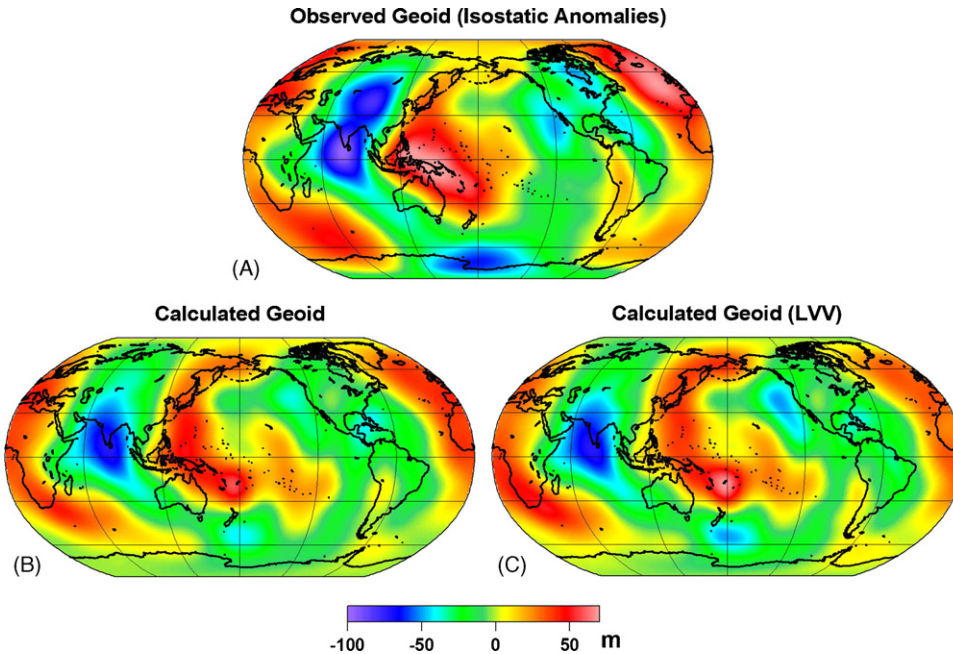


Fig. 3. (a) Isostatic anomalies of the observed geoid (Kaban et al., 2004). (b) Geoid undulations for the model with radially stratified viscosity. (c) Geoid undulations for the model with LVV. Terms C_{20} and C_{40} are excluded from all fields (see text for more explanations).

the geoid for radially stratified viscosity. We have repeated the inversion and resulting velocity-to-density scaling is shown in Fig. 4 (dashed curve). The difference with the initial scaling is quite substantial and reaches 0.06 in the lower mantle. The overall fit to the observed geoid is approximately the same (about 70%) but there are several geoid features presented in the observed geoid, appearing only after introducing the LVV. The most pronounced is the negative anomaly located near the western edge of North America. The extended maximum near South America is also better presented with the LVV. The same is true for the geoid pattern in South Pacific. This preliminary analysis shows that inclusion of LVV in the whole mantle improves mid-range features of the dynamic geoid.

Discrepancies between the dynamic geoid generated by the 3D viscosity models and the initial “radial” model are shown in Fig. 5. We have considered three LVV distributions: the whole mantle LVV and two models, where LVV are restricted to the mantle above the 400 km discontinuity and below it. The difference between the initial (only radial viscosity) dynamic geoid and the geoid with implemented LVV reaches -27 to $+19$ m for the whole mantle LVV,

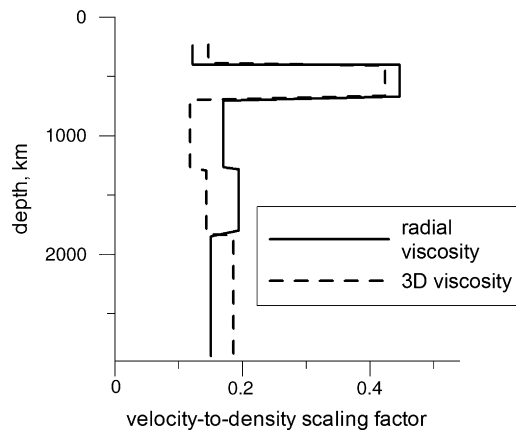


Fig. 4. Velocity-to-density scaling factor vs. depth obtained in a least square adjustment to get a best fit to the observed geoid. Solid line represents the scaling factor for the radially dependent viscosity model. Dashed line is calculated for the model with LVV.

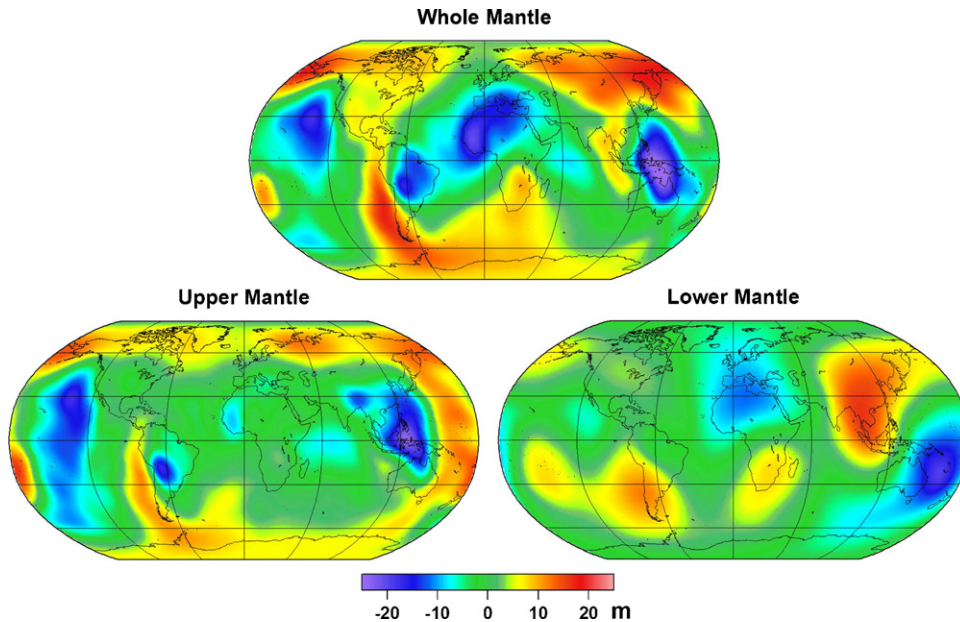


Fig. 5. Discrepancies between a dynamic geoid generated by the 3D viscosity models and initial “radial” model. (Top) whole mantle LVV; (left) LVV in the upper mantle above 400 km; (right) LVV in the lower mantle below 400 km.

while the effects of the upper and lower mantle are equal to -24 to $+16$ and -20 to $+16$ m correspondingly. The differences are exposed mainly at mid-range scale as it was suggested in the preliminary analysis. Noteworthy, these effects are not correlated in general, that might be of significance for future high-resolution dynamic models. It is important that the sum of the upper and lower mantle effects is very close to the effect of the whole mantle LVV, the maximum difference is about 1.3 m. This provides a possibility to model low and upper mantle LVV separately, e.g. using different calculation schemes, which work better in each specific case.

The modifications of the geoid shown in Fig. 5 are chiefly produced by differences in surface dynamic topography since the effect of density variations remains unchanged and the effect of the core-mantle boundary is small. The dynamic topography modifications for the tested viscosity models are shown in Fig. 6. They correspond qualitatively to the changes of the geoid, however we see many small-scale details. These details, which do not remarkably influence geoid, are mainly due to LVV in the upper mantle (Fig. 6, left). The most significant effects are produced by sharp horizontal viscosity contrasts in the upper mantle, e.g. related to the ocean-continent boundaries. Therefore, a relative difference in the amplitudes of the dynamic topography variations due to upper- and lower-mantle viscosity changes is more pronounced than the difference in the dynamic geoid (Fig. 5).

Furthermore, we investigate the effect of LVV on geoid kernels computed for different depths and harmonic degrees. Kernels for the model with LVV are estimated directly from the calculated geoid and internal load representing density anomalies at some depth. The “experimental” curves are compared to the theoretical ones for radially stratified viscosity. In this way we have investigated LVV importance for various depths and spherical harmonic degrees. The experimental kernels depend on spherical harmonic degree only (a response of the geoid to internal load is considered being isotropic), thus the experimental kernels reflect only those effects of LVV correlated with internal load.

The experimental kernels for depths of 200, 400, 800 and 1500 km are shown in Fig. 7 together with the initial kernels for radially stratified viscosity. The discrepancies with the initial kernels are not as significant as could be expected from the geoid differences in Fig. 5. It demonstrates that the most significant geoid changes are not correlated with initial load and may not be taken into account by simple correction of geoid kernels. The most significant changes of the kernels are found for depths of 400 and 800 km, i.e. in the mid-mantle. The effects of the lower and upper mantle LVV are comparable at those depths. For the upper mantle load the effects are also notable at spherical harmonics 2 and 3, it agrees with the results of Karpychev and Fleitout (2000).

Modifications of near-surface mantle velocities (at a depth of 100 km) caused by LVV are shown in Fig. 8. These changes are remarkable and reach 12 mm/year. The impact of the lower mantle LVV is substantial, e.g. the changes in

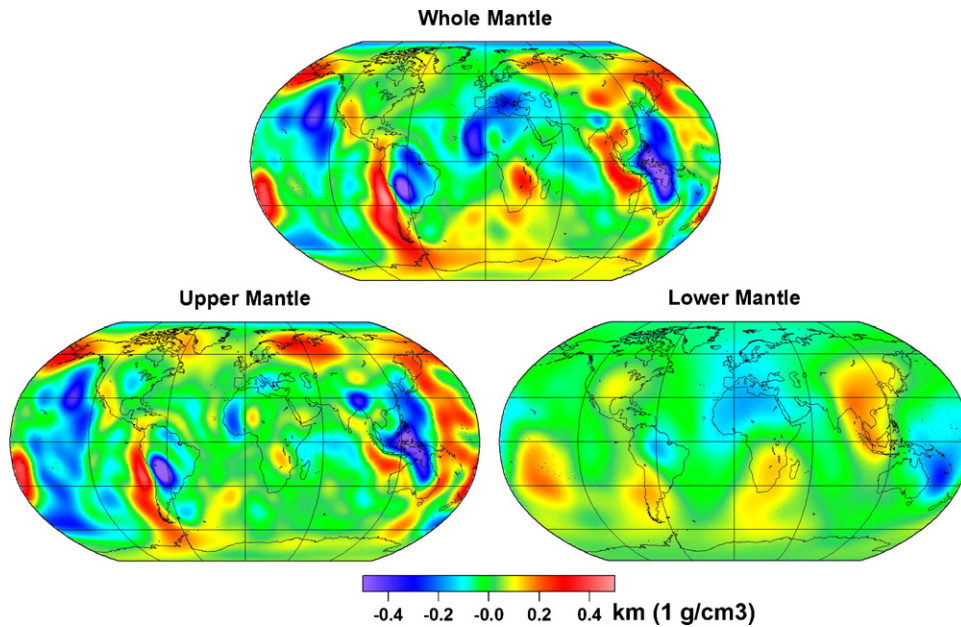


Fig. 6. Discrepancies between the dynamic topography generated by the 3D viscosity models and initial "radial" model. (Top) whole mantle LVV; (left) LVV in the upper mantle above 400 km; (right) LVV in the lower mantle below 400 km. Values in km for density 1 g/cm³.

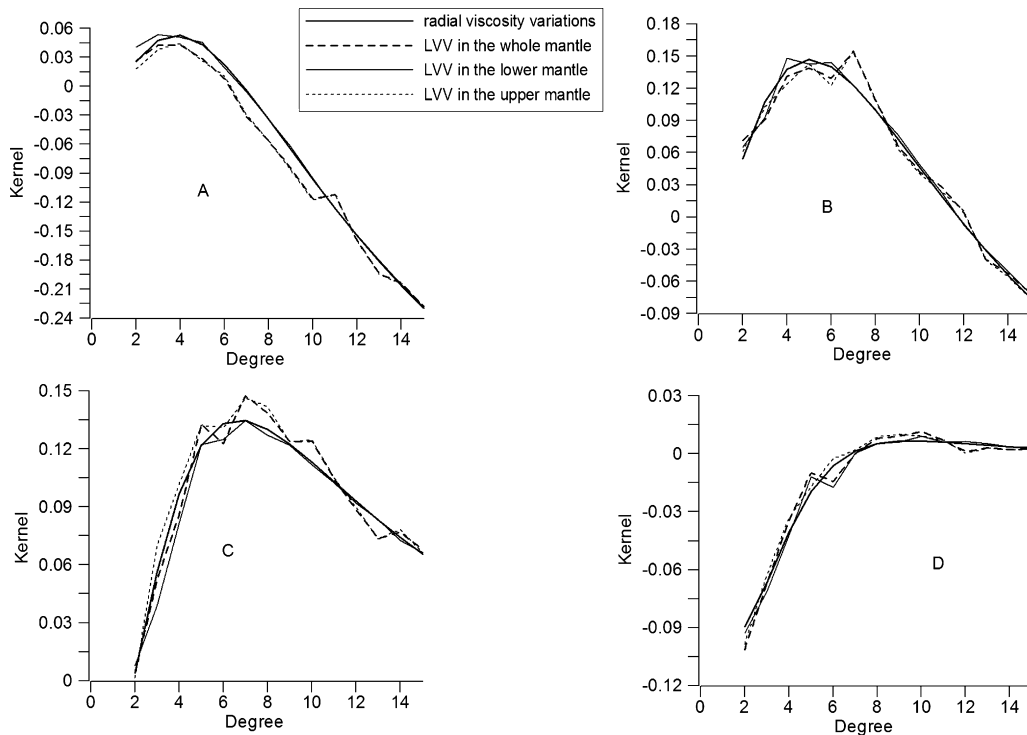


Fig. 7. Geoid kernels for the initial radially stratified model and experimental kernels for the LVV models. The LVV kernels have been calculated directly from a geoid response to internal load. The kernels are calculated for depths of 200 km (a), 400 km (b), 800 km (c) and 1500 km (d). Three LVV models are considered: whole mantle LVV, LVV in the upper mantle above 400 km, LVV in the lower mantle below 400 km.

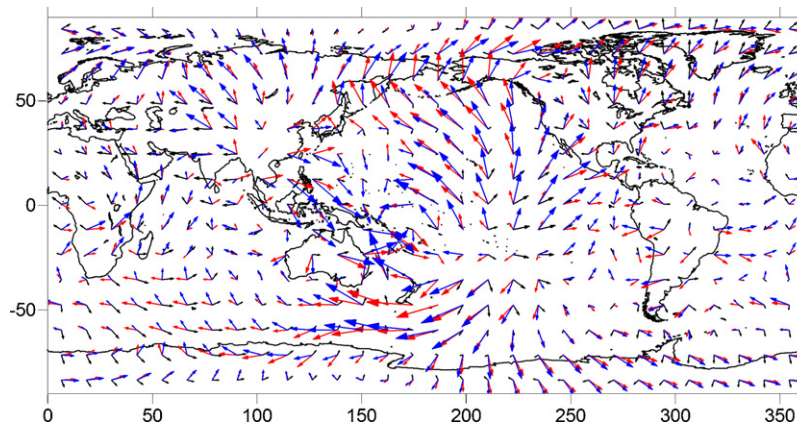


Fig. 8. Transformations (differences with the initial model) of near-surface mantle velocities caused by LVV. Three LVV models are considered: whole mantle LVV (blue), LVV in the upper mantle above 400 km (red), LVV in the lower mantle below 400 km (black). Maximal value is equal to 12.5 mm/year.

the Central Pacific are more than twice as big as in the case of LVV in the upper mantle only (Fig. 8). It is important that on the contrary to the dynamic geoid the effects of the upper and lower mantle LVV on horizontal near-surface velocities are not supplementary; generally the total change exceeds substantially the sum of the effects computed separately. The latter statement is brightly illustrated by the velocity patterns in the Indian Ocean.

It is important to note that the calculated differences in the near-surface mantle flow velocities might not completely correspond to plate velocities because our model does not imply stiff lithospheric plates, which integrate these differences over large areas. Despite individual velocity vectors could be changed due to different boundary conditions, a substantial difference between the lower and upper mantle effects should be still significant. This is evident when we compare the velocity patterns, e.g. for North America: the velocity triplets are rotated but a qualitative difference among different vectors remains similar (Fig. 8).

5. Discussion and conclusions

The main goal of the present paper is to estimate a possible effect of LVV in the whole mantle on the dynamic geoid and mantle velocities. It was demonstrated in the previous studies that large LVV in the upper mantle, primarily associated with cold lithospheric keels and asthenosphere, can substantially change the dynamic geoid and mantle flow velocities induced by mantle density inhomogeneities (e.g. Karpychev and Fleitout, 2000). A possible effect of the lower mantle LVV was considered as insignificant, at least for the largest wavelengths, probably because their amplitude is likely to be less than the difference between the continental roots and hot regions in the upper mantle (Zhang and Christensen, 1993). We have analysed a possible impact of the whole mantle LVV based on complete resolution tomography data.

Using the method of Paulson et al. (2005) we have constructed a 3D mantle viscosity model based on the global seismic tomography model of Ekstrom and Dziewonski (1998) converted to temperatures. Lateral viscosity variations have been calculated according to the temperatures and based on assumptions of depth-dependent homologous temperature. The obtained LVV in the lower mantle are less than those in the subcrustal layer (the total difference is a bit less than two orders of magnitude, Fig. 2), however some anomalous zones in the lower mantle are much bigger in size, than strong but thin LVV in the subcrustal layer. Several large viscosity anomalies extend for more than half of the lower mantle thickness (Fig. 2). Hence their integral effect might be also substantial for global dynamic modelling.

Employing even a simple three-layer radial viscosity profile we are able to explain most of the observed geoid energy (Fig. 3). Although long-wavelength features are fitted reasonably well, there are many mid- and small-scale features, which are different in the model and in the observed fields in Fig. 3. This is especially evident after the removal of the terms C_{20} and C_{40} dominating in the long-wavelength non-hydrostatic geoid. Implementation of the whole mantle 3D viscosity variations apparently improves the model geoid, even without any additional tuning of model parameters.

The difference between the geoid for a radially stratified model and for the model with the whole mantle LVV are as large as -27 to $+19$ m, its value is approximately one-third of the amplitude of the observed geoid anomalies. The effect should be considered as principal, especially taking into account that the employed viscosity model represents perhaps a lower limit for possible viscosity variations. Zhang and Christensen (1993) showed that the impact of LVV might be remarkably increased by insertion of additional vertical viscosity contrasts. Therefore, addition of more viscosity contrasts to the generalized three-layer model will also increase the effect of LVV.

In contrast to geoid, the differences in dynamic topography induced by the upper mantle LVV are larger in amplitude than the changes induced by the lower-mantle LVV (correspondingly: $-0.7 \div +0.4$ and $-0.3 \div +0.2$ km for the density 1 g/cm^3). The higher amplitudes of the changes induced by the upper mantle are exposed in relatively small-scale details, therefore they are more important for regional modelling.

The changes of the dynamic geoid lead to changes in the resulting velocity-to-density scaling factor, which provides the best fit of the model and observed geoid. The differences reach 30% for the lower mantle that can be principal for mineral physics applications.

Amplitudes of the geoid disturbances induced by the lower- and upper-mantle LVV are approximately the same, however the effect of the lower mantle is chiefly exposed on mid-range wavelengths ($l \geq 4$). We have also found that the effects of the upper and lower mantle on the observed geoid are near complementary with respect to the effect of the whole mantle LVV. This conclusion will be checked for various viscosity models. If it is correct, it provides a possibility to work separately with lower and upper mantle LVV, e.g. employing that technique, which is the most effective for each specific case.

The effect of LVV in the whole mantle on near-surface horizontal flow velocities is also found to be significant: the difference with the initial model reaches 12 mm/year. In particularly, the differences relate to toroidal flows that do not exist in the initial model. By contrast to geoid anomalies controlled by vertical flows, the differences of the horizontal flows induced by the lower and upper mantle LVV are essentially non-complementary (Fig. 8). A joint effect of the mantle layers is normally much stronger than the separate effects of the lower and upper mantle LVV.

Despite the considered 3D viscosity model is likely to be oversimplified, we have found that the resulting effect of the whole mantle LVV is significant. The LVV remarkably affect both the dynamic geoid and near-surface flow velocities, being the main parameters currently used to constrain dynamic models. Thus, we may conclude that implementation of whole mantle LVV in the next generation global dynamic models is a vital task.

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Appendix A

Convective flows and geo-potential are governed by the equations expressing conservation of mass (continuity), momentum, energy and gravitational field flux (Poisson's equation). For a spherical Earth model with radius r_0 , it is convenient to solve the set of equations by expanding all the variables in terms of generalized spherical harmonics for degree and order l, m (e.g. Hager and O'Connell, 1981; Corrieu et al., 1995):

$$r \frac{dZ_1^{lm}}{dr} = -(2+k)Z_1^{lm} + LZ_2^{lm} + A^{lm}, \quad (\text{A1})$$

$$r \frac{dZ_2^{lm}}{dr} = -Z_1^{lm} + Z_2^{lm} + Z_4^{lm} \eta^{-1} + B^{lm}, \quad (\text{A2})$$

$$r \frac{dZ_3^{lm}}{dr} = (12+4k)\eta Z_1^{lm} - 6L\eta Z_2^{lm} + Z_3^{lm} + LZ_4^{lm} + k\rho Z_5^{lm} + r^2 \delta \rho^{lm} g_0 + C^{lm}, \quad (\text{A3})$$

$$r \frac{dZ_4^{lm}}{dr} = -(6+2k)\eta Z_1^{lm} + 2(2L-1)\eta Z_2^{lm} - Z_3^{lm} - 2Z_4^{lm} + D^{lm}, \quad (\text{A4})$$

$$r \frac{dZ_5^{lm}}{dr} = Z_5^{lm} + Z_6^{lm}, \quad (\text{A5})$$

$$r \frac{dZ_6^{lm}}{dr} = L Z_5^{lm} - 4\pi G r^3 \delta\rho^{lm}, \quad (\text{A6})$$

$$r \frac{dZ_7^{lm}}{dr} = Z_7^{lm} + Z_8^{lm} \eta^{-1} + E^{lm}, \quad (\text{A7})$$

$$r \frac{dZ_8^{lm}}{dr} = (L - 2)\eta Z_7^{lm} - 2Z_6^{lm} + F^{lm}, \quad (\text{A8})$$

where $L = l(l+1)$, $\delta\rho^{lm} = \delta\rho^{lm}(r)$ a density anomaly, $\eta = \eta(r)$ a radial viscosity profile, $\rho = \rho(r)$ a depth-dependent density (radial function), $g_0 = g_0(r)$ is the unperturbed gravity, G is the gravitational constant.

The Eq. (A1) is derived from the continuity equation, the Eqs. (A2) and (A7) come from the constitutive relationship between stress and strain rate. The Eqs. (A3), (A4), (A8) are obtained from three components of the momentum equation. Eqs. (A5) and (A6) are evident from Poisson's equation. The Eqs. (A7) and (A8) are responsible for the toroidal velocity and stress components and do not appear in the case of radially dependent viscosity.

Components of velocity, stress, gravity and geoid are related to the variables in (A1)–(A8). Variable $Z_1^{lm}(r)$ represents the dimensionless radial velocity for each spherical harmonic mode, variables $Z_2^{lm}(r)$ and $Z_7^{lm}(r)$ are accordingly dimensionless spheroidal and toroidal velocity components, variable $Z_3^{lm}(r)$ is the dimensionless radial stress, variables $Z_4^{lm}(r)$ and $Z_8^{lm}(r)$ represent the dimensionless spheroidal and toroidal shear stress components and variables $Z_5^{lm}(r)$ and $Z_6^{lm}(r)$ are the dimensionless geoid and gravity field.

In the case of only radial viscosity variations terms A – F are equal to zero and the system (A1)–(A6) can be solved in a standard way separately for each spherical harmonic coefficient (e.g. Hager and O'Connell, 1981; Corrieu et al., 1994). For both surface and core-mantle interfaces we use the standard free-slip boundary conditions.

The terms $(A$ – $F)$, which appear in the case of LVV, are the viscous load terms consisting of non-linear products of derivatives of viscosity and derivatives of velocities. The non-linear coupling of various spherical harmonic modes is estimated by an iterative method. The above terms are calculated from the flow velocities obtained in the previous step and this procedure is repeated until convergence is achieved (Zhang and Christensen, 1993).

As the convergence parameters, we use average geoid and velocity magnitudes estimated by the following formulae in the selected area with maximal LVV (70°N–70°S; 80°E–100°E):

$$\frac{1}{N_\theta N_\varphi} \left[\sum_{\theta_1 < \theta < \theta_2} \sum_{\varphi_1 < \varphi < \varphi_2} |\delta N^i| - \sum_{\theta_1 < \theta < \theta_2} \sum_{\varphi_1 < \varphi < \varphi_2} |\delta N^{i-1}| \right] \leq \varepsilon_1, \quad (\text{A17})$$

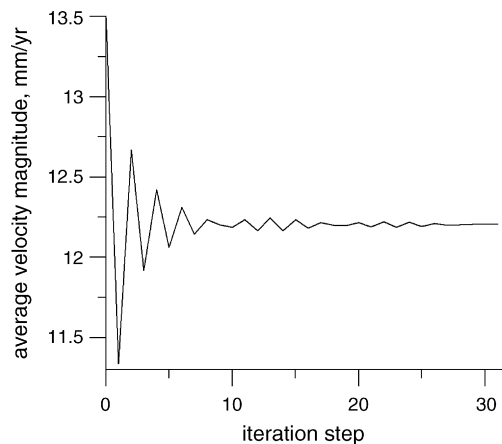


Fig. 9. (A1) Convergence of the iteration process for the whole mantle LVV. The average magnitude of velocities in the selected area with maximal LVV (70°N–70°S; 80°E–100°E) is shown.

$$\begin{aligned}
& \frac{1}{N_r N_\theta N_\varphi} \sum_{r_1 < r < r_2} \sum_{\theta_1 < \theta < \theta_2} \sum_{\varphi_1 < \varphi < \varphi_2} \sqrt{(V_r^i)^2 + (V_\theta^i)^2 + (V_\varphi^i)^2} \\
& - \frac{1}{N_r N_\theta N_\varphi} \sum_{r_1 < r < r_2} \sum_{\theta_1 < \theta < \theta_2} \sum_{\varphi_1 < \varphi < \varphi_2} \sqrt{(V_r^{i-1})^2 + (V_\theta^{i-1})^2 + (V_\varphi^{i-1})^2} \leq \varepsilon_2,
\end{aligned} \tag{A18}$$

where i is a current step number; $N_r N_\theta N_\varphi$ is a cardinal number of a selected area. The convergence is achieved as soon as (A17) and (A18) get less than the predetermined ε_1 and ε_2 . In Fig. 9 (A1) we demonstrate one example of the convergence process for the whole mantle LVV.

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