



available at www.sciencedirect.com



journal homepage: www.elsevier.com/locate/jhydrol



Estimation of the water table throughout a catchment using self-potential and piezometric data in a Bayesian framework

N. Linde ^{a,*}, A. Revil ^a, A. Bolève ^a, C. Dagès ^c, J. Castermant ^{a,b},
B. Suski ^a, M. Voltz ^c

^a CNRS-CEREGE, Europôle de l'Arbois, Université Paul Cézanne, BP 80, 13545 Aix-en-Provence, Cedex 4, France

^b INRA, Unité de Recherche de Géochimie des Sols et des Eaux, BP 80, 13545 Aix-en-Provence, Cedex 4, France

^c INRA, Laboratoire d'Etude des Interactions entre Sol, Agrosystème et Hydrosystème, UMR Agrom-INRA-IRD, Montpellier, France

Received 15 March 2006; received in revised form 4 August 2006; accepted 30 September 2006

KEYWORDS

Self-potential;
Bayesian methods;
Data fusion;
Water table;
Catchment
characterization

Summary Information about spatial variations of the water table that occur throughout catchments is useful to infer large scale flow patterns, but conventional mapping using piezometric data is invasive, slow, and expensive. Water flow in the subsurface generates an electrical current called the streaming current. The resulting self-potential (SP) (electrostatic) signals can be measured non-intrusively, quickly, and inexpensively at the ground surface. We considered two conceptual models to relate SP signals to the water table. The “infiltration model” relates SP signals to the thickness of the vadose zone, while the “water table model” relates SP signals to the distribution of the water table in unconfined aquifers. These models are first calibrated against field data before a Bayesian method is applied to update a kriged map of the water table obtained from piezometric observations using a kriged SP map. The estimated water tables based on the two conceptual models were combined into one final model using concepts from Bayesian Model Averaging. The method was applied to a small agricultural catchment ($\sim 1 \text{ km}^2$) in southern France. The posterior water table estimates were improved mainly on the slopes surrounding the basin. Understanding the variations in the water table on these slopes is important because infiltration in these areas feed the basin with groundwater flow. The Bayesian framework is useful to avoid overconfident predictions when using SP data in hydrogeological estimation because it provides realistic uncertainty estimates. © 2006 Elsevier B.V. All rights reserved.

* Corresponding author. Tel.: +33 4 42 97 17 84; fax: +33 4 42 97 17 85.
E-mail address: linde@cerege.fr (N. Linde).

Introduction

Piezometric data are typically scarce and additional data sources, like non-intrusive geophysical measurements, that provide cheap and reliable information about the water table are in demand (e.g., McLaughlin and Townley, 1996). This is actually one of the five community objectives identified by the International Association of Hydrological Sciences (IAHS) decade on Predictions in Ungauged Basins (PUB), namely “Demonstrate the value of data for hydrological data acquisitions, including alternative data sources, by quantifying the links between data and predictive uncertainty (IAHS, 2003)”. In this study, we estimated the position of the water table and the uncertainty associated with this estimate over a catchment area by combining self-potential (SP) and piezometric data.

In the last decades, new quantitative methods using SP signals have been developed to solve different problems in environmental geosciences. Examples include: the determination of the shape of contaminant plumes (e.g., Naudet et al., 2004); the geometry of ground water flow (e.g., Titov et al., 2000); the interpretation of pumping tests (Rizzo et al., 2004); the determination of the geometry of buried paleochannels (e.g., Revil et al., 2005b); studies of landslides (Perrone et al., 2004); ground water flow in active volcanoes (e.g., Finizola et al., 2002); and glaciology (Kulesa et al., 2003).

The self-potential or spontaneous polarization (SP) method is a passive geophysical method that measures the electrical potential distribution at the ground surface (and possibly in boreholes). The mapping of self-potentials is usually carried out with two non-polarisable electrodes; one is used as a reference, while the other is used to scan variations in the electrical potential at the ground surface. The recorded signals are the superposition of *in situ* signals and telluric currents, which correspond to the main source of noise in addition to cultural activity (50 Hz and harmonics). New methodologies for the acquisition of field data have been developed to improve the signal-to-noise ratio of the recorded data (e.g., Perrier and Raj Pant, 2005). Once the noise is removed, there are two main sources of signals. One source of SP signals is associated with ground water flow through the so-called streaming potential or hydroelectric coupling (e.g., Revil et al., 2005a). The other source is associated with redox potentials (e.g., Naudet et al., 2004).

The streaming potential dominates over the redox potential in most environments that are not heavily contaminated or contain ore bodies. The microscopic origin of the streaming potential signals lies in ionic processes occurring in the vicinity of the surface of minerals in contact with water. Indeed, the electrical double layer coating the surface of minerals implies the existence of a net excess of electrical charge in the pore water (e.g., Revil et al., 2005a). The drag of this-typically positive-excess of charge by the flow of the ground water creates a polarisation of charge at the pore-scale, i.e., the streaming potential, which is one of the cross-coupling effects existing in charged porous materials (Revil et al., 2005a). When redox potentials and man-made signals can be neglected or eliminated, the distribution of the measured electrical potentials provides an electrical

signature of water flow in the sub-surface (e.g., Fournier, 1989; Aubert and Yéné Atangana, 1996).

In this work, we used the first-order approximations of two conceptual models that relate SP signals and the water table. Aubert and Yéné Atangana (1996) assumed that SP signals are mainly due to infiltration of water through the vadose zone. Fournier (1989) and Revil et al. (2003) showed that the variation of the streaming potential coupling coefficient through the water table is also responsible for self-potential signals.

The objectives of this work were: (1) to illustrate that SP data provide an inexpensive and useful data source in hydrological studies at the catchment scale; (2) to apply a Bayesian estimation method, which uses geostatistical techniques, to integrate SP and piezometric data in order to estimate the water table throughout a catchment. Even though SP data have been used in hydrogeological studies (e.g., Fournier, 1989; Aubert and Yéné Atangana, 1996; Revil et al., 2002, 2003, 2005b; Zhang and Aubert, 2003; Rizzo et al., 2004), there have been no attempts: (1) to use a Bayesian estimation method to estimate the elevation of the water table by integrating SP and piezometric data; (2) to estimate prediction uncertainties arising from measurement errors, sparse sampling of data, and uncertainties about the relationship between SP signals and the elevation of the water table; and consequently (3) to use SP and piezometric data to map the water table throughout a catchment.

Self-potential signals and their relation to the water table

In the last two decades, two potential sources of SP signals associated with ground water flow have been identified. The first conceptual model corresponds to slow infiltration of water through the vadose zone (Zablocki, 1978; Jackson and Kauahikaua, 1987; Aubert and Yéné Atangana, 1996). These authors assumed that SP signals are mainly related to the distance along which water percolates vertically through the vadose zone before reaching the water table. This model will be referred to in the text as the infiltration model. The second conceptual model is related to ground water flow in unconfined aquifers (Fournier, 1989; Revil et al., 2002; Revil et al., 2003; Rizzo et al., 2004). According to this model, the strengths of the observed SP signals depend on the contrast of the streaming potential coupling coefficient through the water table (Fournier, 1989) and/or on the contrast of electrical resistivity through the water table (Revil et al., 2003). This model will be referred to in the text as the water table model.

The infiltration model

Given that the vadose zone is uniformly polarized, where each point in space act as a vertically oriented dipole, and that the electrical conductivity distribution in the vadose zone is homogeneous, the resulting self-potential is proportional to the thickness of the vadose zone (Zablocki, 1978; Jackson and Kauahikaua, 1987). Zhang and Aubert (2003) demonstrated that, to the first-order, the following simple

formula can be used to relate SP signals to the thickness of the vadose zone,

$$\varphi(x, y, z) - \varphi_0(x_0, y_0, z_0) = c_V \{ [z(x, y) - h(x, y)] - [z_0(x_0, y_0) - h_0(x_0, y_0)] \}, \quad (1)$$

where $\varphi(x, y, z)$ is the electrical potential at a SP station located at the ground surface; c_V is an apparent streaming potential coupling coefficient; $z(x, y)$ and $z_0(x_0, y_0)$ are the elevation of the self-potential station and the reference station, respectively; $h(x, y)$ and $h_0(x_0, y_0)$ are the corresponding elevations of the groundwater table. This model predicts negative self-potentials at the ground surface if the self-potential reference station is located at the water table. The predicted elevation of the water table (or the elevation of the first impermeable formation) can be obtained from Eq. (1) (Aubert and Y  n   Atangana, 1996) as

$$h(x, y) = z(x, y) - [z_0(x_0, y_0) - h_0(x_0, y_0)] - [\varphi(x, y, z) - \varphi_0(x_0, y_0, z_0)] / c_V. \quad (2)$$

Valley-like depressions of the predicted elevations correspond to preferential ground water flow pathways, whereas ridges correspond to water divides. The infiltration model has been successfully applied to several case studies (e.g., Aubert and Y  n   Atangana, 1996; Zhang and Aubert, 2003) in which the shape of the piezometric surface was confirmed by borehole data. However, Fournier (1989) described several field observations that cannot be correctly explained by Eq. (2).

The streaming potential coupling coefficient can be measured in the laboratory (e.g., Revil et al., 2005a and references therein), but its dependence on water saturation is not well understood (Revil and Cerepi, 2004). The apparent streaming potential coupling coefficient c_V in Eqs. (1) and (2) is in field applications best determined by calibrating the measured SP data against piezometric data.

The water table model

According to the water table model, SP signals originate from contrasts in the coupling coefficient across the water table (e.g., Fournier, 1989; Revil et al., 2003). A first order approximation that relates SP signals to the water table is (Revil et al., 2003),

$$h(x, y) = h_0(x, y) + [\varphi(x, y, z) - \varphi_0(x, y, z)] / c_W, \quad (3)$$

where $c_W = (C_2 - C_1)$ is an apparent streaming potential coefficient, where C_1 and C_2 are the streaming potential coupling coefficients in the vadose zone and the saturated zone, respectively. Consequently, according to the water table model, the self-potential is linearly related to the piezometric head. In the case where there are capillary fringes, it is possible that there is an insignificant contrast of the coupling coefficient between the aquifer and the vadose zone, so $c_W \approx 0$.

Bayesian model

In Section "Self-potential signals and their relation to the water table", we have reviewed two alternative conceptual models relating the shape of the water table to self-poten-

tial data (Eqs. (2) and (3)). In this section, we describe how to use these conceptual models to update prior estimates of the water table obtained from kriged piezometric data. The resulting models provide uncertainty estimates that incorporate the number of data points, the measurement errors, the accuracy of the conceptual models, as well as the consistency between different conceptual models.

Bayes' theorem is an appealing framework for integration of different data types and a priori information (e.g., Rubin, 2003). A Bayesian formulation of data integration is more general than co-kriging (Deutsch and Journ  l, 1998) because a Bayesian formulation allows arbitrary prior distributions and non-linear relationships between primary and secondary variables. Chen et al. (2001) combined collocated geophysical estimates of ground penetrating radar (GPR) velocity, GPR attenuation, and seismic velocity with permeability estimates from flowmeter data to estimate the likelihood of observing a given permeability value given known values of the three geophysical data types. The associated likelihood functions were determined with a normal linear regression method. The prior was a kriged estimate of the permeability structure based on flowmeter data, where the correlation structure had been estimated using available geophysical models and permeability data. The method of Chen et al. (2001) was used to estimate the permeability structure at the South Oyster Bacterial Transport Site, Virginia (Hubbard et al., 2001). Scheibe and Chien (2003) modeled the flow and transport of tracer test data from this fairly homogeneous site and showed that the permeability structure determined by Hubbard et al. (2001) significantly improved transport predictions compared to estimates based on flowmeter data only.

We adapted the method of Chen et al. (2001) to the problem of estimating the geometry of the water table given piezometric and self-potential data by incorporating: (1) likelihood functions that have space-varying variance; (2) likelihood functions where the parameter structure is based on conceptual models (Eqs. (2) and (3)); (3) geophysical data (i.e., the SP data) that are kriged (not inverted) to ensure estimates at all locations of interest; (4) concepts from Bayesian Model Averaging (BMA) to include the uncertainty arising from different possible conceptual models.

When different models are consistent with the available data, it may not be justifiable to rely on a single model. Instead, it might be better to weight the predictions of the various models with weights based on estimated model probabilities. We used concepts from BMA to condition the posterior probabilities to the two conceptual model types $\mathbf{M} = (M_1, M_2)$, where M_1 corresponds to the infiltration model and M_2 corresponds to the water table model.

The estimation method is described in detail in the following section.

Bayesian model

An estimate of the water table $h(x, y)$ using model M_k can be obtained with Bayes theorem as follows:

$$p[h(x, y) | \varphi(x, y), M_k] = A L[\varphi(x, y) | h(x, y), M_k] p[h(x, y)], \quad (4)$$

where A is a normalizing coefficient, $\varphi(x, y)$ is obtained by kriging the SP data, $L[\varphi(x, y) | h(x, y), M_k]$ is the likelihood of observing $\varphi(x, y)$ given $h(x, y)$ and M_k , $p[h(x, y) | \varphi(x, y), M_k]$

and $p[h(x, y)]$ are the posterior and prior probability density functions (pdfs) of the water table, respectively. It should be noted that only collocated estimates are used in Eq. (4), which simplifies the computations without significantly affecting the resulting models.

The prior pdf $p[h(x, y)]$ is obtained by kriging the available piezometric data. The likelihood functions $L[\varphi(x, y)|h(x, y), M_k]$ are first estimated at the locations where we have collocated SP data φ_{col} and piezometric data h_{col} , where the parameters c_v (Eq. (2)) and c_w (Eq. (3)) can be estimated using linear regression. The likelihood functions are assumed to have a Gaussian distribution where the estimation variances $(\sigma_k^{reg})^2$ from regressions of the collocated data are used to estimate the variance of the likelihood functions at the collocated locations. The variance of the likelihood functions at other locations $(\sigma_k^{est}(x, y))^2$ are estimated by adding an additional error term that reflect the uncertainty associated with the estimation variance of the kriged SP estimates away from the SP stations, as follows:

$$(\sigma_k^{est}(x, y))^2 = (\sigma_k^{reg})^2 + (\sigma_\varphi^{krige}(x, y))^2 - C_0, \quad (5)$$

where $(\sigma_\varphi^{krige}(x, y))^2$ is the estimation variance obtained from ordinary kriging of the SP data and C_0 is the corresponding nugget (i.e., the uncorrelated contribution to the total kriging variance). The contribution of C_0 is already incorporated into $(\sigma_k^{reg})^2$ because the collocated SP and piezometric data are in practice separated by a short distance. Eq. (4) is solved numerically by discretization.

The posterior probability that a model M_k is correct is estimated from Bayes theorem as follows:

$$p[M_k|\varphi_{col}] = A_1 L[\varphi_{col}|M_k] p[M_k], \quad (6)$$

where A_1 is a normalizing coefficient, $p[M_k]$ is the prior probability that M_k is the correct model type and the likelihood function $L[\varphi_{col}|M_k]$ describes the likelihood that the observed collocated SP data φ_{col} are observed given that model type M_k is the correct model. It is assumed that the data errors in φ_{col} are normally distributed with a variance that corresponds to the nugget C_0 of the kriged SP data. The likelihood function $L[\varphi_{col}|M_k]$ is given by

$$L[\varphi_{col}|M_k] = \frac{1}{\sqrt{2\pi C_0}} \exp \left[-\frac{\sigma_{col}^2}{2C_0} \right] \quad (7)$$

where σ_{col}^2 is the variance of the differences between the predicted and measured SP data at the collocated locations and where we have assumed that the errors in the head observations are negligible. In Bayesian Model Averaging, the likelihood is evaluated for all possible sets of model parameters that the parameterization of the model type can take (Draper, 1995), however, we only calculate the likelihood function (Eq. (7)) for the least-squares estimates of model M_k .

The sum of the prior probabilities $p(M_k)$ is one:

$$\sum_{k=1}^K p(M_k) = 1. \quad (8)$$

We have no strong evidence a priori about which model type is preferable and we make the neutral choice that all models are a priori equally likely (Draper, 1995):

$$p(M_1) = p(M_2) = 1/2. \quad (9)$$

The posterior mean $E[h(x, y)|\varphi(x, y)]$ and variance $\text{var}[h(x, y)|\varphi(x, y)]$ of the BMA model are (Draper, 1995),

$$E[h(x, y)|\varphi(x, y)] = \sum_{k=1}^K E[h(x, y)|\varphi(x, y), M_k] p(M_k|\varphi_{col}), \quad (10)$$

$$\begin{aligned} \text{var}[h(x, y)|\varphi(x, y)] &= \sum_{k=1}^K \text{var}[h(x, y)|\varphi(x, y), M_k] p(M_k|\varphi_{col}) \\ &\quad + \sum_{k=1}^K \{E[h(x, y)|\varphi(x, y), M_k] \\ &\quad - E[h(x, y)|\varphi(x, y)]\}^2 p(M_k|\varphi_{col}). \end{aligned} \quad (11)$$

The posterior mean is simply a weighted mean of the different model predictions, but the posterior variance includes an additional term that quantifies the deviations of the individual models from the posterior mean.

Application to the Roujan catchment

Study site

The Roujan catchment is located in southern France (Fig. 1). It covers a surface area of approximately 91 ha. This catchment was chosen because it is a test site where research on vadose zone and ground water flow has been performed over the last fifteen years (e.g., Voltz et al., 1997; Ribolzi et al., 2000; Louchart et al., 2001). It belongs to the Long Term Environmental Research Laboratory OMERE (Observatoire Méditerranéen de l'Environnement rural et de l'Eau) (Voltz and Albergel, 2002). The site has been strongly altered by human activity, with terraced slopes and a major network of ditches collecting the runoff water. Its land use consists mainly of vineyards and shrubs. In geological terms, the substrata derive from marine, lacustrine or fluvial sediments. The soils of the catchment range from clayey to loamy. Most parts of the basin exhibit water tables whose depth fluctuates largely both in space and time. The vadose zone can reach thicknesses of more than 10 m. The Miocene marine deposits form a layer of marls which can be considered impervious and corresponds to the lower boundary of most groundwaters of the catchment. Presently, 28 piezometers have been installed and the piezometric levels are recorded every week. The electrical conductivity of the ground water varies between 0 and 0.30 S/m (at 25 °C). The soil is quite rich in clay minerals and its cation exchange capacity varies between 5 and 17 meq/g. High resolution electrical resistivity tomographies (not shown here) show a mean resistivity of $20 \pm 5 \Omega\text{m}$ with no resistivity contrasts between the vadose zone and the unconfined aquifer in agreement with laboratory measurements (see Table 1 in Suski et al. (2006)). However, the electrical resistivity distribution in the first 50 cm of the soil appears to be highly heterogeneous during the summer.

The self-potential survey

The SP survey was conducted during two days in June 2005 using Metrix MX-20 multimeters and Cu/CuSO₄ non-polarisable electrodes.

In order to get some understanding of the scatter of the data due to near-surface effects, we first performed ten to

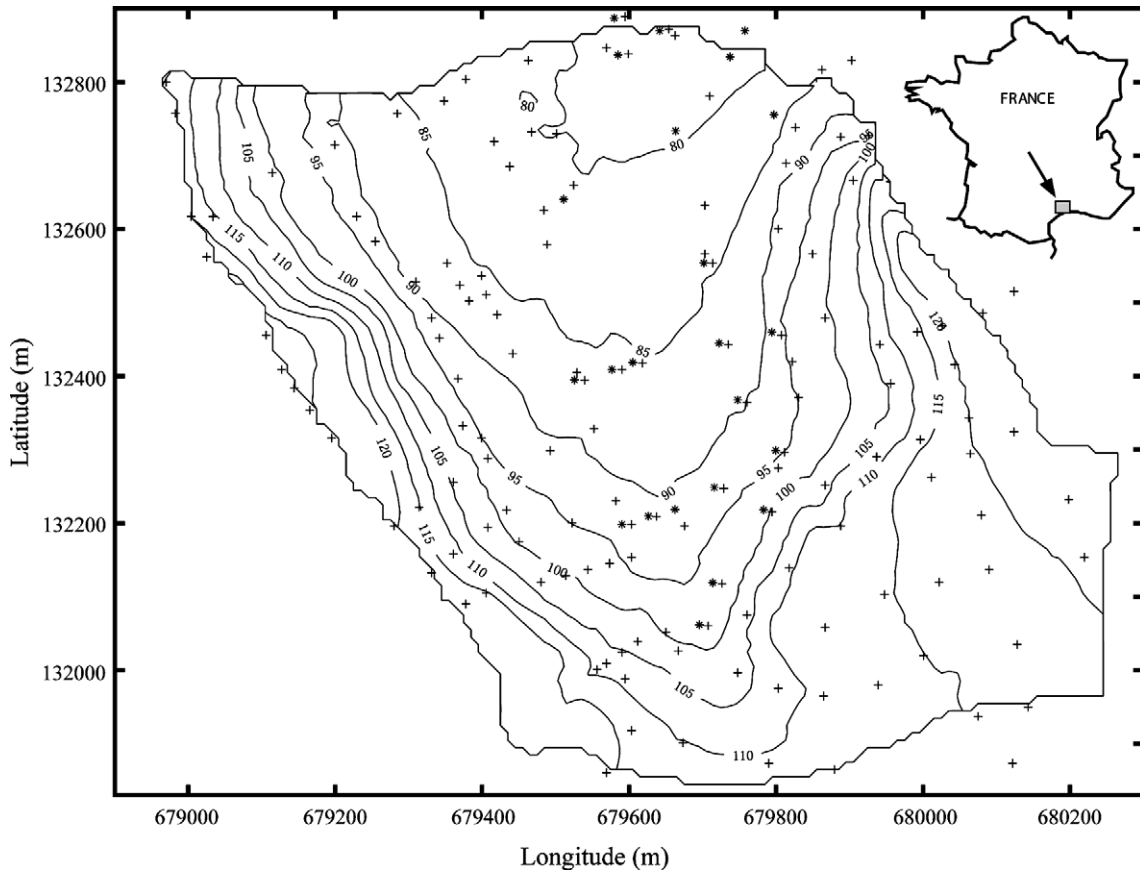


Figure 1 Elevation map with positions of piezometers (*) and SP stations (+). The location of Roujan is indicated in the upper right part of the figure.

thirteen self-potential measurements around each piezometer. We determined the mean and then we removed the mean from the data set. Finally, we summed up all the mea-

surements (286 measurements in total) and averaged the data per classes. The result is reported Fig. 2. It supports the idea that the distribution of the self-potential measurements around a measurement station is described by a Gaussian probability distribution,

$$p(\varphi) = \frac{1}{(2\pi)^{1/2} \sigma_\varphi} \exp \left[-\frac{(\varphi - \bar{\varphi})^2}{2\sigma_\varphi^2} \right], \quad (12)$$

where $\bar{\varphi}$ is the mean of the distribution ($\bar{\varphi} = 0$ here) and σ_φ^2 is the associated variance. For the self-potential measurements made at Roujan, we obtained $\sigma_\varphi = 1.1$ mV (Fig. 2). We believe that this variability mainly originates from the heterogeneity of the electrical resistivity in the upper 50 cm of the ground.

Distribution of the coupling coefficient

We performed laboratory measurements of the streaming potential coupling coefficient for a set of samples extracted from the basin of Roujan. The samples were taken at different depths, sampling both the vadose zone and the aquifer. All the experiments were performed at saturation using water from the aquifer of Roujan (~ 0.1 S/m at 25 °C). The streaming potential coupling coefficient varies from -5.6 to -3.2 mV/m (see Table 1 in Suski et al. (2006)). These values are representative of field conditions because the

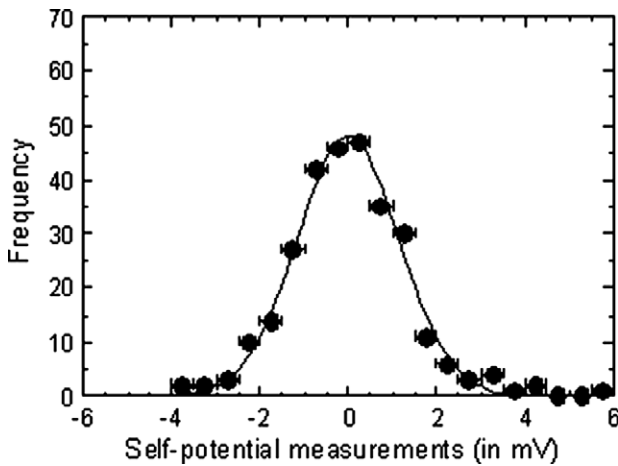


Figure 2 Probability distribution of the self-potential measurements. The measurement of the self-potential at a given station appears to be the realization of a random process given by a Gaussian distribution. A total of 286 measurements are used to determine this distribution.

streaming potential coupling coefficient is only very weakly dependent on temperature (e.g., see Fig. 9 in [Revil et al. \(2005a\)](#)).

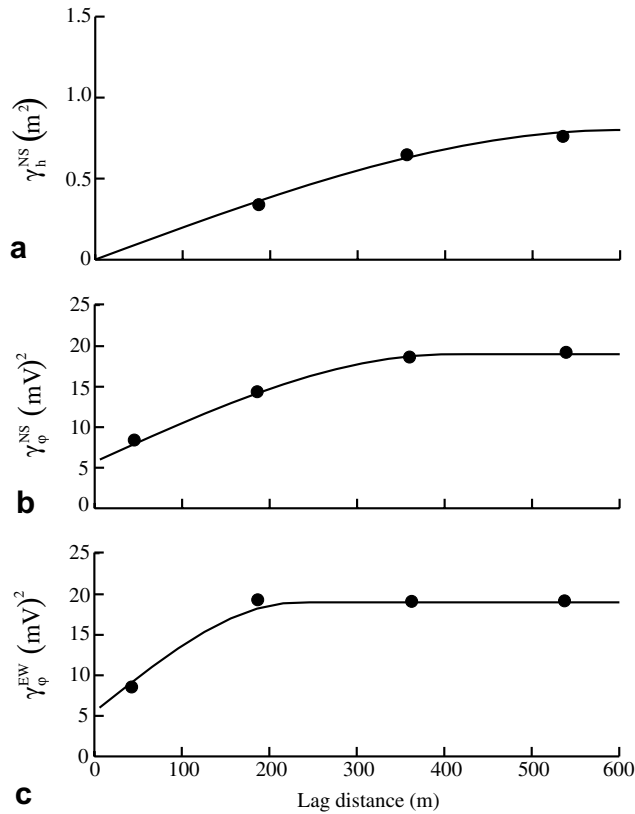


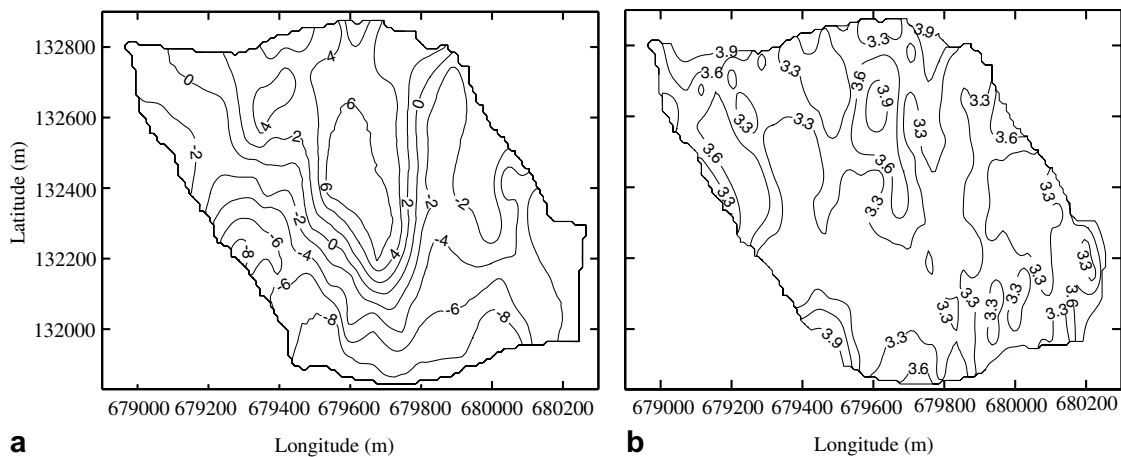
Figure 3 Experimental (filled circles) and theoretical (solid line) semi-variograms are shown for the detrended piezometric levels in the north–south direction (a) and for the detrended SP data in both the north–south (b) and the east–west (c) direction.

Kriging of piezometric and SP data

We mapped the self-potential signals over the entire catchment area with 140 SP stations, where four to six self-potential measurements have been performed around each SP station. The experimental and theoretical semi-variograms of the detrended piezometric data in the north–south direction, and the experimental and theoretical semi-variograms of the detrended SP data in the north–south (b) and east–west (c) directions are shown in [Fig. 3a–c](#), respectively. The trend model used has a cone shape with its apex at the northernmost piezometer (see [Fig. 1](#)). The slope of the trend model for the heads were found by linear regression and was 1.4%. An isotropic spherical semi-variogram model with a sill of $0.8 m^2$ and an effective range of 600 m was used to fit the residual piezometric data. The distribution of the piezometers (see [Fig. 1](#)) did not allow reliable estimates of the semi-variogram of the piezometric data in the east–west direction and the range was assumed to be the same as in the north–south direction.

The residual SP data were fitted with a nugget of $6 (mV)^2$ and a spherical semi-variogram model with an effective range of 400 m in the north–south direction and 175 m in the east–west direction.

Ordinary kriging ([Deutsch and Journal, 1998](#)) of the detrended SP and piezometric data were performed to obtain maps of the SP signals ([Fig. 4a](#)) and water table ([Fig. 5a](#)). The trend model is commonly assumed to be perfectly known and this is a good assumption in the north–south direction in the central part of the study area, but there is no reason to assume that the cone shape used to represent the head data is correct far away from available head data (i.e., on the western and eastern slopes surrounding the catchment). We believe that the choice of the type of trend model is rather arbitrary, for example, a linear trend model with only a north–south component would essentially provide the same size of the head residuals. We assumed that the difference between the trend model that assumed a cone shape and a linear trend model in the north–south direction corresponded to the largest possible error in the choice of the trend model. This difference was assumed



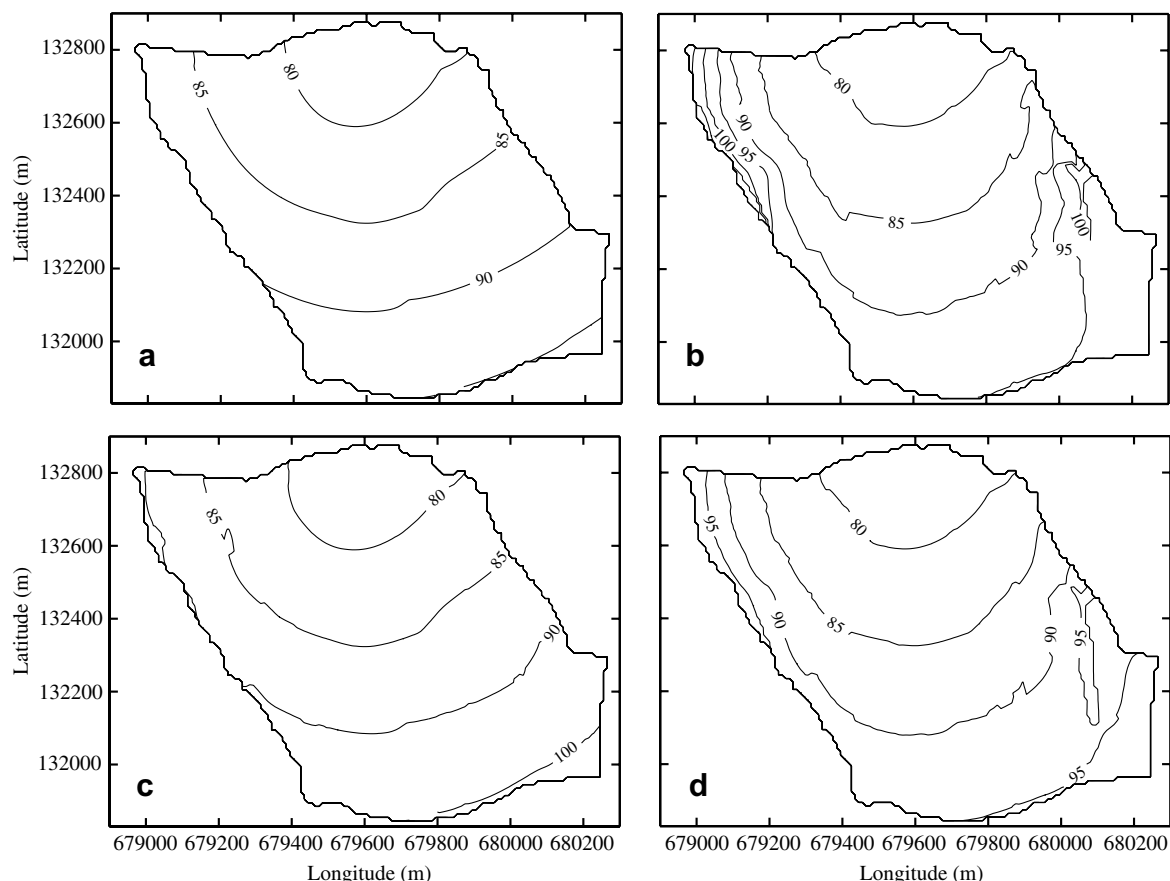


Figure 5 The prior model of hydraulic head (a) was updated with the infiltration model (b) and the water table model (c). The infiltration and water table models were averaged using the BMA approach (d).

to correspond to two standard deviations of the assumed normally distributed error of the cone shaped trend model. We can thus add an additional term to the kriging errors in order to provide error estimates that are more reasonable away from the available head data. This additional variance term is scaled by the kriging variance divided by the sill in order to keep the estimation errors low close to the piezometers. The corresponding kriging errors are shown in Figs. 4b and 6a, respectively.

Likelihood functions

The likelihood functions $L[\varphi(x, y)|h(x, y), M_k]$ (see Eq. (4)) were determined by linear regression. A scatter plot between the measured thickness of the vadose zone and the collocated kriged SP estimates (Fig. 7a) reveals a linear trend with a correlation coefficient of -0.86 , where c_v is -0.95 mV/m (see Eq. (2)). A scatter plot between the elevation of the water table and the collocated kriged SP estimates (Fig. 7b) reveals a weaker linear trend with a correlation coefficient of -0.58 , where c_w is -0.29 mV/m (see Eq. (3)). The apparent coupling coefficients c_v and c_w are unusually low. For example, Revil et al. (2003) and Naudet et al. (2004) reported values of c_w of -3.2 mV/m and -10.6 mV/m, respectively. The low values of c_v and c_w are also indicated by the low values of the streaming potential coupling coefficient C (see Table 1) with a mean value of

-4.3 mV/m, which is inversely related to the electrical conductivity of the pore water (Fig. 8). The low values of the apparent coupling coefficients are attributed both to the relatively high conductivity of the pore water (0.1 S/m), which results in a relatively low streaming potential coupling coefficient and to the low electrical conductivity of the aquifer materials (i.e., approximately 20 Ω m) that results in lower electrical potentials at the ground surface compared with more resistive aquifer materials.

The resistivity of the ground (vadose zone and aquifer), determined from a geoelectric survey, is rather uniform $\sim 20 \pm 5$ Ω m in agreement with the petrophysical measurements reported by Suski et al. (2006) in their Table 1. It follows that we can apply the Bayesian estimation method (Section "Bayesian model") to update the prior water table map with the SP map.

Estimation of the water table

For each model (Eqs. (2) and (3)) and location, we determine the position of the water table (Fig. 5b and c) and the uncertainty of the estimates using Eq. (4) (Fig. 6b and c). Finally, the posterior mean of the position of the water table incorporating both models is estimated from Eq. (10) (Fig. 5d). The updated models estimate steep gradients associated with the slopes of the catchment (see elevation map (Fig. 1)), gradients that were not resolved in the prior

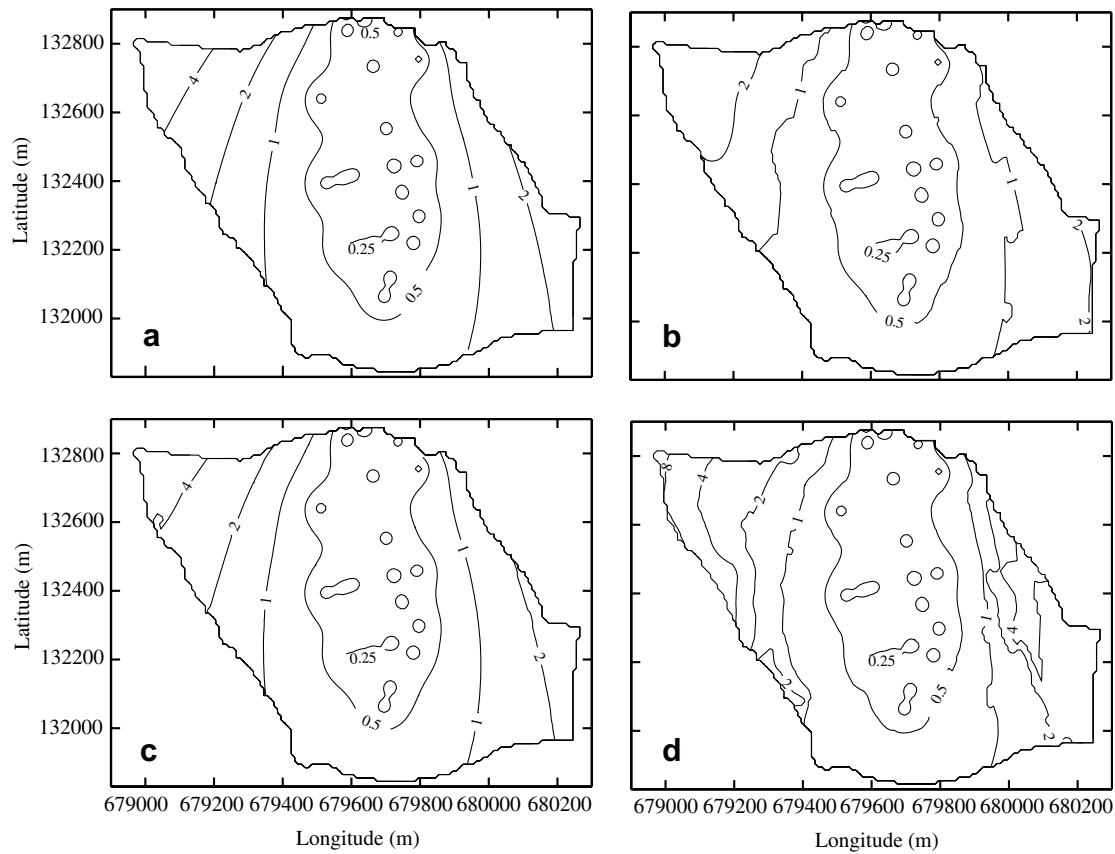


Figure 6 The kriging errors of the prior hydraulic head increase outside the basin where the piezometers are located. The estimation errors of the updated models based on the infiltration model (b) and the water table model (c) are lower compared with the prior model, especially outside the basin. The weighted estimate of the infiltration model and the water table model have larger uncertainty estimates (d) compared with the individual models because conceptual uncertainty is included.

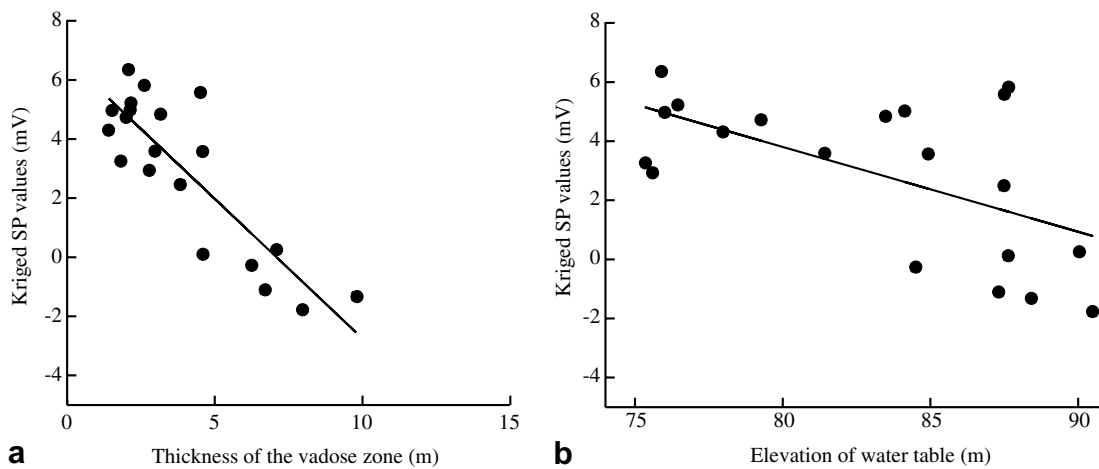


Figure 7 Relationship between the self-potential data measured in the field and the water table. (a) Correlation between the self-potential data (in mV) and the thickness of the vadose zone (in m) (correlation coefficient is -0.86). (b) Correlation between the self-potential data (in mV) and the piezometric heads (in meters above sea level) (correlation coefficient is -0.58).

model (Fig. 5a) because of the distribution of the piezometers. The estimated spatial variations in the water table are larger in the model based on the infiltration model compared with the model based on the water table model,

something that can be explained by the higher sensitivity of the self-potential data to changes in the thickness of the vadose zone (see Fig. 7a) compared with changes in the elevation of the water table (see Fig. 7b). The weighted

Table 1 Measurements of the streaming potential coupling coefficient C and electrical resistivity ρ

Samples	Depth interval (cm)	T (°C)	σ_f (25 °C) ($S\ m^{-1}$)	C (mV/m)	C (2) (mV/m)	ρ ($\Omega\ m$)
A0	80–100	24	0.115	-4.79 ± 1.38	-5.5 ± 1.4	16.83
A2	110–130	26	0.086	-4.24 ± 0.58	-3.6 ± 0.5	17.23
A3	140–160	25	0.117	-2.83 ± 0.06	-3.3 ± 0.1	19.31
A4	160–180	25	0.116	-4.82 ± 0.75	-5.6 ± 0.8	20.69
A5	180–190	26	0.116	-4.00 ± 0.14	-4.6 ± 0.1	18.77
A6	190–210	24	0.110	-2.93 ± 0.18	-3.2 ± 0.2	23.28

Measurements of C were obtained by imposing a known hydraulic gradient across the samples while measuring the resulting differences in electrical potential. Electrical resistivity was measured with the samples saturated by ground water from the study area. All the samples were extracted from a drillhole located in the vicinity of piezometer #6.

(1) Corrected at $\sigma_f = 0.100\ S\ m^{-1}$.

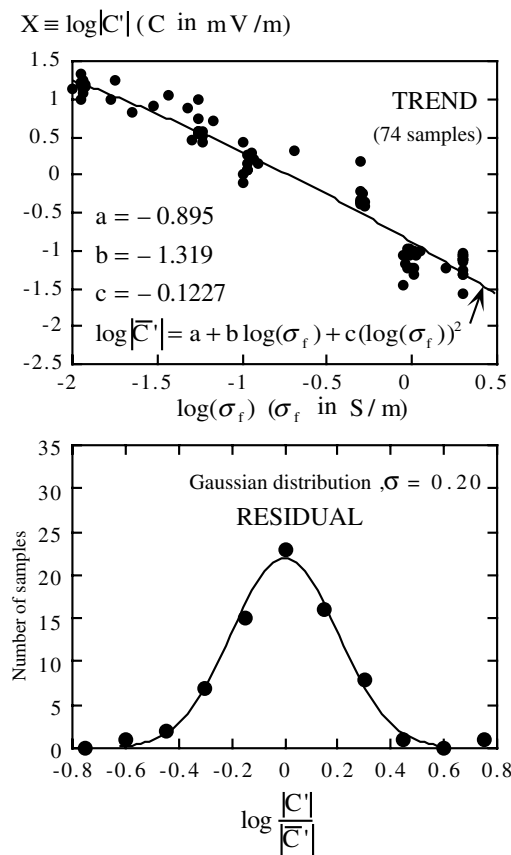


Figure 8 (a) A compilation of 74 laboratory and field measurements of the streaming potential coupling coefficient (in mV/m) (Revil et al., 2003; Naudet et al., 2003; Rizzo et al., 2004; Suski et al., 2006) and its dependence on the electrical conductivity of the pore water that is fitted with a second order polynomial. (b) At a given ground water conductivity, the streaming potential coupling coefficient is given by a log normal distribution.

estimates give a large weight to the infiltration model because of its higher probability of being the correct model.

The estimation uncertainties of the prior model (Fig. 6a), posterior model based on the infiltration model (Fig. 6b), posterior model based on the water table model (Fig. 6c), and the total uncertainty estimated from Eq. (11) (Fig. 6d)

are shown. The improvements in the estimation errors for the updated models are most significant away from the piezometers, e.g., the maximum estimation error of the model based on the infiltration model is 2 m compared with more than 4 m for the prior model. The estimation errors are smaller for the models based on the infiltration model compared with the water table model because of the lower variances of the associated likelihood functions. The weighted models have larger estimation errors because uncertainties regarding the proper model type are included. Areas in the weighted models with much higher estimation errors compared with the individual models indicate areas where the two individual models deviate significantly. It should be noted that the uncertainty arising from applying the calibrated conceptual models outside of the area sampled by the piezometers is not included in the estimation method and these errors might be considerable.

Further improvements in estimating the water table with self-potential data

The estimation error of using the self-potential data only to estimate the elevation of the water table is approximately two meters at Roujan when using the infiltration model (see Fig. 6b) and the question arises if future surveys can be conducted such that the estimation errors decrease substantially, say to one meter. Adding more self-potential data will improve the estimates as seen by the SP semi-variograms (see Fig. 3b–c), but just to a certain extent (i.e., to the estimation error at a given SP station). There are three obvious ways to improve the estimates presented in this work: (1) estimate the electrical conductivity of the pore water at each SP station; (2) measure the temperature of the electrolyte in the electrodes; (3) monitor the SP stations at high-frequencies and transform the signals into the frequency domain to remove distortions from telluric currents and cultural activity.

1. The electrokinetic coupling coefficient is inversely proportional to the electrical conductivity of the pore water (e.g., Revil et al., 2003). Variations in the electrical conductivity of the pore water are therefore partly responsible for the scatter in the estimated likelihood functions and for the nugget in the semi-variograms of the SP signals (see Fig. 3b–c). Variations in the electrical conductivity of the pore water could be incorporated in

the estimation, but it would then be necessary to sample the electrical conductivity of the pore water at the SP stations.

2. The Cu/CuSO₄ electrodes have a temperature drift of $\sim 0.8 \text{ mV}/^\circ\text{C}$ and this effect is considerable given the low amplitudes of the SP signals at this site even if the drift is corrected for by repeated measurements at the reference station. A temperature reading of the electrolyte at each SP station and continuous readings at the reference station would make it possible to remove this effect. Alternatively, use of the more stable Petiau electrodes should improve our measurements, according to the supplier (SDEC, Reignac sur Indre, France), the temperature drift of the Petiau electrodes are only $\sim 0.2 \text{ mV}/^\circ\text{C}$.
3. The recorded signals are composed by in situ signals and telluric currents, which correspond to the main source of noise in addition to cultural activity (50 Hz and harmonics). For monitoring purposes, the removal of cultural noise and telluric currents can be accomplished by filtering the SP measurements using the magnetic variations recorded with a magnetometer and a reference station for the self-potential survey (Kawakami and Takasugi, 1994).

The first two additions to the survey procedure can be carried out without slowing down the survey significantly, whereas the third addition is probably too time-consuming for this type of application. In addition, the physical models behind the infiltration model could be improved by taking the distribution of the water content into account, something we have not attempted in this work. The apparent coupling coefficients in the Roujan catchment are at the lower end of the range usually observed (-1 to -10 mV/m). This is probably the consequence of the relatively high electrical conductivity of the pore water and the aquifer materials. Consequently, the signal-to-noise ratio is likely to be higher at sites where the electrical conductivity of the pore water and the geological formation are lower (see also Section "Likelihood functions").

It should be noted that the self-potential method can be used, with much lower estimation uncertainty, to estimate temporal variations in the water table below a given station. Suski et al. (2006) found in an infiltration experiment carried out within the Roujan catchment that the relative temporal variations of the piezometric levels could be estimated with a precision of 20 cm when using a fixed network of electrodes.

Concluding statements

In this work, we estimated the geometry of the ground water table at a catchment scale by integrating SP and piezometric data following a Bayesian approach. The method was applied to data collected at the test site of Roujan, Hérault, in southern France, where 140 SP measurements and 22 piezometric observations were collected in June 2005. We found that the kriged SP data correlated better with the thickness of the vadose zone (correlation coefficient of -0.86) than with the elevation of the water table (correlation coefficient of -0.58); thereby, indicating that the self-potential signal mainly originated from the vadose

zone as predicted by the infiltration model by Zhang and Aubert (2003). However, predictions based on both regression models were combined into one final estimate, where the estimation uncertainties also reflect the differences in the predictions between the infiltration model and the water table model by Revil et al., 2003. The apparent coupling coefficients $c_v = -0.95 \text{ mV/m}$ (see Eq. (2)) and $c_w = -0.29 \text{ mV/m}$ (see Eq. (3)) are unusually low at this site and applications at other sites are likely to enjoy higher signal-to-noise ratios and the updated head estimates would be further improved compared with this study. This work opens the door to cheap and routine mapping of the water table at km-scale by combining few piezometric observations with a larger amount of SP data.

Our proposed method is recommended in cases where little is known about the catchment in terms of lithofacies and their permeabilities, the infiltration rates and its spatial distribution, and so on. If a conceptual hydrogeological model is available, it would be much more appropriate to condition the ground water model to available hydraulic head data through inverse modeling (e.g., McLaughlin and Townley, 1996), where the estimated hydraulic heads at the SP locations would be included in the data vector, but where the associated data errors would be fairly large. Water table maps that are constructed through inverse modeling represent a more physically realistic head surface (e.g., conservation of mass would be guaranteed) and the resulting head surface would be consistent with the boundary conditions applied and any available permeability data. The water table models presented in this work only represent a state estimation (i.e., an image) when no underlying groundwater model is available. Despite these shortcomings, we think that water table maps based on SP and hydraulic head data provide a cheap, fast, and useful tool to help to understand the underlying hydrogeological controls in poorly characterized catchments throughout the globe.

Acknowledgements

We thank Claude Doussan (INRA Avignon) and Frédéric Perrier (CEA-DASE) for fruitful discussions about self-potential signals in the vadose zone. We thank Fabienne Trolard, Guilhem Bourrié, Guillaume Coulouma, François Garnier, Olivier Huttel and Stefan Finsterle for their support. We thank the financial support of INRA, the French Direction de la Recherche for a post-doc grant to N. Linde, PACA Region Ph.D. grant for J. Castermant, and ECCO-PNRH (French National Program in Hydrogeology). Comments from two anonymous reviewers helped us to significantly improve the manuscript.

References

- Aubert, M., Yéné Atangana, Q., 1996. Self-potential method in hydrogeological exploration of volcanic areas. *Ground Water* 34 (6), 1010–1016.
- Chen, J., Hubbard, S., Rubin, Y., 2001. Estimating the hydraulic conductivity at the South Oyster Site from geophysical tomographic data using Bayesian techniques based on the normal linear regression model. *Water Resour. Res.* 37 (6), 1603–1613.
- Deutsch, C.V., Journel, A.G., 1998. *GSLIB: A geostatistical software library and user's guide*, 2nd ed. Oxford Univ. Press, London.

- Draper, D., 1995. Assessment and propagation of model uncertainty. *J. Roy. Stat. Soc. Ser. B* 57 (1), 45–97.
- Finizola, A., Sortino, F., Lénat, J.-F., Valenza, M., 2002. Fluid circulation at Stromboli volcano (Aeolian Islands, Italy) from self-potential and soil gas surveys. *J. Volcanol. Geotherm. Res.* 116 (1–2), 1–18.
- Fournier, C., 1989. Spontaneous potentials and resistivity surveys applied to hydrogeology in a volcanic area: case history of the Chaîne des Puys (Puy-de-Dôme, France). *Geophys. Prosp.* 37, 647–668.
- Hubbard, S.S., Chen, J., Peterson, J., Mayer, E.L., Williams, K.H., Swift, D.J., Mailloux, B., Rubin, Y., 2001. Hydrogeological characterization of the South Oyster bacterial transport site using geophysical data. *Water Resour. Res.* 37 (10), 2431–2456.
- IAHS, 2003. IAHS Decade on Predictions in Ungauged Basins (PUB). PUB Science and Implementation Plan 2003–2012.
- Jackson, D.B., Kauahikaua, J., 1987. Regional self-potential anomalies at Kilauea Volcano. *USGS Professional Paper* 1350, 947–959.
- Kawakami, N., Takasugi, S., 1994. SP monitoring during the hydraulic fracturing using the TG-2 well. EAGE 56th Meeting and Exhibition, Vienna, Austria, Paper 1004, 6–10 June.
- Kulesa, B., Hubbard, B., Brown, G., 2003. Cross-coupled flow modeling of coincident streaming and electrochemical potentials and application to subglacial self-potential data. *J. Geophys. Res.* 108 (B8), 2381. doi:10.1029/2001JB001167.
- Louchart, X., Voltz, M., Andrieux, P., Moussa, R., 2001. Herbicide transport to surface waters at field and watershed scales in a Mediterranean vineyard area. *Journal of Environmental Quality* 30, 982–991.
- McLaughlin, D., Townley, L.R., 1996. A reassessment of the groundwater inverse problem. *Water Resour. Res.* 32 (5), 1131–1161.
- Naudet, V., Revil, A., Bottero, J.-Y., Bégassat, P., 2003. Relationship between self-potential (SP) signals and redox conditions in contaminated groundwater. *Geophys. Res. Lett.* 30 (21), 2091. doi:10.1029/2003GL018096.
- Naudet, V., Revil, A., Rizzo, E., Bottero, J.-Y., Bégassat, P., 2004. Groundwater redox conditions and conductivity in a contaminant plume from geoelectrical investigations. *Hydrol. Earth Syst. Sci.* 8 (1), 8–22.
- Perrier, F., Raj Pant, S., 2005. Noise reduction in long term self-potential monitoring with traveling electrode referencing. *Pure Appl. Geophys.* 162, 165–179.
- Perrone, A., Iannuzzi, A., Lapenna, et al., 2004. High resolution electrical imaging of the Varco d'Izzo earthflow (southern Italy). *J. Appl. Geophys.* 56 (1), 17–29.
- Revil, A., Cerepi, A., 2004. Streaming potential in two-phase flow condition. *Geophys. Res. Lett.* 31 (11), L11605, doi:10.1029/2004GL020140.
- Revil, A., Hermitte, D., Voltz, M., Moussa, R., Lacas, J.-G., Bourrié, G., Trolard, F., 2002. Self-potential signals associated with variations of the hydraulic head during an infiltration experiment. *Geophys. Res. Lett.* 29 (7), 1106, doi:10.1029/2001GL014294.
- Revil, A., Naudet, V., Nouzaret, J., Pessel, M., 2003. Principles of electrography applied to self-potential electrokinetic sources and hydrogeological applications. *Water Resour. Res.* 39 (5), 1114. doi:10.1029/2001WR000916.
- Revil, A., Cary, L., Fan, Q., Finizola, A., Trolard, F., 2005b. Self-potential signals associated with preferential ground water flow pathways in a buried paleo-channel. *Geophys. Res. Lett.* 32, L07401. doi:10.1029/2004GL022124.
- Revil, A., Leroy, P., Titov, K., 2005a. Characterization of transport properties of argillaceous sediments. Application to the Callovo-Oxfordian Argillite. *J. Geophys. Res.* 110, B06202. doi:10.1029/2004JB003442.
- Ribolzi, O., Andrieux, P., Valles, V., Bouzigues, R., Bariac, R., Voltz, M., 2000. Contribution of groundwater and overland flows to storm flow generation in a cultivated Mediterranean catchment. Quantification by natural chemical tracing. *J. Hydrol.* 233, 241–257.
- Rizzo, E., Suski, B., Revil, A., Straface, S., Troisi, S., 2004. Self-potential signals associated with pumping-tests experiments. *J. Geophys. Res.* 109, B10203. doi:10.1029/2004JB003049.
- Rubin, Y., 2003. *Applied Stochastic Hydrogeology*. Oxford University Press.
- Scheibe, T.D., Chien, Y.J., 2003. An evaluation of conditioning data for solute transport prediction. *Ground Water* 41 (2), 128–141.
- Suski, B., Revil, A., Titov, K., Voltz, M., Dagès, C., Huttel, O., 2006. Monitoring of an infiltration experiment using the self-potential method. *Water Resour. Res.* 42, W08418. doi:10.1029/2005WR004840.
- Titov, K., Loukhmanov, V., Potapov, A., 2000. Monitoring of water seepage from a reservoir using resistivity and self-polarization methods: case history of the Petergoph fountain water supply system. *First Break* 18, 431–435.
- Voltz, M., Albergel, J., 2002. OMERE: Observatoire Méditerranéen de l'Environnement Rural et de l'Eau – Impact des actions anthropiques sur les transferts de masse dans les hydrosystèmes méditerranéens ruraux Proposition d'Observatoire de Recherche en Environnement. French Ministry of Research, Paris, 25p.
- Voltz, M., Andrieux, P., Bouzigues, R., Moussa, R., Ribolzi, O., Joseph, C., Trambouze, W., 1997. Main Hydrological processes in a farmed catchment of the mediterranean area. In: Viville, D., Littlewood, I. (Eds.), *Ecohydrological Processes in Small Basins*. 6th ERB conference Strasbourg, 24–26 Septembre 1996, IHP-V Technical Documents in Hydrology no. 14, UNESCO, Paris, pp. 111–115.
- Zablocki, C.J., 1978. Streaming potentials resulting from the descent of meteoric water. A possible source mechanism for Kilauean self-potential anomalies. *Geotherm. Res. Council, Transactions* 2 (2), 747–748.
- Zhang, G.-B., Aubert, M., 2003. Quantitative interpretation of self-potential anomalies in hydrogeological exploration of volcanic areas: a new approach. *Near Surf. Geophys.*, 69–75.