

The influence of viscous and latent heating on crystal-rich magma flow in a conduit

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ABSTRACT

The flow dynamics of crystal-rich high-viscosity magma is likely to be strongly influenced by viscous and latent heat release. Viscous heating is observed to play an important role in the dynamics of fluids with temperature-dependent viscosities. The growth of microlite crystals and the accompanying release of latent heat should play a similar role in raising fluid temperatures. Earlier models of viscous heating in magmas have shown the potential for unstable (thermal runaway) flow as described by a Gruntfest number, using an Arrhenius temperature dependence for the viscosity, but have not considered crystal growth or latent heating. We present a theoretical model for magma flow in an axisymmetric conduit and consider both heating effects using Finite Element Method techniques. We consider a constant mass flux in a 1-D infinitesimal conduit segment with isothermal and adiabatic boundary conditions and Newtonian and non-Newtonian magma flow properties. We find that the growth of crystals acts to stabilize the flow field and make the magma less likely to experience a thermal runaway. The additional heating influences crystal growth and can counteract supercooling from degassing-induced crystallization and drive the residual melt composition back towards the liquidus temperature. We illustrate the models with results generated using parameters appropriate for the andesite lava dome-forming eruption at Soufrière Hills Volcano, Montserrat. These results emphasize the radial variability of the magma. Both viscous and latent heating effects are shown to be capable of playing a significant role in the eruption dynamics of Soufrière Hills Volcano. Latent heating is a factor in the top two kilometres of the conduit and may be responsible for relatively short-term (days) transients. Viscous heating is less restricted spatially, but because thermal runaway requires periods of hundreds of days to be achieved, the process is likely to be interrupted. Our models show that thermal evolution of the conduit walls could lead to an increase in the effective diameter of flow and an increase in flux at constant magma pressure.

Key words: fluid dynamics, finite-element methods, magma, magma flow, viscosity, volcanic activity.

1 INTRODUCTION

Understanding the flow of magma from a crustal reservoir via a conduit to the free surface is a key determinant of the dynamics of volcanic eruptions. The heat budget of the magma is an important factor in this. The flow dynamics of fluids with temperature-dependent viscosities, such as magmas, can vary enormously. In an extreme case Pinkerton & Stevenson (1992) observed that the viscosities of magma can change by a factor of 10^{13} as the magma cools by 200°C . The heat liberated by viscous dissipation is potentially a major contributor. Magmas are generally multiphase fluids, with crystal nucleation and growth during conduit ascent playing a significant role in the eruption dynamics (Melnik & Sparks 1999). The release

of latent heat during such a phase change will also contribute to the overall heat budget and, more importantly here, contribute locally to the rheological behaviour of the magma. For example, latent heating has been invoked as a factor in the surprising fluidity of very highly siliceous rhyolite lava flows (Manley 1992). Whilst viscous heating has been considered in other magma conduit models (e.g. Costa & Macedonio 2003; Mastin 2005), the effects of latent heating have received less attention. Here we present an analysis of both effects.

Our principal motivation is to understand silicic effusive eruptions, such as the andesitic lava dome eruption of Soufrière Hills Volcano, Montserrat. From November 2005 to the present this eruption has produced numerous lava domes of andesitic composition with a limited range (57–61 per cent) of SiO_2 contents with the

groundmass representing a low-silica rhyolite melt (71–72 per cent SiO_2). Mafic inclusions make up approximately 1 per cent by volume of the erupted material. This fact suggests a process of magma mixing and heating from hotter, basaltic magma at the base of the crustal magma reservoir supplying the eruption (Rutherford & Devine 2003). Soufrière Hills andesitic magma is already rich in crystals within the magma reservoir prior to eruption, where it is thought to have a crystallinity of approximately 50–65 per cent (Sparks *et al.* 2000). At the start of lava extrusion in late 1995 and early 1996, the lava had a highly crystalline groundmass with only 5–15 per cent residual rhyolitic glass, and dome growth was typically observed as the extrusion of spines at low volumetric rates (Barclay *et al.* 1998). Samples from subsequent periods of more rapid dome growth have tended to have higher glass content (up to 30 per cent) although the overall glass content range is wide (5–30 per cent) and there is always some glass remaining in the groundmass (Rutherford & Devine 2003). The degree of crystallization depends upon the rate of magma ascent to the surface as recorded by the groundmass. A high flux can result in the suppression of crystal growth, which maintains a lower viscosity and, therefore, a lower conduit resistance, which then enhances flux further (Melnik & Sparks 1999).

The temperature of the andesite in the Soufrière Hills Volcano magma reservoir determined from analysis of hornblende phenocrysts and associated Fe-Ti oxides (Barclay *et al.* 1998) is estimated to be $830 \pm 10^\circ\text{C}$, at a pressure of 130 ± 25 MPa. However, there is also evidence of localized heating. For example, an increase in TiO_2 in the outer 20–30- μm -wide rims of titanomagnetite grains suggests temperatures of up to 900°C , 60–70 $^\circ\text{C}$ hotter than the predicted magma storage temperature (K. Cashman, 2006, personal communication.). The mafic inclusions may have supplied this heat. However, many hornblende phenocrysts in the erupted andesite show no evidence of disequilibrium breakdown that occurs with temperature rising beyond 870°C at a pressure of 130 MPa (Rutherford & Devine 2003). This suggests that only small batches of the Soufrière Hills andesite were raised above temperatures beyond the hornblende stability field and for short periods of time (Rutherford & Devine 2003).

In addition to zoning in titanomagnetite crystals, there are also calcic rims to plagioclase phenocrysts and calcic plagioclase microclites which require an increase in temperature or pressure (Couch *et al.* 2003a). Also pargasitic (high-temperature) hornblende microphenocrysts are found in samples from the Vulcanian explosions which occurred in 1997 (Rutherford & Devine 2003). Recent research on lavas from Mount St Helens, which produced a series of domes in the 1980s and again since 2004, suggests another explanation for the type of disequilibrium features seen in the Soufrière Hills lava (Blundy *et al.* 2006). Slow magma ascent results in decompression-driven crystallization and this process was used to explain the solidification of the spine extruded at Mount Pelée by Gilbert (1904). Degassing-induced crystallization can generate local and transient temperature fluctuations due to latent heat release (Couch *et al.* 2003b). Ion-microprobe measurements of dissolved H_2O in phenocryst-hosted melt inclusions from pumice erupted from May to October 1980 at Mount St Helens Volcano show that all microclites and a significant proportion of phenocrysts were formed from near-isothermal decompression (Blundy & Cashman 2005). There is no evidence of mafic inclusions in Mount St Helens lavas and, therefore, temperature increases could be predominantly from latent heat release. Extensive shallow crystallization observed in dome samples from Soufrière Hills Volcano suggests that latent heat would also be liberated. Keszthelyi (1995) observed the tem-

perature at the base of pahoehoe lava flow lobes extruded on Kilauea Volcano, Hawaii. After approximately 2 min of rapid cooling, the temperature of the lava increases by 20–50 $^\circ\text{C}$ over the following 2–3 min. The delay was attributed to crystallization kinetics and the heating to latent heat release. Because the initial cooling is so rapid, the magma is forced into a supercooled state with few crystals grown. When nucleation does begin it is as a sudden pulse which then liberates a large amount of latent heat. For andesitic magma the latent heat of solidification is approximately $3.5 \times 10^5 \text{ J kg}^{-1}$ and, therefore, it is possible to liberate approximately 4 $^\circ\text{C}$ (350 kJ kg^{-1}) for every 1 vol.% of crystal growth. Thus, it is predicted that for Soufrière Hills lava, latent heat could potentially increase the temperature by as much as 30–40 $^\circ\text{C}$ during very rapid degassing and crystallization (Couch *et al.* 2003a).

In this paper we consider flow in an axisymmetric (cylindrical) conduit of infinitesimal and finite length to simulate flow of a cooling magma in a conduit. The non-linear governing equations are solved using the finite element method. The model is parametrized with values appropriate for Soufrière Hills Volcano, but the results have general relevance although care is needed in applying these results to other volcanic systems. The lava rheology is described by an empirical expression for the viscosity considering crystal and water content, as well as temperature dependence appropriate for lava dome building eruptions. Newtonian and non-Newtonian (i.e. specifically a shear thinning rheology as considered in this paper) magma flow properties are modelled to understand how rheological changes affect the flow, crystallinity and temperature fields. Either adiabatic or isothermal boundary conditions are used at the conduit wall to represent the two end-member states. An isothermal boundary condition most appropriately represents the conduit wall at the start of an eruption. As the eruption continues, the conduit walls will heat up and tend towards an adiabatic condition. We consider first the effects of viscous heating alone, and then in combination with crystal-growth to account for the latent heat released. Crystal growth is considered using the theory of Hort (1998) as developed by Melnik & Sparks (2005), which account for changes in magma liquidus in the melt phase. Complex eruptive behaviour has been attributed to volatile loss and crystal growth by Melnik & Sparks (1999) and in this research we try to better constrain the additional influence of latent heat.

2 COMPUTATIONAL MODEL

We consider the calculation of velocity, temperature and crystallinity of the magma in a 1-D infinitesimal segment of a perfectly cylindrical conduit along a profile extending from the centre to the edge of the conduit. Our models maintain a constant mass flux because it would not be appropriate to use a constant pressure in a conduit segment since in later models we consider flow along a conduit length and here the pressure gradient is not expected to remain constant. However, lava dome-forming eruptions typically experience steady state extrusion rates over extended periods of time (Harris *et al.* 2003). The very small Reynolds numbers experienced in these flows ($\text{Re} \ll 1$) mean that we can assume the flow remains laminar and that $\delta P / \delta r = 0$. The temperature of the conduit wall at the start of the model is assumed to be in thermal equilibrium with the magma and set equal to a temperature of 1123 K. Although we only consider the flow dependence on the temperature, strain-rate and crystal concentrations in time in the radial direction (and in the vertical direction in the finite-vertical-extent model), the model equations are still too complicated to allow analytical solutions. We produce numerical solutions using the finite element code Finley

Table 1. Model parameters and values

Symbol	Parameter	Value
σ	Shape coefficient for crystals	1.0
Ω	Coefficient introduced by Melnik & Sparks (2005)	30
ϕ	Initial crystal volume fraction in magma reservoir	0.50
ϕ_p	Initial phenocryst volume fraction in magma reservoir	0.50
ϕ_m	Initial microlite volume fraction in magma reservoir	0.0
U_0	Maximum growth rate	10^{-9} m s^{-1}
I_0	Maximum nucleation rate	$30 \times 10^{-9} \text{ m}^3 \text{ s}^{-1}$
T	Initial magma and conduit wall temperature	1123 K approx 850 °C
T_U	Undercooling for maximum crystal growth rate	60 K
T_I	Undercooling for maximum crystal nucleation rate	90 K
ρ	Magma density	2350 kg m^{-3}
α_S	Water solubility coefficient	$4.11 \times 10^{-6} \text{ Pa}^{-1/2}$
L	Latent heat capacity for magma	$3.5 \times 10^5 \text{ J kg}^{-1}$
C_P	Heat capacity for magma	$918.0 \text{ W m}^{-1} \text{ K}^{-1}$
κ	Diffusivity for magma	$2.46 \text{ J kg}^{-1} \text{ K}^{-1}$
R	Radius of conduit	15 m

(Davies *et al.* 2004; Gross *et al.* 2007). The model equations and computational techniques are discussed below and the parameters and values used are given in Table 1.

2.1 Momentum and heat equations

Here, for completeness, we outline the general axisymmetric form of the governing equations. The reduced equations used in the calculations are obtained by dropping the terms containing derivatives with respect to the z -coordinate. The constitutive equation for a Newtonian, viscous material reads:

$$\sigma'_{ij} = 2\eta D'_{ij}, \quad (1)$$

where

$$\sigma'_{ij} = \sigma_{ij} + P\delta_{ij} \quad \text{and} \quad P = -\frac{1}{3}\sigma_{kk}$$

$$D'_{ij} = D_{ij} - \frac{1}{3}D_{kk}\delta_{ij}. \quad (2)$$

Here σ_{ij} is the stress, η is the viscosity, D_{ij} is the symmetric part of the velocity gradient, the so-called strain-rate, δ_{ij} is the Kronecker delta, P is the pressure (i.e. positive in compression) and η is the viscosity of the fluid. The momentum equations in axisymmetrical coordinates read:

$$(r\sigma_{rr})_{,r} + r\sigma_{rz,z} - \sigma_{\theta\theta} + rf_r = 0$$

$$r\sigma_{zz,z} + (r\sigma_{rz})_{,r} + rf_z = 0. \quad (3)$$

Insertion of eq. (1), (2) into (3) yields:

$$[r(2\eta v_{r,r} - P)]_{,r} + r[\eta(v_{r,z} + v_{z,r})]_{,z} - 2\eta \frac{v_r}{r} + P + rf_r = 0$$

$$r(2\eta v_{z,z} - P)_{,z} + [r\eta(v_{r,z} + v_{z,r})]_{,r} + rf_z = 0. \quad (4)$$

The boundary condition at the walls is that of no-slip. We have implemented the complete model as outlined in this section into our finite element code using the partial differential equation scripting device eScript (Gross *et al.* 2007) as described in Section 2.4.

In addition to the above momentum equations we have to consider the heat eq. (5):

$$\rho c_p \dot{T} = \frac{1}{r} [k(rT_{,j})_{,j} + \tau \dot{\gamma} + L\rho(\dot{\phi}_{ph} + \dot{\phi}_{mc})]. \quad (5)$$

L is the latent heat of crystallization, ρ is the density of the bulk magma, ϕ_{ph} and ϕ_{mc} are the corresponding volume fractions for

phenocrysts and microlites, respectively, (that will be discussed in more detail in Section 2.3) and the superscripted dot designates the material time derivative, i.e.:

$$\dot{T} = T_{,t} + v_j T_{,j}. \quad (6)$$

The term $\tau \dot{\gamma}$ in eq. (5) is the viscous dissipation, where $\tau = \sqrt{\frac{1}{2}\sigma'_{ij}\sigma'_{ij}}$ is the second deviatoric stress invariant and $\dot{\gamma} = \sqrt{2D'_{ij}D'_{ij}}$ is the second deviatoric invariant of the strain-rate (equivalent shearing). Viscous heating is important in fluids in which a local temperature increase from viscous friction produces a decrease in viscosity. As a consequence of this viscosity decrease there can be an enhanced flow rate for a constant applied pressure gradient. This enhanced flow rate increases the strain rate and above a critical pressure gradient the flow can accelerate. A *Gruntfest* number G' (Gruntfest *et al.* 1964) represents a dimensionless combination of parameters given by:

$$G' = \frac{aR^4 P^2}{4k\eta_0}. \quad (7)$$

At a critical value of G' the flow will experience a thermal runaway in which the temperature-dependence of the viscosity is represented by $\eta = \eta_0 e^{-a(T-T_0)}$. A thermal runaway can be experienced in any fluid that has a temperature-dependent viscosity (Newtonian or non-Newtonian), however the form of eq. (7) will change depending upon the viscosity relationship. In eq. (7), R is the radius of the pipe and k is the thermal conductivity of the fluid. For a cylindrical pipe with isothermal walls, the critical Gruntfest number for flow acceleration is about 8.0 (Gruntfest *et al.* 1964).

The pressure gradient is adjusted during each time step to ensure that the prescribed mass flux is conserved. In reality the extrusion rate could be expected to increase due to a corresponding decrease in viscosity. However, to model this transient effect is beyond the limitations of our current model and future efforts will entail developing 2-D and 3-D transient conduit models. The time step in the model is also adjusted at each time step to ensure the Courant condition is satisfied. For our 1-D infinitesimal segment models only the second of the eqs (4) are needed, with $P = 0$, $f_z = -\nabla_z P$ and the advection terms in the heat eq. (5) can be dropped, that is, $\dot{T} = T_{,t}$.

2.2 Rheology equations

The melt viscosity of the magma is calculated using an empirical equation developed by Hess and Dingwell (1996), where c is the water concentration in weight percent:

$$\log \eta_m = -3.545 + \left(0.833 \ln c + \frac{9601 - 2368 \ln c}{T - (195.7 + 32.25 \ln c)} \right). \quad (8)$$

The crystal volume fraction has a very large influence upon the effective viscosity and this is represented as a relative viscosity (Costa 2005):

$$\eta = \eta(\phi) \eta_m \quad (9)$$

$$\eta(\phi) = \left\{ 1 - \alpha \operatorname{erf} \left(\frac{\sqrt{\pi}}{2} \phi \left[1 + \frac{\beta}{(1-\phi)^\gamma} \right] \right) \right\}^{-B/\alpha}$$

This function contains the error function $\operatorname{erf}(x)$ and three coefficients α and β and γ . We use the coefficient values given by Costa (2005) and as shown in Table 1 to produce the effective viscosities as modelled from data available in the literature.

For increasing crystal volume fraction it is not possible to assume that the viscosity change is entirely from crystal interaction. The stress-strain relationship for melt will become increasingly non-linear and, therefore, cannot be described with a Newtonian flow model. Rheological measurements of rhyolite show that the melt becomes non-Newtonian at subliquidus temperatures that can be best approximated by a power-law or pseudo-plastic viscosity model (Pinkerton & Stevenson 1992). Plastic fluids have a yield strength, with flow viscosity decreasing with shear rate, a pseudo-plastic material. We model magma flow using the Carreau model which describes a pseudo-plastic fluid. The reason we do not consider a power-law model is because it breaks down in regions where the shear rate is zero and this would be the case at the conduit centre (Phan-Thien 2002).

$$\eta(\dot{\gamma}) = \eta_\infty + \frac{\eta_0 - \eta_\infty}{(1 + \Gamma^2 \dot{\gamma}^2)^{(1-n)/2}}. \quad (10)$$

Here η_0 is the zero shear rate viscosity that we take to be equal to η in eq. (9), the relaxed Newtonian viscosity. η_∞ is the infinite shear rate viscosity that we set to zero, n is the power-law index and Γ is a time constant. The inverse of the time constant gives the minimum shear-strain rate necessary before the flow experiences any shear thinning. Webb & Dingwell (1990) show that igneous melts experience a departure from Newtonian behaviour for shear-strain rates exceeding $5 \times 10^{-5} \text{ s}^{-1}$. This corresponds to a relaxation time constant (Γ) of 20 000 s, as used in our model. For a power-law index of 1 the Carreau model describes a Newtonian fluid and for $0 < n \leq 1$ the model is shear thinning. Pinkerton & Stevenson (1982) used a power-law index of 0.8 to describe the decrease in apparent viscosity at high strain rates for a rhyolitic melt and we use this value in our model.

2.3 Crystal growth and latent heating formulation

An increase in temperature of the magma may arise from the release of the latent heat during crystallization. During magma ascent, crystals form primarily due to the change in pressure as opposed to a temperature change. Water exsolution with decompression increases the liquidus temperature of the melt phase as a result of a progressive change in chemical composition of the melt, resulting in supercool-

ing. Crystal growth begins when the temperature of the magma becomes lower than its liquidus temperature, and therefore, exsolution of water can induce crystallization (Cashman & Blundy 2000). The water content in the magma melt phase is given by Henry's solubility law, $c = \alpha_s \sqrt{P}$, where α_s is the solubility coefficient and P is the pressure. Petrological studies at Soufrière Hills indicate that the andesite magma in the reservoir has a dissolved water content of 4.3 per cent (Barclay *et al.* 1998). Gardner *et al.* (1999) found that decompressing rhyolitic melts results in equilibrium degassing for low decompression rates $< 0.025 \text{ MPa s}^{-1}$. Over a 5-km long conduit with a maximum pressure of 140 MPa, the vertical ascent rate of the magma must remain below 0.9 m s^{-1} for degassing to be in equilibrium or near-equilibrium conditions. This is generally the case for effusive, crystal-rich eruptions such as at Soufrière Hills Volcano.

The formation of bubbles from the exsolved gases reduces the bulk density but impedes the free flow of the liquid component. These effects can be modelled at the bubble scale (Blower *et al.* 2001). However, the processes of bubble aggregation and migration as gas flows through preferred pathways at or near the conduit walls is potentially a major factor determining the number and role of bubbles in the magma. Rather than introduce a separate, ill-constrained, model component to handle this, we choose to ignore the role of bubbles in the magma other than in their implicit role in microlite formation. Since we do not model bubbles, or a variable density, our treatment is not directly comparable to the Melnik & Sparks (2005) 1-D models.

The treatment we use to model crystal growth induced by isothermal degassing is the same as that used by Melnik & Sparks (2002a) and follows Hort (1998). Magma flow in lava dome-forming eruptions is already rich in phenocrysts and microphenocrysts in the magma reservoir prior to eruption. For Soufrière Hills Volcano the magma in the reservoir is thought to comprise 35–45 vol.% phenocrysts and 15–20 vol.% microphenocrysts (Sparks *et al.* 2000), giving an initial crystallinity between 50 and 65 per cent. Phenocrysts are crystals that have formed in the magma reservoir and can grow during ascent, whilst microlites are new crystals that form during ascent. Phenocryst and microlite volume fractions are represented by ϕ_{ph} and ϕ_{mc} , respectively, and the total crystal volume fraction is $\phi = \phi_{\text{ph}} + \phi_{\text{mc}}$. For Soufrière Hills Volcano, nucleation is the dominant crystallization process so that magma reservoir-formed crystals experience little growth during ascent (Couch *et al.* 2003a). Degassing-induced crystallization is modelled with crystal growth and nucleation rates introduced as functions of undercooling. The effective liquidus temperature is T_m , which changes during crystallization due to the progressive chemical change of the melt according to:

$$T_m = (T_{\text{liq}} - T)(1 - \phi/\phi_{\text{eq}}) + T, \quad (11)$$

where

$$T_{\text{liq}} = a_T + b_T \ln(P) + c_T \ln(P)^2 + d_T \ln(P)/P^2. \quad (12)$$

T is the temperature in Kelvins, P is the pressure in Pascals and the coefficients a_T , b_T , c_T and d_T are taken from Melnik & Sparks (2005) and given in Table 1. The equilibrium crystallinity in the melt phase is given by:

$$\phi_{\text{eq}} = \frac{A(P)[T - T_{\text{liq}}(P)]}{B(P) - T}, \quad (13)$$

where $A(P)$ and $B(P)$ are functions of the pressure as discussed in Melnik & Sparks (2005). In our model we assume that the influence of the bubble volume fraction on the evolution of crystal growth

is negligible and also that the mass densities of the microlites and the phenocrysts are constant. With these assumptions the evolution equations for the volume fractions of the phenocrysts and microlites can be written as

$$\dot{\phi}_{\text{ph}} = 3 \left(\frac{4\pi N_{\text{ph}} \phi_{\text{ph}}^2}{3} \right)^{1/3} U(t)(1 - \phi), \quad (14)$$

and

$$\dot{\phi}_{\text{mc}} = 3\sigma(1 - \phi)U(t) \int_0^\omega I(\omega) \left[\int_\omega^t U(\eta) d\eta \right]^2 d\omega, \quad (15)$$

respectively. In the above relationships, N_{ph} is the number density of the phenocrysts in the condensed phase, I is the nucleation rate, U is the linear crystal growth rate. Crystallization proceeds through two processes, crystal nucleation and growth, both dependent upon undercooling and at the liquidus temperature both I and U are equal to zero (Hort 1998). ω and η are integration parameters, and σ is the shape coefficient for the crystals. All the parameters for these equations are given in Table 1.

2.4 Computational techniques

The code has a Python-based user interface allowing the user to state the model equations in the form of systems of first and second order differential equations. The flow equations are computed in a framework represented spatially by the axisymmetric coordinates of a finite element mesh using the *Finley* finite element kernel library. However the software can also solve the model equations in 1-D, 2-D and 3-D. The modelling library *escript* has been developed as a module extension of the scripting language Python to facilitate the rapid development of 3-D parallel simulations on the Altix 3700 (Davies *et al.* 2004). The finite element kernel library, *Finley*, has been specifically designed for solving large-scale problems on cc-NUMA architectures and has been incorporated as a differential equation solver into *escript*. In *escript* Python scripts orchestrate numerical algorithms which are implicitly parallelized in *escript* module calls, without low-level explicit threading implementation by the *escript* user.

The *escript* Python module provides an environment to solve initial boundary value problems (BVPs) problems through its core finite element library *Finley*. A steady, linear second-order BVP for an unknown function u is processed by *Finley* in the following templated system of PDEs (expressed in tensorial notation):

$$-(A_{ijkl}v_{k,l})_{,j} - (B_{ijk}v_k)_{,j} + C_{ikl}v_{k,l} + D_{ik}v_k = -X_{ij,j} + Y_i, \quad (16)$$

where the Einstein summation convention is used. *Finley* accepts a system of natural boundary conditions given by:

$$n_j(A_{ijkl}v_{k,l} + B_{ijk}v_k) + d_{ik}v_k = n_jX_{ij} + y_i \text{ on } \Omega_i^N, \quad (17)$$

where n denotes the outer normal field of the domain and A, B and X are as for eq. (16). d and y are coefficients defined on the boundary. The Dirichlet boundary condition is also accepted:

$$u_i = r_i \text{ on } \Omega_i^D, \quad (18)$$

where r_i is a function defined on the boundary. *Finley* computes a discretization of eq. (16) from the variational formulation. The variational problem is discretized using isoparametric finite elements on unstructured meshes. Available element shapes are line, triangle, quadrilateral, tetrahedron and hexahedron of orders one and two.

With both the *escript* and *Finley* technologies, complex models and very large simulations can be rapidly scripted and run easily. The code is fully portable, but optimized at this stage for the local SGI ALTIX super cluster.

Putting eq. (4) in the shape of the PDE eq. (16) yields:

$$A_{ijkl} = r\eta \left[\left(\text{pen} - \frac{2}{3} \right) \delta_{ij}\delta_{kl} + (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \right] \quad (19)$$

$$B_{ijk} = \eta \left(\text{pen} - \frac{2}{3} \right) \delta_{ij}\delta_{k0}$$

$$C_{ikl} = \eta \left(\text{pen} - \frac{2}{3} \right) \delta_{kl}\delta_{i0}$$

$$D_{ik} = \frac{2}{r} \eta \left(\text{pen} - \frac{2}{3} \right) \delta_{i0}\delta_{k0}$$

$$X_{ij} = rP^t\delta_{ij}$$

$$Y_i = P^t\delta_{i0} + \frac{\pi a^4 \rho_a g r}{\eta_a Q} \delta_{i1},$$

where the indexes are 0 for the r coordinates and 1 for the z -coordinates.

3 RESULTS

We consider three model scenarios. The first is magma flow in an infinitesimal conduit segment with a constant (space and time) mass flux and only one heat source, namely shear heating. Second, we consider shear heating with latent heat release again for an infinitesimal conduit segment. Lastly, we consider magma flow in a finite conduit segment with a constant fluid flux and the influence of latent heat release. This last model is used to effectively simulate magma flow in a finite conduit by updating the pressure field in the segment and updating the material properties within the segment from information at the previous time step. We use a constant density for the bulk magma in all our simulations for simplicity and define the pressure to be equal to the overpressure driving the magma to the free surface plus the weight of the magma. For all our model simulations we consider two boundary conditions for the conduit walls: isothermal and adiabatic. It can be expected that at the beginning of an eruption the conduit walls will be cooler than the ascending magma and here isothermal conditions would be most appropriate. However, at later times in the eruption, when the conduit walls become heated, the conduit walls will tend towards an adiabatic state.

3.1 Magma flow with constant mass flux and no crystal growth

We model magma flow in an infinitesimal axisymmetrical conduit segment for the empirical rheology outlined in Section 2 with a constant crystallinity and constant water content. Table 2 shows the different crystal and water contents in the magma component considered. Assuming a driving pressure of 140 MPa at the base of the conduit, it is possible to estimate the approximate depths at which these crystallinities will occur within the conduit from the pressure field. For crystallinity in excess of 80 per cent the magma behaves in a more solid-like fashion and cannot be treated as a Newtonian fluid. Therefore, our model crystallinities are limited to the range 50–75 per cent (Watts *et al.* 2002). The magma is driven by

Table 2: Magma properties for simulating magma flow in an infinitely long conduit without latent heating

Crystal content (per cent)	Pressure (MPa)	Water content in melt (per cent)	Initial viscosity (Pa s)	Approximate depth (m)
60	140	4.86	5.0×10^5	5000
65	84.58	3.78	1.7×10^6	3020
70	53.15	3.00	1.1×10^7	1898
75	36.38	2.48	2.0×10^7	1300

a pressure gradient acting along the axis of the conduit over a range of fluxes suitable for Soufrière Hills Volcano. We maintain a constant extrusion rate by iteratively adjusting the driving pressure gradient at each time step. The boundary condition at the conduit walls is that of no-slip and the wall temperature is maintained at a constant value of 1123 °K, equal to the initial temperature of the magma, or allowed to behave adiabatically. The radius of the conduit segment in our infinitely long domain is 15 m, an appropriate dimension for Soufrière Hills Volcano (Sparks *et al.* 2000; Barclay *et al.* 1998). We observe how the flow profile and temperature change over time and how this affects the driving pressure. We use the reduced time suggested by Gruntfest *et al.* (1964) of $\tau = \frac{\kappa t}{\rho C_p R^2}$, where t is the time in seconds, and for $\tau \approx 1$ (approximately 6.3 yr in this case) the solution either evolves to its steady state or enters a transient phase.

Fig. 1 shows the temporal evolution of the temperature and z -axis velocity field across the (half) conduit for two different extrusion rates for magma with a crystallinity of 70 per cent for Newtonian and non-Newtonian flow and isothermal and adiabatic boundaries. For all eight cases the effects of viscous heating near the margin of the conduit are initially apparent. For model results with adiabatic walls this initially raised temperature region at the conduit spreads across the conduit to develop an approximately uniform temperature field across the conduit at later times. For isothermal conduit walls there is also an initial peaked temperature region near the conduit wall and this also disappears with time to create a relatively constant temperature in the centre of the conduit with a steep thermal gradient near the conduit walls. Non-Newtonian flow for all the models maintains relatively lower temperatures, because an enhanced strain rate is translated into a lower viscosity, resulting in lower shear stresses. The heating is more pronounced for the higher extrusion rate for all the models due to the higher shear stresses experienced along the conduit wall. The initial velocity profiles for the Newtonian and non-Newtonian rheological models are very similar, a consequence of the non-Newtonian rheology departing only slightly from a Newtonian rheology at low strain-rates (eq. 10).

In Fig. 1 for adiabatic conduit walls the flow velocity profiles initially change from a parabola to one closer to plug-like when the temperature is enhanced at the conduit walls compared to the centre of the conduit. Over time as the temperature tends towards an isothermal condition the flow profile changes back into a parabolic shape. With an isothermal boundary condition the flow profile is always moving away from the initial parabolic into a stretched parabolic flow profile with a much higher vertical velocity in the centre of the conduit than close to the conduit walls. If we consider these two boundary conditions to represent the beginning of an eruption and flow at a more mature stage, then it may be envisaged that conduit flow initially has a stretched parabolic profile, for the isothermal case, that slowly evolves into a parabolic profile as the conduit walls heat up. In our model, for an extrusion rate of $8 \text{ m}^3 \text{ s}^{-1}$ the conduit flow profile departs substantially from parabolic flow after a time of 0.1τ , approximately 230 d. Hence, there is likely to be a competition between the time to heat the conduit walls and the time for shear heating to influence the flow profile. Since these models have

a constant extrusion rate they must overestimate the time for the flow profile to change, since an increase in temperature leads to a decrease in viscosity that will reduce the driving pressure gradient.

With isothermal conduit walls during the early stages of flow a low viscosity zone will develop close to the conduit walls. Costa & Macedonio (2003) considered the thermal and mechanical effects caused by viscous heating in a 2-D channel of magma. They found that, because of the low thermal conductivity of magma, heat dissipation away from the zone of viscous shearing was low, as indicated by a high Nahme number (equal to $\mu_0 v^2 \beta / k$, where β is a rheological parameter) and for high magma flow rates, viscous heating can modify Poiseuille flow into a plug flow with a hotter layer near the wall. Their models were of basaltic magma, which has a significantly lower crystallinity and viscosity than the crystal-rich, high-viscosity magmas considered here. For our models, cooling is likely to prevail over viscous heating at low fluxes when an isothermal boundary condition is used.

In Fig. 2 we observe how the magma flow properties and conduit wall boundary conditions affect the driving pressure and temperature over time. All models have a crystallinity of 70 per cent and an extrusion rate of $6 \text{ m}^3 \text{ s}^{-1}$. Fig. 2(a) shows the maximum and minimum temperatures across the conduit. For isothermal conduit wall conditions the temperature increase is largest when the flow is Newtonian. The non-Newtonian flow model we use is not very sensitive to the strain-rate but it can maintain relatively lower temperatures, by up to 100 °C in the example we consider. For all the models considered the driving pressure decreases dramatically due to the increase in temperature and the corresponding decrease in viscosity (Fig. 2b). Note that for Newtonian and non-Newtonian flow regimes with adiabatic conduit walls the maximum temperatures are identical in time. This is because it is the boundary condition that most significantly affects the flow properties, rather than the rheology of the magma.

Fig. 3 shows the final total pressure gradient for four different constant crystal volume fractions with the temperature in equilibrium within the conduit, after a significant amount of time has elapsed. For a crystal content maintained at 60 per cent and isothermal conduit walls, the driving pressure initially increases with extrusion rate. However, it does not increase as rapidly as would be expected for Hagen–Poiseuille flow due to the effects of shear heating. At an extrusion rate of approximately $6 \text{ m}^3 \text{ s}^{-1}$ the pressure gradient required actually starts to decrease. For the same crystallinity, models with adiabatic conduit walls require an approximately constant pressure gradient for all extrusion rates, because the temperature within the conduit increases significantly. At higher crystal volume fractions (Figs 3b–d) it becomes energetically favourable to maintain these higher flow rates due to a decrease in pressure gradient. Increasing crystallinity thus tends to increase the flux at which a thermal runaway may occur but only after a significant period of time. As the conduit boundary condition tends to the adiabatic case at later times during an eruption, the required pressure gradient will tend to remain the same for all extrusion rates due to the thermal heating available. Crystallinity is likely to increase towards the

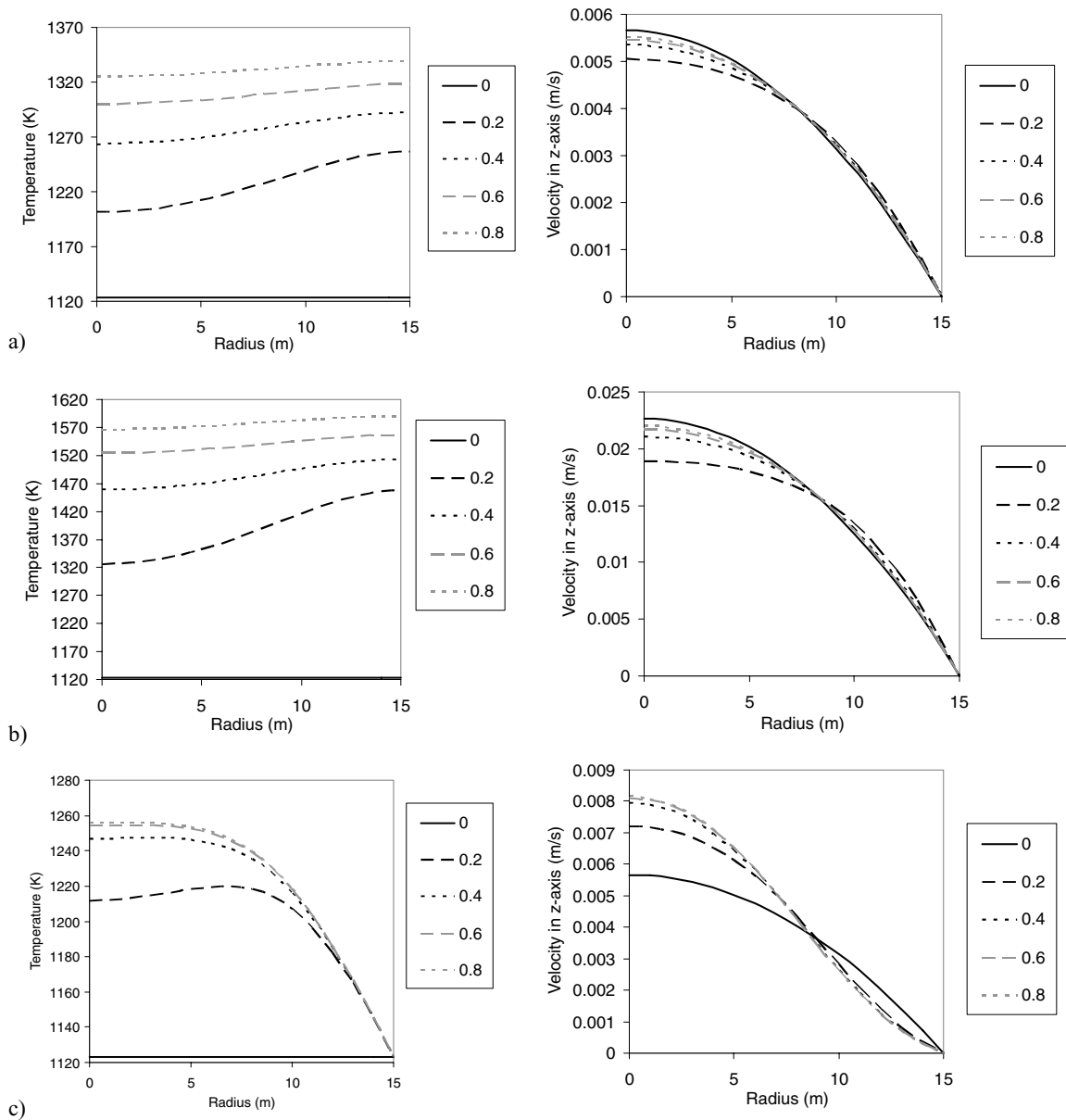


Figure 1. Temperature (left-hand panels) and velocity (right-hand panels) profiles across the half conduit in reduced time steps (numbered lines correspond to different reduced time values) simulated for eight different model scenarios. Crystallinity is 70 per cent for all the models runs. Newtonian flow with adiabatic conduit walls with a flux of $2 \text{ m}^3 \text{ s}^{-1}$ (a) and $8 \text{ m}^3 \text{ s}^{-1}$ (b). Newtonian flow with isothermal conduit walls with a flux of $2 \text{ m}^3 \text{ s}^{-1}$ (c) and $8 \text{ m}^3 \text{ s}^{-1}$ (d). Non-Newtonian flow with adiabatic conduit walls with a flux of $2 \text{ m}^3 \text{ s}^{-1}$ (e) and $8 \text{ m}^3 \text{ s}^{-1}$ (f). Non-Newtonian flow with isothermal conduit walls with a flux of $2 \text{ m}^3 \text{ s}^{-1}$ (g) and $8 \text{ m}^3 \text{ s}^{-1}$ (h).

conduit exit because of degassing-induced crystallization. Therefore, it is in the upper conduit in which a thermal runaway would be most likely to occur for a constant flux. However, the driving pressures would need to be relatively high and sustained over periods of the order of hundreds of days. Such sustained flows are not common at Soufrière Hills for periods more than a few tens of days and several other processes may intervene to disrupt the development of such a thermal runaway.

Non-Newtonian flow acts to make the flow more stable by decreasing the viscosity along the conduit margins and preventing such large shear stresses from forming. This will be more pronounced for flows with more extreme departures from Newtonian behaviour than modelled here.

3.2 Magma flow with constant mass flux and crystal growth

Here we model magma flow in an infinitesimal axisymmetrical conduit segment for an empirical rheology with microlite crystal growth producing latent heating. The initial crystal content is assumed to be a constant value within the magma segment. The initial crystal content is out of equilibrium with the prescribed pressure, although the water content is in equilibrium with this pressure. Thus it is assumed that the magma has been instantaneously degassed and that the growth of crystals has not yet started. This is the same as considering magma that has been transported to higher level within a conduit, similar to decompression experiments in which we assume

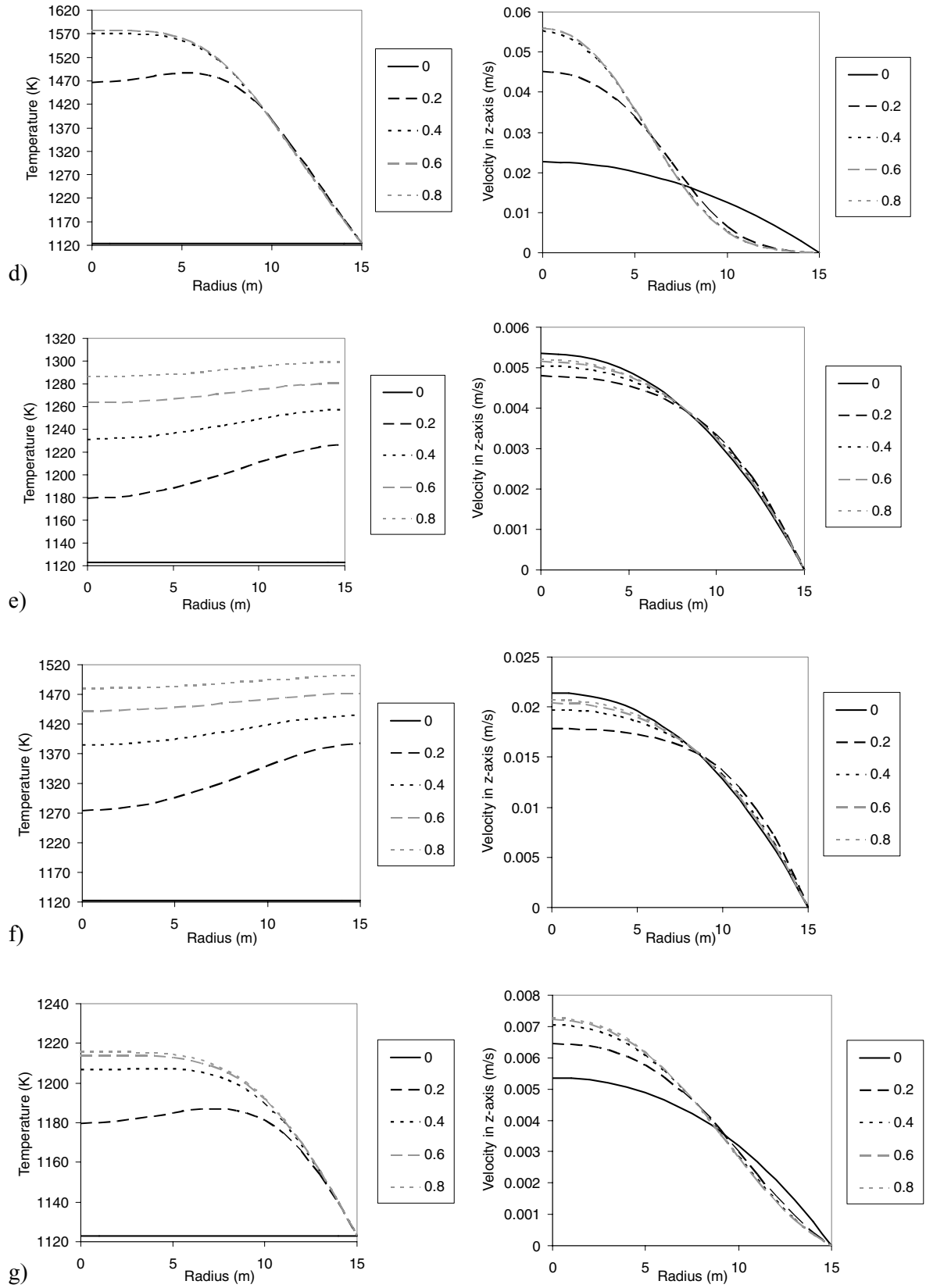


Figure 1. (Continued.)

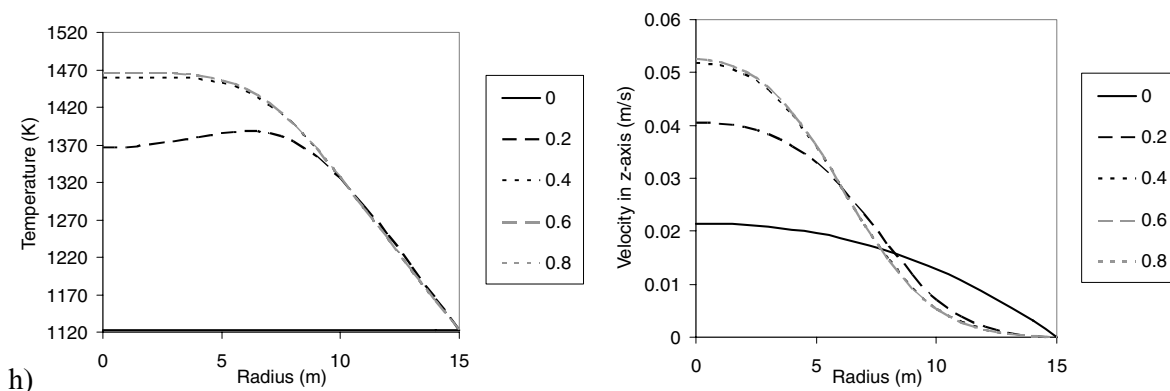


Figure 1. (Continued.)

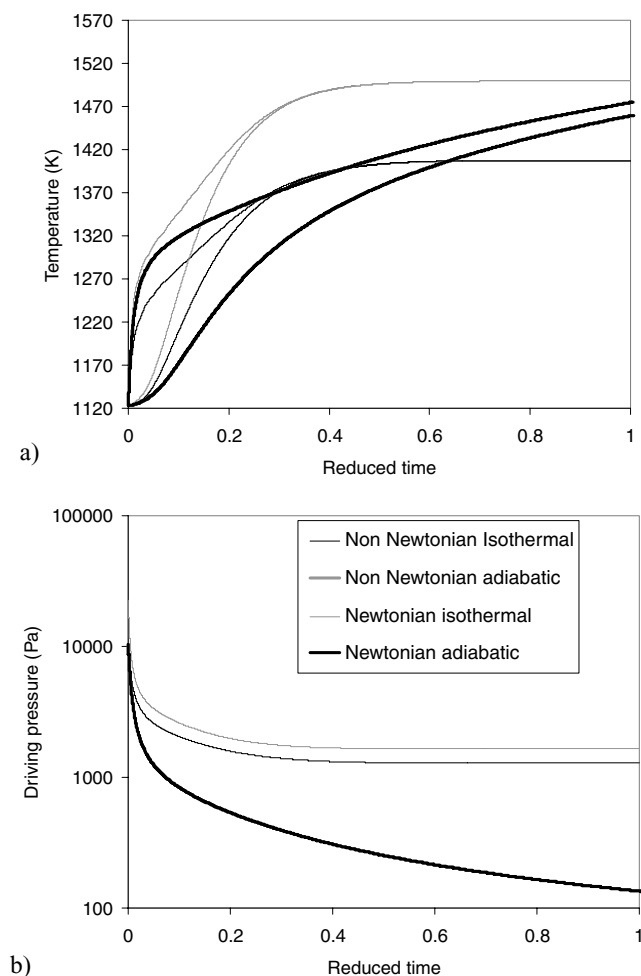


Figure 2. Maximum and minimum temperature, shown as two lines with the same thickness and shading (a), and overpressure gradient (b), against reduced time for four extrusion rates (the crystallinity is 70 per cent and the water content is in equilibrium with this crystallinity). Note that for Newtonian and non-Newtonian flow regimes with adiabatic conduit walls (thick black and grey lines) the maximum temperatures are identical in time. This is because it is the boundary condition that most significantly affects the flow properties, rather than the rheology of the magma.

that the magma has been degassed at time $t = 0$. However, this degassing period occurs over a period less than the critical degassing value given by Gardner *et al.* (1999) to ensure that the water content is in equilibrium. As for Section 3.1, magma flow in the segment is driven by a pressure gradient acting along the axis of the conduit. The initial crystal content is composed entirely of phenocrysts (calculated for a magma reservoir pressure of 140 MPa) and microphenocrysts and the additional parameters used in the model are given in Table 3:

This model represents an instantaneous depressurizing event that affects the upper conduit, equivalent for example, to the removal of a dome 300 m high and emptying of the conduit by 410 m (i.e. a pressure decrease of ~ 17 MPa). This example, therefore, represents extreme depressurization to highlight any cross-conduit variation in crystallinity. The process of crystal growth due to rapid degassing is likely to be similar for other depths in the conduit although the volume of crystal growth will be different.

Fig. 4 shows the temporal evolution of the temperature and velocity field at two different extrusion rates for magma with a crystallinity of 70 per cent for Newtonian and non-Newtonian flow and the two temperature boundary conditions. For all eight cases the effects of heating near the margin of the conduit are apparent. For model results with adiabatic walls (a, b, e and f) the temperature field remains enhanced at the conduit walls. Over the timescales we consider, the temperature does not reach an approximate isothermal profile as was observed in Fig. 1. This is partly a consequence of crystal growth which will be described next. For adiabatic conduit walls the flow profile departs from a parabolic shape into a more plug-like shape. Non-Newtonian flow in the adiabatic conduit wall models show a more plug-like form than for the Newtonian case, despite the temperature difference between the conduit centre and walls being smaller. This is a consequence of the shear-thinning nature of the fluid, allowing shear to become localized along the conduit walls.

For model results with isothermal conduit walls (c, d, g and h) the temperature is enhanced away from the walls. The peak in temperature migrates towards the centre of the conduit over time. The flow departs from a parabolic profile to a flow with a plug-like shape in the centre of the conduit but with reduced vertical flow near the conduit wall. For these simulations there will be an initial increase in temperature due to latent heat release, but after a time shear heating becomes significant and the temperature field develops peaks near the conduit walls. Crystal growth at the fluxes we consider here occurs much more rapidly than the time for shear heating to

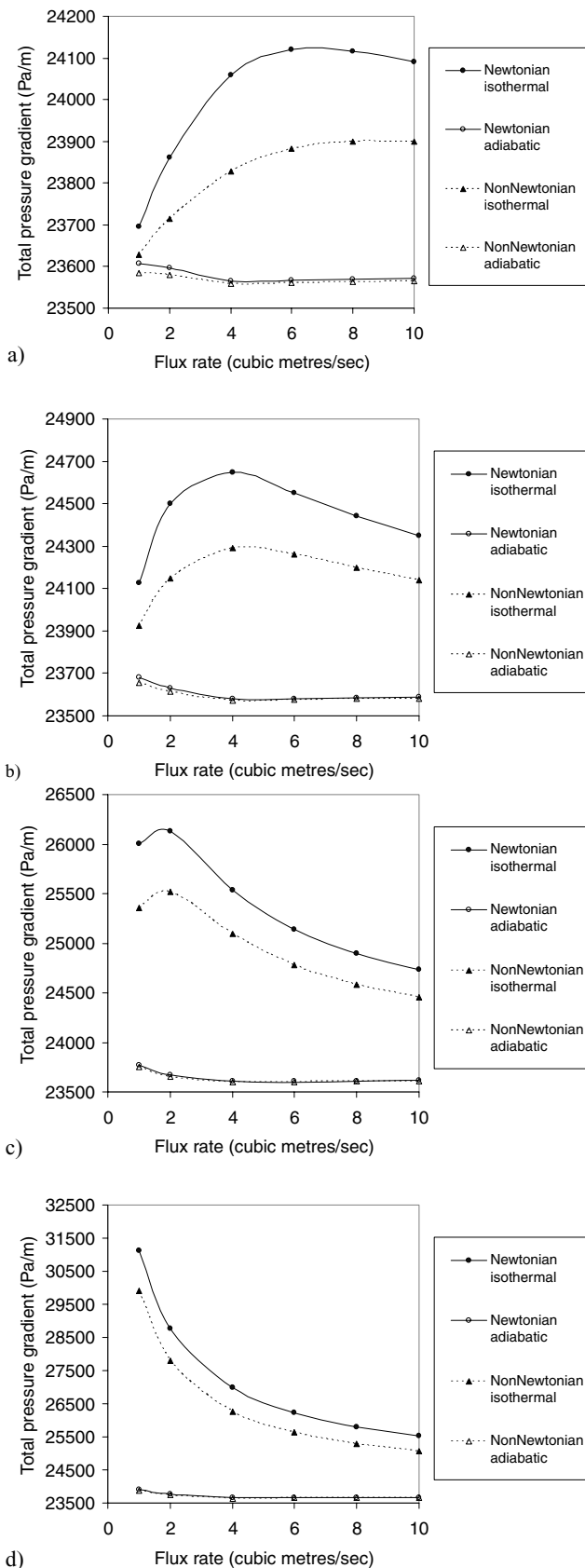


Figure 3. Final total pressure gradient against extrusion rate for different crystallinities (a, 60 per cent; b, 65 per cent; c, 70 per cent and d, 75 per cent).

have a significant counteracting effect upon the temperature field and resulting crystal volume field.

More interesting is the crystal growth distribution across the conduit (Fig. 5). For isothermal conduit wall boundary conditions the largest increase in crystallinity is at the cool conduit wall, where it reaches an equilibrium value of 75 per cent. For an adiabatic boundary condition the crystallinity is lowest at the conduit walls. The initial release of latent heat suppresses crystal growth in the centre of the conduit. If shear heating has a significant effect upon the temperature field during crystal growth, then the crystallinity may be suppressed further in the higher temperature region near the wall. Hence there will be a maximum in crystal volume fraction at the conduit wall, a minimum in crystallinity next to this, and a more gradual increase in crystallinity towards the centre of the conduit. Over time the temperature increases due to shear and latent heat release in the conduit and this essentially fixes the crystal field since it is no longer undercooled (Fig. 6). When the flux is large and shear heating is significant, the crystallinity gradient is highest close to the conduit wall.

Considering again Fig. 4, it is possible to see that for isothermal conduit walls, an enhanced crystallinity at the conduit walls and in the centre of the conduit acts to cause the flow profile to become close to plug-like. Over time the shear heating-induced peak migrates towards the centre of the conduit due to enhanced crystallinity near the conduit walls confining flow. For non-Newtonian flow the temperature peak is less pronounced. For adiabatic conduit walls the crystallinity increase in the centre of the conduit helps to maintain a plug-like flow profile. For all the model results shown in Fig. 5 the effects of shear heating become important once equilibrium crystallinity has been reached. If we return to our assumption that the two boundary conditions used represent the beginning of an eruption and a mature stage, then it may be envisaged that initially the effective width of the conduit will be less because of a high crystal volume fraction at the walls. Over time the flow profile widens to fill the entire conduit as the conduit walls heat up.

Considering the temperature and crystallinity profile at $\tau = 0.01$ and 0.06, it is possible to extract the viscosity field as shown in Fig. 6. The simulations with higher extrusion rates show an enhanced decrease in viscosity close to the conduit wall due to the influence of shear heating. For adiabatic conduit walls (Figs 6a and b) viscosity is highest at the centre of the conduit due to the growth of crystals, which results in a plug-like flow profile (Fig. 4). For isothermal conduit walls (Figs 6c and d) the viscosity at the conduit wall is highest because it has the lowest temperature and highest crystal volume fraction. The minimum in viscosity for flow with isothermal conduit walls moves towards the centre of the conduit with time following the peak in temperature (Fig. 4). These models suggest that the viscosity could vary by up to three orders of magnitude across the conduit, but this value will be more pronounced for flows significantly influenced by shear heating.

As the magma ascends there will be a competition between a decrease in melt viscosity due to shear heating and an increase in effective viscosity due to crystal growth. The factors determining which process is more significant (latent heat versus shear heat) are the decrease in pressure leading to crystal growth and the flux. However, in a finite conduit these processes will not be mutually exclusive. A high flux will suppress the growth of crystals maintaining a lower viscosity, whereas a lower extrusion rate will allow the growth of crystals and an increase in viscosity. Since shear heating is dependent on the strain rate and the viscosity, the amount of latent heat released is not a simple function of the extrusion rate.

Table 3: Model conditions to simulate magma flow in an infinitely long conduit with latent heating

Initial crystal content (per cent)	Initial applied pressure (MPa)	Final applied pressure (MPa)	Water content in melt in equilibrium with applied pressure (per cent)	Total crystal content in equilibrium with applied pressure (per cent)	Approximate initial depth in conduit (m)	Approximate final depth in conduit (m)
70	53.15	36.38	2.49	75	1898	1299

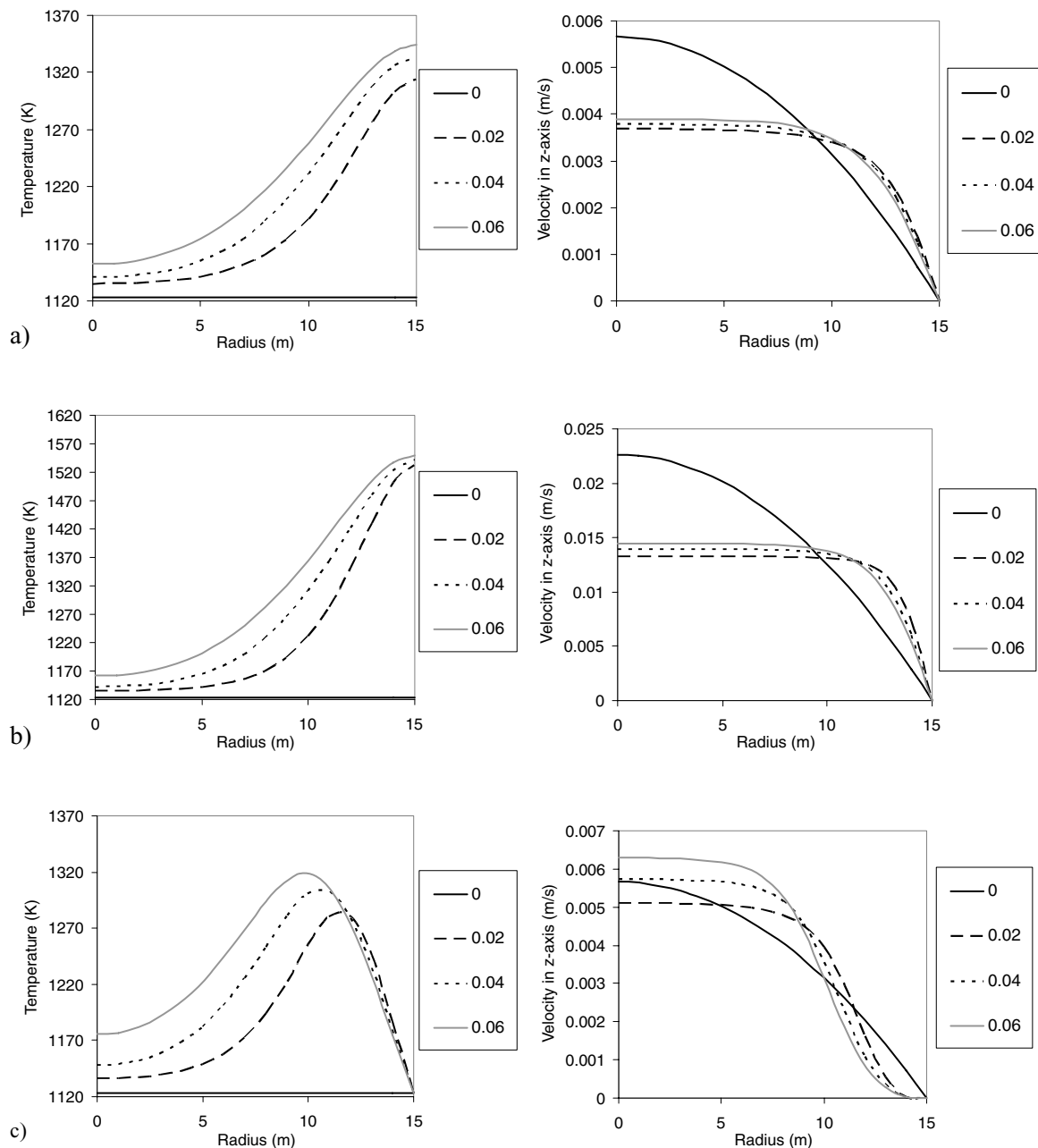


Figure 4. Temperature and velocity conduit profiles for different reduced times (numbered lines) for eight different model runs. Crystallinity is initially 70 per cent for all the models runs and increases in time due to a pressure decrease as detailed in Table 2. Newtonian flow with adiabatic conduit walls with a flux of $2 \text{ m}^3 \text{ s}^{-1}$ (a) and $8 \text{ m}^3 \text{ s}^{-1}$ (b). Newtonian flow with isothermal conduit walls with a flux of $2 \text{ m}^3 \text{ s}^{-1}$ (c) and $8 \text{ m}^3 \text{ s}^{-1}$ (d). Non-Newtonian flow with adiabatic conduit walls with a flux of $2 \text{ m}^3 \text{ s}^{-1}$ (e) and $8 \text{ m}^3 \text{ s}^{-1}$ (f). Non-Newtonian flow with isothermal conduit walls with a flux of $2 \text{ m}^3 \text{ s}^{-1}$ (g) and $8 \text{ m}^3 \text{ s}^{-1}$ (h).

3.3 Magma flow in a finite conduit for a constant flux with latent heating

Here we test a more realistic model of magma flow that accounts for a temperature gradient along a conduit of finite length. We apply the

same momentum and energy equations as presented in Section 3.2, but to a finite length conduit, by modelling a constant flux for mass conservation and by iteratively adjusting the driving pressure gradient at each time step across a section of magma. This is achieved by adjusting the upstream pressure with respect to the previous time

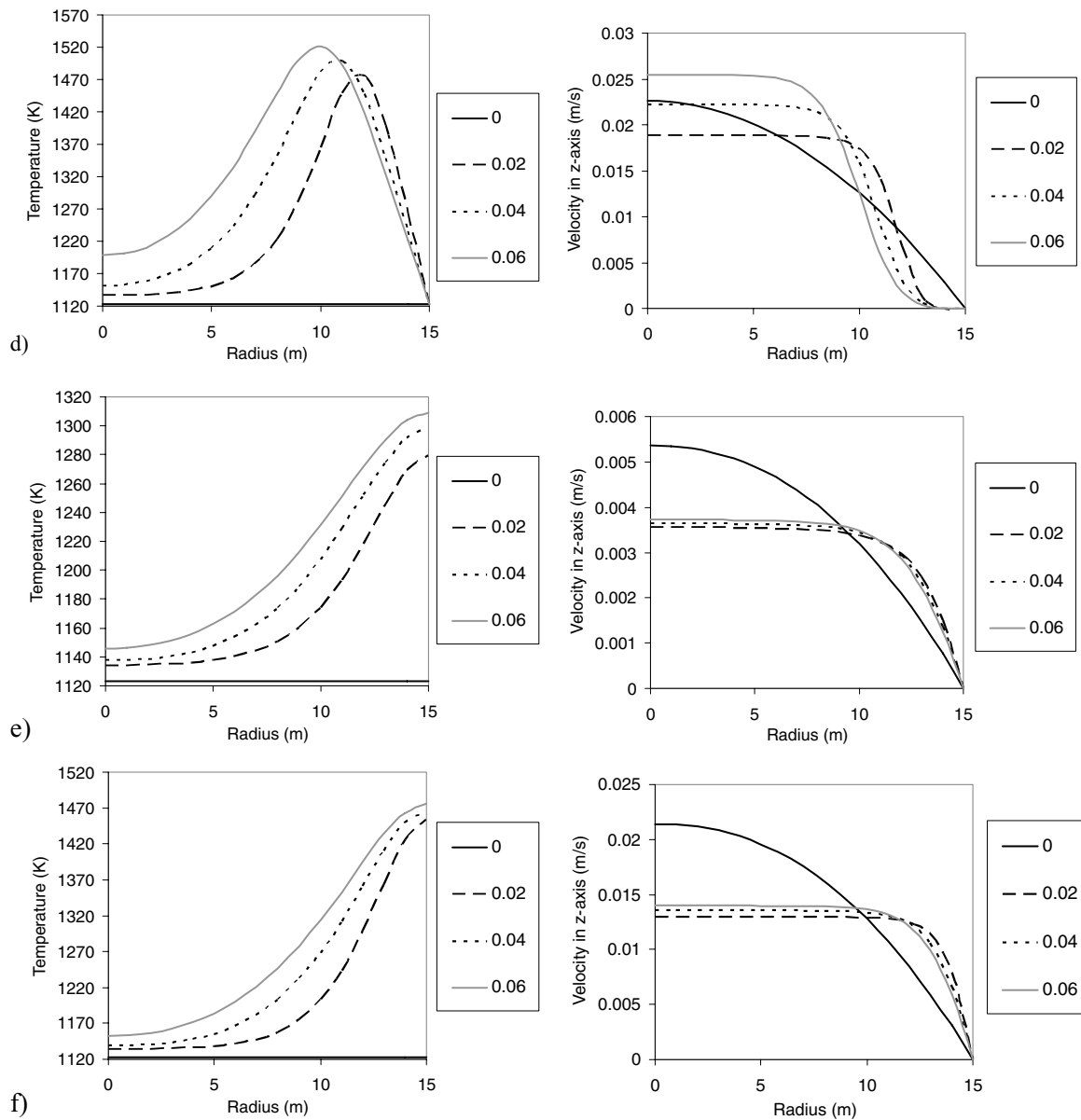


Figure 4. (Continued.)

step and by adjusting the downstream pressure to maintain a constant extrusion rate. As before the boundary temperature conditions used at the conduit walls are isothermal and adiabatic to represent the start of the eruption and later times, respectively, although early in the eruption the conduit wall temperature may be less than the initial magma temperature. For the adiabatic conduit wall condition we make the assumption that the magma is transported faster than the heat can diffuse vertically in the magma column. The boundary flow condition is that of no-slip and the temperature of the magma is initially constant at 1123 K. The conduit is 5 km in length, appropriate for Soufrière Hills Volcano (Devine *et al.* 1998). Using a bulk magma density of 2350 kg m^{-3} and using an acceleration due to gravity equal to 10 m s^{-2} , gives a magmastatic pressure of 117.5 MPa at the magma reservoir exit. It is assumed that a maximum overpressure of approximately 27.5 MPa could exist within the reservoir before the walls would fail and so an initial pressure in the magma reservoir is chosen to be 145 MPa for all model simulations.

This is slightly higher than the 130 MPa pressure inferred for the andesite magma reservoir of SHV from the $P_{\text{H}_2\text{O}}$ values measured in fluid inclusion studies (Barclay *et al.* 1998). The pressure along the conduit will decrease as the magma ascends and this will result in water exsolution (assumed to be in equilibrium with the pressure) which will trigger crystallization and the release of latent heat. The initial crystal content in the magma chamber is 50 per cent.

We calculate the steady-state crystal volume fraction in the ascending magma from the modelled velocity and pressure fields for a fixed extrusion rate, because it is too complex to model a transient crystal field along the entire conduit using this model approach. This would require knowing the initial crystallinity and water content at different depths within the conduit or assuming a constant initial crystallinity for the entire conduit. Instead we subdivide the vent into thin slices along its axis and within each slice we assume Hagen–Poiseuille flow. The height of the vent is fixed so that the pressure and the pressure gradient, assumed to be spatially constant within

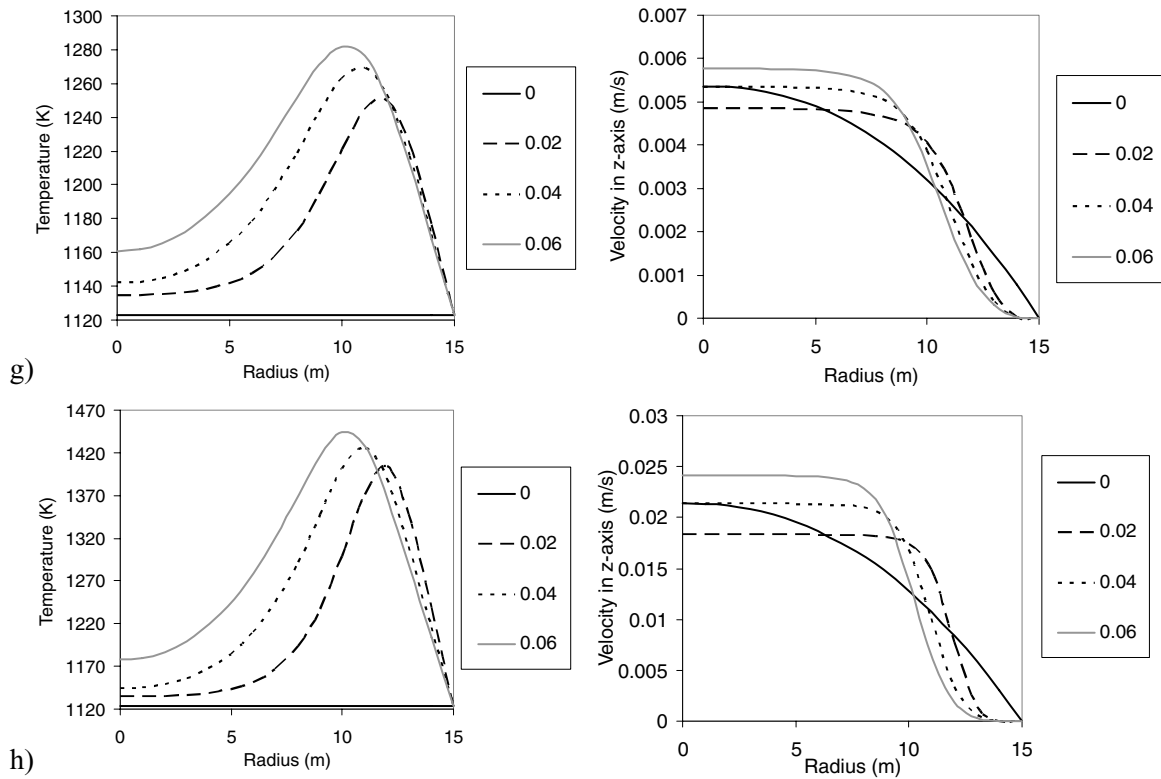


Figure 4. (Continued.)

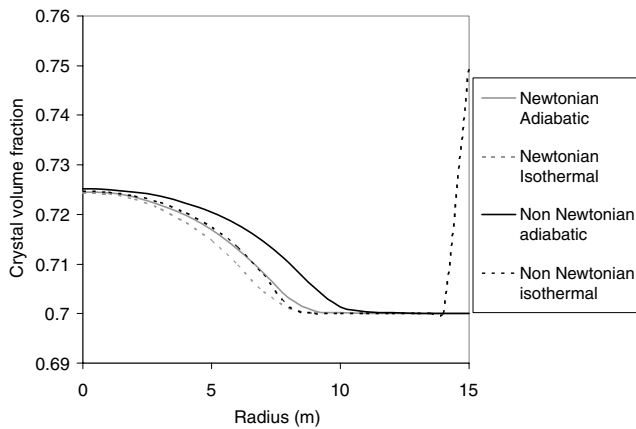


Figure 5. Final crystal volume fractions across the conduit for four model runs. Crystallinity is initially 70 per cent at $t = 0$ for all the models runs and increases in time due to a pressure decrease as detailed in Table 2. All model runs have a flux of $2 \text{ m}^3 \text{ s}^{-1}$.

each slice, are not known a priori. The pressure gradient within each slice has to be determined iteratively because we have assumed a constant (known) extrusion rate. For the first time step the pressure at the base of the segment is equal to the magma chamber pressure. The pressure at the top of the segment is adjusted iteratively to maintain the required extrusion rate. For the next segment the pressure at the base of the segment is equal to the previous pressure at the top of the segment and the process is repeated. Neighbouring slices interact through the fluxes $v_i \phi$ and $v_i T$ of the crystal contents (eqs 14 and 15) and the temperature (eq. 5) which we assume applied at the lower boundary of each segment. The crystal contents and the temperature are then brought slicewise to a steady state (equi-

librium) by treating the ‘time’ as a numerical relaxation parameter. Iterations may be necessary after steady state is reached since the viscosity depends on both, crystallinity and temperature.

Since we assume the boundary condition of no-slip at the conduit walls, the crystal volume fraction there will be in equilibrium with the pressure field at all times. The assumption that $v_r = 0$ is also made and, therefore, crystals are only advected in the z -axis. This modelling technique is appropriate for high Peclet number flows, such that heat is not advected downward into the conduit. We make the assumption that the length of the modelled segment is small enough so that changes in pressure and crystallinity in the vertical direction are very small within the segment and thus it is possible to calculate the total crystal growth within the segment.

Fig. 7 shows the steady-state crystal volume fraction along the conduit length for a flux of $2.0 \text{ m}^3 \text{ s}^{-1}$. Most of the crystal growth occurs in the upper 2000 m of the conduit, as also shown by Melnik & Sparks (2002a) in their 1-D numerical models. However, our axisymmetric models provide information on the variation in crystallinity across the conduit. Newtonian and non-Newtonian flow properties produce almost identical results since the ascent time is too low for non-Newtonian effects to become important at this relatively low flux. With adiabatic conduit walls, at the conduit exit the crystal volume fraction varies between 53 and 61 per cent (60 per cent for the non-Newtonian case) and the final pressure is approximately 28 MPa. For isothermal conduit walls at the conduit exit the crystal volume fraction varies between 56 and 77 per cent (75 per cent for the non-Newtonian case) and the final pressure is approximately 26 MPa. These are large overpressures at the conduit exit, but by fixing the extrusion rate and initial crystal content, large overpressures at the free surface can be generated mathematically. Volcanologically, this would be equivalent to a very large lava dome sitting above the conduit exit. It is possible to model the pressure at

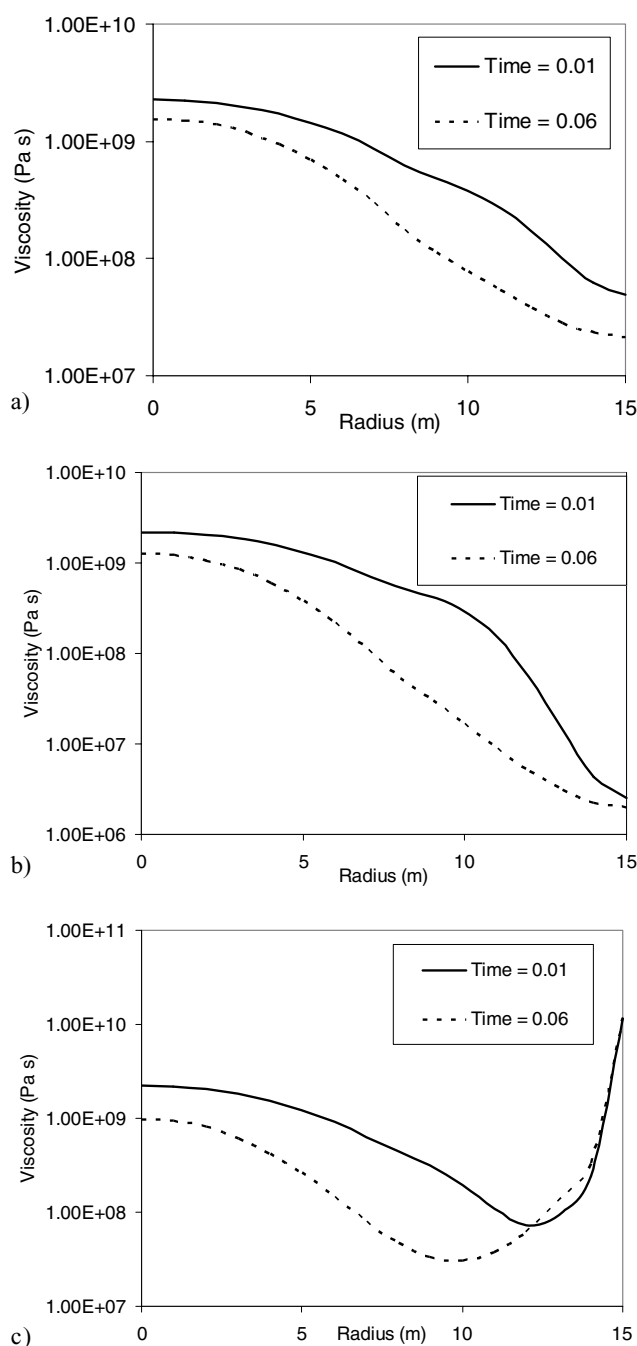


Figure 6. Final viscosity across the conduit with respect to the radius for four model runs for Newtonian flow. Adiabatic conduit walls with a flux of $2 \text{ m}^3 \text{ s}^{-1}$ (a) and $8 \text{ m}^3 \text{ s}^{-1}$ (b). Isothermal conduit walls with a flux of $2 \text{ m}^3 \text{ s}^{-1}$ (c) and $8 \text{ m}^3 \text{ s}^{-1}$ (d).

the conduit exit to equal atmospheric pressure; however this would result in different initial magma chamber pressures for the different rheological models and boundary conditions used. Using the same magma chamber pressure for the different rheological models and boundary conditions allows for a more rigorous comparison of results. At the conduit exit the total crystal volume fraction varies by approximately 8 per cent in the radial direction across the conduit for adiabatic boundary conditions and 21 per cent for isothermal conduit walls. The point of inflection for the minimum crystal volume fraction is at a depth of 2560 m for adiabatic conduit walls and

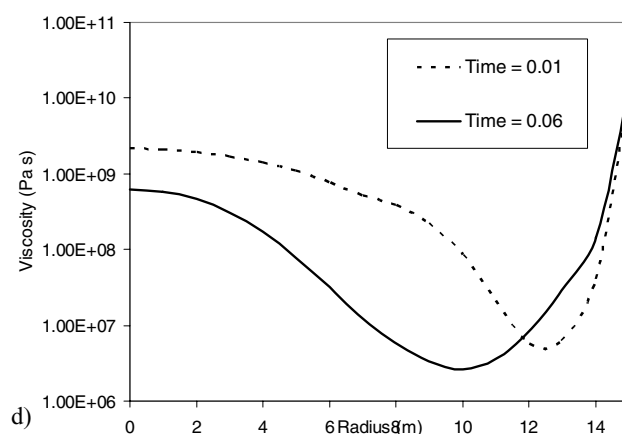


Figure 6. (Continued.)

4450 m for isothermal conduit walls corresponding to the point at which the crystals reach their maximum growth rate.

Fig. 8 shows the temperature and crystal volume fraction in the radial direction at different depths in the conduit. For adiabatic conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ the crystal volume fraction is lowest close to the conduit walls where the temperature is highest. The increase in temperature is from latent heat release which maintains high temperatures at the stationary walls. The highest crystal volume fraction is at the centre of the conduit where the temperature is lowest. For the higher flux of $3 \text{ m}^3 \text{ s}^{-1}$ there is a more pronounced minimum in the temperature field and a maximum in the crystal volume fraction close to the conduit walls. Between the centre of the conduit and the conduit walls the crystal volume fraction increases up to a radius of about 12 m.

A similar trend is observed for magma flow in a conduit with isothermal conduit walls. For a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (Figs 8c and g) the crystal volume fraction is highest at the conduit wall because the conduit wall crystallinity is always in equilibrium with the pressure field because of zero vertical velocity and is enhanced because of the cool boundary conditions. The temperature in the centre of the conduit is increased by the release of latent heat. At these low fluxes the temperature field is approximately constant between the centre of the conduit and a radial distance of 12 m. At the conduit exit the temperature field has a slight peak at a radial distance of approximately 13 m because the amount of heat released in the upper conduit is too great to be fully transported into the conduit centre. For the higher flux of $3 \text{ m}^3 \text{ s}^{-1}$ (Figs 8d and h) there is a minimum in the crystallinity field close to the conduit walls. Between the centre of the conduit and the conduit walls the crystal volume fraction increases up to a radius of about 13 m. In the centre of the conduit, where the vertical velocity is highest, there will be less time for the magma to crystallize and the crystal volume fraction will be lowest. The peak in temperature close to the conduit wall is due to the largest growth of crystals at the cool wall, resulting in a large amount of latent heat. Due to isothermal conduit wall boundary conditions the temperature can only increase close to the wall. The temperature field is most pronounced for the highest flux because latent heat released along the conduit walls will be unable to be diffused into the core of the magma column. This process also acts to suppress crystal growth in this region resulting in a local minimum in crystallinity at a radius of approximately 14 m.

There is little difference in the crystal volume fractions and temperature fields for Newtonian and non-Newtonian flows (Fig. 9).

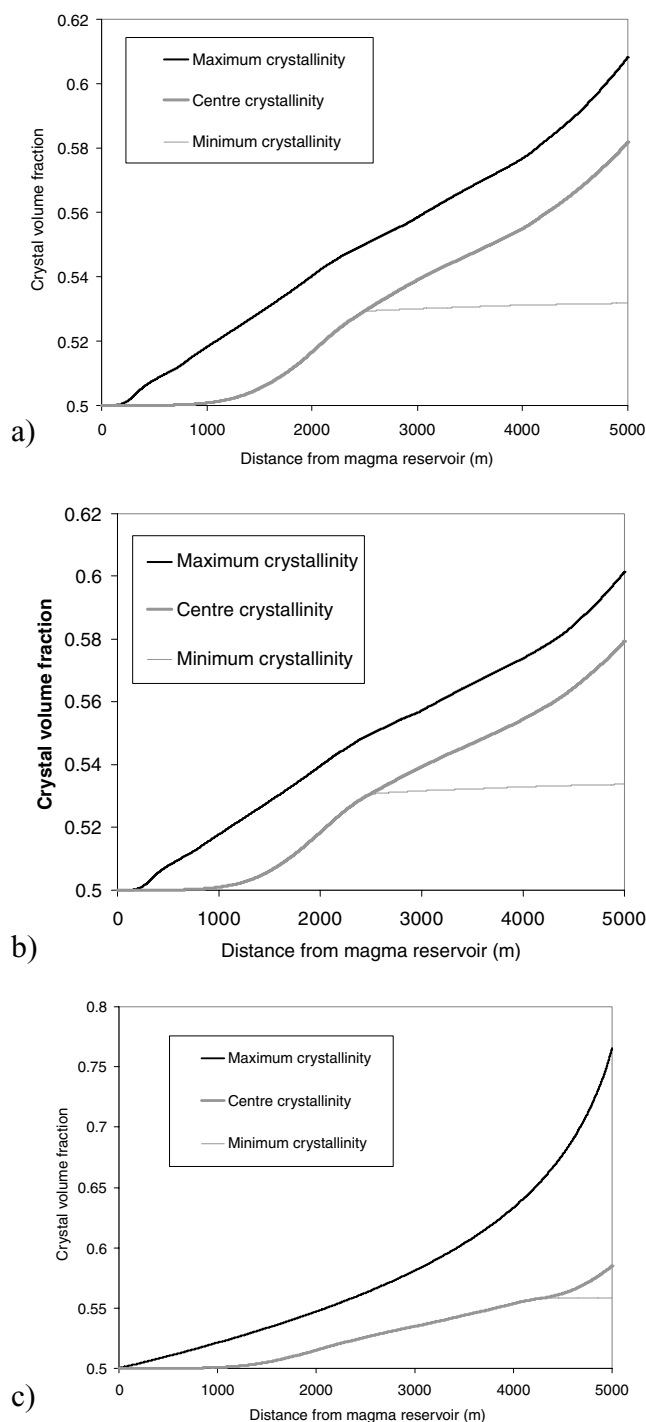


Figure 7. Variation in crystallinity along the conduit for a flux of $2.0 \text{ m}^3 \text{ s}^{-1}$. Curves refer to the maximum crystal volume fraction, the crystal content at the centre of the conduit and the minimum crystal volume fraction. Model runs are for adiabatic conduit walls with Newtonian (a) and non-Newtonian flow (b) properties, and isothermal conduit walls with Newtonian (c) and non-Newtonian flow properties (d).

Non-Newtonian flow properties result in a slightly lower crystal volume fraction and lower temperature because shear thinning reduces the pressure gradient applied to the segment. Flows with the lowest flux have the smallest variation in crystallinity and hence the viscosity variation across the conduit is also smallest. For low fluxes the temperature field at the conduit exit is almost constant across

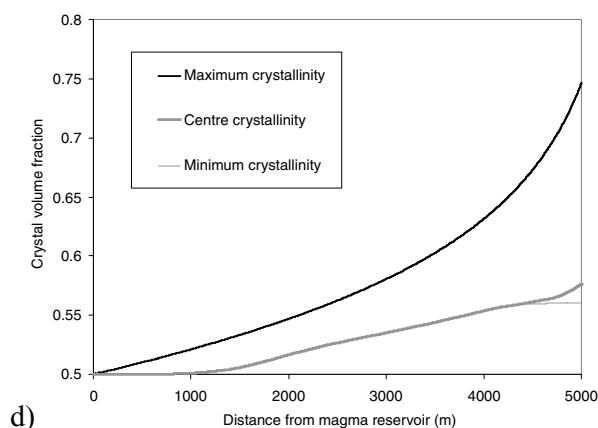


Figure 7. (Continued.)

the conduit radius except near the colder walls. This is because the latent heat released is diffused into the conduit core.

Fig. 10 shows flow profiles across the conduit exit for the results shown in Fig. 8. For adiabatic conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (Fig. 10a) the flow profile departs very slightly from parabolic as the magma ascends within the conduit. Flow is enhanced close to the conduit walls and reduced in the centre of the conduit. For a flux of $3 \text{ m}^3 \text{ s}^{-1}$ (Fig. 10b) the flow profile departure from a parabolic shape is more pronounced and tends towards a plug-like shape. For isothermal conduit wall boundary conditions the flow profile also departs from a parabolic shape into a stretched parabola form (Fig. 10c) with the vertical velocity almost decreased to zero close to the conduit walls.

A low crystallinity and viscosity in the centre of the conduit can act as a positive feedback mechanism to enhance ascent rates, since rapidly ascending magma will suppress the growth of crystals in the core which in turn reduces the viscosity. This positive feedback mechanism may be responsible for changes in crystal content, but only over relatively long timescales. The very high crystallinity at the conduit walls for high fluxes means that the flow is almost stationary for some distance from the walls (Fig. 10c), effectively narrowing the conduit width. This could have large implications for the flow dynamics as shown by Melnik & Sparks (2002a) who modelled an increase in the extrusion rate of approximately 50 per cent for a decrease in conduit diameter from 30 to 28 m for the same overpressure.

Fig. 11 compares the flow profiles and viscosities at the conduit exit for the different models for fluxes of $0.5 \text{ m}^3 \text{ s}^{-1}$ (a and b) and $3 \text{ m}^3 \text{ s}^{-1}$ (c and d). Non-Newtonian flow properties result in enhanced flow features. That is, for adiabatic conduit walls the non-Newtonian flow has evolved closer to plug-like flow than for the Newtonian flow case. For isothermal conduit walls for a flux of $3 \text{ m}^3 \text{ s}^{-1}$ the flow profile is a more exaggerated stretched parabola shape for non-Newtonian flow than for Newtonian flow. The viscosity change across the conduit is less for non-Newtonian conduit flow than for Newtonian flow due to its shear thinning character.

Along the conduit, the latent heat released can raise the magma temperature by approximately 93°K for the extrusion rates shown in Fig. 12. With adiabatic conduit walls the magma temperature increases smoothly with distance from the conduit walls for both extrusion rates modelled. For isothermal conduit walls at the higher extrusion rate of ($3 \text{ m}^3 \text{ s}^{-1}$) the maximum temperature is generally equal to the temperature at the centre of the conduit until a distance of 4640 m from the conduit reservoir (Fig. 12d). At this point the

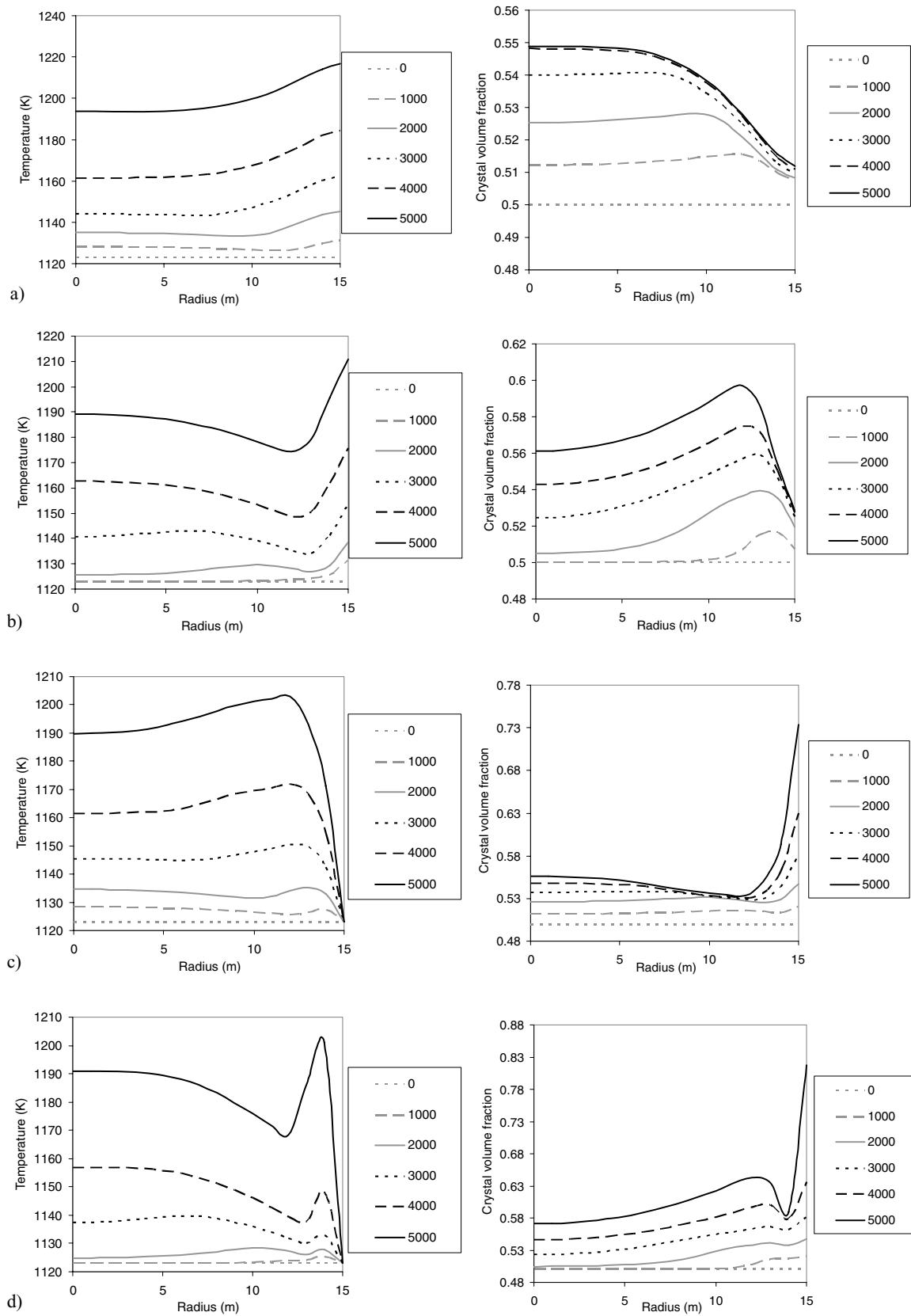


Figure 8. Crystal volume fraction and temperature plots across the conduit for an initial crystallinity of 50 per cent and an initial reservoir pressure of 145 MPa. Newtonian flow with adiabatic conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (a) and $3 \text{ m}^3 \text{ s}^{-1}$ (b). Newtonian flow with isothermal conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (c) and $3 \text{ m}^3 \text{ s}^{-1}$ (d). Non-Newtonian flow with adiabatic conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (e) and $3 \text{ m}^3 \text{ s}^{-1}$ (f). Non-Newtonian flow with isothermal conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (g) and $3 \text{ m}^3 \text{ s}^{-1}$ (h). The numbers in the key correspond to the distance in metres from the magma reservoir exit.

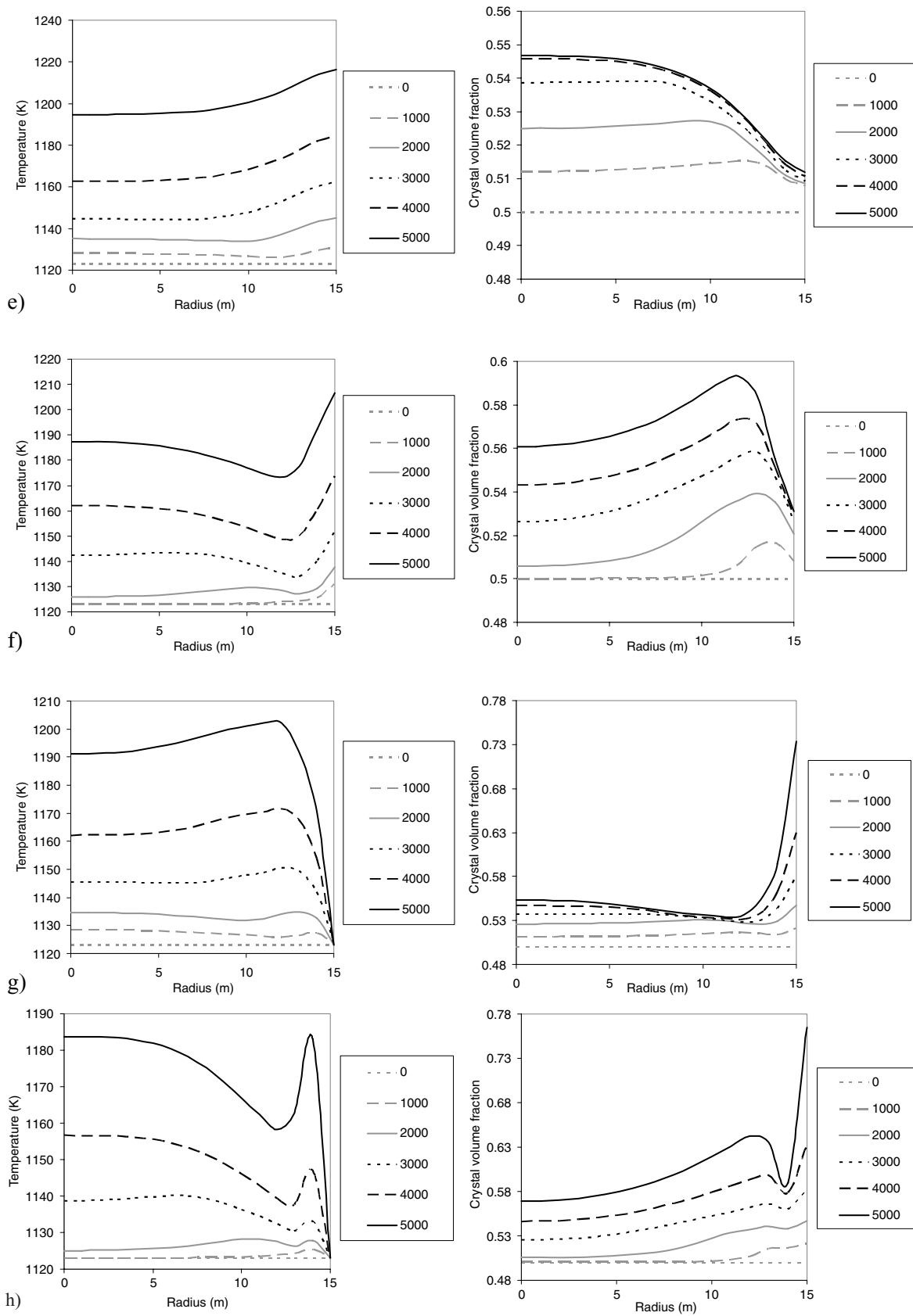


Figure 8. (Continued.)

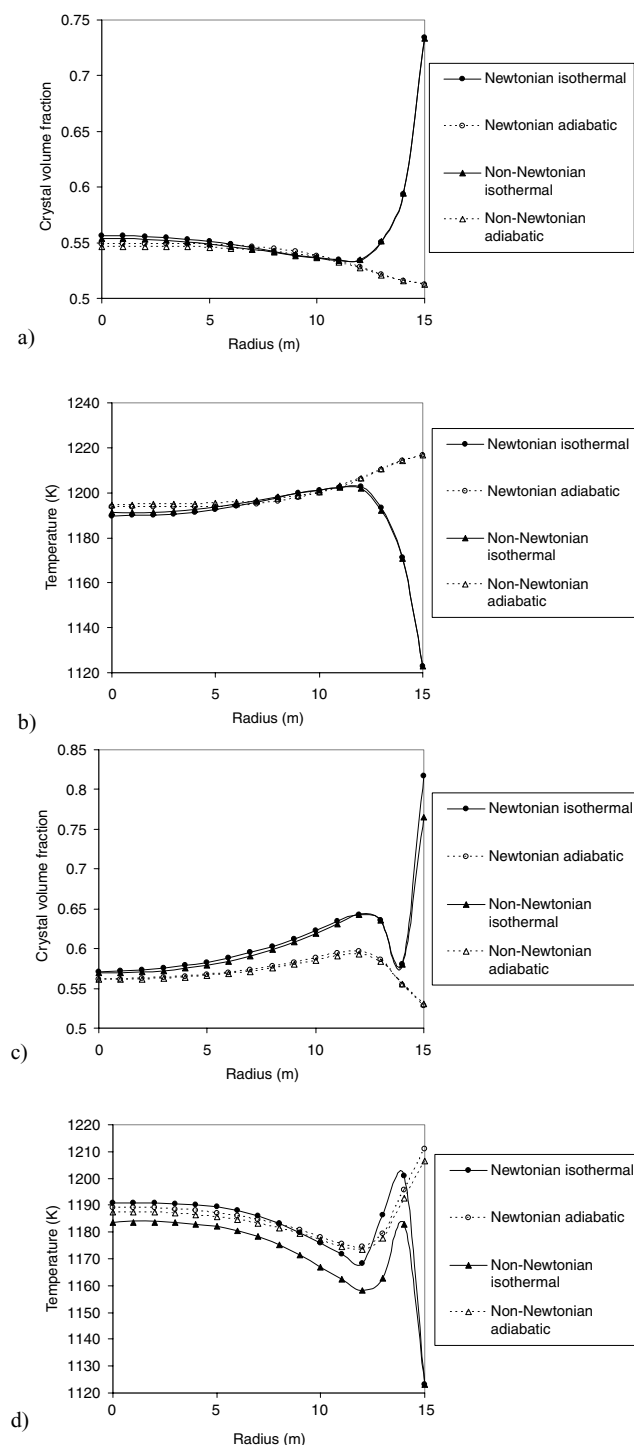


Figure 9. Comparison of crystal volume fraction and temperature field at the conduit exit. (a) Crystal volume fraction for a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (a and b) and $3 \text{ m}^3 \text{ s}^{-1}$ (c and d).

temperature maximum increases rapidly and does not equal the temperature in the centre of the conduit. Latent heat released requires time to diffuse into the centre of the conduit. Beyond a distance of 4640 m the amount of crystallization and latent heat released is too large to diffuse into the core of the conduit and this results in the peaked temperature profile shown in Fig. 8.

The experiments of Geschwind & Rutherford (1995) into the crystallization of rhyolitic melts of similar composition to Montser-

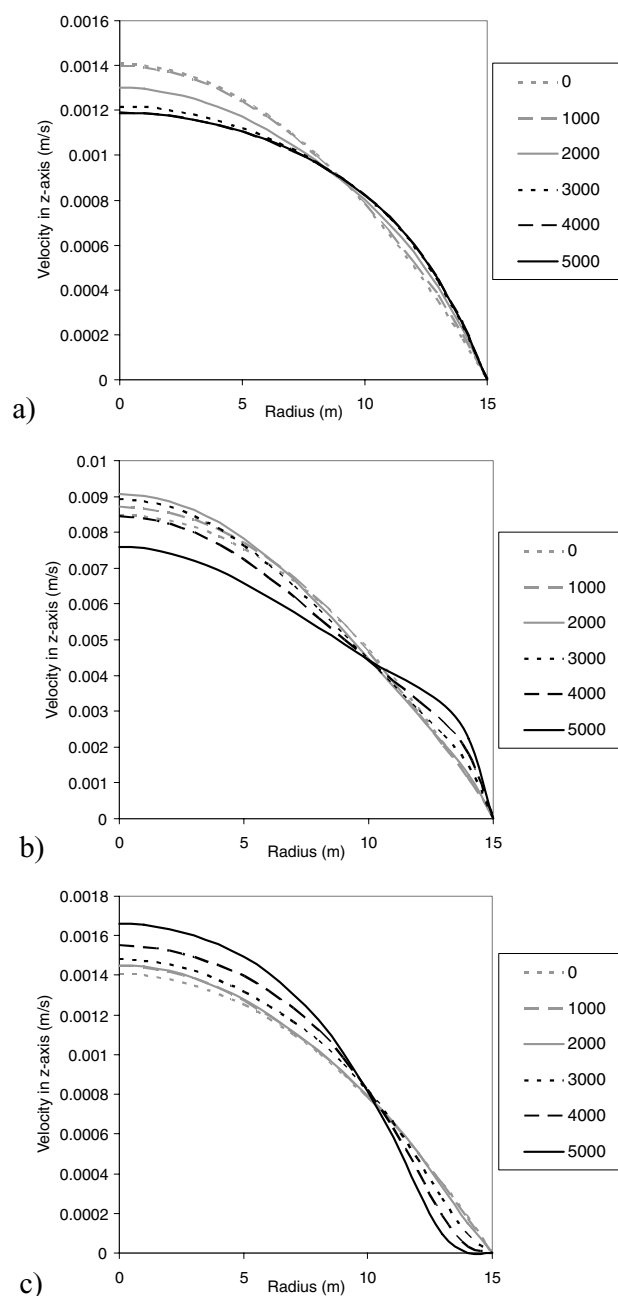


Figure 10. Flow profile across the conduit exit for magma ascending in a conduit with an initial crystal volume fraction of 0.5 and a reservoir pressure of 145 MPa. Newtonian flow with adiabatic conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (a) and $3 \text{ m}^3 \text{ s}^{-1}$ (b). Newtonian flow with isothermal conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (c) and $3 \text{ m}^3 \text{ s}^{-1}$ (d). Non-Newtonian flow with adiabatic conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (e) and $3 \text{ m}^3 \text{ s}^{-1}$ (f). Non-Newtonian flow with isothermal conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (g) and $3 \text{ m}^3 \text{ s}^{-1}$ (h). The numbers in the key correspond to the distance in metres from the magma reservoir exit.

rat, initially with 4 per cent water, indicate undercooling of 150–200 °C as a consequence of degassing. We find a similar range of values for the isothermal conduit wall boundary condition models (Fig. 13). With adiabatic conduit walls a maximum undercooling of 50–80 °C is achieved. For a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ and isothermal conduit walls (Fig. 13c) the undercooling in the centre of the conduit at depth is higher than that experienced at the conduit wall due

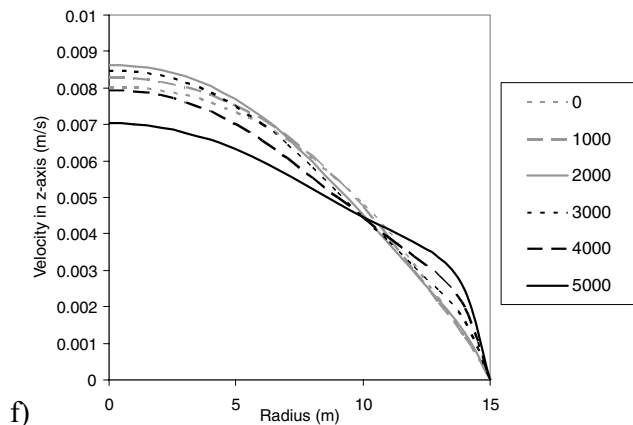
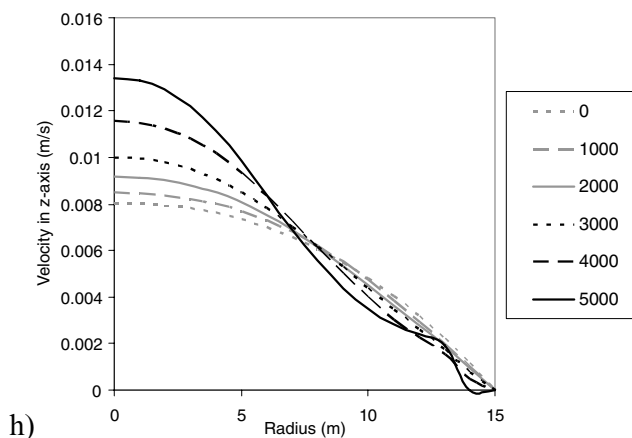
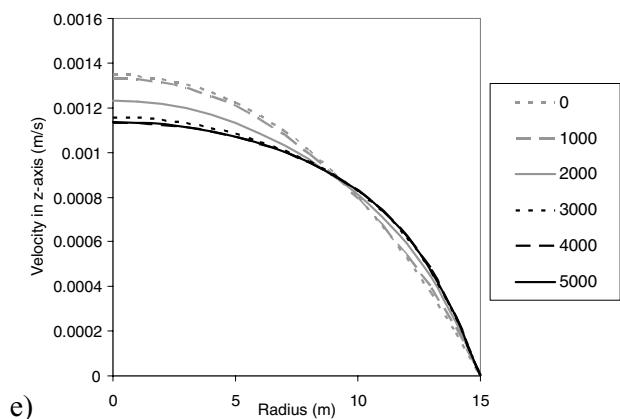
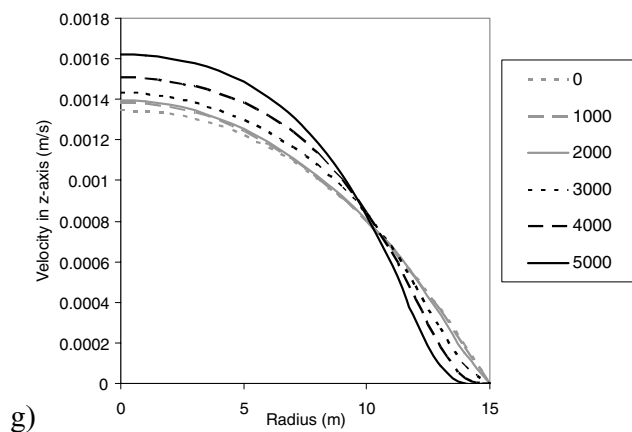
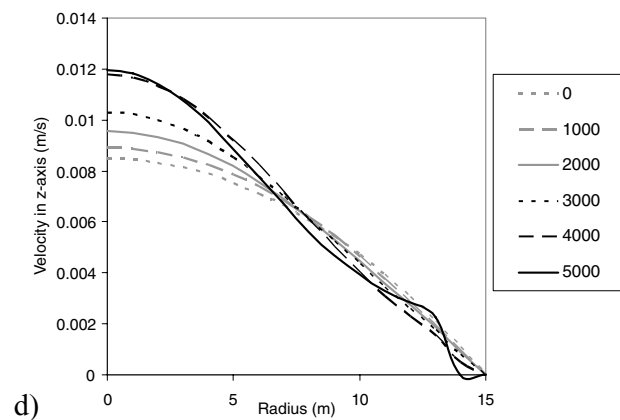


Figure 10. (Continued.)

to the cooler conduit walls allowing the crystallinity to be closer to equilibrium initially. In the centre of the conduit the temperature increase due to latent heating has not penetrated into the core allowing the undercooling to be greater here. After a distance of approximately 500 m from the magma reservoir, the undercooling in the centre is less than that experienced at the conduit wall due to the increase in temperature in the conduit centre and the maintained lower crystal volume fraction. That is, the crystallinity is suppressed at the conduit walls and here the undercooling will be greatest. The undercooling at the conduit exit for this flux varies between 56 and 122 °C. For a flux of 3 m³ s⁻¹ the undercooling follows a similar trend with the crossover distance being 2000 m from the magma

Figure 10. (Continued.)

reservoir and the undercooling at the conduit exit being between 74 and 141 °C (Fig. 13d).

For adiabatic conduit walls and a flux of 0.5 m³ s⁻¹ (Fig. 13a) the maximum undercooling is essentially always experienced at the centre of the conduit where the temperature is lowest. For the higher flux of 3 m³ s⁻¹ the maximum undercooling is only at the centre of the conduit until a distance of 2400 m from the magma reservoir. After this distance the undercooling is at a maximum corresponding to the temperature minimum shown in the profiles of Fig. 8(b).

4 DISCUSSION

Our conduit flow models show that there may be an effective competition between shear heating and crystal growth (latent heat release). A thermal runaway can be suppressed for the range of fluxes we have considered due to the growth and subsequent influence of crystals (Fig. 3). Thus we can conclude that the growth of crystals acts to stabilize the flow field at the timescales observed in crystal-rich magma flows. Fluctuations in discharge rate at Soufrière Hills Volcano occur over many timescales Sparks *et al.* (1998). Short-period changes in flux (days to tens of days) could be related to crystal growth suppression in the centre of the conduit due to enhanced temperatures from latent heat release. We expect, therefore, that latent heating-induced crystallization will have a larger effect upon the viscosity than shear heating over these shorter periods. For longer time periods or higher fluxes, shear heating may play a role. Due to the high Peclet number, the characteristic distance over which viscous heating becomes relevant is likely to be longer than the length of

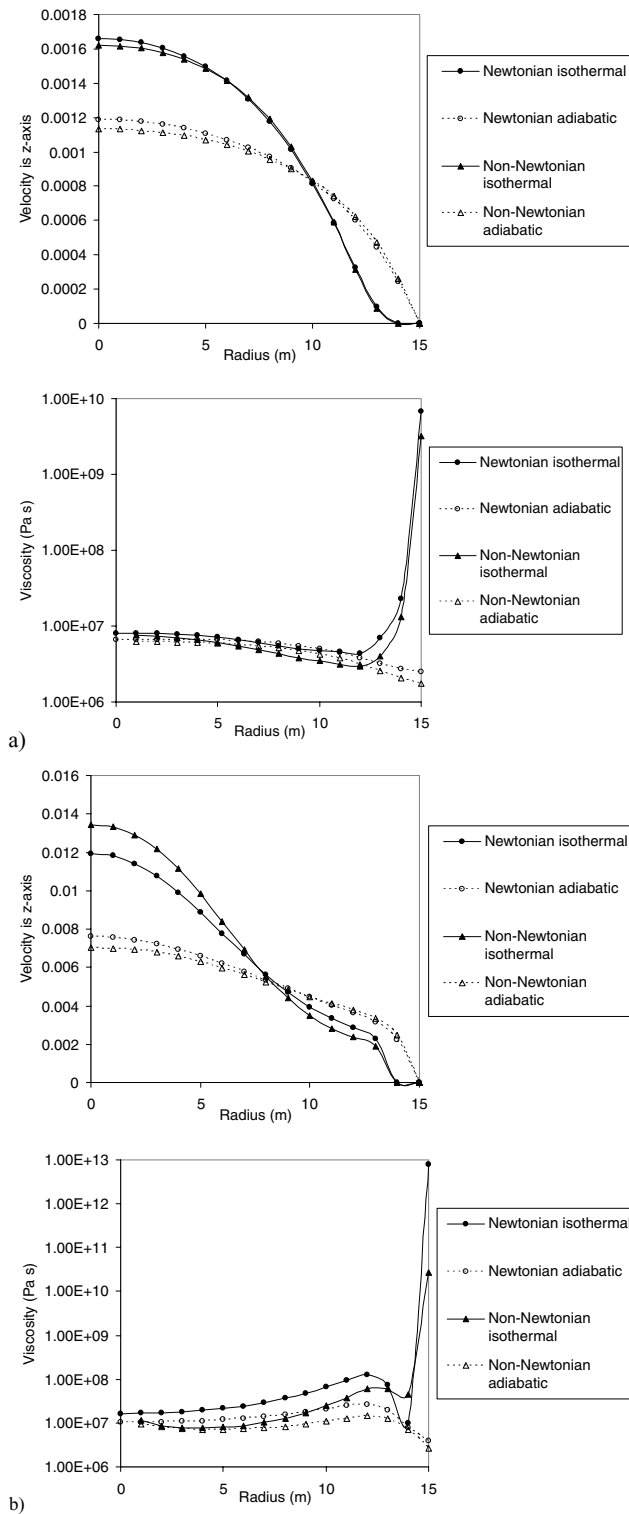


Figure 11. Comparison of flow and viscosity profiles at the conduit exit, for a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (a) and $3 \text{ m}^3 \text{ s}^{-1}$ (b).

the conduit for Soufrière Hills Volcano (Costa & Macedonio 2003). However, even moderate increases in temperature are likely to have important consequences, for example in the suppression of crystal growth. Given enough time and higher fluxes, viscous heating may alter the cross-conduit magma properties but it is unlikely to produce a thermal runaway. The actual behaviour of Soufrière Hills

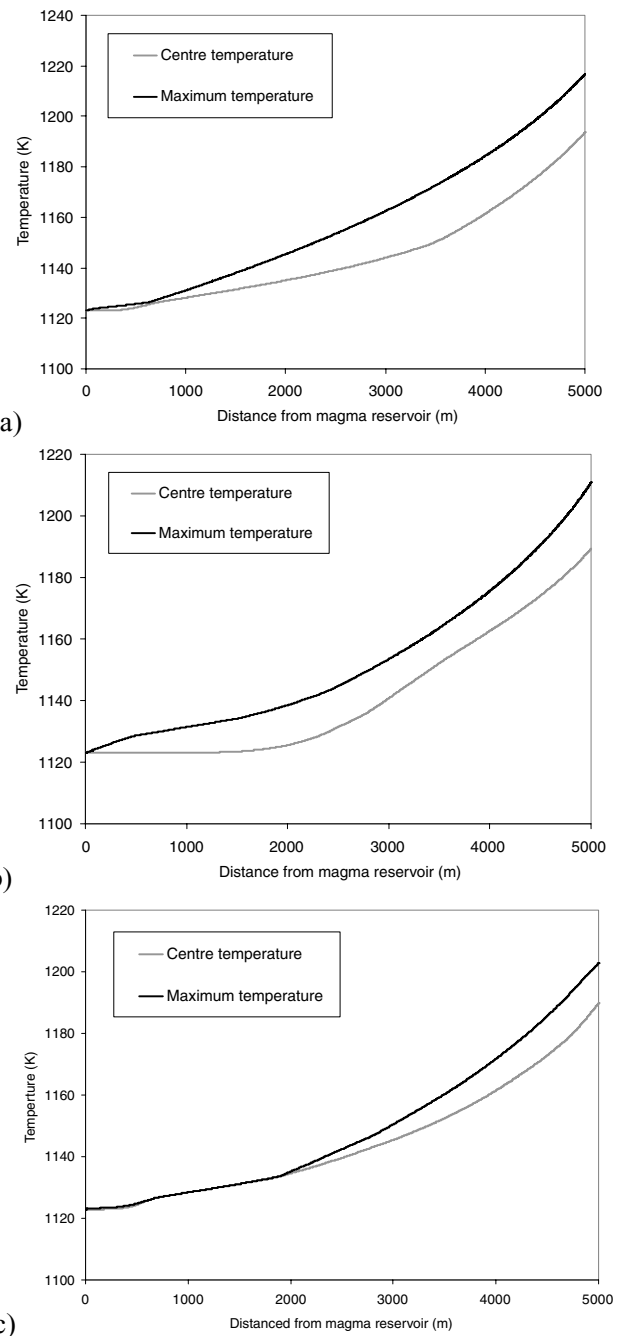


Figure 12. Effects of latent heat release on temperature for four models. Newtonian flow with adiabatic conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (a) and $3 \text{ m}^3 \text{ s}^{-1}$ (b). Newtonian flow with isothermal conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (c) and $3 \text{ m}^3 \text{ s}^{-1}$ (d).

Volcano has approached sustained fluxes for several months in late 1997 and again in 2006, but even then there were short-term fluctuations that suggest that other mechanisms were dominating any long-term shear heating effects.

The effects of viscous heating in our models of infinitesimal and finite conduit segments are qualitatively comparable to the generic models of Costa & Macedonio (2003). However, for the range of parameters chosen to represent the Soufrière Hills Volcano conduit we find that the evolution of the flow profile from parabolic to plug-like does not take place. Instead, the parabolic shape becomes more

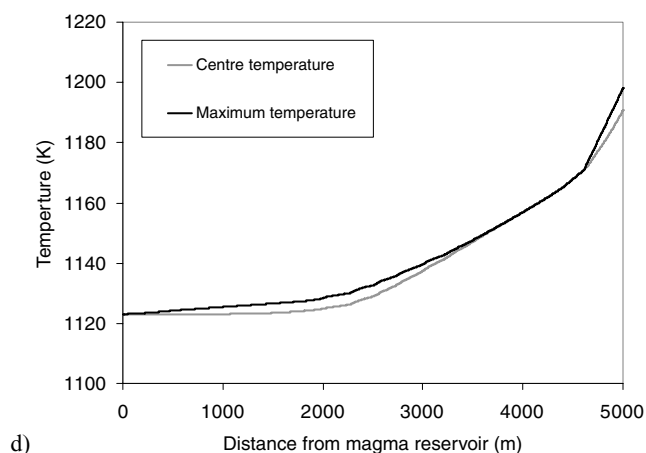


Figure 12. (Continued.)

peaked in the central part of the conduit with a minor effective narrowing of the width of flow at higher fluxes. However, when combined with latent heat effects, the resultant model conduit flow profiles are more plug-like, with increased crystallinities in the centre of the flow. In the equivalent finite conduit models the flow profiles evolve upwards towards the surface to profiles that are more similar to those of the viscous heating-only models. A novel and most interesting features of our analysis are the model predictions for the crystal contents along and across the conduit under different conditions and times. Crystal growth only becomes significant in the top 2 km of the conduit, consistent with the experimental observations on the latent heating of samples from Mt. St Helens which largely occurred at pressures below 50 MPa (Blundy *et al.* 2006, Fig. 2). Across the conduit the crystallization is concentrated near the cold walls, increasingly so for higher fluxes, and in the finite conduit case shown in Fig. 11 a distinct boundary gradient of crystal growth exists in the outermost 1 m of the magma column. For high fluxes the very high crystallinity at the conduit walls with respect to the flow in the centre of the conduit means that the flow is near-stationary for some distance from the walls, effectively narrowing the conduit width. This has implications for the flow dynamics and can act as a means to generate accelerating fluxes.

Although the effects of shear heating and latent heat release on conduit flow seem to be rather similar, their spatial and temporal domains of influence are much less so. Crystallization-induced heating is largely confined to the uppermost 2 km. It requires transit time periods of the order of 10 d (Couch *et al.* 2003a,b) to produce the microlites, and the effect on viscosity and flow is felt over this order of time. Our model of latent heat release due to instantaneous depressurization has occurred several times at Soufrière Hills Volcano, due to wholesale collapse of the lava dome, most notably on 12 July 2003 and 20 May 2006. Viscous heating during shear flow affects the whole conduit, but more so at upper levels and at higher extrusion rates. It takes periods of the order of several hundred days for thermal runaway to develop. Individually, shear heating and latent heating are not capable of producing plug-like flow profiles in our models, but in combination after a substantial time period they may do so.

Thermal feedback through viscous heating in magma conduits (dykes) was originally advocated by Fuji & Uyeda (1974) as a potential mechanism for the development of explosive fragmentation of magma. Mastin (2005) invoked viscous dissipation to explain the,

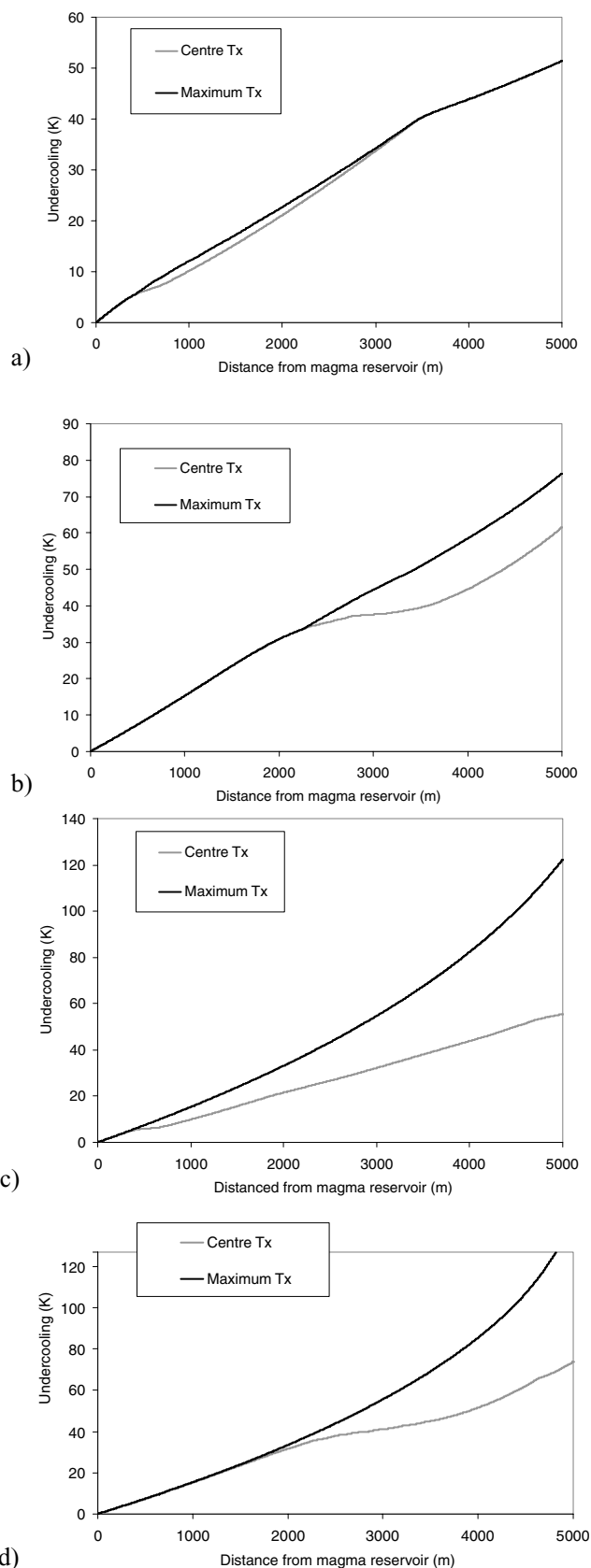


Figure 13. Undercooling along the conduit for four model runs. Newtonian flow with adiabatic conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (a) and $3 \text{ m}^3 \text{ s}^{-1}$ (b). Newtonian flow with isothermal conduit walls with a flux of $0.5 \text{ m}^3 \text{ s}^{-1}$ (c) and $3 \text{ m}^3 \text{ s}^{-1}$ (d).

relatively long, duration of Plinian eruptions and the binary character of some pumice samples produced by them via a reduction in the depth of the fragmentation surface in the conduit. Polacci *et al.* (2001) analysed different pumice samples (white and grey) from the eruption of Mount Pinatubo. They observed that white pumice samples have a higher vesicularity, more deformed vesicles and no microlites in the groundmass. The grey pumice samples had a lower vesicularity, broken crystal fragments and microlites are abundant in the groundmass. They proposed that these different types of pumice evolved from the same magma but formed in different regions of the conduit; the white pumice formed in the centre of the conduit where the temperature and strain-rates were lowest, whereas the grey pumice formed near the conduit walls where the temperature and strain-rates were highest. They concluded that any variation in temperature and strain-rate was a consequence of viscous heating. For the extrusion rates observed during this eruption it is likely that viscous heating would have been important; however latent heat release may also have played a role. A higher microlite content in the grey pumice samples suggests that latent heat could have been liberated which then maintained lower melt viscosities suppressing further crystal growth. Our model results in Fig. 7 show that the viscosity variation across the conduit can be significantly influenced by crystallinity (latent-heat) and shear heating. For enhanced extrusion rates the variation in crystallinity at the conduit walls can also be large, resulting in a peak in the temperature field next to the conduit walls. A lower melt viscosity near the conduit walls could result in high strain-rates and the break-up of crystals.

The Soufrière Hills eruption has been essentially effusive with some Vulcanian explosions, but no Plinian explosions. In 1997 these Vulcanian explosions were periodic (approximately 10 hr) for several weeks (Druitt *et al.* 2002). The explosion (Clarke *et al.* 2002) and conduit (Melnik & Sparks 2002b) model characteristics were explained by a pressurized cap forming above the magma column. Shear heating is much too slow a process to have played a role in these repetitive explosions.

One of the most intriguing aspects of the Soufrière Hills eruption is the initial episode of generally accelerating extrusion rate from November 1995 ($<1 \text{ m}^3 \text{ s}^{-1}$) to March 1998 ($>5 \text{ m}^3 \text{ s}^{-1}$), with a mean value of $4.1 \text{ m}^3 \text{ s}^{-1}$ (Sparks *et al.* 1998). Our use of isothermal and adiabatic boundary conditions for our models may well represent the change in thermal state of the real conduit walls from a cool state at the start of the eruption in 1995 to one near to magmatic temperatures by 1998. The effect of lower temperatures and higher crystallinity at the conduit wall in the early (isothermal) days relative to the later (adiabatic) time is to widen the effective flow area and increase the flux for a constant magma pressure. A similar acceleration in extrusion rate was also observed between August 2005 and January 2006, after a two-year hiatus in lava effusion. Alternatively, the acceleration period, about 2.5 yr, is similar to the time for runaway instability to develop in a flow with shear heating as shown in Figs 2 and 9. Could this accelerating flux have been produced by slowly evolving shear heating? Given that there were huge short-term perturbations in this overall behaviour, including the explosion of 17 September 1996 that evacuated the conduit to a depth of about 4 km (Robertson *et al.* 1998), it would appear that a continuously evolving state of shear heating can be ruled out.

Our analysis has ignored the explicit effects of bubbles, density variations, compressibility, gas escape, heat loss and brittle fracture. The general roles of bubble growth and microlites in modifying conduit magma viscosity are now well known (Sparks 1997) and have led to models of shallow pressurization with multiple steady-state

solutions (Melnik & Sparks 2002a). Costa & Macedonio (2002) also argued for a similar multiple-state behaviour based on cooling-induced viscosity rise. The source of this shallow pressurization, measured by tiltmeters (Voight *et al.* 1999), may be focused in the marginal shear zones of the topmost part of the conduit rather than within the body of the conduit (Green *et al.* 2006). At greater depths, about 1500 m below the top of the lava dome, there seems to be a semi-permanent trigger source for low frequency seismicity. Neuberg *et al.* (2006) argued that this source is the location of brittle failure of magma in the shear zone. It also provides a mechanism to channel gas away from the magma body (Collier & Neuberg 2006). Any future models of the topmost 1.5 km of the conduit at Soufrière Hills Volcano must accommodate the observed phenomena of periodic shallow pressurization and deeper quasi-permanent, low-frequency seismicity.

Most of the conduit modelling based on Soufrière Hills Volcano, including the models presented here, has assumed a cylinder with a length of 5 km and radius of 15 m. This is unlikely to be accurate. Couch *et al.* (2003b) suggested that the conduit had a much smaller cross-section at depth based on the experimental observation that pumice samples with no degassing-induced crystallization had to rise through the conduit in 4–8 hr at most. One way to achieve this is for the cylinder to change to a more tabular, dyke-like body at depth. Such a narrower conduit at depth would be more susceptible to shear and shear heating, though any wall margin cooling effects would be less at greater depths. If this were to be the case, it would serve to reinforce the relative dominance of shear heating/high rise velocities at depth and latent heating at shallower levels already noted in our models. At the lava dome surface, observations of lava spines with variable diameters in the range 25–50 m (Watts *et al.* 2002) suggest that either the spines can expand within the dome itself or that the conduit near its exit may not be of constant bore. One way to achieve the latter is to ream out the conduit during explosions [the occurrence of large spines at the Soufrière Hills lava dome increased noticeably after the explosion of 17 September 1996, (R. Watts, 2006, personal communication)]. Alternatively, secondary rotational flows derived from viscous heating effects modelled by Costa & Macedonio (2005) could be a mechanism that would lead to the thermal or mechanical erosion of the walls of the conduit. Any widening of the conduit locally or temporarily will increase the flux at constant pressure as the relative wall effects are reduced. The magnitude of this will depend on the temperature budget of the flow and has not been modelled by us. Once the conduit is enlarged, can it become narrower again? It is worth stressing that at Soufrière Hills Volcano whatever the effects of such changes over the shorter term, they have not been such that the conduit has evolved noticeably over the past 11 yr.

5 CONCLUSIONS

Our models show that potentially, both viscous heating and crystallization latent heating can be major factors in the flow regime of the magma conduit beneath Soufrière Hills Volcano, and probably at other silicic volcanoes. The model flow profiles across the conduit for viscous heating evolve from initial parabolic flow to become extended in the centre of the flow with the velocities decreasing to near zero close to the walls. When latent heating is added, the models show more plug-like profiles. The domains of influence of the two effects are more distinct. Viscous heating occurs along the whole length of the conduit but only becomes significant at high fluxes. Because crystallization is caused by depressurization in the top

2 km of the conduit, the effects on flow of the attendant release of latent heat are largely confined there. If fluxes are high then viscous heating is enhanced but crystallization is suppressed. The thermal runaway feedback process of shear heating reducing the viscosity and increasing shear at constant pressure occurs in our Soufrière Hills models, but only after periods of several hundred days. The slow acceleration of flux predicted by such models was seen during the 1995–1998 period but was interrupted by several, shorter but major perturbations that should have destroyed any continuity of the process. Conditions of near constant flux for periods of more than a few weeks have been rare at Soufrière Hills. The magmatic overpressure in the reservoir that drives the eruption is much steadier than the flux. For a constant pressure gradient, the models show that the crystal growth tends to suppress tendencies for any thermal runaway. The surface flux boundary condition to these models (the extrusion rate) is generally too poorly known to provide accurate constraints for models over days to weeks. The evolution of variable profiles of crystal content and growth across the conduit is a distinctive outcome of our models. This will be an important factor in more realistic models that additionally take into account gas exsolution and transport and brittle failure at the conduit margins.

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