

Anomalous diffusion of electromagnetic eddy currents in geological formations

Chester J. Weiss¹ and Mark E. Everett²

Received 27 April 2006; revised 3 April 2007; accepted 17 May 2007; published 25 August 2007.

[1] Controlled-source electromagnetic (EM) induction in some geological formations is shown here to be compactly described by an anomalous subdiffusion process. Such a process, which is not universal, is governed by a fractional diffusion equation or alternatively the convolutional form of Ohm's law. A subdiffusing eddy current vortex, or electromagnetic smoke ring, propagates in such a way that its position of median intensity overruns its position of peak intensity. This behavior is not allowed in classical diffusion but is a simple consequence of diffusion within a stationary fractal medium. A similar analysis has been applied to understand heavy-tailed traveltime distributions that appear in certain hydrological time series. The tell-tale signature of anomalous electromagnetic diffusion is a slope β of the magnetic zero-crossing moveout curve that is constant with transmitter-receiver (RX) offset and significantly different from unity. Neither lateral heterogeneity nor uniaxial anisotropy can generate such a constant-slope moveout curve with an economy of model parameters. Controlled-source EM data from two sites in Texas and one in New Mexico are used in this study to test the eddy current subdiffusion hypothesis.

Citation: Weiss, C. J., and M. E. Everett (2007), Anomalous diffusion of electromagnetic eddy currents in geological formations, *J. Geophys. Res.*, 112, B08102, doi:10.1029/2006JB004475.

1. Introduction

[2] The purpose of this paper is to demonstrate that, in certain situations, the diffusion of electromagnetic (EM) eddy currents into geological materials may be compactly described by a fractional or anomalous diffusion equation that generalizes the conventional Maxwell equations governing EM induction in the earth. Furthermore, we argue that recognizing the anomalous diffusion of electric currents in the subsurface from its tell-tale signature in observed EM responses can lead to fresh insights into the interpretation of EM geophysical data. The argument is supported by analysis of ground-based time domain electromagnetic (TDEM) data acquired at several sites in New Mexico and Texas.

1.1. Electrical-hydraulic (eh) Relationship

[3] It has been established that the EM response of the subsurface sometimes provides a good indication of the presence and movement of electrically conductive pore fluids. The EM method is particularly suited for tasks such as monitoring changes in fluid reservoirs or characterizing zones of high salinity. For example, recent applications of crosswell EM technology have demonstrated that EM responses can be used to monitor changes

in reservoirs during enhanced oil recovery operations [Wilt and Alumbaugh, 2003]. Along the Red River in Texas, EM data acquisition from airborne, ground-based, and borehole platforms has mapped, respectively, wide-area soil salinization and the depth extent, thickness, and volume of a saline plume associated with oilfield brine production [Paine, 2003]. These EM data were also able to resolve the subsurface transport of the brine from its source in the Permian uplands to the Pleistocene river terrace.

[4] The above two examples are relatively extreme cases in which the subsurface fluid reservoir is either deliberately placed into a dynamic, nonequilibrium state or has been greatly perturbed by previous human activity. Under natural conditions, EM methods can also provide valuable information such as the preferred orientation of hydraulically conductive fracture sets [Skinner and Heinson, 2004]. A recent 2.5-dimensional inversion has demonstrated the capabilities of controlled-source electromagnetic data for imaging heterogeneous, water-bearing fracture zones beneath a thick conductive overburden in Brazil [Sasaki and Meju, 2006]. Despite these successes, it is clear that EM methods do not have sufficient lateral resolution or depth sensitivity to provide a complete description of the major hydraulic pathways.

[5] The differential equations governing subsurface fluid flow are similar to the equations governing EM induction in geological materials. In fact, induced electric currents flow in response to voltage gradients in the same manner that fluid flows in response to pressure gradients. Electric current and fluid flow are related in principle since both

¹Department of Geosciences, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, USA.

²Department of Geology and Geophysics, Texas A&M University, College Station, Texas, USA.

occur within the same pore volumes and on the same pore surfaces. However, there are important differences between fluid flow paths and electric current paths in a network of rough-walled fractures due, in large part, to differences in the aperture dependence of hydraulic and electric conduction [Brown, 1989; Madadi *et al.*, 2003].

[6] On account of the foregoing argument plus myriad other poorly understood factors that control the bulk electrical and hydraulic properties of mineral assemblages and heterogeneous rock formations, the putative electrical-hydraulic (eh) relationship has long eluded a simple characterization. An attempt, based on the fractal geometry of fracture networks and the spectral analysis of resistivity logs, recently was made to explain an observed eh correlation in the fractured welded tuffs of Nevada [Purvanche, 2003]. The challenge remains for EM geophysicists to construct a comprehensive theory that can explain EM responses in terms of subsurface hydraulic pathways in heterogeneous geological media.

[7] It should be noted that a huge literature has developed on eh correlations based on laboratory measurements of individual rocks and soil samples. An excellent review is provided by Lesmes and Friedman [2005]. Laboratory studies fall somewhat outside the scope of this article since (1) we are concerned here exclusively with field-based studies, (2) due to issues such as length-scale-dependent heterogeneity, the laboratory results have not yet proven applicable to the interpretation of field EM geophysical data in a completely satisfactory manner, and (3) the eh relationship is still not fully understood in the laboratory.

1.2. Anomalous Diffusion: Fundamentals

[8] Given the impossibility of using EM geophysical responses to precisely image individual subsurface hydraulic pathways, yet cognizant of the potential scientific and societal benefits of a breakthrough in our understanding of the hydrogeological implications of EM data, we have turned to recent advances in hydrology for guidance. Concepts related to anomalous diffusion, originally formulated to better understand electric charge transport in condensed matter physics [Scher and Montroll, 1975], have been developed substantially of late in many disciplines including hydrology [e.g., Scher *et al.*, 2002; Kim *et al.*, 2007 and references therein]. These new developments, given the close physical connection between electric and fluid transport, might find relevance in the interpretation of EM geophysical data. This is the topic that we explore further in this paper.

[9] In hydrology, it has long been recognized that there exist a variety of flow paths within a catchment that rainfall can take before it enters the stream. Chloride in rainwater provides a natural chemical signal that can be used to trace the possibilities. Routes could involve subsurface flow, runoff, or lateral flow through the vadose zone. Kirchner *et al.* [2000] compared time series of rainfall and streamflow chloride concentrations from several catchments. The nature of the delayed response of the output streamflow concentration to the input rainfall concentration suggested to Kirchner *et al.* that catchments serve as fractal low-pass filters. Essentially, the time series of streamflow chloride concentration is described by the convolution of a heavy-tailed distribution of traveltimes

from rainfall to streamflow with the time series of rainfall chloride concentration.

[10] Catchments that behave in the foregoing manner are said to retain a long chemical memory of their past inputs. They deliver low levels of chlorine to streams for very long times and, as such, do not possess a mean or characteristic flushing time. Power spectra of observed streamflow chloride concentration exhibit a $1/f$ -type scaling, reflecting the relative absence of short-timescale components. These considerations imply that traveltime distributions $h(t)$ are typical of a (heavy-tailed) power law form $h(t) \sim t^{-m}$ with $m \sim 0.5$.

[11] This phenomenon, known as fractal stream chemistry, has been placed on a firm theoretical footing by Scher *et al.* [2002]. Their physical model for fluid transport in catchments is based on the earlier developed idea of a continuous time random walk (CTRW) [Scher and Montroll, 1975] undertaken by electric charge carriers in an amorphous solid. In the hydrological context, the random walkers are hypothetical rainfall particles and the amorphous solid is the catchment.

[12] The defining characteristic of a CTRW (see also review by Metzler and Klafter [2000]) resides in the aggregate behavior of the random walkers. In classical diffusion, the peak and the median of a propagating pulse $G(x, t)$ of random walkers are located at the same position and travel at the same velocity (Figure 1a). The pulse effectively passes a given point in space after a well-defined time interval. The time it takes for the random walker to hop between successive sites is classically distributed as $\psi(t) \sim \exp(-\lambda t)$ where λ is the characteristic transition rate.

[13] In anomalous diffusion, on the other hand, the median of the pulse $A(x, t)$ travels with a velocity that decreases with time and separates from the peak which remains nearly fixed at the point of origin (Figure 1b). A continuous low-intensity stream of random walkers passes the given point in space for an extremely long time. This behavior reflects the heavy-tailed hopping time distribution $\psi(t) \sim t^{-(1+\alpha)}$ with $0 < \alpha < 1$. Such a distribution does not have a characteristic transition rate. Both the pulse mean position $x(t)$ and the pulse standard deviation $s(t)$ scale as t^α (Scher *et al.*, 2002).

1.3. Fractional Diffusion Equation

[14] It has been shown in a very clear manner by Metzler and Klafter [2000] that the CTRW with hopping time distribution $\psi(t) \sim t^{-(1+\alpha)}$ is equivalent to the time-fractional diffusion equation

$$\frac{\partial}{\partial t} A(x, t) = {}_0D_t^{1-\alpha} \left[v_\alpha \frac{\partial^2}{\partial x^2} A(x, t) \right] \quad (1)$$

where the Riemann-Liouville, or fractional differentiation [Oldham and Spanier, 1974], operator ${}_0D_t^{1-\alpha}$ is written in terms of the convolution

$${}_0D_t^{1-\alpha} A(x, t) = \frac{1}{\Gamma(\alpha)} \frac{\partial}{\partial t} \left[\int_0^t \frac{A(x, t')}{(t-t')^{1-\alpha}} dt' \right] \quad (2)$$

and v_α is a generalized diffusion constant. In the case of $\alpha \rightarrow 1$, equation (1) reduces to the classical diffusion

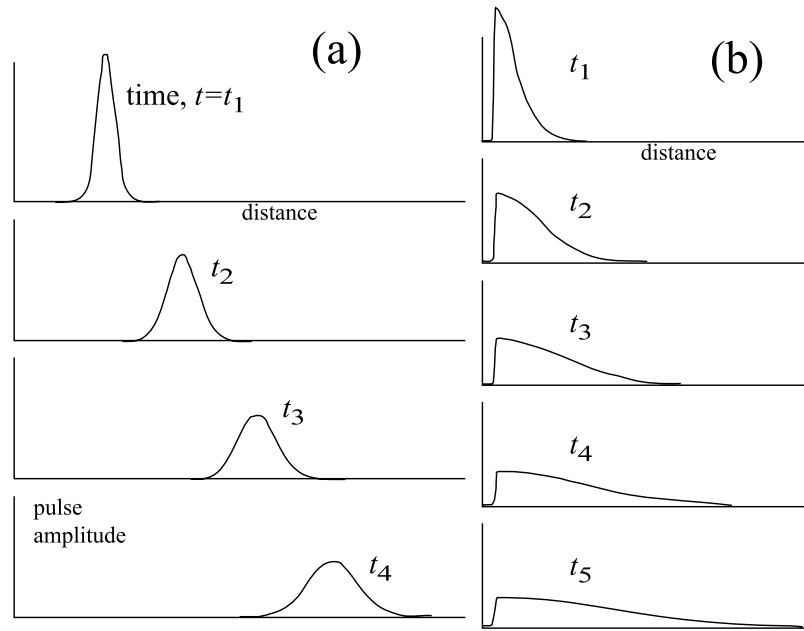


Figure 1. (a) Propagation of a Gaussian pulse $G(x,t)$ undergoing classical diffusion. (b) Propagation of a CTRW pulse $A(x,t)$ undergoing anomalous diffusion (after the work of *Scher and Montroll* [1975]).

equation $\partial A/\partial t = v \partial^2 A/\partial x^2$, as it should, where v is the classical diffusion coefficient. Fractional diffusion equations in a geophysical context have been studied by *Paradisi et al.* [2001], *Caputo* [2003], and many others. There are well-known analytic solutions to equation (1) in terms of Fox hypergeometric functions [*Schneider and Wyss*, 1989]. Numerical solutions to equation (1) are also becoming commonplace [*Tadjeeran et al.*, 2006].

[15] Note that the Gaussian propagator $G(x,t)$ in Figure 1a is a solution to the classical diffusion equation while the CTRW propagator $A(x,t)$ in Figure 1b is a solution to the time-fractional diffusion equation (1). We aim in the paper to motivate the adoption of equation (1) as a potentially useful mathematical description of EM induction in certain geological media. As a first step, we analyze equation (1) in view of EM induction to gain some preliminary insights into its physical implications. Recall that the classical diffusion coefficient is $v = (\mu_0 \sigma)^{-1}$, where μ_0 is the magnetic permeability of free space and σ is the electrical conductivity. We assume throughout that the geological medium is nonmagnetic. Note further that v is in $[m^2/s]$, is constant in time, and is related to the time-varying mean position $\bar{x}(t)$ of the Gaussian propagator by $v = \bar{x}^2/t$. This is consistent with the well-known result that the mean-squared displacement $\bar{x}^2$ scales linearly with t in classical diffusion.

[16] In the anomalous case, the EM induction generalized diffusion coefficient takes on the new constant value $v_\alpha = \bar{x}^2/t^\alpha$, in $[m^2/s^\alpha]$ [*Metzler and Klafter*, 2000]. Since $0 < \alpha < 1$, we always have $v_\alpha < v$; hence our description of EM induction is an example of subdiffusion. The foregoing dimensional analysis also reveals that an anomalous electrical conductivity may be defined according to $\sigma_\alpha = \sigma t^{\alpha-1}$. Thus the anomalous diffusion of eddy currents may be viewed as classical diffusion with an electrical conductivity that continuously grows according to an inverse fractional power of time.

1.4. Anomalous Diffusion: Electromagnetic Induction

[17] Let us now briefly review some salient points about EM induction and subsurface-induced current flow. A controlled-source EM geophysical response is a surface measurement of the diffusing electromagnetic field that is generated by the subsurface-induced electric currents. The latter are caused by the electromotive force (emf) provided by an external EM transmitter under the control of the experimenter. The geometry of induced electric current pathways in the subsurface is prescribed by the fundamental empirical laws of Faraday, Ampère, and Ohm. Knowledge of the subsurface electrical conductivity distribution $\sigma(\mathbf{r})$, along with a capability for forward modeling of the governing equations, is required in order to determine in principle the electric current pathways. Overviews of controlled-source electromagnetic induction are given in many places [e.g., *West and Macnae*, 1991; *Everett and Meju*, 2005; *Fitterman and Labson*, 2005].

[18] Detailed computations based on different sets of assumptions [*Brown*, 1989; *David*, 1993] indicate that electric current and fluid flow take different pathways through a fractured or porous medium. Fluid flow, for example, is strongly affected by roughness of a fracture surface while electric current is more smoothly distributed over the entire width of the fracture surface [*Brown*, 1989, Figure 5]. Both sets of computations reveal that electrical and hydraulic pathways, while certainly not identical, are nevertheless highly correlated. This is not surprising given the similarity of the governing differential equations.

[19] The hydrological example described above indicated that rain falling within a catchment can travel to the stream via a wide range of possible routes. Analysis of chloride tracer concentrations suggests that streamflow is the convolution of past rainfall inputs with the traveltime distribution function for a single rainfall pulse. The traveltime distribution is heavy-tailed. By analogy, we suggest here

the induced electrical currents that flow along diverse pathways within a heterogeneous subsurface. Accordingly, we propose that an analog of the fractional diffusion equation (1) may provide a useful description of EM induction in heterogeneous geological media. A first-principles derivation of the EM induction equivalent of the fractional diffusion equation (1), for the case of a horizontal loop lying on the surface of a uniform half-space, is provided in the Appendix A.

[20] The relevance of a fractional diffusion equation for EM induction originates in the supposition that the EM response is a superposition of responses from individual geological structures that exist at multiple length scales and the mutual induction between them. A classical diffusion equation may be adequate to describe geological media, which can be approximated by blocks of smoothly varying material properties. However, it is the superposition of individual EM responses from induced currents flowing in layers upon layers of hierarchical structures that combines to form the measured EM response and is ultimately governed by a fractional diffusion equation.

[21] We regard the underlying geological medium as inherently rough with coherent structures that are recognizable across multiple length scales, as in fractal geometry. This scenario now has abundant field and laboratory evidence and theoretical support (for example, works of *Barton and Zoback* [1992], *Pilkington and Todoeschuck* [1993], *Bailey and Smith* [2005], *Aharonov and Rothman* [1996], *Radlinski et al.* [1999], *Schlager* [2004], *Acocella and Neri* [2005], and *Purkis et al.* [2005] to cite just a few of the great many examples in the literature). An important implication for EM geophysics is that a rough geological medium points toward the existence of great numbers of potential routes for the induced electric current to flow within the subsurface.

[22] Interpretation of electromagnetic geophysical data in terms of a fractal description of subsurface electrical conductivity is not new. *Everett and Weiss* [2002] showed that profiles of frequency domain controlled-source EM responses are inherently rough, yet reproducible, wiggle for wiggle, if the measurements are repeated. The remarkably consistent signatures indicate that the EM responses are due, not to some instrumental or random noise, but rather to the extremely rough underlying conductivity structure of the ground. The EM response profiles are, in fact, examples of fractional Brownian motion [*Mandelbrot and Van Ness*, 1968], a type of fractal signal.

[23] *Bahr et al.* [2002] have shown that a representation of the middle and lower crust by a network of resistors with fractal geometry can explain deep-probing, natural source electric field array data from Scandinavia. Furthermore, their fractal resistor network supports earlier suppositions about the coexistence of ductile and brittle material in the lower crust, which in turn has important rheological implications for models of crustal formation.

[24] The remainder of the paper is organized as follows. In section 2, we outline the design of our field experiments to test the hypothesis that EM responses might be governed in some instances by a fractional diffusion equation. In section 3, we describe the TDEM observations from sites in New Mexico and Texas. In section 4, we discuss the new interpretation of the acquired TDEM data in terms of conventional interpretations including layered models, hete-

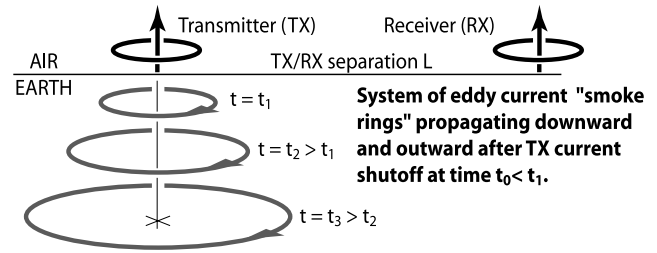


Figure 2. Conceptual representation of the transient EM sounding experiment investigated here.

rogenicity, and anisotropy. In section 5, we discuss the results and their implications for a fractional diffusion model of EM induction in geological materials. The conclusions are presented in this section of the paper.

2. Experiment Design

[25] As mentioned above, the time domain electromagnetic (TDEM) sounding method is a standard geophysical exploration tool for mapping subsurface variations in electrical conductivity due to buried man-made objects and natural variations in rock properties [e.g., *Fitterman and Labson*, 2005]. We employ the TEM47 system manufactured by Geonics Ltd. and described in their Web site www.geonics.com.

[26] In the configuration used here (Figure 2), a circular loop of wire is laid out on the ground and connected to a power supply which drives the loop with a fast (microsecond timescale) ramp-off current waveform. The fast ramp-off, which can be approximated by an abrupt step-off for our purposes, provides the electromotive force, or emf, that causes eddy currents to flow in the underlying, electrically conductive geology. The locus of the maximum intensity of the eddy current distribution diffuses downward and outward with time like a smoke ring [*Lamb*, 1932, p. 210; *Nabighian*, 1979] blown by the transmitter loop. A secondary magnetic field is established by the diffusing vortex of eddy currents. The time derivative of the secondary magnetic field appears as a transient decaying voltage in the accompanying receiver (RX) loop placed on Earth's surface. The RX loop voltage is sampled at discrete time instants, then integrated over discrete logarithmically spaced time gates, and stored to disk.

2.1. Magnetic Zero-Crossing Time

[27] As shown by *Nabighian* [1979] and many others, the transient step-off vertical magnetic field $H_z(t)$ measured at the RX loop undergoes a sign reversal when the radius of the outward-moving eddy current vortex becomes roughly equal to the TX-RX separation distance L . The vortex radius is defined as the location of maximum current intensity. From Figure 1, it is clear that a classically diffusing eddy current vortex will move out from the TX loop in a different manner than an anomalously diffusing vortex. Furthermore, the shape of the vortex will be different. In the classical case (Figure 1a), the current vortex maintains its Gaussian or bell-shaped appearance at all times. In the anomalous case (Figure 1b), the median intensity of the vortex propagates faster than the mean intensity and the shape becomes

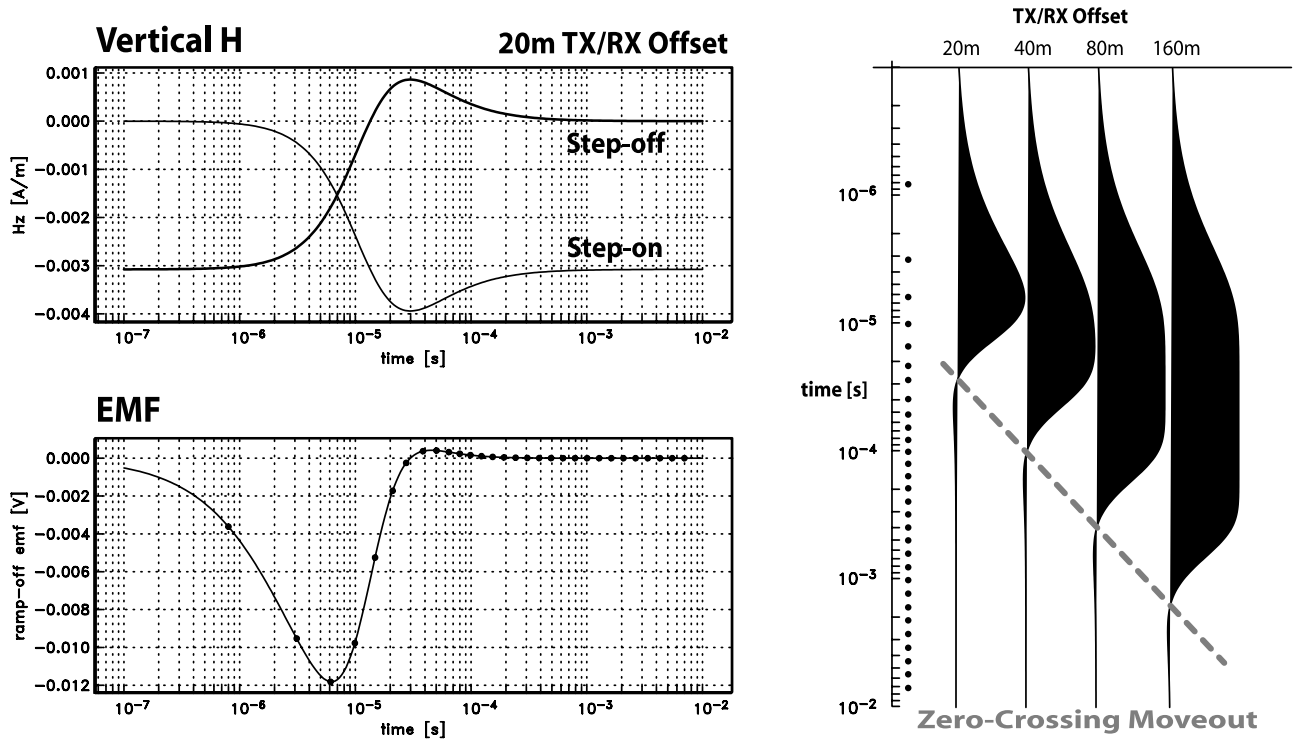


Figure 3. (top left) Time evolution of the magnetic field $H_z(t)$ at a receiver loop corresponding to the step-off transmitter waveform for the case of classical diffusion. The step-on response is also shown for comparison. (bottom left) Induced voltage in a receiver loop for the step-off waveform. (right) Induced voltage transients, normalized by their respective peak amplitudes for four different values of TX-RX separation distance L . The small circles indicate the set of time gates at which the RX voltage is stored by the Geonics PROTEM47 system. The gray line indicates the classical $\tau_H(L)$ moveout curve.

distorted as shown. These considerations apply to a spatially uniform diffusion coefficient $\nu = (\mu_0\sigma)^{-1}$ in the classical case and a spatially uniform generalized diffusion coefficient $\nu_\alpha = (\mu_0\sigma_\alpha)^{-1} = (\mu_0\sigma t^{\alpha-1})^{-1}$ in the anomalous case.

[28] We refer to the instant at which the $H_z(t)$ measured at the RX loop undergoes its sign reversal as the magnetic zero-crossing time, designated by τ_H , in [s]. We hereinafter interpret the magnetic zero-crossing time τ_H as a good indication of the elapsed time, after TX step-off, for the maximum intensity of the subsurface eddy current vortex to pass directly beneath the overlying horizontal RX loop. This is the key assumption of our subsequent analysis. Classically, the zero-crossing time is nearly, although not precisely, the instant at which the maximum intensity of the eddy current vortex crosses the vertical axis defined by the RX loop [Nabighian, 1979]. This is why we refer to it as a magnetic zero-crossing time and not an eddy current moveout time, τ_E . Nevertheless, we select τ_H as our proxy for τ_E since τ_H is readily identified in field recordings of $H_z(t)$. To summarize, τ_H is taken as an estimate of the time that the peak value of the anomalous pulse in Figure 1b, representing the strength of the eddy current vortex, passes beneath the RX loop.

[29] The implication of the foregoing discussion is that measurements of τ_H as a function of TX-RX separation distance L could help to discriminate between classical and anomalous diffusion of subsurface eddy currents. While it is clear that τ_H always increases with L in both the classical

and anomalous cases, the form of the function $\tau_H(L)$ does depend on the style of the diffusion. Recall in the classical case that the mean-squared displacement $\underline{x}^2$ of a Gaussian propagator scales linearly with t . In EM induction, the distance the eddy current vortex has propagated from the TX loop is defined as its mean displacement $\underline{x}(t)$. Our key assumption from the preceding paragraph is that the mean displacement of the eddy current evaluated at the magnetic zero-crossing time is $\underline{x}(\tau_H) = L$. The expectation for classical EM diffusion is therefore the scaling relation $\tau_H \sim L^2$. Using similar reasoning, in anomalous diffusion, we have the scaling $\underline{x}^2 \sim t^\alpha$, and hence the expectation for EM induction in the anomalous case is a scaling behavior of the form $\tau_H \sim L^{2/\alpha}$ with $0 < \alpha < 1$.

[30] In Figure 3, we compute the time evolution of $H_z(t)$ in an RX loop for a step-off current energization of the TX loop over a uniformly conducting half-space occupying the region $z > 0$, with the vertical coordinate z oriented positive downward. A step-on response is also shown for comparison, although we do not consider it further. Also shown in Figure 3 (right) is the magnetic zero-crossing moveout curve $\tau_H(L)$. It is confirmed by the straight-line behavior of the τ_H vs. L^2 plots in Figure 4 that, in fact, $\tau_H \sim L^2$. This remains true whether we use the RX voltage, which is the quantity measured by the TEM47 instrument, or the more fundamental quantity $H_z(t)$. The RX voltage sensed by the induction coil receiver is proportional to the time derivative of $H_z(t)$. The analysis is made for two different

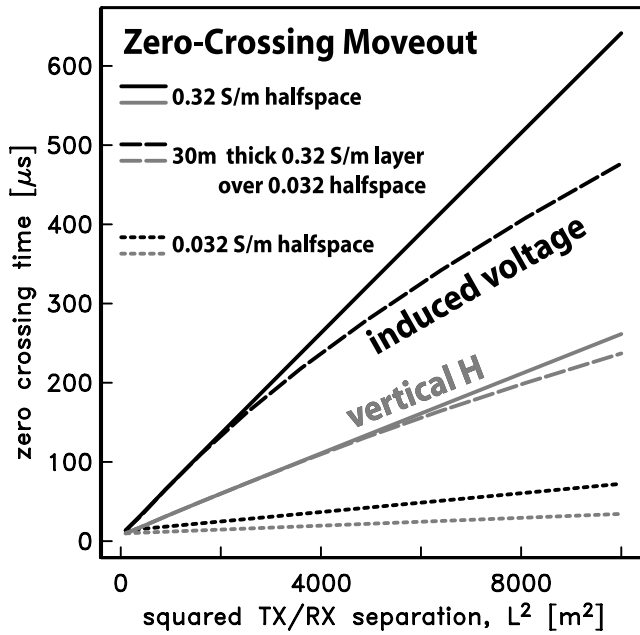


Figure 4. Classical power law behavior $\tau_H \sim L^2$ of the zero-crossing time τ_H for vertical magnetic field $H_z(t)$ and RX voltage measurement. Two examples of a uniformly conducting half-space are shown (solid lines and dotted lines). The classical behavior, i.e., a straight-line plot, breaks down for a layered Earth conductivity model (dashed lines).

conductivities ($\sigma = 0.32$ and 0.032 S/m). Notice that the straight-line behavior breaks down for the layered Earth model since, in that case, the diffusion coefficient $v = (\mu_0\sigma)^{-1}$ is no longer spatially uniform but depends on a conductivity $\sigma(z)$ that varies with the z -coordinate. The results shown in Figures 3 and 4 confirm the well-established result that $\tau_H \sim L^2$ in the case of classical EM diffusion.

2.2. Field Sites

[31] To test the hypothesis that anomalous eddy current diffusion might operate in actual geological formations, TEM47 transient RX voltage waveforms, as a function of TX-RX separation distance L , were collected at three geologically distinct areas in New Mexico and Texas (Figure 5). The sites span a range of stratigraphic complexities. The Sandia Park site is underlain by ~ 400 -m-thick interbedded marine limestone and siliciclastics [Kelly and Northrop, 1975; Ferguson *et al.*, 1999]. The Brazos County site is characterized by ~ 20 m of alluvial floodplain deposits [Wroblewski, 1996] overlying ~ 300 m of interbedded mudstones and clayey sandstones [Yancey and Davidoff, 1991]. Last, the Galveston Island site is a classic example of barrier island development, in this case a ~ 8 -m-thick beach sand lying unconformably over interbedded sands and shales [Cole and Anderson, 1982].

[32] The foregoing geological information allows us to estimate the likely geoelectric complexity at each site. For example, the thickness of the individual beds at the Sandia Park site is similar to the length scale of our TEM47 experiment; thus the site should be amenable to mathematical approximation by a uniform half-space conductivity model. The Brazos County site requires a more thorough treatment using a conductivity model perhaps consisting of a fractal-like hierarchical sequence of alternating conductive and resistive layers to reflect its varied geological history of floodplain depositional and erosional cycles. The Galveston site, with its thin surficial layer of beach sand, should be modelable by a uniformly conducting layer over a terminal half-space representative of underlying Pliocene fine-scale sediments.

3. Observations

[33] At each of the three sites, a convenient site for working was chosen and the TX loop (8 turns, 5.0 m diameter, ~ 3 A current) was laid out on the ground at a fixed location. The RX coil was then moved outward from

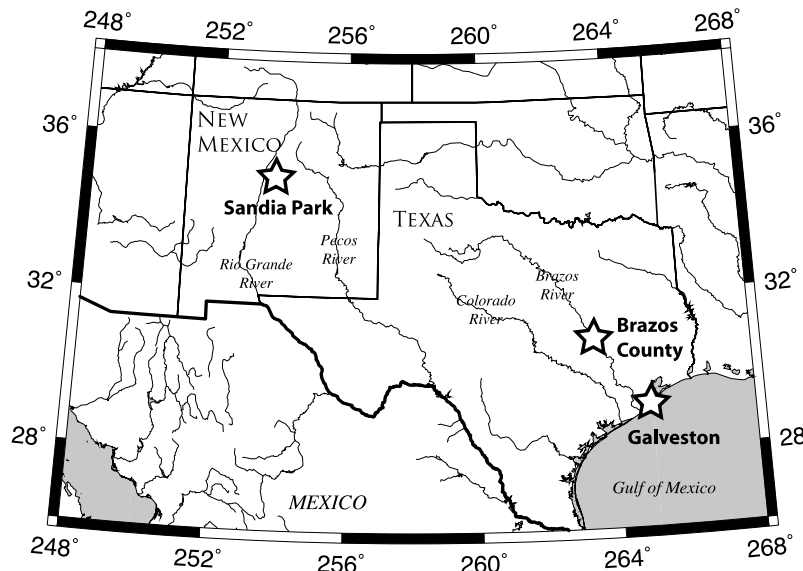


Figure 5. Site locations for TEM47 data acquisition.

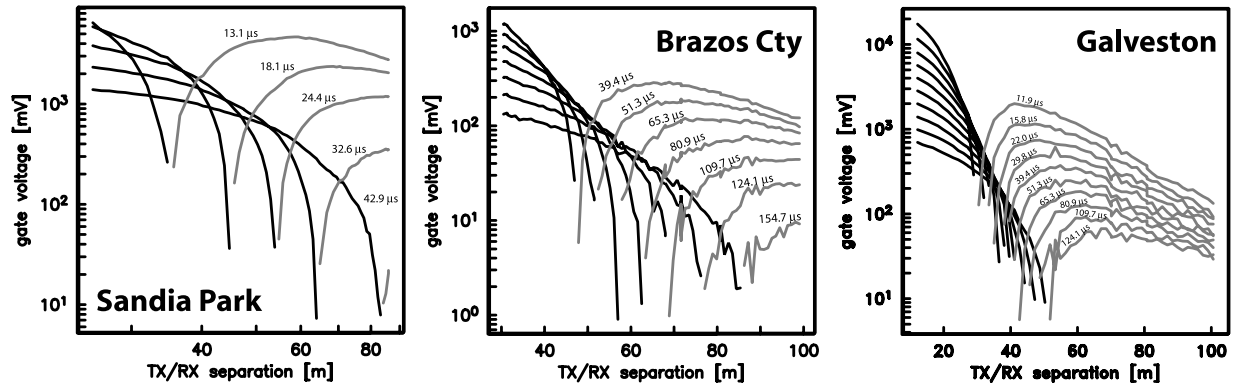


Figure 6. Time gate gathers of TEM47 RX voltage (negative polarity in black; positive in gray) for the New Mexico and Texas sites indicated in the Figure 5. See text for details.

the TX loop along a straight profile from $L \sim 10$ m to ~ 100 m in 1.0-m station increments. At each RX station, a TEM47 RX voltage transient waveform was acquired. The waveform is recorded at 30 logarithmically spaced time gates spanning the time range ~ 10 – 150 μ s after the TX ramp-off, as described earlier.

[34] It is conventional in TDEM analysis to display separately the transient waveforms acquired from each RX station along a profile. However, we find it more useful for our purposes to display the data in a different format. We have gathered the voltage readings from each RX station according to common a time gate. In other words, we construct time gate gathers for each site in which RX voltage is plotted, for the various time gates, as a function of TX-RX separation L .

[35] Representative time gate gathers of the TEM47 RX voltage from the three sites are shown in Figure 6. The curves indicate a clearly observable zero-crossing time τ_H that grows with increasing TX-RX separation L . For example, with a TX-RX separation of $L = 35$ m at the Sandia Park site, the magnetic zero-crossing time is $\tau_H = 13.1$ μ s, while at $L = 45$ m separation, it is $\tau_H = 18$ μ s. In general, the TX-RX separation L associated with the magnetic zero-crossing time τ_H is indicated by the transition from black to gray for each curve. While error estimates are not directly available from the TEM-47 system, the data are generally reproduced to a few percent or less on repeat surveys. The error bars are not shown to simplify the presentation.

[36] The geological formation at Sandia Park is a very thick (compared to the TEM47 exploration depth) sequence of marine limestone and siliciclastics. The expectation for this type of lithology is a relatively low electrical conductivity σ in comparison with the clay-rich Brazos floodplain sediment and the seawater-dominated Galveston beach sands. The relative conductivities can be assessed by examining the behavior of time gate gather curves. In general, the larger the spacing between adjacent zero-crossings in the time gather curves, the lower the conductivity of the subsurface. This is because subsurface eddy current vortices move out faster from the TX loop in resistive media than in conductive media, the diffusion speed scaling as $1/\sqrt{\sigma}$.

[37] The wide intervals between successive zero-crossings in the Sandia Park time gate gather curves indicate a

relatively low electrical conductivity σ compared with the other two sites. A cursory inspection of the Brazos County and Galveston data reveals a moderate conductivity at the Brazos County floodplain and a relatively high conductivity at the Galveston beach site, as expected. The high conductivity at Galveston is likely due to the presence of conductive seawater saturating the pore space at the base of the sand layer.

[38] Our analysis now returns to the RX voltage transients recorded at the individual RX stations. The magnetic zero-crossing time τ_H at a given RX station can be estimated, to sufficient precision for our purposes, by linearly interpolating the transient voltage response between the values that bracket the all-important sign reversal. This analysis has been performed for all RX stations at all three sites. The behavior of τ_H as a function of the squared TX-RX separation L^2 for the three sites is shown in Figure 7. We refer to these as moveout curves.

[39] Recall that the scaling of the moveout curves is $\tau_H \sim L^2$ in the case of classical diffusion and $\tau_H \sim L^{2/\alpha}$, with $0 < \alpha < 1$, in the case of anomalous diffusion. Let us define the slope of the moveout curve as $\beta = 1/\alpha$. In that case, moveout curve with slope $\beta = 1$ is consistent with classical diffusion. A linear moveout curve with $\beta > 1$ (such that $\alpha < 1$) is consistent with an anomalous diffusion process, in particular, a subdiffusion process. A nonlinear moveout curve indicates EM induction that cannot be explained by eddy current diffusion in a geological medium having a uniform (classical or anomalous) diffusion coefficient.

[40] Note that the slopes of the moveout curves vary considerably among the three sites. The data at Sandia Park appear to be typical of classical diffusion. The Brazos County data (gray and black dots) are clearly consistent with an anomalous diffusion process, with well-defined slopes of $\beta = 1.20$ and 1.26 for two data sets acquired at different dates with different TEM47 instruments. The later data set extended the earlier data set to larger values of TX-RX separation L . Finally, a more complicated pattern is observed at Galveston in which the moveout curve $\tau_H(L^2)$ is definitely nonlinear. The three or four cusps that can be observed throughout the Sandia Park moveout curve are processing artifacts caused by uncertainty in the linear interpolation of RX voltage responses at early time gates. This cause of this artifact is a consequence of the relatively

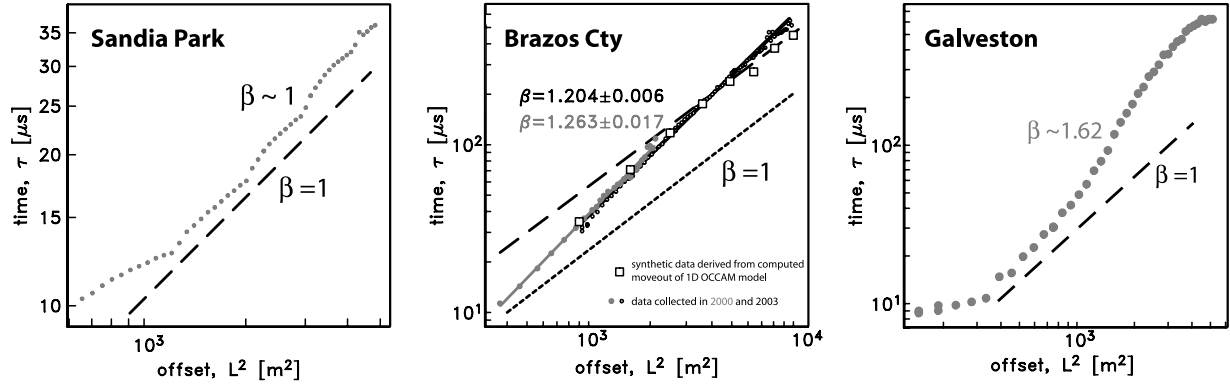


Figure 7. Log-log plots (dotted lines) of the magnetic zero-crossing time τ_H as a function of the squared TX-RX separation L^2 for the three sites. A slope of $\beta = 1$ (dashed lines) indicates classical diffusion. Included in the Brazos County data are zero-crossing times (squares) for a smooth conductivity depth profile $\sigma(z)$ consistent with the measured RX voltage transients.

low conductivity σ at Sandia Park and does not affect the other moveout curves.

4. Interpretation: Layered Models, Heterogeneity and Anisotropy

4.1. Layered Models

[41] The observed TEM47 responses at Sandia Park and Galveston are readily explained by classical eddy current diffusion. Whereas the case for Sandia Park is fairly clear-cut since the slope of the moveout curve $\beta \sim 1$, the case for Galveston requires further analysis. We have computed theoretical TEM47 responses using an analytic forward code for one-dimensional electrical conductivity models $\sigma(z)$ consisting of a layer over a half-space. Two scenarios are considered: the first involves a conductive layer over a resistive half-space, the second is a resistive layer over a conductive half-space. The two conductivities, in both scenarios, are $\sigma = 0.32$ (conductor) and 0.032 S/m (resistor). The layer thickness is allowed to vary between 1 and 30 m.

[42] The magnetic zero-crossing time curves $\tau_H(L^2)$ computed for the two-layered $\sigma(z)$ models are shown in Figure 8. Comparing these curves to the Galveston measured curve

$\tau_H(L^2)$ in Figure 7 reveals that the Galveston observations can be mostly explained by a resistive layer of ~ 10 – 20 m thickness over a conductive half-space. The nonlinear behavior of the Galveston $\tau_H(L^2)$ curve at large L^2 is not reproduced by the two-layer model but it is likely that a second resistive zone at depth could account for it.

[43] The Galveston observations rule out the hypothesis that anomalous eddy current diffusion in a “stationary fractal” medium is occurring at that site. We define a stationary fractal medium as one which can be characterized by an anomalous diffusion coefficient v_α that does not depend on position. It might be noted, however, that both the observations shown in Figure 7 and the ensemble of curves shown in Figure 8 demonstrate that $\tau_H(L^2)$ moveout data give rise to an apparent fractional diffusion. This is evidenced by a linear moveout with slope $\beta > 1$. This case occurs if the TDEM data are available only over a limited range of L such that nonlinearity in the $\tau_H(L^2)$ curve cannot be resolved.

[44] We now explore whether the Brazos County data (Figure 7, middle) require an explanation in terms of anomalous diffusion in a stationary fractal medium. Certainly, the observed $\tau_H(L^2)$ moveout curve is linear with a

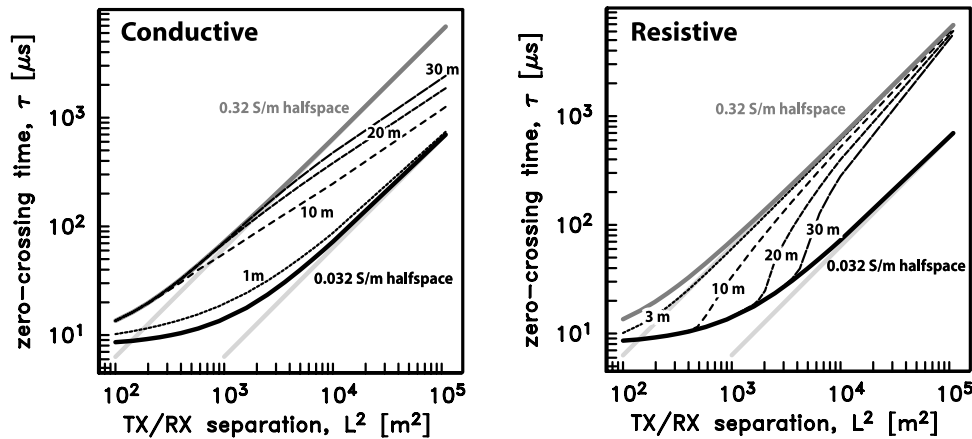


Figure 8. Magnetic zero-crossing time curves $\tau_H(L^2)$ computed for two-layered Earth conductivity models $\sigma(z)$ using a one-dimensional analytic forward code: (left) a conductive layer over a resistive half-space; (right) a resistive layer over a conductive half-space. Curves are computed for varying layer thickness, as indicated. Light ray lines indicate a slope of $\beta = 1$.

slope $\beta > 1$. The TEM47 RX voltages at the Brazos site were inverted using an Occam style layered inversion [Constable *et al.*, 1987] to recover the smoothest profile $\sigma(z)$ that fits the data below a specified tolerance.

[45] A synthetic $\tau_H(L^2)$ moveout curve (square symbols in Figure 7, middle) was extracted from the computed TEM47 responses of the smoothest model. Notice that the synthetic moveout curve matches the short-offset observations, including the linear behavior with a slope $\beta \sim 1.2$. The long-offset behavior ($L > 55$ m) of the synthetic moveout curve based on the smooth conductivity depth profile tends toward the classical limit $\beta \rightarrow 1$. The observed Brazos moveout curve maintains, however, its anomalous $\beta \sim 1.2$ slope to the longest TX-RX offsets that were recorded in the experiment.

[46] A possible explanation for the persistence of the anomalous moveout of the Brazos County $\tau_H(L^2)$ data could be the presence of multiscale geological heterogeneity associated with the river floodplain sediments over which the TEM47 observations were made. The observed linear moveout with slope $\beta \sim 1.2$ is easily reproduced by replacing integer order derivatives in the governing EM diffusion equation with fractional ones.

[47] As we have shown, a smooth multilayered conductivity profile $\sigma(z)$, based on the conventional Maxwell diffusion equations, generates a classical response that approximates the observed linear moveout over a certain range of TX-RX offsets. In general, however, an N -layered model requires $2N-1$ parameters (layer conductivities and thicknesses) to reproduce the linear $\tau_H(L^2)$ moveout curve. The central advantage of the fractional diffusion approach is that just a single parameter, β , is required to explain the same set of observations. Recall from earlier discussions in the text that the β parameter describes the anomalous diffusion of the subsurface eddy current vortex intensity. In particular, β is a measure of how the peak position and the width of the vortex scale with time.

[48] The compact description of EM induction at Brazos in terms of a single model parameter is the chief reason that we offer the fractional diffusion interpretation. The conventional Maxwell diffusion equation based on integer order derivatives requires a multitude of parameters in order that the underlying conductivity model replicates the observed linear $\tau_H(L^2)$ moveout. Note that the classical Occam solution departs at large TX-RX offsets from linearity. There is good physical justification for the fractional diffusion approach. A smooth Occam model is a piecewise-constant representation of the subsurface conductivity $\sigma(z)$ profile. This parameterization is simply a convenient mathematical idealization of the geologic heterogeneity of floodplain deposits. The fractional diffusion approach that we advocate naturally accommodates a rough geological structure at all length scales, in fact, that is its *raison d'être*.

4.2. Heterogeneity

[49] We have interpreted the anomalous $\tau_H(L^2)$ moveout behavior at the Brazos river floodplain, with its slope of $\beta \sim 1.2$, in terms of EM induction in a stationary fractal medium. It is important to investigate whether the same behavior of the $\tau_H(L^2)$ moveout curve could be caused by an underlying anisotropic or three-dimensional heterogeneous electrical conductivity structure. This possibility has

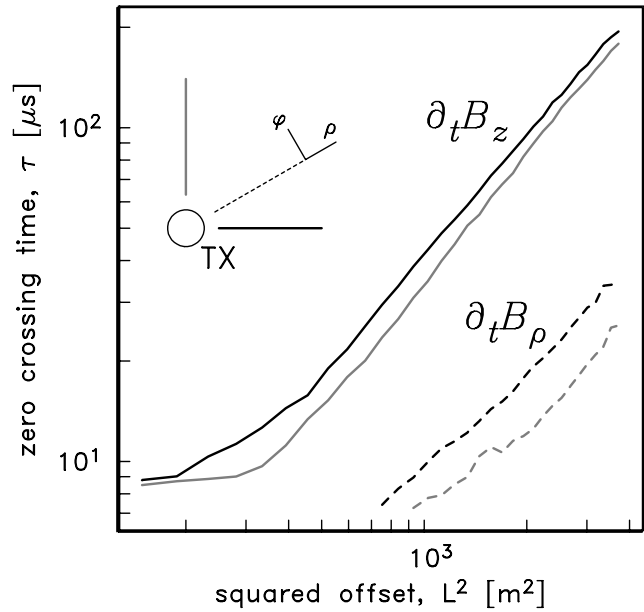


Figure 9. (*inset*) Two perpendicular TEM47 measurement profiles (line 0 in black; line 1 in gray) with a common TX loop position at the Brazos floodplain site. North points toward the top of the inset. (*main figure*) Magnetic zero-crossing moveout curves $\tau_H(L^2)$ for vertical and radial field components.

been tested by a second round of experimentation at the Brazos site. We have recorded all three components of the transient magnetic field, using three mutually orthogonal induction coil receivers, along with two perpendicular profiles. The experimental setup is shown in the inset of Figure 9, in which the gray and black lines represent the two orthogonal measurement profiles.

[50] The observed signals are the voltages, as a function of TX-RX offset, induced in receiver coils whose axes are oriented, respectively, in the vertical z and radial ρ directions. The latter is directed along the line between the TX loop and RX station. The observed voltages are proportional to $\partial B_z/\partial t$ and $\partial B_\rho/\partial t$, respectively. As shown in Figure 9, the slope of the $\tau_H(L^2)$ moveout curve does not depend on either the coil orientation or on the profile direction. Different profiles, which sample different regions of the subsurface, or different coil orientations, which sample different aspects of the eddy current vortex, should generate different slopes if the slope is controlled by a laterally heterogeneous electrical conductivity. Instead, the Brazos floodplain appears to act as a stationary fractal medium regardless of the TX-RX profile or RX coil orientation. The parallel slopes of the vertical and radial $\tau_H(L^2)$ moveout curves seem to argue against lateral heterogeneity as a cause of the apparent anomalous diffusion at the Brazos site.

[51] An indication of lateral heterogeneity at the Brazos site is given in Figure 10. The azimuthal response $\partial B_\phi/\partial t$ due to a horizontal TX loop should vanish over a uniform or plane-layered Earth. At all stations, however, an azimuthal response is observed. Although considerably smaller than the vertical $\partial B_z/\partial t$ and radial $\partial B_\rho/\partial t$ responses, the nonzero azimuthal response indicates that the subsurface is laterally heterogeneous to some extent. However, as shown above, the subsurface heterogeneity is insufficient to appreciably

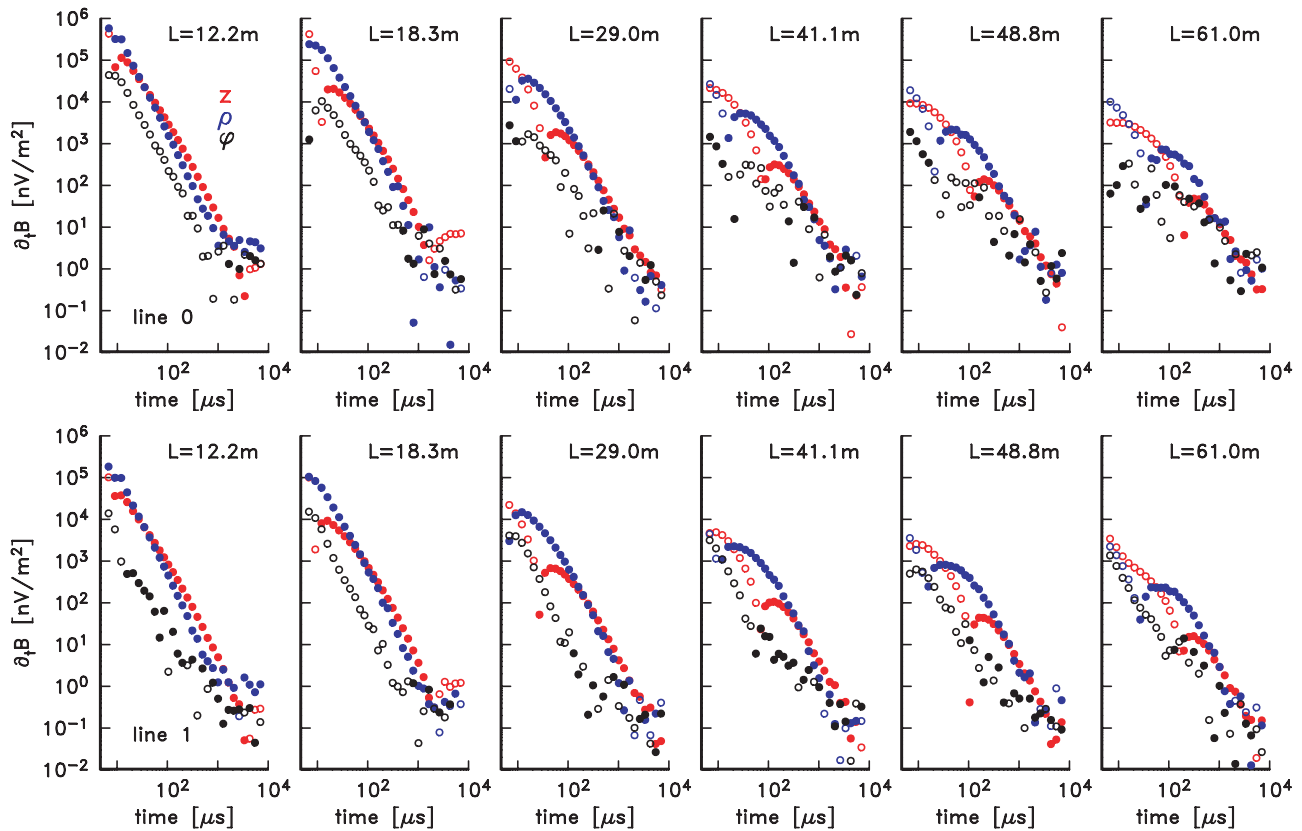


Figure 10. TEM47 transients recorded at the Brazos site for various TX-RX offsets L . All three RX components (vertical $\partial B_z/\partial t$, radial $\partial B_\rho/\partial t$, and azimuthal $\partial B_\phi/\partial t$) are shown. The top panel displays measurements along the *black* line while the bottom panel corresponds to the gray line shown in Figure 9, inset.

affect the slope of the radial and vertical moveout curves. The conclusion we draw is that the Brazos site appears to be a stationary fractal medium despite the demonstrable presence of some lateral heterogeneity.

4.3. Anisotropy

[52] The possible role of lateral anisotropy in generating an apparent anomalous diffusion at the Brazos County site has also been investigated. Consider a spatially homogeneous uniaxial medium characterized by an electrical conductivity tensor of the form $\sigma = \text{diag}(\sigma_\perp, \sigma_\parallel, \sigma_\parallel)$ where $\sigma_\perp$ is the electrical conductivity across the strike of the anisotropic structure and $\sigma_\parallel$ is the along-strike electrical conductivity. An overview of possible geological structures that could generate such an electrical conductivity tensor is provided by *Everett and Constable* [1999].

[53] We have computed magnetic zero-crossing $\tau_H(L^2)$ curves for the uniaxial medium. The calculations are based on the Fourier transformation of the relevant analytic solution in the frequency domain [*al-Garni and Everett, 2003*]. Similar calculations have appeared recently in the work of *Collins et al.* [2006] who examined the TEM47 response of a foliated metamorphic schist modeled as a layered uniaxial medium.

[54] In Figure 11, the $\tau_H(L^2)$ curves for various values of $\sigma_\perp$ and $\sigma_\parallel$ are shown. For a ready comparison between the curves, but without loss of generality, the geometric mean conductivity $\sigma_m = \sqrt{\sigma_\parallel \sigma_\perp}$ is held fixed at $\sigma_m = 0.1$ S/m. It is

straightforward to observe that the $\tau_H(L^2)$ is classical with slope $\beta = 1$, regardless of the anisotropic ratio $\lambda = \sqrt{\sigma_\parallel/\sigma_\perp}$. In this way, we have demonstrated that a uniaxial medium characterized by a tensor $\sigma = \text{diag}(\sigma_\perp, \sigma_\parallel, \sigma_\parallel)$ cannot account for the observed anomalous diffusion at the Brazos site, the hallmark of which is the slope $\beta \neq 1$.

[55] We conclude this section by remarking that it should be possible to generate a heterogeneous and/or anisotropic Earth model of sufficient complexity to precisely reproduce the observed $\tau_H(L^2)$ curve at the Brazos site. However, the electrical conductivity distribution $\sigma(\mathbf{r})$ of such a model would likely require many parameters to describe. We perform this task with a single model parameter β by supposing that the EM induction process is one of the anomalous diffusion within a stationary fractal medium.

5. Discussion and Conclusion

[56] Fractional derivatives naturally arise when one considers transport processes within spatially hierarchical materials [*Nigmatullin, 1984; 1986; Metzler and Klafter, 2000*]. The spatial hierarchy of the electrical conductivity structure of geological materials, while not universal, has previously been observed over the Brazos river floodplain deposits [*Everett and Weiss, 2002*]. The fractional diffusion equation (1) suggested by the Brazos County data has a simple physical interpretation based on the anomalous diffusion of the subsurface eddy current vortex. The process is

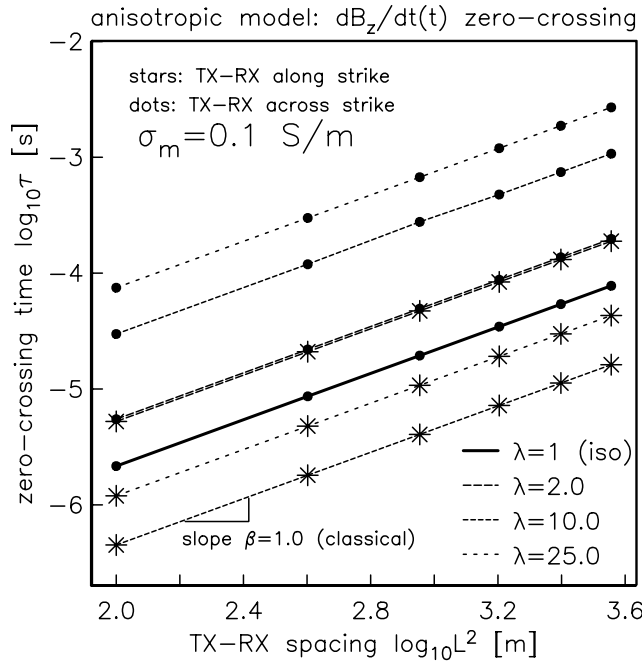


Figure 11. Magnetic zero-crossing time curves $\tau_H(L^2)$ computed for uniaxial Earth conductivity models of the form $\sigma = \text{diag}(\sigma_{\perp}, \sigma_{\parallel}, \sigma_{\parallel})$ with fixed mean conductivity $\sigma_m = \sqrt{\sigma_{\parallel}\sigma_{\perp}}$ and variable anisotropic ratios $\lambda = \sqrt{\sigma_{\parallel}/\sigma_{\perp}}$. Both along-strike (stars) and across-strike (dots) TX-RX profiles are shown. In all cases, the slope of the moveout is classical, with $\beta = 1.0$.

entirely analogous to the one described in the classic paper by *Scher and Montroll* [1975] in which the median of a diffusing pulse overruns its peak value.

[57] The fractional diffusion description of EM induction, governed by equation (1), is entirely compatible with the classical integer order Maxwell equations encompassing the convolutional form of Ohm's law, $\mathbf{J} = \sigma * \mathbf{E}$ [*Jackson*, 1975], where $*$ denotes the time-convolution operation. The compatibility of the two approaches is readily understood since a fractional derivative is nothing more than a convolution integral [*Oldham and Spanier*, 1974]. In short, the geometric complexity of the medium can be encapsulated either as a fractional derivative in the governing diffusion equation, as we have done here, or as a convolutional constitutive relation. Here we present direct field observations consistent with the former approach but fully recognize that others may wish to retain the classical integer order Maxwell equations and load the medium complexity into Ohm's constitutive relation. It should be recognized, however, that neither Maxwell nor Ohm was concerned with geological or other spatially hierarchical media when he formulated his equations.

[58] The fractional diffusion equation (1) describes EM induction in a one-dimensional stationary fractal Earth in which eddy current vortices, or smoke rings in the language of *Nabighian* [1979], diffuse with anomalous velocity. The Brazos County data indicate anomalously long magnetic zero-crossing times τ_H compared with the classical expectation. The resultant subdiffusion process at Brazos County implies that a fractional derivative of order $0 < \alpha < 1$ is appropriate. Our findings do not require the medium to

require any special electrical properties such as memory or polarizability; it is a purely a geometric effect.

[59] The time domain EM induction data presented here indicate that classical diffusion is not always the most compact physically based description when interpreting geophysical interrogations of the subsurface. Our data instead suggest that a fractional diffusion equation, founded on a simple but powerful physical justification, may apply in some cases. The fractional diffusion approach has advantages over the classical approach in terms of an economy of model parameters and a more realistic description of the rough structure of geologic materials. Anomalous diffusion is familiar to electromagnetic geophysicists under the guise of a convolutional form of Ohm's constitutive relation. The physical interpretation of anomalous electromagnetic diffusion is that of EM induction in a stationary fractal medium. It is hoped that the concepts presented in this paper will stimulate further research along these lines into innovative approaches for the interpretation of controlled-source EM data.

Appendix A: Derivation of TDEM Fractional Diffusion Equation

[60] Consider a horizontal loop source lying on the surface of a uniform half-space. The electric field $\mathbf{E}$ due to current energization of the loop is

$$\mathbf{E} = E_{\phi}(\rho, \varphi) \hat{\phi}. \quad (\text{A1})$$

[61] In equation (A1), the electric field is expressed in cylindrical coordinates (ρ, ϕ, z) with radial, azimuthal, and vertical directions, respectively. The corresponding unit vectors are denoted as $\hat{\rho}$, $\hat{\phi}$, and $\hat{z}$. As indicated in equation (A1), the symmetry of the problem dictates that the electrical field is purely azimuthal and depends on (ρ, z) only.

[62] The classical EM induction equations that govern the spatiotemporal evolution of the electric field are the usual ones, Ampere's law, Faraday's law, Ohm's law, and the magnetic constitutive equation:

$$\nabla \times \mathbf{H} = \mathbf{J} \quad (\text{A2})$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B} \quad (\text{A3})$$

$$\mathbf{J} = \sigma \mathbf{E} \quad (\text{A4})$$

and

$$\mathbf{B} = \mu_0 \mathbf{H} \quad (\text{A5})$$

where the magnetic intensity and flux density fields are given by $\mathbf{H}$ and $\mathbf{B}$, respectively, μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ H/m), σ is the electrical conductivity, and $\mathbf{J}$ is the electric current density [*Jackson*, 1975]. Solving for the electric field $\mathbf{E}$ from these equations yields the classical Maxwell vector diffusion equation

$$\frac{\partial \mathbf{E}}{\partial t} = -\frac{1}{\mu_0 \sigma} \nabla \times \nabla \times \mathbf{E}. \quad (\text{A6})$$

[63] In recognition of the symmetry of the problem, equation (A1) can be applied such that equation (A6) reduces to the scalar diffusion equation

$$\frac{\partial E_{\phi}(\rho, z)}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla_{2D}^2 E_{\phi}(\rho, z), \quad (\text{A7})$$

where ∇_{2D}^2 is the two-dimensional cylindrical Laplacian operator

$$\nabla_{2D}^2 E_{\phi}(\rho, z) = \frac{\partial}{\partial \rho} \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho E_{\phi}) \right] + \frac{\partial^2 E_{\phi}}{\partial z^2}. \quad (\text{A8})$$

[64] Equation (A7) represents the classical diffusion equation governing EM induction in the Earth, applicable for azimuthally symmetric problems such as the case of a loop source lying on a uniform half-space. The fractional diffusion equation relevant for this class of EM induction problems is obtained by the application of the time-fractional derivative or Riemann-Liouville operator [Oldham and Spanier, 1974] ${}_0D_t^{1-\alpha}$ defined in equation (2). The result is as follows:

$$\frac{\partial}{\partial t} E_{\phi}(\rho, z, t) = {}_0D_t^{1-\alpha} \left[\frac{1}{\mu_0 \sigma} \nabla_{2D}^2 E_{\phi}(\rho, z, t) \right] \quad (\text{A9})$$

[65] An anomalously diffusing, azimuthally symmetric electric field evolves in space and time according to equation (A9) rather than the classical equation (A7). The physical justification for introducing the Riemann-Liouville operator is described in the text of the paper and is based on the seminal work by Scher and Montroll [1975]. The eddy current vortex intensity is proportional to the electric field amplitude.

[66] **Acknowledgments.** Sections of the work were conducted and supported by the Sandia National Laboratories, Albuquerque, NM. Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000. The authors appreciate the thoughtfulness of the reviewers' comments and those of the associate editor. Their insights have contributed greatly to the evolution of this manuscript.

References

- Acocella, V., and M. Neri (2005), Structural features of an active strike-slip fault on the sliding flank of Mt. Etna (Italy), *J. Struct. Geol.*, **27**, 343–355.
- Aharonov, E., and D. H. Rothman (1996), Growth of correlated pore-scale structures in sedimentary rocks: A dynamical model, *J. Geophys. Res.*, **101**, 2973–2988.
- al-Garni, M., and M. E. Everett (2003), The paradox of anisotropy in electromagnetic loop-loop responses over a uniaxial halfspace, *Geophysics*, **68**, 892–899.
- Bahr, K., M. Smirnov, E. Steveling, and BEAR Working Group (2002), A gelation analogy of crustal formation derived from fractal conductive structures, *J. Geophys. Res.*, **107**(B11), 2314, doi:10.1029/2001JB000506.
- Bailey, R. J., and D. G. Smith (2005), Quantitative evidence for the fractal nature of the stratigraphic record: Results and implications, *Proc. Geol. Assoc.*, **116**, 129–138.
- Barton, C., and M. Zoback (1992), The self-similar distribution and properties of macroscopic fractures at depth in crystalline rock in the Cajon Pass scientific drill hole, *J. Geophys. Res.*, **97**, 5181–5200.
- Brown, S. R. (1989), Transport of fluid and electric current through a single fracture, *J. Geophys. Res.*, **94**, 9429–9438.
- Caputo, M. (2003), Diffusion with space memory modelled with distributed order space fractional differential equations, *Ann. Geophys.*, **46**, 223–234.
- Cole, M., and J. Anderson (1982), Detailed grain size and heavy mineralogy of sands of the Northeastern Texas Gulf coast: Implications with regard to coastal barrier development, *Gulf Coast Assoc. Geol. Soc. Trans.*, **32**, 555–563.
- Collins, J. L., M. E. Everett, and C. B. Johnson (2006), Detection of near-surface horizontal anisotropy in a weathered metamorphic schist at Llano Uplift (Texas) by transient electromagnetic induction, *Phys. Earth Planet. Inter.*, **158**, 159–173.
- Constable, S., R. Parker, and C. Constable (1987), Occam's inversion: A practical algorithm for generating smooth models from electromagnetic sounding data, *Geophysics*, **52**, 289–300.
- David, C. (1993), Geometry of flow paths for fluid transport in rocks, *J. Geophys. Res.*, **98**, 12,267–12,278.
- Everett, M. E., and S. Constable (1999), Electric dipole fields over an anisotropic seafloor: Theory and application to the structure of 40 Ma Pacific Ocean lithosphere, *Geophys. J. Int.*, **136**, 41–56.
- Everett, M. E., and C. J. Weiss (2002), Geological noise in near-surface controlled-source electromagnetic data, *Geophys. Res. Lett.*, **29**(1), 1010, doi:10.1029/2001GL014049.
- Everett, M. E., and M. A. Meju (2005), Controlled-source electromagnetic induction: Background and recent advances, in *Hydrogeophysics*, edited by Y. Rubin and S. Hubbard, pp. 157–183, Springer, New York.
- Ferguson, C., J. Timmons, F. Pazzaglia, K. Karstom, G. Osburn, and P. Bauer (1999), Geology of Sandia Park Quadrangle, Bernalillo and Sandoval Counties, New Mexico, map, New Mexico Bureau of Mines and Mineral Resources, Socorro, N. M.
- Fitterman, D. V., and V. F. Labson (2005), Electromagnetic induction methods for environmental problems, in *Near-Surface Geophysics*, edited by D. W. Butler, pp. 301–355, SEG Press, Tulsa Okla.
- Jackson, J. D. (1975), *Classical Electrodynamics*, 848 pp., John Wiley, Hoboken, N. J.
- Kelly, V., and S. Northrop (1975), *Geology of Sandia Mountains and Vicinity, New Mexico, Mem. N. M. Bur. Mines Miner. Resour.*, vol. 29, 136 pp., New Mexico Bureau of Mines and Mineral Resources, Socorro, N. M.
- Kim, J. W., E. Perfect, and H. Choi (2007), Anomalous diffusion in two-dimensional Euclidean and prefractal geometrical models of heterogeneous porous media, *Water Resour. Res.*, **43**, W01405, doi:10.1029/2006WR004951.
- Kirchner, J. W., X. Feng, and C. Neal (2000), Fractal stream chemistry and its implications for contaminant transport in catchments, *Nature*, **403**, 524–527.
- Lamb, H. (1932), *Hydrodynamics*, 6th ed., 738 pp., Cambridge Univ. Press, New York.
- Lesmes, D. P., and S. P. Friedman (2005), Relationships between the electrical and hydrogeologic properties of rocks and soils, in *Hydrogeophysics*, edited by Y. Rubin, and S. Hubbard, Springer, New York.
- Madadi, M., C. D. Van Sice, and M. Sahimi (2003), Fluid flow and conduction in two-dimensional fractures with rough, self-affine surfaces: A comparative study, *J. Geophys. Res.*, **108**(B8), 2396, doi:10.1029/2002JB002332.
- Mandelbrot, B. B., and J. W. Van Ness (1968), Fractional Brownian motions, fractional noises and applications, *SIAM Rev.*, **10**, 422–437.
- Metzler, R., and J. Klafter (2000), The random walk's guide to anomalous diffusion: A fractional dynamics approach, *Phys. Rep.*, **339**, 1–77.
- Nabighian, M. N. (1979), Quasi-static transient response of a conducting half-space—An approximate representation, *Geophysics*, **10**, 1700–1705.
- Nigmatullin, R. (1984), To the theoretical explanation of the “universal response”, *Phys. Status Solidi, B Basic Res.*, **123**, 739–745.
- Nigmatullin, R. (1986), The realization of the generalized transfer equation in a medium with fractal geometry, *Phys. Status Solidi, B Basic Res.*, **133**, 425–430.
- Paine, J. G. (2003), Determining salinization extent, identifying salinity sources, and estimating chloride mass using surface, borehole and airborne electromagnetic induction methods, *Water Resour. Res.*, **39**(3), 1059, doi:10.1029/2001WR000710.
- Oldham, K. B., and J. Spanier (1974), *The Fractional Calculus*, Elsevier, New York.
- Paradisi, P., R. Cesari, F. Mainardi, and F. Tampieri (2001), Fractional Fick's law for non-local transport processes, *Physica, A*, **293**, 130–142.
- Pilkington, M., and J. P. Todoeschuck (1993), Fractal magnetization of continental crust, *Geophys. Res. Lett.*, **20**, 627–630.
- Purkis, S. J., B. M. Reigl, and S. Andrefouet (2005), Remote sensing of geomorphology and facies patterns on a modern carbonate ramp (Arabian Gulf, Dubai, UAE), *J. Sediment. Res.*, **75**, 861–876.

- Purvance, D. T. (2003), Roles apparent resistivity and phase play in an aquifer's electrical-hydraulic conductivity relation, *Geophys. Res. Lett.*, 30(4), 1170, doi:10.1029/2002GL016131.
- Radlinski, A. P., E. Z. Radlinska, M. Agamalian, and G. D. Wignall (1999), Fractal geometry of rocks, *Phys. Rev. Lett.*, 82, 3078–3081.
- Sasaki, Y., and M. A. Meju (2006), A multidimensional horizontal-loop controlled-source electromagnetic inversion method and its use to characterize heterogeneity in aquiferous crystalline rocks, *Geophys. J. Int.*, 166, 59–66.
- Scher, H., and E. W. Montroll (1975), Anomalous transit-time dispersion in amorphous solids, *Phys. Rev., B Solid State*, 12, 2455–2477.
- Scher, H., G. Margolin, R. Metzler, J. Klafter, and B. Berkowitz (2002), The dynamical foundation of fractal stream chemistry: The origin of extremely long retention times, *Geophys. Res. Lett.*, 29(5), 1061, doi:10.1029/2001GL014123.
- Schlager, W. (2004), Fractal nature of stratigraphic sequences, *Geology*, 32, 185–188.
- Schneider, W. R., and W. Wyss (1989), Fractional diffusion and wave equations, *J. Math. Phys.*, 30, 134–144.
- Skinner, D., and G. Heinson (2004), A comparison of electrical and electromagnetic methods for the detection of hydraulic pathways in a fractured rock aquifer, Clare Valley, South Australia, *Hydrogeol. J.*, 12, 576–590.
- Tadjeeran, C., M. M. Meerschaert, and H.-P. Scheffler (2006), A second-order accurate numerical approximation for the fractional diffusion equation, *J. Comput. Phys.*, 213, 205–213.
- West, G. F., and J. C. Macnae (1991), Physics of the electromagnetic induction exploration method, in *Electromagnetic Methods in Applied Geophysics*, vol. 2, edited by M. N. Nabighian, pp. 5–45, SEG Press, Tulsa, Okla.
- Wilt, M., and D. Alumbaugh (2003), Oil field characterization and monitoring using electromagnetic geophysical techniques, *J. Pet. Sci. Eng.*, 39, 85–97.
- Wroblewski, C. (1996), An aquifer characterization at the Texas A&M University Brazos river hydrological field site, Burleson County, Texas, MS thesis, Texas A&M Univ., Tex.
- Yancey, T., and A. Davidoff (1991), Paleogene sequence stratigraphy in the Brazos river valley, Texas, *Gulf Coast Association of Geological Societies Field Trip Guidebook*, 104 pp.

M. E. Everett, Department of Geology and Geophysics, Texas A&M University, College Station, TX 77843, USA. (everett@geo.tamu.edu)

C. J. Weiss, Department of Geosciences, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA. (cjweiss@vt.edu)