Mathematical Geology, Vol. 38, No. 8, November 2006 (© 2007)
DOI: 10.1007/s11004-006-9060-x

The Effect of Spatial Heterogeneity in Soil Properties on Erosion Pattern: A Conceptual Model and Computer Simulations[1]

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KEY WORDS: erosion rate, spatial structure, random field.

INTRODUCTION

There exist many models for soil erosion, ranging from the essentially analytical to the essentially empirical. They seek primarily to relate erosion rate to its various driving and determining factors, of which there are many (e.g., Nearing, 1991; Wilson, 1993; Winterwerp and van Kesteren, 2004). The objective of this work is to investigate the effect, not of the factors themselves, but of the statistical properties of those factors.

It is with this outlook that we investigate the effect that heterogeneity of both hydrodynamic and cohesive resistance forces has on the pattern of erosion. This has been the least studied issue in erosion physics, despite indication (e.g., Moglen and Bras, 1995) that spatial variation is an important factor in resultant topography and therefore should be incorporated into erosion models at a fundamental level.

We do this in the context of a relatively simple analytical formulation for erosion so that the effects of the heterogeneity are more visible. For this same reason, while the variables we use represent actual physical quantities, they are implemented in non-dimensional form.

We consider the reduced-dimension case for simplicity: our simulation domain consists of an uniform rectangular 2D grid, 512 units in each the x (horizontal) and z (vertical) directions.

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SPATIAL STRUCTURE

The resistance and hydrodynamic fields will be randomly generated subject to desired distributions and fractal properties (e.g., Dodds and Rothman, 2000; Otnes and Enochsen, 1978; Power and Tullis, 1991; Turcotte, 1997).

$R(x, z)$, the random field of cohesive resistance force, is defined on the aforementioned rectangular grid, and persists throughout any given simulation. R has a log-Normal distribution, with unit mean and standard deviation that we leave open. This means that

$$\mu_R = e^{\mu_Q + \sigma_{Q^{2/2}}} = 1 \tag{1}$$

and

$$\sigma_R^2 = \mathrm{e}^{\sigma_{Q^2}} \left(\mathrm{e}^{\sigma_{Q^2}} - 1\right), \tag{2}$$

where $Q = \log(R)$ and is Normally distributed with mean μ_Q and standard deviation σ_Q. R has a power spectral density slope of -2 (in both spatial dimensions) for all frequencies above k, and a slope of 0 for all below it. The parameter k therefore controls correlation length. This spectral schematization follows from direct measurements (e.g., Armstrong, 1986).

Denoting the soil surface by $z_S(x, t)$, the hydrodynamic force acting on it is represented by F, which we subdivide into deterministic (F_D) and stochastic (F_S) components:

$$F(x, t) = F_D(z_S(x, t)) + F_S(x, t). \tag{3}$$

For F_D, we consider a viscous shear force that erodes fine particles of the soil vertically upward. We assume that flow velocity increases linearly with height from the roughness troughs (Nikora and others, 2004), and that the viscous boundary layer is of constant thickness. The shear force acting on the soil surface is therefore proportional to the local velocity, and we can express F_D as:

$$F_D = 2\frac{z_S - z_{\min}}{z_{\max} - z_{\min}} \cos(\theta), \tag{4}$$

where $z_{\min}$ and $z_{\max}$ are $\min_x(z_S)$ and $\max_x(z_S)$ respectively, and $\tan(\theta) = dz/dx$. The coefficient 2 is included so that F_D has an approximate mean of 1, which maintains that $F \approx R$. It is important to note that, in this formulation of F_D, we are considering the domain as the length-scale for the flow, in that it is the surface within the domain that determines the spatially averaged velocity profile.

F_S is a random field with a power spectral density slope of -2, along the surface of the soil and in time; it is distributed Normally with a mean of zero and a standard deviation, for each time step, proportional to the mean value of F_D at that time step. That is,

$$\sigma_{F_S} = p_F \mu_{F_D}, \tag{5}$$

so that

$$\sigma_F^2 = \sigma_{F_D}^2 + p_F^2 {\mu_{F_D}}^2. \tag{6}$$

Erosion processes of this type have their natural analogue in the so-called "surface erosion" of weakly consolidated estuarine mud (e.g., Mehta and others, 1989), where the bed surface is lowered by "flock-by-flock" detachment of bottom sediment.

EROSION MODEL AND SIMULATIONS

In this model, erosion occurs as:

$$\frac{\partial z}{\partial t} = \begin{cases} -(F-R)^{0.5} & \text{for} \quad F > R \\ 0 & \phantom{\text{for}} \quad F \le R \end{cases}. \tag{7}$$

This simplified formulation, in particular, the 0.5 exponent, follows from analytic considerations and examples from our previous work (Sidorchuk and others, 2004a,b).

The heterogeneity of the total system is thus described by the parameters σ_R Eq. (2), p_F Eq. (6) and k. We perform simulations with $\sigma_R = 0.1$, 0.5, 1, with $p_F = 0.1$, 0.5, 1, and with $k = 1$, 6, 40; that is, 27 scenarios in total. For each scenario, we begin with $z_S(x) = 0$, run the model 1000 times, each for 500 timesteps, calculating ensemble averages of relevant outputs. The outputs are the statistical moments and the power spectral density of z_S. We give the statistical moments as functions of time; we give the power spectral density of z_S as at the final timestep.

RESULTS AND DISCUSSION

The overall erosion rate is indicated by the final value of the mean of z_S. Results consistently showed a decrease in erosion rate with increased k, that is, with decreased correlation length (Fig. 1a). With respect to parameters σ_R and p_F, no result is clear. It does seem, however, that erosion rate increases when the two have similar values; the highest rate occurred with $\sigma_R = 0.1$ and $p_F = 0.1$, and the lowest rate occurred with $\sigma_R = 1$ and $p_F = 0.1$.

The mean of z_S, followed over the 500 timesteps, generally exhibited positive curvature. This is due to the early erosion of low-resistance areas followed by the slower erosion of higher-resistance areas.

An equilibrium z_S profile did not develop, in the sense of saturation of its standard deviation (Fig. 1b). It generally continued to rise, at least within the chosen time domain for simulations, that is, 500 timesteps. No significant trends were apparent in its dependence on σ_R and p_F, although there is a clear dependence on k (Fig. 1b).

Skewness coefficients of z_S were largely erratic, apart from the observation of overall negativity (Fig. 1c). We presume this negativity is due to fast erosion of narrow depressions in the soil compared to the remainder of the soil surface experiencing a more moderate erosion rate.

The spectral amplitudes of the soil surface (Fig. 1d) clearly reflected the spectral structure on which the particular scenarios derived. That is, the frequency structure of the final profile is very similar to the frequency structure of the soil

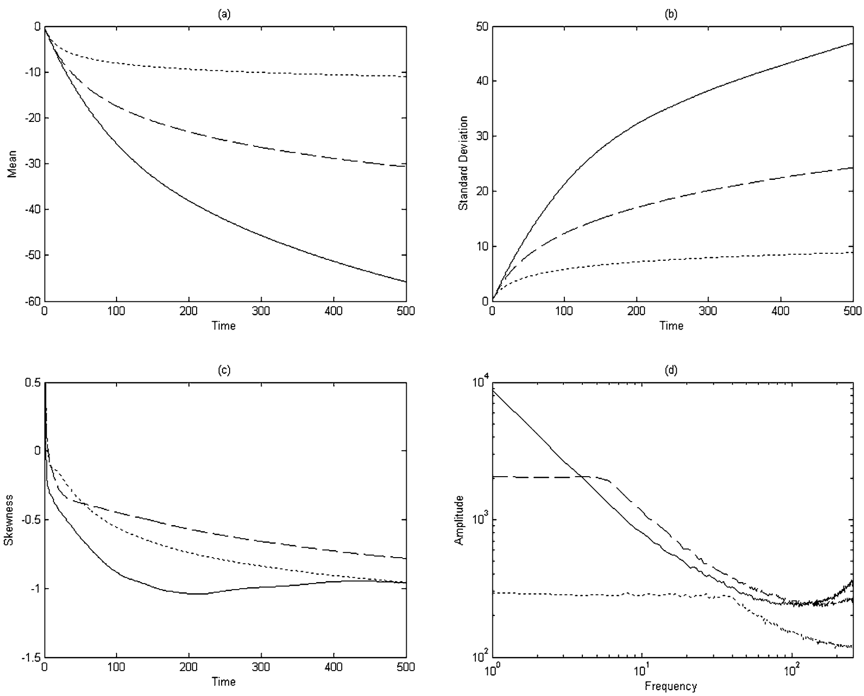

Figure 1. Simulation results for case $\sigma_R = 1$ and $p_F = 0.5$: (a), (b), (c) and (d) show the mean, standard deviation, skewness and spectra of z_S respectively. In all plots: the solid line represents full red noise, $k = 1$; the dashed line represents red noise above $k = 6$; the dotted line represents red noise above $k = 40$.

resistance field: the transitions from white to red noise are apparent at the same k values. The only real divergence from this is that there is generally increased high-frequency noise, even to the extent of positive power-spectral slopes for wavenumbers above approximately 200. We presume that this further evidence of narrow depressions eroded into the soil.

Though narrow eroded models can and do occur in nature, we generally consider the occurrence of them here to be an effect of the simplified formulation for erosion rate that we have used in the model. It is possible that Eq. (7) may be better suited to models in which a wider range of erosive mechanisms is incorporated.

CONCLUSION

We have developed a computational model that exposes the effect of spatial heterogeneity on erosion pattern by the use of non-dimensional variables and a simplified erosion formulation. Many simulations were run, encompassing a wide range of physical scenarios. The most important results, from the authors' perspective, are: (1) that the key heterogeneity parameter influencing the erosion rate and soil surface roughness is the correlation length-scale in soil properties; (2) the importance of the similarity in values of σ_R and p_F, that is, the standard deviations of the hydrodynamic and resistance forces, in determining the overall erosion rate; and (3) that the spectral structure persists from the soil resistance force to the soil surface profile.

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