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Restoration of seismic parameters and electrical conductivity by the diffraction tomography method

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Abstract

The results of numerical simulation on restoration of the local elastic inhomogeneities and the local perturbations of electrical conductivity are considered. The restoration is implemented by the diffraction tomography method with the help of elastic and electromagnetic sounding wavefields. The perturbations of electrical conductivity are restored by "diffusive" electromagnetic field. In both cases, seismic and electromagnetic, the direct problem is solved by the finite difference method and the restoration is implemented in the time domain using the first order Born approximation.

Keywords: Seismic wave; Electromagnetic wave; Diffraction tomography; Numerical simulation; Restoration

1. Introduction

Geophysical diffraction tomography is an imaging technique (Devaney, 1984) that makes use of a large volume of the input data (recorded traces) to produce the image of underground medium parameters with a high spatial resolution. The seismic and electromagnetic waves as a sounding signal may bring an information about the seismic parameters and electrical conductivity of underground medium (Ryzhikov and Troyan, 1994; Zhou et al., 1993; Mauriello and Patella, 1999). The inverse problem on restoration of such parameters with the help of seismic and electromagnetic sounding signals is nonlinear. Linearision of the inverse problem, for example, with the use of the first-order Born approximation allows us to obtain the solution at enough small computational burden, but it brings the limitations in restoration of the medium parameters (in non-weak contrast inhomogeneities we may not get the satisfactory accuracy of restoration). We consider the restoration of medium parameters by the diffraction tomography method with the help of the first-order Born approximation (Devaney, 1984; Ryzhikov and Troyan, 1994; Troyan and Kiselev, 2001). A single formalism allows us to describe the restoration procedure using seismic waves and electromagnetic waves of the diffusion type exited by the point and area sources, including infinite sources. The inversion is implemented in a time domain with a finite shape of the sounding signal. The desired parameters are found by the algebraic method with the appropriate regularisation, which is based on a priori information about smoothness of the desired parameter as a spatial function and on a priori information about possible location and sizes of the target object.

The results of numerical simulation on restoration of the seismic parameters and electrical conductivity for the local inhomogeneities with a simple and complex geometry are considered. The examples of restoration of local inhomogeneity of electrical conductivity are represented in Troyan and Kiselev (2001). In this article an analogous numerical simulation is implemented for other observation scheme and for other size of the restoration region, that leads to

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higher quality of restoration and allows to determine more correctly the horizontal size of inhomogeneity under restoration. The result of restoration using complete electric field vector is shown.

The study of the changes of the geophysical parameters of the earth crust in seismoactive zones is a relevant moment in investigations oriented to the prediction of the earthquakes. Alterations of the elastic parameters and electrical conductivity can be considered as an object of monitoring to obtain the precursory information.

2. Basic formulas, elastic case

The source $\hat{\boldsymbol{f}} \equiv \hat{\boldsymbol{f}}(\boldsymbol{x},t)$ located in the point $(x = x_s, z = z_s)$ of Cartesian system of coordinates $(x, y, z; \boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3)$, produces the wave field $\boldsymbol{u} \equiv \boldsymbol{u}(x,z,t) \equiv \boldsymbol{u}(\boldsymbol{x},t)$ which satisfies the equation

$$\rho\frac{\partial^2\boldsymbol{u}}{\partial t^2} = \hat{\boldsymbol{f}} + (\lambda+\mu)\nabla\nabla\cdot\boldsymbol{u} + \mu\Delta\boldsymbol{u} + \nabla\lambda\nabla\cdot\boldsymbol{u} + \nabla\mu\times\boldsymbol{u} + 2(\nabla\mu\cdot\nabla)\boldsymbol{u},$$

where λ, μ are the Lame parameters, and ρ is mass density.

The velocities of longitudinal (v_p) and shear (v_s) waves, expressed through the quantities λ, μ, ρ, read as

$$v_p = \sqrt{(\lambda+2\mu)/\rho}, \quad v_s = \sqrt{\mu/\rho}.$$

We introduce the differences $\delta\lambda = \lambda - \lambda_{\mathrm{rf}}$, $\delta\mu = \mu - \mu_{\mathrm{rf}}$ and $\delta\rho = \rho - \rho_{\mathrm{rf}}$ of the values λ, μ, ρ for the unknown medium which are connected with the wave field $\boldsymbol{u}(\boldsymbol{x},t)$

$$L\boldsymbol{u} = -\hat{\boldsymbol{f}},$$
$$L\boldsymbol{u} \equiv (\lambda+\mu)\nabla\nabla\cdot\boldsymbol{u} + \mu\Delta\boldsymbol{u} + \nabla\lambda\nabla\cdot\boldsymbol{u} + \nabla\mu\times\boldsymbol{u} + 2(\nabla\mu\cdot\nabla)\boldsymbol{u} - \rho\frac{\partial^2\boldsymbol{u}}{\partial t^2}$$

and the values $\lambda_{\mathrm{rf}}(\boldsymbol{x})$, $\mu_{\mathrm{rf}}(\boldsymbol{x})$, $\rho_{\mathrm{rf}}(\boldsymbol{x})$ for the known, reference (rf) medium for which the wave field is $\boldsymbol{u}_{\mathrm{rf}}(\boldsymbol{x},t)$

$$L_{\mathrm{rf}}\boldsymbol{u}_{\mathrm{rf}} = -\hat{\boldsymbol{f}},$$
$$L_{\mathrm{rf}}\boldsymbol{u}_{\mathrm{rf}} \equiv (\lambda_{\mathrm{rf}}+\mu_{\mathrm{rf}})\nabla\nabla\cdot\boldsymbol{u}_{\mathrm{rf}} + \mu_{\mathrm{rf}}\Delta\boldsymbol{u}_{\mathrm{rf}} + \nabla\lambda_{\mathrm{rf}}\nabla\cdot\boldsymbol{u}_{\mathrm{rf}} + \nabla\mu_{\mathrm{rf}}\times\boldsymbol{u}_{\mathrm{rf}} + 2(\nabla\mu_{\mathrm{rf}}\cdot\nabla)\boldsymbol{u}_{\mathrm{rf}} - \rho_{\mathrm{rf}}\frac{\partial^2\boldsymbol{u}_{\mathrm{rf}}}{\partial t^2}$$

Assuming $\delta\lambda$, $\delta\mu$ and $\delta\rho$ small we can write

$$L_{\mathrm{rf}}\delta\boldsymbol{u} \approx -\delta L\boldsymbol{u}_{\mathrm{rf}}, \tag{1}$$

where $\delta\boldsymbol{u} = \boldsymbol{u} - \boldsymbol{u}_{\mathrm{rf}}$ is the difference field. The right hand side of (1)

$$\delta L\boldsymbol{u}_{\mathrm{rf}} = \nabla\times(\delta\mu\nabla\times\boldsymbol{u}_{\mathrm{rf}}) + 2\sum_{j=1,3}\nabla\cdot(\delta\mu\nabla\boldsymbol{u}_{\mathrm{rfj}})\boldsymbol{e}_j + \nabla(\delta\lambda\nabla\cdot\boldsymbol{u}_{\mathrm{rf}}) - \delta\rho\frac{\partial^2\boldsymbol{u}_{\mathrm{rf}}}{\partial t^2}$$

can be considered as a source of that field.

We shall represent the components of the difference field δu_i from (1) at the observation point of $\boldsymbol{x} = \boldsymbol{x}_r$ as follow,

$$\delta u_i(\boldsymbol{x}_s,\boldsymbol{x}_r,t) = \int_S\int_0^\infty \tilde{\boldsymbol{u}}_i(\boldsymbol{x},\boldsymbol{x}_r,t-\tau)\cdot\delta L\boldsymbol{u}(\boldsymbol{x},\boldsymbol{x}_s,\tau)\mathrm{d}\tau\mathrm{d}\boldsymbol{x}, \tag{2}$$

where S is the region of restoration; $\boldsymbol{u} \equiv \boldsymbol{u}(\boldsymbol{x},\boldsymbol{x}_s,t)$ and $\tilde{\boldsymbol{u}} \equiv \tilde{\boldsymbol{u}}(\boldsymbol{x},\boldsymbol{x}_r,t)$ are the wave fields for the appropriate sources. After introducing the tomography functionals $p_i^\rho(\boldsymbol{x},\boldsymbol{x}_r,\boldsymbol{x}_s,t)$, $p_i^\lambda(\boldsymbol{x},\boldsymbol{x}_s,\boldsymbol{x}_r,t)$, $p_i^\mu(\boldsymbol{x},\boldsymbol{x}_s,\boldsymbol{x}_r,t)$ (Ryzhikov and Troyan, 1994) the components of the difference field δu_i (2) can be written down as

$$\delta u_i(\boldsymbol{x}_s,\boldsymbol{x}_r,t) = \int_S[\delta\lambda(\boldsymbol{x})p_i^\lambda(\boldsymbol{x},\boldsymbol{x}_s,\boldsymbol{x}_r,t) + \delta\mu(\boldsymbol{x})p_i^\mu(\boldsymbol{x},\boldsymbol{x}_s,\boldsymbol{x}_r,t) + \delta\rho(\boldsymbol{x})p_i^\rho(\boldsymbol{x},\boldsymbol{x}_s,\boldsymbol{x}_r,t)]\mathrm{d}\boldsymbol{x}. \tag{3}$$

Using the linear relations between $\delta\lambda(\boldsymbol{x})$, $\delta\ \rho(\boldsymbol{x})$ and $\delta\mu(\boldsymbol{x})$

$$\delta\lambda(\boldsymbol{x}) = c_\lambda\delta\mu(\boldsymbol{x}), \quad \delta\rho(\boldsymbol{x}) = c_\rho\delta\mu(\boldsymbol{x})$$
$$(c_\lambda = \mathrm{const}, \quad c_\rho = \mathrm{const}),$$

the Eq. (3) can be rewritten as

$$\delta u_i(\boldsymbol{x}_s,\boldsymbol{x}_r,t) \approx \int_S[c_\lambda p_i^\lambda(\boldsymbol{x},\boldsymbol{x}_s,\boldsymbol{x}_r,t) + p_i^\mu(\boldsymbol{x},\boldsymbol{x}_s,\boldsymbol{x}_r,t) + c_\rho p_i^\rho(\boldsymbol{x},\boldsymbol{x}_s,\boldsymbol{x}_r,t)]\delta\mu(\boldsymbol{x})d\boldsymbol{x}. \tag{4}$$

After discretization of the Eq. (4) the system of equations for determination of $\delta\mu$ (vector $\mathbf{d}_\mu$), c_λ and c_ρ can be written as

$$P(c_\lambda,c_\rho)\mathbf{d}_\mu = \mathbf{d}_u, \tag{5}$$

where $\mathbf{d}_u$ are the samples of the scattered field. The final version of this system after introducing the regularising terms reads as

$$[P^T(c_\lambda,c_\rho)P(c_\lambda,c_\rho) + \alpha_1(B_x^TB_x + B_z^TB_z) + \alpha_2C^TC + \alpha_3D^TD]\mathbf{d}_\mu = P^T(c_\lambda,c_\rho)\mathbf{d}_u, \tag{6}$$

where α_1, α_2, α_3 are the regularization coefficients; matrices B_x and B_z are the finite difference images of the second partial derivatives with respect to x and z correspondingly; C and D are the penalty matrices for non-zero values of $\mathbf{d}_\mu$ correspondingly at boundary and near boundary points of the restored region S.

3. Basic formulas, electromagnetic case

Numerical simulation to restore the local inhomogeneities of electrical conductivity $\sigma = \sigma(\mathbf{x})$, located in the uniform space (electrical conductivity $\sigma = \mathrm{const}$, electrical permittivity $\varepsilon' = \varepsilon\varepsilon_0 = \mathrm{const}$, magnetic permittivity $\mu' = \mu\mu_0 = \mathrm{const}$) is implemented for 2-D problem. The link between the electrical field $\boldsymbol{E} = \boldsymbol{E}(\mathbf{x},t)$ and magnetic field $\boldsymbol{H} = \boldsymbol{H}(\mathbf{x},t)$ in conductive media, excited by a current density $\boldsymbol{j}_{\mathrm{ex}} = \boldsymbol{j}_{\mathrm{ex}}(\boldsymbol{x},t)$, is determined by the Maxwell equations, written as

$$\nabla\times\boldsymbol{E} = -\mu'\frac{\partial\boldsymbol{H}}{\partial t}, \quad \nabla\times\boldsymbol{H} = \boldsymbol{j} + \boldsymbol{j}_{\mathrm{ex}}, \qquad \boldsymbol{j} = \sigma\boldsymbol{E}.$$

Electrical field $\boldsymbol{E} = \boldsymbol{E}(\mathbf{x},t)$ in the medium, containing the local inhomogeneity of $\sigma = \sigma(\mathbf{x})$, is given by a solution of the following equation:

$$L\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{j}_{\rm ex}, \quad L\boldsymbol{E} \equiv \frac{1}{\mu'}\nabla\times\nabla\times\boldsymbol{E} + \sigma\frac{\partial \boldsymbol{E}}{\partial t}.$$

The reference medium (rf) is supposed to be known ($\sigma_{\rm rf}$), and electrical field $\boldsymbol{E}_{\rm rf}$ satisfies the equation

$$L_{\rm rf}\boldsymbol{E}_{\rm rf} = -\frac{\partial}{\partial t}\boldsymbol{j}_{\rm ex}, \quad L_{\rm rf}\boldsymbol{E}_{\rm rf} \equiv \frac{1}{\mu'}\nabla\times\nabla\times\boldsymbol{E}_{\rm rf} + \sigma_{\rm rf}\frac{\partial \boldsymbol{E}_{\rm rf}}{\partial t}.$$

We assume that the magnitude of the value $\delta\sigma = \sigma - \sigma_{\rm rf}$ makes it possible to write an approximate equality

$$L_{\rm rf}\delta\boldsymbol{E} \approx -\delta L\boldsymbol{E}_{\rm rf},$$

where $\delta\boldsymbol{E} = \boldsymbol{E} - \boldsymbol{E}_{\rm rf}$ is the difference field. Thus the value

$$\delta L\boldsymbol{E}_{\rm rf} = \delta\sigma\frac{\partial \boldsymbol{E}_{\rm rf}}{\partial t}$$

can be considered as a source of this field. The components of the difference field δE_i can be written down as

$$\delta E_i(\boldsymbol{x}_s, \boldsymbol{x}_r, t) = \int\!\!\int_S\int_0^\infty \widetilde{\boldsymbol{E}}_i(\boldsymbol{x}, \boldsymbol{x}_r, t-\tau)\cdot\delta L\boldsymbol{E}(\boldsymbol{x}, \boldsymbol{x}_s, \tau)\mathrm{d}\tau\mathrm{d}\boldsymbol{x}.$$

Wave fields $\boldsymbol{E}(\boldsymbol{x}, \boldsymbol{x}_s, t)$ and $\widetilde{\boldsymbol{E}}_i(\boldsymbol{x}, \boldsymbol{x}_r, t)$ satisfy respectively the following equations:

$$L_{\rm rf}\boldsymbol{E} = -\hat{\boldsymbol{f}} = \delta(z - z_s)f(t)\boldsymbol{e}_1 \quad \text{and} \quad L_{\rm rf}\widetilde{\boldsymbol{E}}_i = -\hat{\boldsymbol{f}}_i \quad (i = 1, 3), \tag{7}$$

where the sources $\hat{\boldsymbol{f}}_i$ are given by the next formula

$$\hat{\boldsymbol{f}} \equiv \hat{\boldsymbol{f}}_i \equiv \hat{\boldsymbol{f}}_i(x_r, z_r, t) = \delta(x - x_r)\delta(z - z_r)\delta(t)\boldsymbol{e}_i.$$

Introducing the tomography functionals

$$p_i^\sigma(\boldsymbol{x}, \boldsymbol{x}_r, \boldsymbol{x}_s, t) = \int_0^\infty \widetilde{\boldsymbol{E}}_i(\boldsymbol{x}, \boldsymbol{x}_r, t-\tau)\cdot\frac{\partial}{\partial\tau}\boldsymbol{E}(\boldsymbol{x}, \boldsymbol{x}_s, \tau)\mathrm{d}\tau,$$

the components of the difference field can be written as

$$\delta E_i(\boldsymbol{x}_s, \boldsymbol{x}_r, t) \equiv \delta E_i = \int_S \delta\sigma p_i^\sigma \mathrm{d}\boldsymbol{x}. \tag{8}$$

After discretization of Eq. (8) we obtain the system of linear equations. This system and its solution, using the appropriate regularisation, are analogous to the elastic case (Eqs. (5) and (6)).

4. Restoration of inhomogeneities

Restoration of elastic inhomogeneity. Numerical simulation is implemented in 2-D case. The direct problem is solved by the finite difference method. We consider restoration (v_p value) of the symmetric (Fig. 1) and non-symmetric (Fig. 2) inhomogeneities comparable in size with the wavelength of the sounding signal, with contrast 20% relatively

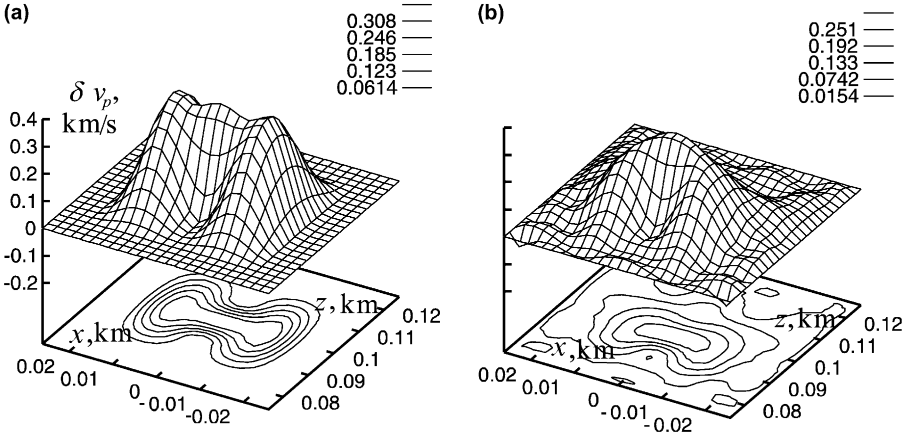

Fig. 1. Restoration of v_p for symmetric elastic inhomogeneity. (a) The model; (b) the result of restoration.

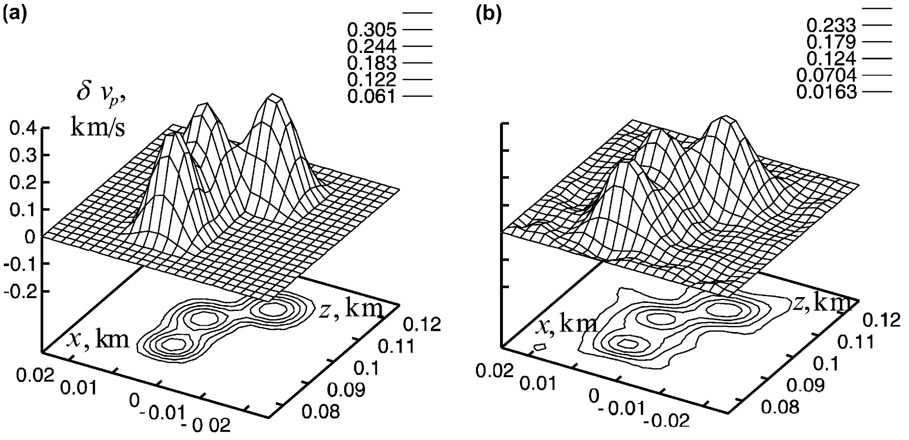

Fig. 2. Restoration of v_p for non-symmetric elastic inhomogeneity. (a) The model; (b) the result of restoration.

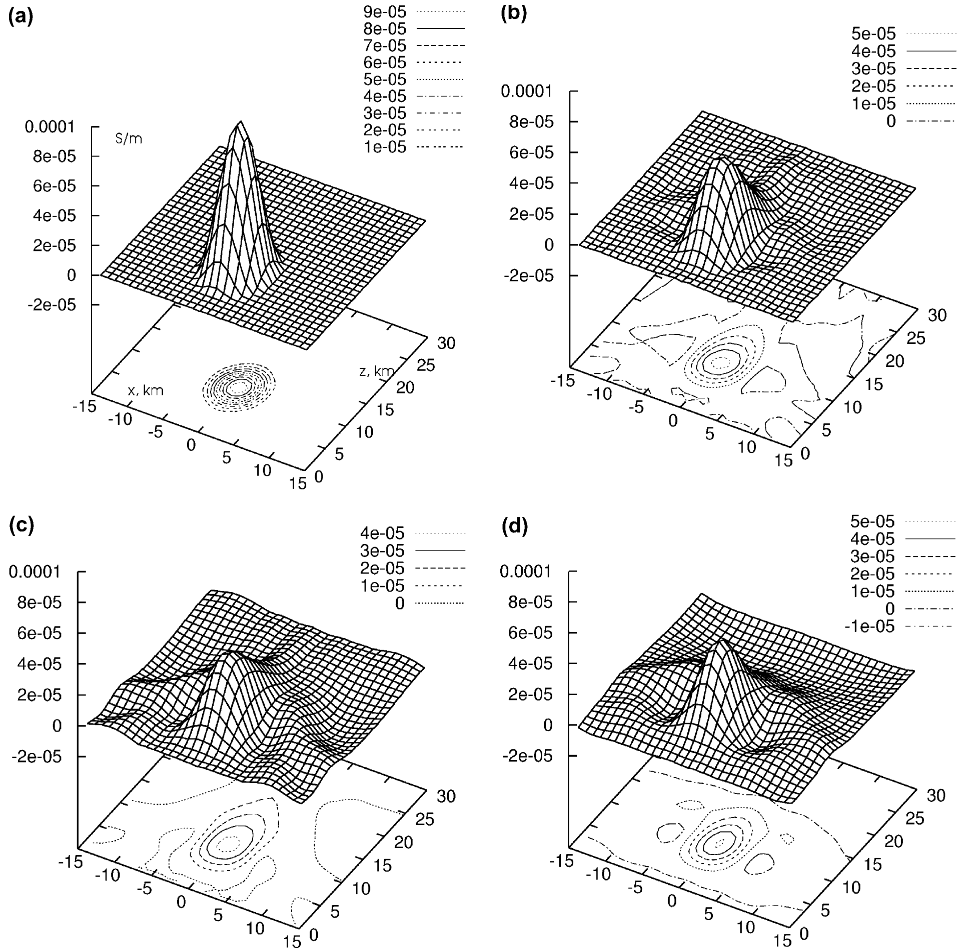

Fig. 3. Restoration of electrical conductivity. (a) The model; (b)–(d) the results of restoration by the use of seven observation points ($x = -7.5$, -5.0, -2.5, 0.0, 2.5, 5.0, 7.5 km).

to the reference medium. The source and receivers (9 source-receiver pairs) are located in the horizontal profile ($z = 0$). The relative error of the restoration is approximately 20%.

Restoration of electrical conductivity. Numerical simulation is implemented in 2-D case. The direct problem is solved by the finite difference method. The source from the right-hand side of the left equation from (7) produces the non-stationary plane wave with apparent frequency 10 Hz and with non-zero horizontal component of the electrical field E_1. The model and the results of restoration are represented in Fig. 3. The sounding plane wave propagates in the uniform space with electrical conductivity $\sigma = 10^{-4}$ S/m. The maximum value of electrical conductivity of the inhomogeneity and its cross size are $\sigma = 10^{-4}$ S/m and 10 km, respectively Fig. 3(a). The centre of inhomogeneity is located at the point ($x = 0$ km, $z = 10$ km) with a distance of 10 km from the observation line ($z = 0$ km). The restoration by the diffraction tomography using both components of the electrical field Fig. 3(b) gives the error of 50%. The restoration using E_1 and E_3 components is represented in Fig. 3(c) and (d), respectively.

5. Conclusions

We conclude from the numerical simulation that the realisation of the diffraction tomography method, under appropriate observation schemes, allows satisfactory accuracy for restoration of seismic velocity in the case of not too large contrast inhomogeneities of size of λ with the use of a small number of the source-receiver pairs. The relative error in this case is approximately equal to the contrast of inhomogeneity relatively to the reference medium. Similar restoration algorithm is implemented for restoration of local inhomogeneity of electrical conductivity with electromagnetic “diffusive” sounding signal. Using a priori information about the sizes and location of inhomogeneity can lead to a significant improvement of the restoration.

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