

Driving force of lateral permeable gas flow in magma and the criterion of explosive and effusive eruptions

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Abstract

The release of gas from ascending magma is one of the most important factors that controls the style of volcanic eruptions. It is proposed in this paper that the viscous resistance of decompressed magma to gas expansion coupled with the lateral gradient of the ascent velocity can work as a driving force of the permeable gas flow that may transport gas in magma toward the country rock. Based on this idea, the rate of gas loss from magma is calculated in terms of the mean ascent velocity, the gas volume fraction and the 'degassing factor' that is proportional to the magma viscosity and the permeability in the limit of the large gas volume fraction. This formulation of gas loss is applied to a simulation of the magma ascent process to examine the eruption style. For the simulation we employ a model of one-dimensional, non-stationary conduit flow in which volatiles are assumed to move upward with the same speed as the liquid magma and to deposit on bubbles in an amount exceeding the solubility. The simulation reveals that the gas volume fraction in the erupting magma sharply drops from nearly unity to significantly lower values as the dimensionless degassing factor increases and crosses a critical value that is only slightly dependent on various conditions. This consequence yields a simple criterion of the eruption style that displays the ranges of explosive and effusive eruptions on the viscosity–permeability plane separated by a boundary line that shifts following the size and geometry type of the conduit.

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1. Introduction

A volcanic eruption sometimes produces continuous liquid lava effusively and sometimes ejects various sizes of magma fragments explosively. Explosive eruptions involve the process of magma fragmentation that transforms a bubbly liquid flow of magma into a gassy flow mixed with magma fragments in the conduit (Woods, 1995). The mechanism of magma fragmentation is thus an important factor that determines the eruption style (Sparks, 1978; Alidibirov and Dingwell, 1996; Papale, 1999; Sahagian, 1999). Another important factor is the amount of volatiles

that are lost from magma during its ascent motion. Namely, an eruption is made effusive and explosive, respectively, when relatively significant and insignificant amounts of volatiles escape from the ascending magma.

Volatiles in magma may migrate in various ways, including through diffusion and bubble migration. Among them permeable gas flow is probably one of the most effective mechanisms for gas transportation in magma. The coalescence of bubbles in magma is thought to connect the dispersed gas phase and to produce some networks of bubbles that may provide the gas flow with permeable paths (Klug and Cashman, 1996). Accordingly, many studies have focused on methods for estimating the permeability of magma and its dependence on

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the gas volume fraction theoretically (Saar and Manga, 1999; Blower, 2001) and experimentally (Klug and Cashman, 1996; Klug et al., 2002; Takeuchi et al., 2005; Mueller et al., 2005).

Gas migration in magma has both a vertical component along the ascent motion and a lateral component toward the boundary walls. The vertical gas migration, which is caused by a difference between the ascent velocity of gas and that of the liquid magma, redistributes the gas concentration within the conduit and influences the depth and nature of magma fragmentation (Yoshida and Koyaguchi, 1999; Melnik et al., 2005). The vertical gas migration can even reduce the volume fraction of bubbles due to gas filtration and make the eruption effusive (Melnik and Sparks, 1999). On the other hand, the lateral gas migration reduces the amount of gas in magma at each depth and provides another basic mechanism for the control between explosive and effusive eruptions (Eichelberger et al., 1986; Jaupart and Allegre, 1991; Woods and Koyaguchi, 1994). In this paper we consider the lateral gas transport in magma, assuming that it may play a more important role because of the much shorter path length required for gas removal. An important problem we encounter in this context is the question of what drives the lateral permeable gas flow. To answer this question, we here propose a driving force that originates from the viscous resistance of decompressed magma to gas expansion, and we formulate the corresponding degassing rate.

Once the degassing rate is formulated we can make a computer simulation of the magma ascent process to find the criterion of the eruption style for this specific mechanism of degassing. Many models applicable to the simulation are actually available (Sahagian, 2005). In particular, a model of one-dimensional stationary magma flow has frequently been used for the study of magmatic and eruptive processes (Wood, 1995; Woods and Koyaguchi, 1994; Papale, 2001; Mastin, 2005; Melnik et al., 2005; Macedonio et al., 2005), while non-stationary magma flow has also been analyzed (Melnik and Sparks, 2002; Proussevitch and Sahagian, 2005). In this paper we employ a non-stationary model because we are interested in how the pre-eruptive process leads to different eruption styles. Studies of non-stationary magma flow are necessary to reveal some features of eruptive processes that involve the uncertainties and complexities of non-linear systems (Sparks, 2003).

2. Escape of gas through permeable flow

Coalescence of bubbles in vesiculating magma is considered to create permeability which serves to connect the dispersed gas phase (Klug and Cashman, 1996).

Laboratory measurements of permeability for some artificial silicate melts that have been decompressed at high temperatures actually confirm that permeable networks of bubbles can be created in ascending magmas (Burgisser and Gardner, 2005; Takeuchi et al., 2005). Permeable gas flow thus can be an important mechanism contributing to the release of volatiles from magma. It is expected that substantial escape of gas through the permeable flow to the country rock is likely to result in silent effusion of lava rather than explosive eruptions in which abundant gas inevitably fragments magma (Eichelberger et al., 1986; Jaupart and Allegre, 1991). In this section we propose a driving force of the lateral permeable gas flow and determine the rate of magma degassing.

Let us first suppose that the ascending magma column is of a vertical, planar shape even if almost the same argument can be applied to a cylindrical conduit after minor modification, as shown below. We assume that gas in the magma moves laterally along the x direction with its volumetric flow rate q and escapes into the adjacent country rock. If the gas is subjected to the pressure p_g and moves as a permeable flow following Darcy's Law (Turcotte and Schubert, 1982), the degassing rate G , i.e., the mass of gas that is lost from a unit volume of the magma in a unit time, is represented by

$$G = \frac{\partial}{\partial x}(\rho_g q) \quad q = -\frac{\kappa}{\eta_g} \frac{\partial p_g}{\partial x} \quad (1)$$

where ρ_g , η_g , and κ are the density, viscosity and permeability of the gas phase, respectively. The effect of the vertical gas flow is not considered in this treatment.

The pressure gradient that may drive the permeable gas flow is assumed here to originate from the bubble pressure that is enhanced due to the viscous resistance of magma to gas expansion in ascending and decompressed magma (Fig. 1a). The same mechanism of bubble pressure enhancement was considered in the model of Melnik (2000) and Melnik et al. (2005) to lead to the magma fragmentation in a vertical magma flow, but we here focus on the lateral gradient of the ascent velocity and gas pressure (Fig. 1b). For mathematical simplicity we assume that the gas is confined in spherical bubbles with the same radius b , neglecting the volumes of connecting paths. Due to the viscous resistance of magma, the bubble expansion cannot immediately follow the decompression but is delayed with the following pressure difference $p_g - p$ between the bubble gas and the ambient liquid magma:

$$p_g - p = \frac{4\eta}{b} \frac{db}{dt} \quad (2)$$

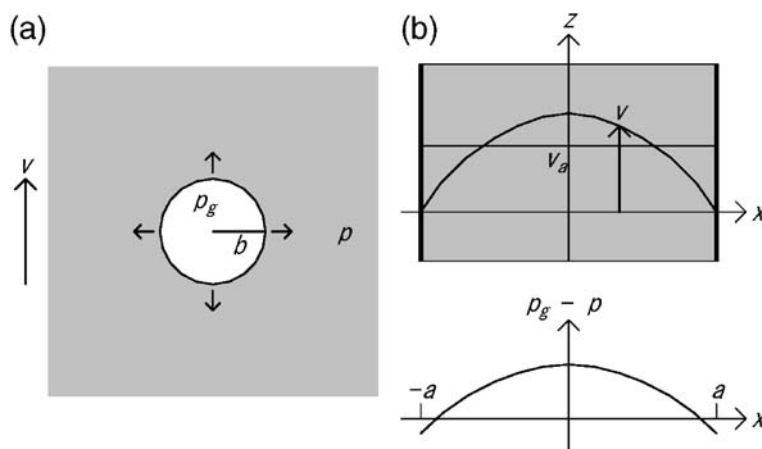


Fig. 1. The escape of gas from magma through the permeable flow driven by the lateral gradient of the ascent velocity. (a) The gas pressure p_g enhanced due to viscous resistance to bubble expansion in ascending and decompressed magma. (b) The magma ascent velocity v and the pressure difference $p_g - p$ between the gas and the liquid magma shown as a function of the lateral coordinate x over the thickness $2a$ of a planar conduit.

where η is the viscosity of the liquid magma. This relation is obtained from a simple expression of the radial velocity and the normal stress component at the bubble wall surrounded by a spherically symmetric fluid flow.

To rewrite Eq. (2) we introduce the mass of the gas phase M contained in a unit volume of the magma:

$$M = \rho_g \psi = \frac{4\pi}{3} b^3 \rho_g N \quad (3)$$

where ψ is the volume fraction of the gas phase and N is the number of bubbles in the unit volume. In evaluating the change of b from Eq. (3) we may neglect the change of N assuming that the nucleation of bubbles occurs mostly in a deeper conduit and does not play an important role in shallower processes involving significant degassing. In fact, N generally changes in proportion to the bulk magma density due to expansion of the magma but this change is much smaller than the changes of b and ρ_g . With this assumption Eq. (2) can be written in terms of the time derivatives of M and ρ_g . For ρ_g we use the equation of state for an ideal gas:

$$\rho_g = \frac{p}{RT} \quad (4)$$

where T is the absolute temperature and R is the universal gas constant divided by the molecular weight of the gas molecule. In Eq. (4) the pressure difference between p_g and p is neglected because this difference gives only a minor change of the gas density value and has no significant effect on the expression of the degassing rate. On the other hand, the change of M arises due to both the gas loss represented by Eq. (1) and the

new deposition of dissolving gas molecules on bubbles due to the decompression. To evaluate the second contribution, we make a simple assumption that the gas molecules exceeding the solubility deposit so as to meet the equilibrium condition. Summing up all these contributions to Eq. (2) yields

$$p_g - p = -\frac{4\eta}{3\rho_g \psi} \left\{ G + \left[\frac{\psi}{RT} + \rho_l (1 - \psi) \frac{dm_d}{dp} \right] \frac{dp}{dt} \right\} \quad (5)$$

where ρ_l is the density of the liquid magma and m_d is the solubility, i.e., the mass of gas molecules that can dissolve in a unit mass of the liquid magma.

The vertical pressure change associated with the decompression is proportional to the magma ascent velocity v , as

$$\frac{dp}{dt} = -g\rho_r v \quad v = \frac{3}{2} v_a \frac{a^2 - x^2}{a^2} \quad (6)$$

where g is the gravitational acceleration and ρ_r is the reference density that represents the pressure gradient in the magma. The value of ρ_r may be approximated by the density of the country rock as long as the magma pressure nearly follows the lithostatic distribution. In Eq. (6) v is given by the velocity in a Poiseuille flow between vertical parallel walls separated by $2a$ with its mean value v_a (Fig. 1b). Eq. (5) is now regarded as a differential equation for p_g with respect to x because the term G in its right-hand side is the second derivative of p_g in Eq. (1). According to Eq. (6) v has an explicit and strong dependence on x . If the implicit dependence of other variables on x is

neglected, the differential Eq. (5) can be satisfied by a simple solution:

$$p_g - p = \frac{G}{\rho_g \psi} \left[-\frac{4\eta}{3} + \frac{\eta_g \psi}{2\kappa} (a^2 - x^2) \right] \quad (7)$$

with the following expression of G independent of x :

$$G = \frac{4g\rho_r\eta\kappa v_a}{a^2\eta_g\psi} \left[\frac{\psi}{RT} + \rho_l(1 - \psi) \frac{dm_d}{dp} \right] \quad (8)$$

The degassing rate G given above is proportional to η , κ and v_a and dependent on ψ and p in a more complex way.

An almost identical treatment can be applicable to a cylindrical conduit of the radius a . In this case Eqs. (1) and (6) are slightly modified understanding that x represents the radial distance from the center of the cylinder. As a result of this modification, the factor 4 in (8) and the factor 1/2 in the second term of Eq. (7) are replaced by 32/3 and 1/4, respectively, for the cylindrical geometry.

It is noted that the pressure difference in Eq. (7) becomes negative near the boundary walls at $x=\pm a$ where the effect of decompression (the second term) is smaller than that of the gas loss (the first term). When gas has a smaller pressure than the liquid magma, permeable networks must be conserved by some mechanisms other than gas expansion. Near the boundary walls, fractures can be generated in the magma in the glass transition by brittle failure because of the high shear-strain rates and low temperatures realized there (Gonnerman and Manga, 2003). In fact, there is evidence for shear fractures in the textures of rhyolite conduits (Tuffen and Dingwell, 2005; Tuffen et al., 2003). Repeated failures of magma can also be an important trigger mechanism of the low-frequency earthquakes observed in many active volcanoes (Neuberg et al., 2006). It is thus likely that paths for gas loss are conserved near the boundary walls as a result of such shear fractures.

In addition, Massol and Jaupart (1999) showed that a horizontal gradient of the bulk magma pressure can be produced by the vertical gradient of ascent velocity or bulk magma viscosity. The ascent velocity clearly increases with a decrease of the bulk magma density associated with the expansion of bubble gas. In this respect it is inferred that the lateral variation of gas pressure in bubbles we are considering in Eq. (2) would be significantly greater than that of the bulk magma pressure because the average of pressure over incompressible liquid magma must significantly reduce the original pressure anomaly in bubbles. According to a scaling analysis, the ratio of the pressure variation in Eq. (7) to that of the bulk magma pressure obtained by Massol and Jaupart (1999) is roughly equal to the

magma chamber pressure divided by the pressure at each flow point. Therefore, the horizontal variation of bubble pressure is more important, particularly for shallow conduits. There is still a possibility, however, that the gradient of the bulk magma pressure may play a significant role in lateral gas flow if the viscosity gradient is quite large.

3. Magma flow model

To examine the degassing mechanism formulated above we make a computer simulation of the magma ascent process. For the simulation we employ a model of one-dimensional conduit flow in which the ascent velocity, the bulk magma density and other quantities are averaged over the conduit cross-section at each depth (Fig. 2). This approximation has been employed in many previous studies of both stationary (Woods and Koyaguchi, 1994; Woods, 1995; Papale, 2001; Mastin, 2005; Melnik et al., 2005; Macedonio et al., 2005) and non-stationary (Melnik and Sparks, 2002; Proussevitch and Sahagian, 2005) magma flow. To resolve the time-dependent evolution toward an eruption we here employ a non-stationary model, such that variables may depend on the time t as well as the vertical coordinate z . The conduit is fed with magma from a chamber at its bottom $z=0$. The magma flow is bounded by the magma head at $z=z_m$, which can move upward until it reaches the surface vent at $z=h$ and causes an eruption.

Explosive eruptions involve magma fragmentation, which transforms a bubbly liquid flow into a gassy flow mixed with magma fragments. So far several criterions

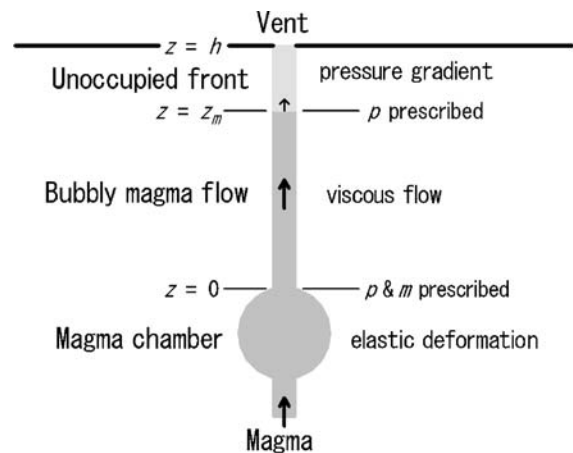


Fig. 2. The model of time-dependent magma flow used for the simulation. Bubbly magma flow in the conduit is constrained by the initial and boundary conditions specified in terms of the pressure p and the mass ratio of the total volatiles to the liquid magma m . The magma head at $z=z_m$ can be moved up to the surface vent at $z=h$.

with different physical bases, such as gas volume fraction, strain rate (Papale, 1999) and brittle failure (Alidibirov and Dingwell, 1996; Zhang, 1999; Marti et al., 1999), have been proposed to describe the fragmentation. Whether brittle or ductile fragmentation will occur is probably related to the different conditions of timescale and magma strength (Ichihara et al., 2002). On the other hand, a simple criterion in which fragmentation occurs above a critical gas volume fraction in magma is known empirically and has frequently been used for the analysis of magma flow. The critical volume fraction of 75–83% is derived from a geometrical constraint on the packing of spherical bubbles (Sparks, 1978). Some observations of pumice samples suggest that magma may fragment at the vesicularity of about 64% if it is sufficiently sheared, while it may expand to higher vesicularity with connected permeable network of bubbles (Gardner et al., 1996).

In fact the fragmentation is not explicitly taken into account in calculating magma flow in the present treatment. One reason for this is technical, i.e., the upper gassy flow containing magma fragments is characterized by a very short timescale involving the sound speed and is difficult to accommodate to slow liquid flow in the lower conduit. The treatment is made more complicated if the gassy flow region involves inhomogeneous degassing (Papale, 2001). A more essential reason is that it is desirable to keep our conclusion as independent of individual fragmentation criteria as possible. If our purpose is restricted to predicting the eruption style, this can be achieved without a strong reliance on a specific fragmentation condition. Namely, we may regard an eruption as explosive or effusive when the gas volume fraction calculated for bubbly magma flow is sufficiently large or small, respectively. The eruption style judged in this way turns out to be insensitive to a critical value of the gas volume fraction, as the results of our simulation show later. Therefore, we can practically obtain the criterion of eruption style almost independently of the fragmentation condition even if the calculated flow that lacks the upper gassy part cannot correctly represent the actual state for explosive eruptions.

Our treatment of non-stationary bubbly magma flow is essentially the same as others (Melnik and Sparks, 2002; Proussevitch and Sahagian, 2005) but more or less simplified, neglecting inertia terms, temperature changes and disequilibrium thermodynamic effects. The degassing rate G formulated above is taken into account in the time-dependent mass conservation as

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial z}(\rho v) - G \quad \frac{\partial M_g}{\partial t} = -\frac{\partial}{\partial z}(M_g v) - G \quad (9)$$

for the whole magma and the volatiles contained in it, respectively. Here ρ is the bulk magma density and M_g is the mass of the total volatiles that are either dissolved or contained as bubbles in a unit volume of the magma. It is assumed here that the volatiles move up at the same speed v as the liquid magma.

If the permeability is given as a function of the gas volume fraction as

$$\kappa = \kappa_1 \left(\frac{\psi - \psi_c}{1 - \psi_c} \right)^n \quad \text{for } \psi > \psi_c \quad (10)$$

with the constants κ_1 , ψ_c and n (Saar and Manga, 1999; Blower, 2001; Mueller et al., 2005), the degassing rate G in Eq. (8) is rewritten as

$$G = Dv \left(\frac{\psi - \psi_c}{1 - \psi_c} \right)^n \frac{\psi + e(1 - \psi)}{\psi} \quad (11)$$

where

$$D = \frac{4\alpha g \rho_r \eta \kappa_1}{3a^2 \eta_g RT} \quad e = \rho_l RT \frac{dm_d}{dp} \quad (12)$$

The constant κ_1 is the permeability in the limit of $\psi = 1$, and the degassing factor D defined in Eq. (12) plays an important role in this paper. Following the definition of variables in this section, v_a in Eq. (8) is simply replaced by v in Eq. (11). In Eq. (12), α is a numerical factor that depends on the conduit geometry; it is 3 for a planar conduit with the width $2a$ and 8 for a cylindrical column of the radius a (see the remark about the cylindrical conduit in the Last section).

Because magma has a very high viscosity, the inertia terms can be neglected in the equation of motion (i.e., the momentum conservation law) so that we have

$$v = -\frac{1}{f} \left(\frac{\partial p}{\partial z} + g\rho \right) \quad f = \frac{\alpha \eta_f}{a^2} \quad (13)$$

Here f is the “wall friction” that is expressed for a Newtonian fluid in terms of the magma viscosity η_f and the same geometrical parameters a and α as in Eq. (12). It is noted that the conduit is assumed to be rigid in Eq. (13). The equation of state is given as

$$\rho = (1 - \psi)\rho_l + \psi\rho_g \quad \rho_l = \rho_r \left(1 + \frac{p - p_r}{K_l} \right) \quad (14)$$

In Eq. (14) the pressure dependence of the liquid magma density ρ_l is approximated by a linear relation with the bulk modulus K_l and the reference density ρ_r at the reference pressure p_r (we may assume $p_r = g\rho_r h$ for simplicity). The pressure dependence of ρ_g that completes Eq. (14) has been given in Eq. (4).

The gas volume fraction of the magma is represented as

$$\psi = \frac{\rho}{\rho_g} \frac{m - m_1}{1 + m} \quad m = \frac{M_g}{\rho - M_g} \quad (15)$$

in terms of the mass ratio m of the total volatiles to the liquid magma and the mass ratio m_1 for the dissolving part. Assuming that the volatiles are in chemical equilibrium, the dissolving mass is given by

$$m_1 = m_d(m > m_d) \quad m_1 = m(m < m_d) \quad m_d = \left(\frac{p}{p_s}\right)^\gamma \quad (16)$$

Here the pressure dependence of the solubility m_d is described by the last equation with constants p_s and γ (Burnham, 1975).

We now have a complete set of equations for our simulation. Namely, the two equations in Eq. (9) coupled with other equations can be regarded as differential equations to calculate the time dependence of the variables. If the distributions of ρ and M_g along z are prescribed at a certain time t , the value of m is first determined at each point of z from the second equation in Eq. (15). Then p and ψ are obtained so as to meet Eqs. (14) (4) (16) and the first equation in Eq. (15). We can thus evaluate v from Eq. (13), G from Eq. (11) with the third equation of Eq. (16) and finally the time derivatives of ρ and M_g in Eq. (9). Therefore we can integrate Eq. (9) with respect to t and obtain ρ and M_g along with other variables as a function of t .

In this calculation it is more convenient to regard p and m as the primary variables in place of ρ and M_g . With this understanding a solution of the differential equations is constrained by the initial and boundary conditions that are specified in terms of p and m . Because some equations we are dealing with contain the first derivative of m and the second derivative of p with respect to z , we may specify the values of p and m at $z=0$ and the value of p at $z=z_m$ for the boundary condition (Fig. 2). At $z=0$ we here prescribe suitable constant values for p and m even if our simulation scheme is designed to allow the bottom pressure to vary in accordance with the magma budget and elastic deformation in the chamber. At $z=z_m$, the pressure is prescribed so as to represent a suitable pressure gradient that may characterize the frontal zone overlying the magma. The magma head location z_m is moved with the same speed v as the magma at this point. After the magma head reaches the surface vent at $z=h$, its location is fixed there with p equated with the atmospheric pressure.

For the initial distribution of p at $t=0$ we simply assume a linear relation that may connect the two boundary values at $z=0$ and $z=h$. For m , we like to see the effect of volatiles that are poured with magma from the chamber into the conduit, which is initially gas free. A sharp drop of m in the conduit, however, tends to make the numerical calculation unstable and so we assume a ramp shape of the initial distribution of m that linearly decreases up to zero to a shallower point of the conduit. We may start the calculation at $t=0$ from the initial distribution of m that is nearly in contact with the solubility curve somewhere in the conduit. This is because, before the contact occurs, the distributions of all variables simply shift upward with the ascending background flow, keeping their forms almost unchanged. The initial location of z_m is set at the highest point of the volatile-rich zone with finite m values. For $t>0$, the pressure above z_m that is kept the same as the initial distribution specifies the upper boundary condition of the conduit flow, and the flow state including p below z_m is calculated from the above-mentioned equations. The location of z_m changes with the calculated ascent velocity at $z=z_m$.

4. Parameters for the numerical calculation

Some parameters that appear in the model for our simulation involve large uncertainty. In particular, the nature of permeable gas flow in magma has not yet been constrained very well. If the inertial effects play an important role in the gas flow, the Darcy's law itself should be modified (Rust and Cashman, 2004). Furthermore, actual gas flow in natural liquid magma may behave differently than predicted from permeability measurements for the quenched samples. For these reasons, we prefer to restrict our purpose to finding some fundamental features of the problem and to make our simulation as simple as possible.

With respect to the permeable gas flow, much interest has been focused on the relation between the permeability κ and the gas volume fraction ψ . The relation (10) was theoretically derived with parameter values such as $n=2$ and $\psi_c=0.3$ from the model of granular materials (Saar and Manga, 1999) and the analogy with a network in an electrical circuit (Blower, 2001). On the other hand, Klug and Cashman (1996) proposed a relation with $n=3.5$ and $\psi_c=0$, fitting the data of permeability and porosity available for silicate compounds, but there is still some controversy about the presence or absence of the threshold ψ_c (Klug et al., 2002; Takeuchi et al., 2005). Based on their new data for decompressed lavas, Mueller et al. (2005) suggested that both relations,

one with $n=3$ and $\psi_c=0$ and the other with $n=2$ and $\psi_c=0.3$, are able to fit to the data within their large scatter. In this paper we mainly show the results of simulation for $n=3$ and $\psi_c=0$ with some additional results for $n=2$ and $\psi_c=0.3$ to find the effect of the permeability relation.

Other parameters may be treated in a simplified manner comparable to the permeability. Even if the magma viscosity depends on temperature, pressure, and the presence of dissolved volatile components and gas bubbles, we assume that the viscosity is a constant independent of z and t . In addition, the local viscosity η of the liquid magma surrounding the bubbles plays a more important role in our analysis than the bulk magma viscosity η_f in Eq. (13) that controls the ascent motion. Since we neglect the effect of temperature change, we may regard the degassing factor D in Eq. (12) as well as the wall friction f in Eq. (13) as constant, noting that the gas viscosity η_g is almost entirely dependent on temperature.

For the solubility of volatiles, the power law with $\gamma=1/2$ is often employed in Eq. (16) understanding that it is applicable to water vapor (Burnham, 1975). In our simulation, however, we assume $\gamma=1$ to simplify the analysis. This assumption allows the parameter e in Eq. (12) to be reduced to a constant. If $\gamma=1/2$ is assumed instead, e must be treated as a variable that depends on the pressure p and becomes very large for small values of p . Such anomalous behavior of e seriously disturbs the state of magma in a shallow conduit. This unexpected effect is probably suppressed by over-saturation of gas molecules in nature but can be avoided by the simple assumption of $\gamma=1$ in the simulation.

For the numerical calculation we introduce the dimensionless variables and constants that are scaled by the units in Table 1. These units are systematically derived from the base units, ρ_r , h and f , for the density, the vertical coordinate and the wall friction, respectively. The constants having the dimension of pressure, such as p_s , K_1 and K_g , are also scaled by p_r . Note that the variables ψ and m and the constant e are dimensionless from the onset. The dimensionless equations written in terms of the scaled variables have the same forms as the original but the constants h , ρ_r , p_r , f , g and a can be set to unity. Some representative values of the scaling units are shown in Table 1. It is noted that the viscosity changes over a wide range from basaltic to rhyolitic magmas so that the scaling units of time, velocity and the degassing rate could vary over a wide range through f .

In the numerical calculations whose results are shown below the material constants and the initial and

Table 1

Units used to scale variables and constants

Quantity	Unit to scale		Typical value
Variable			
Density	ρ	ρ_r	2600 kg/m ³
Pressure	p	$p_r = g\rho_r h$	130 MPa
Velocity	v	$g\rho_r/f$	26 (2.6 × 10 ⁻⁴) m/s
Coordinate	z	h	5 km
Time	t	$p_f/g^2\rho_r^2$	190 (1.9 × 10 ⁷) s
Degassing rate	G	$g^2\rho_r^3/p_f$	14 (1.4 × 10 ⁻⁴) kg/m ³ s
Constant			
Wall friction	f	$f = \alpha\eta_r/a^2$	10 ³ (10 ⁸) kg/m ³ s
Degassing factor	D	$g\rho_r^2/p_r$	0.52 kg/m ⁴

In estimating typical values of the units, ρ_r , h and f are first assumed as well as $g=10$ m/s². The typical values of f may correspond to $\eta_r=10^2$ Pa s and $a^2/\alpha=1$ m² ($\eta_r=10^9$ Pa s and $a^2/\alpha=100$ m²).

boundary conditions are specified in the following way. The elastic properties of the magma and the H₂O vapor may be roughly represented without great ambiguity by $K_1/p_r=100$ and $\rho_r RT/p_r=10$, respectively, assuming $K_1=15$ GPa (Murase and McBirney, 1973) and $T=1200$ K. The gross behavior of the solubility may be approximately described for the vapor in the magma by $p_s=4$ GPa with the assumption of $\gamma=1$ (McMillan, 1994), which leads to $p_s/p_r=30$ and $e=0.3$. On the other hand, the degassing factor D should be permitted to change over a wide range so as to represent the great variability of the magma viscosity and permeability. For the boundary condition, we can set the dimensionless pressure at the surface, p_h/p_r , to be 0.001, corresponding to the atmospheric pressure. The pressure p_o and the mass ratio of volatiles m_o in the supplied magma are treated as variable parameters.

For the numerical calculation the range of the dimensionless coordinate ($0 < z/h < 1$) is divided into equally spaced grid points. The discrete variables at the internal grid points between the magma head and the conduit bottom are calculated as a function of time by integrating the dimensionless differential equations with the Runge–Kutta method. The highest grid point is selected as the point that is above and the nearest to the magma head location and is shifted upward when the magma head crosses it. The boundary values of variables other than those prescribed by the boundary condition (p at the top and p and m at the bottom) are obtained from extrapolating the values of these variables on some internal grids. The number of grids covering the whole conduit is usually set to be 50. When this grid size is taken, the dimensionless time step for the integration should be of the order of 10⁻⁶ or smaller so as to make the calculation convergent. It takes about one

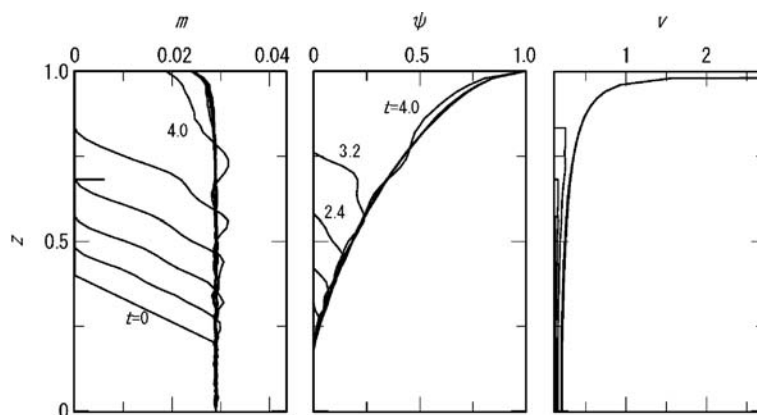


Fig. 3. Distribution of the mass ratio of the total volatiles m , the gas volume fraction ψ and the magma ascent velocity v along the vertical coordinate z at some selected steps of time t for $n=3$, $\psi_c=0$, $D=0.01$, $p_o=1.1$ and $m_o=0.029$. Other parameter values are given in the text. All variables and constants are represented in the dimensionless scales defined in Table 1.

half hour to obtain a single solution of the time-dependent magma flow using a personal computer.

5. Results of the simulation

In this section some results of the numerical calculations are presented using the dimensionless variables that are denoted here by the same notations as the original. Fig. 3 demonstrates one of the typical results of calculated time-dependent magma flow. In this figure spatial distributions of the mass ratio of the total volatiles m (left), the gas volume fraction ψ (center) and the magma ascent velocity v (right) are shown at every dimensionless time step of 0.8. The area occupied by bubbly magma gradually expands upward as the magma head moves up from the depth at which m crosses the solubility curve at $t=0$. For this calculation the dimensionless parameters are prescribed as $D=0.01$, $p_o=1.1$ and $m_o=0.029$ with $n=3$ and $\psi_c=0$ for the permeability relation in addition to those specified in the Previous section.

Fig. 3 shows a case with insignificant degassing. According to the distribution of m , the magma loses only a small amount of volatiles. As the magma ascends and the pressure is reduced, ψ increases remarkably due to continued generation and expansion of the gas phase. Finally, the magma, which has a high gas volume fraction close to unity, erupts when it reaches the surface vent at $z=1$. The ascent motion is greatly accelerated due to significant expansion of the underlying magma, so that the velocity of erupting magma becomes quite large at the surface. Because of this high gas volume fraction in the erupting magma, this case is considered to represent an explosive eruption even if the process of magma fragmentation is not explicitly taken into account in the present calculation.

The same calculation results are displayed in a different way in Fig. 4, where the ascent velocities, v_o at $z=0$ and v_h at $z=1$, the magma head location z_m , and the gas volume fraction ψ_h of the erupting magma are given as a function of time t . When the magma head reaches the surface at $z=1$ an eruption starts with finite values of v_h and ψ_h . The magma flow finally approaches a stationary state because the boundary conditions are all fixed. In this case, ψ_h becomes as high as 0.996 and v_h is about 400 times greater than v_o in the final state.

In contrast to the case in Figs. 3 and 4, which represents an explosive eruption, Figs. 5 and 6 demonstrate another

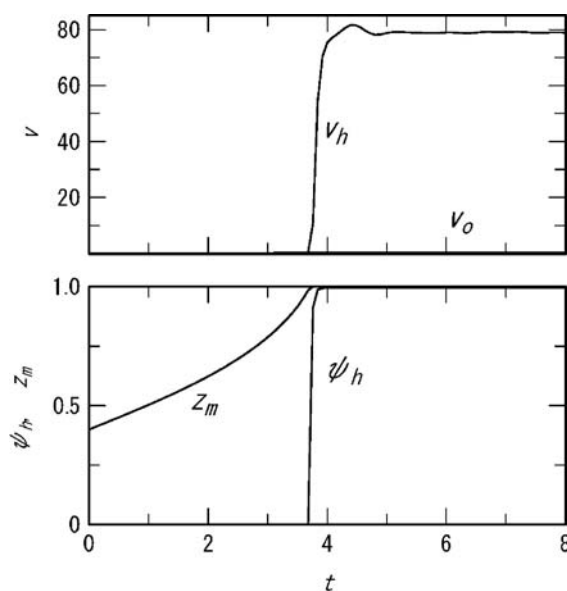


Fig. 4. Changes of the magma ascent velocities, v_o at $z=0$ and v_h at $z=1$, the gas volume fraction of the erupting magma ψ_h and the magma head location z_m with time t for the same case as in Fig. 3.

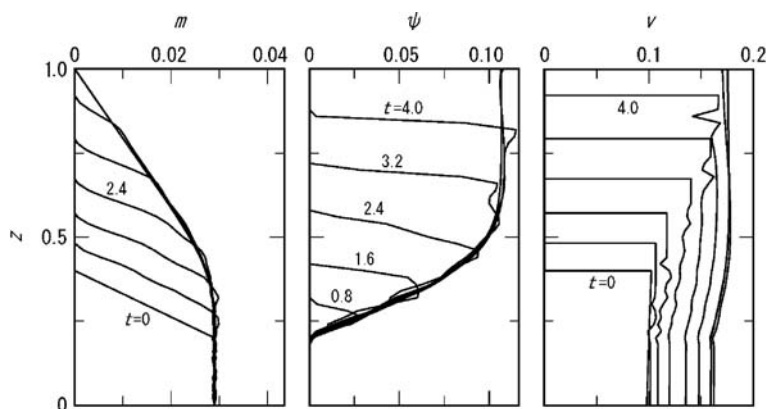


Fig. 5. Distribution of the mass ratio of the total volatiles m , the gas volume fraction ψ and the magma ascent velocity v along the vertical coordinate z at some selected steps of time t for $D=10$. The parameter values other than D are the same as in Fig. 3.

typical result of a calculation involving significant gas loss. In this case $D=10$ is prescribed instead with the other conditions kept same. Here most volatiles are lost from the magma as soon as they deposit on bubbles in the gas phase. After bubbles form, m is only slightly higher than the solubility (Fig. 5, left) and ψ cannot be larger than a certain value even near the surface (center). The ascent motion is not accelerated very much (right) so that v_o and v_h do not differ significantly (Fig. 6). Because of this low gas volume fraction during the ascent motion this case is understood to represent an effusive eruption.

Fig. 7 gives the final gas volume fraction in the erupting magma, $\psi_{h,\infty}$, as a function of the dimension-

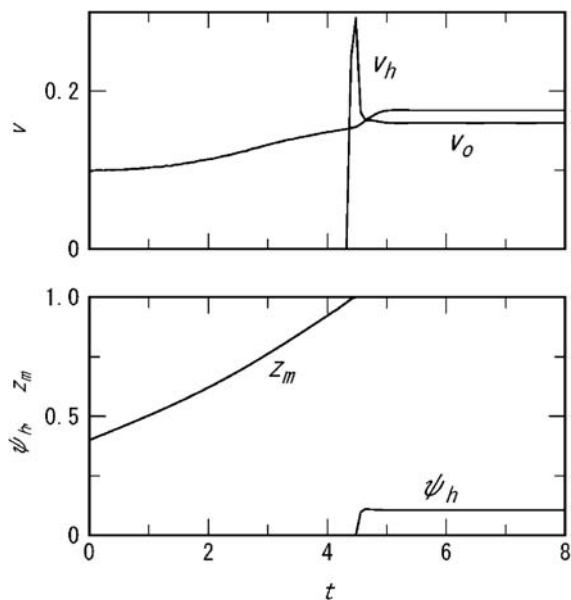


Fig. 6. Changes of the magma ascent velocities, v_o at $z=0$ and v_h at $z=1$, the gas volume fraction of the erupting magma ψ_h and the magma head location z_m with time t for the same case as in Fig. 5.

less degassing factor D . Here the parameters other than D are fixed at the same values as have been adopted in Figs. 3–6. Together with this result another curve corresponding to the permeability relation with $n=2$ and $\psi_c=0.3$ is added for comparison. According to Fig. 7, $\psi_{h,\infty}$ is close to unity for relatively small values of D while it approaches ψ_c for somewhat greater values of D . These two states correspond to explosive and effusive eruptions, respectively. It is noted that the two states are separated rather sharply by a relatively small change of D . This feature is considered to result from coupling between the gas loss and the ascent motion. Namely, insignificant degassing allows great acceleration of ascent motion that may prevent the magma from losing the gas phase effectively during a short ascent time in return, and vice versa.

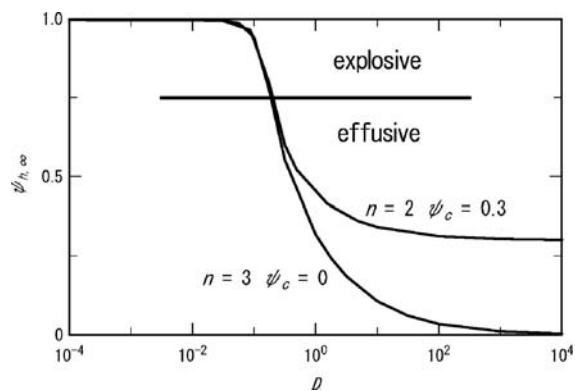


Fig. 7. The final gas volume fraction of the erupting magma $\psi_{h,\infty}$ as a function of the degassing factor D for $p_o=1.1$, $m_o=0.029$ and the other parameter values specified in the text. The results for the two permeability relations with $n=3$ and $\psi_c=0$ and $n=2$ and $\psi_c=0.3$ are compared. All variables and constants are represented in the dimensionless scales defined in Table 1.

According to Fig. 7 the eruption style strongly depends on the dimensionless degassing factor D . It can be further pointed out that this relation between D and $\psi_{h,\infty}$ is not very sensitive to other conditions. For example, Fig. 8 shows that the pressure p_o at the conduit bottom has little effect on $\psi_{h,\infty}$ except for the intermediate values of D between the explosive and effusive ranges. Therefore, p_o does not influence the dependence of $\psi_{h,\infty}$ on D and the eruption style substantially even if p_o controls the ascent velocity and the time t_e required before an eruption starts. Similarly, Fig. 9 demonstrates that the mass ratio m_o of supplied volatiles does not affect $\psi_{h,\infty}$ significantly even if it may shift the depth at which the bubble formation starts. There the three-fold change of m_o results in an only 10% change of $\psi_{h,\infty}$ even in this intermediate condition between explosive and effusive eruptions.

6. Criterion of the eruption style

In this section the dimensionless degassing factor is denoted by D' to discriminate it explicitly from the original dimensional form D defined in Eq. (12). Recalling the unit for scaling in Table 1, D' is written as

$$D' = \left(\frac{4p_r}{3RT\rho_r} \right) \left(\frac{\eta}{\eta_g} \right) \left(\frac{\alpha\kappa_1}{a^2} \right) \quad (17)$$

This expression of D' consists of multiplication of the three dimensionless factors. The first factor involving T , p_r and ρ_r can be regarded almost as a constant of about

0.1. The second factor changes over a wide range in proportion to the magma viscosity η , whereas the gas viscosity η_g in the denominator is almost completely dependent on temperature alone and is estimated to be about 10^{-5} Pa s. The third factor again changes over a wide range in proportion to the permeability κ_1 defined in the limit of the gas volume fraction ψ to unity. This factor also contains the effect of the size and geometry type of the magma conduit: $\alpha=3$ for a planar conduit with the width $2a$ and $\alpha=8$ for a cylindrical conduit of the radius a .

On the other hand, our simulation above predicted that the eruption style is mainly controlled by D' . Namely, an eruption is expected to be explosive or effusive if D' is smaller or greater, respectively, than its critical value D'_c . According to Fig. 7 with some supplemental information from Figs. 8 and 9, D'_c is estimated to be about 0.1 even if it may more or less depend on some other conditions, including the pressure and volatile concentration in the supplied magma as well as the permeability relation. The criterion of the eruption style is thus well represented by the diagram in Fig. 10, which displays the ranges of explosive and effusive eruptions on the η – κ_1 plane separated by a boundary line that may shift with the size and geometry type of the conduit.

This criterion of the eruption style physically reflects the amount of the volatiles that remains in magma without being lost through the permeable gas flow during the time spent for the magma ascent. An eruption should be explosive or effusive for relatively lower or

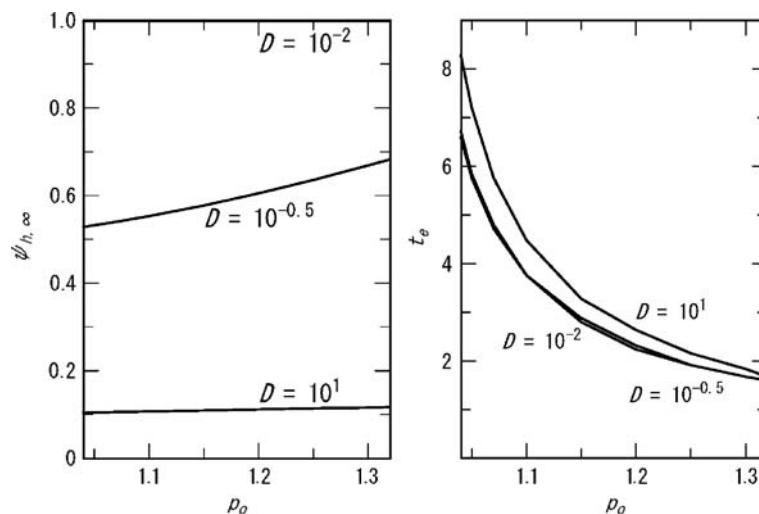


Fig. 8. The effect of the bottom pressure p_o on the final gas volume fraction of the erupting magma $\psi_{h,\infty}$ and the time spent from the start of vesiculation to the eruption t_e shown for three selected values of the degassing factor D . The parameter values are fixed as $m_o=0.029$, $n=3$ and $\psi_c=0$ with the others specified in the text. All variables and constants are represented in the dimensionless scales defined in Table 1.

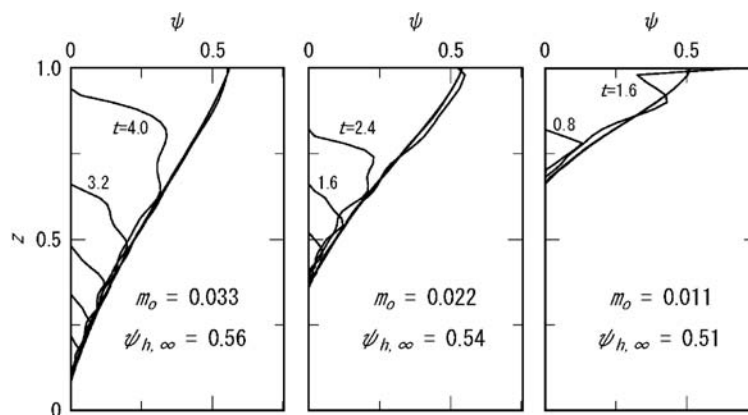


Fig. 9. Distribution of the gas volume fraction ψ at some selected time steps for three cases with different mass ratios of the supplied total volatiles, m_o . The final gas volume fraction of the erupting magma $\psi_{h,\infty}$ is compared among the three cases. The parameter values are fixed as $D=10^{-0.5}$, $p_o=1.1$, $n=3$ and $\psi_c=0$ with the others specified in the text. All variables and constants are represented in the dimensionless scales defined in Table 1.

higher permeability, respectively, simply because a higher permeability facilitates the escape of gas from magma. A higher magma viscosity creates a more favorable condition for effusive eruptions because it produces a higher gas pressure due to greater viscous resistance to gas expansion and promotes the permeable gas flow. A larger conduit size decreases the lateral gradient of ascent velocity and gas pressure and so shifts an eruption to the explosive side. For a similar reason cylindrical conduits create a more favorable condition for effusive eruptions than fissures even if the effect is small. On the other hand, the criterion does not explicitly depend on the bulk magma viscosity η_f . It may be expected that a greater ascent velocity caused by a smaller viscosity should reduce the time taken for the ascent motion and hinder the gas loss. In fact the in-

creased ascent velocity simultaneously enhances the lateral gradient of gas pressure that promotes the permeable gas flow so as to cancel out the favorable effect on explosive eruption.

7. Discussion

Our criterion of the eruption style (Fig. 10) predicts that effusive eruptions should occur in the condition of relatively higher viscosity. This conclusion seems to conflict with the knowledge that low-viscous basaltic magmas usually participate in silent effusive eruptions while explosive eruptions are generally produced by more felsic magmas having higher viscosities. The effect of viscosity in this criterion may be partly compensated by the effect of permeability, because measured permeability tends to be greater for basalts (Saar and Manga, 1999) than dacites and rhyolites (Eichelberger et al., 1986; Klug and Cashman, 1996; Mueller et al., 2005). In addition, permeable gas flow may be made easier in deformable paths (McKenzie, 1984), which are more likely to occur in basaltic magmas. It is noted that a typical type of eruption for low-viscous magmas is the strombolian eruption, which emits lava fragments somewhat explosively. Furthermore, basaltic magmas sometimes produce even strongly explosive plinian eruptions with tephra deposits (Gurenko et al., 2005). Since felsic magmas also produce lava flow or lava dome we can conclude that the viscosity is not a unique determinant of the feasibility of magma fragmentation. Nevertheless, felsic magmas tend to cause more violent explosions, probably because they have apparently greater strengths and allow higher pressures to be stored in them.

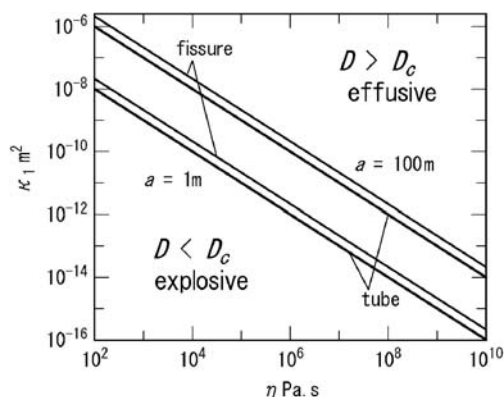


Fig. 10. The criterion of eruption style displayed on the plane of the magma viscosity η and the gas permeability κ_1 in the limit of the gas volume fraction to unity. The boundary line that separates the ranges of explosive and effusive eruptions shifts with the size $2a$ and geometry type (planar fissure or cylindrical tube) of the conduit.

According to the same criterion explosive eruptions should tend to occur in conduits of larger sizes. This prediction may be interpreted to mean that explosive eruptions are associated with greater eruption rates, as has been pointed out previously (Eichelberger et al., 1986; Jaupart and Allegre, 1991; Woods and Koyaguchi, 1994), because the magma flux generally becomes greater with an increasing conduit size. In fact the eruption rate is proportional to the ascent velocity as well as the area of the conduit cross-section and our analysis indicates that the conduit size makes the more important contribution. As mentioned above, ascent velocity changes the lateral gradient of gas pressure as well as the time spent for the ascent motion so that it may not significantly influence the efficiency of gas escape as a whole.

Using Figs. 8 and 9, we can conclude that the eruption style is less sensitive to the pressure and amount of volatiles at the conduit bottom than to the degassing factor. Nevertheless, these conditions of the supplied magma can significantly influence eruption styles under certain critical conditions. In fact the transition between explosive and effusive eruptions can be caused by a change of the magma pressure (Jaupart and Allegre, 1991). Even if the actual transition of the eruption style occurs in various ways (Sparks, 2003), a typical trend would be the evolution from an explosive to effusive eruption during a short sequence of eruptive events. And in fact, in many events, such as the 1783 Asama, 1914 Sakurajima, 1980 Mt. St. Helens and 1991 Pinatubo eruptions, it has been observed that explosive plinian eruptions are followed by lava flows or the growth of lava domes. Within the framework of our simulation this trend is explained by either the released pressure due to magma discharge from a finite chamber capacity or the reduced amount of volatiles supplied from a stratified magma chamber. The same conclusion actually has been obtained in the treatment of steady-state magma flow, where the transition is attributed to a jump between two states with different flow rates (Melnik, 2000; Slezin, 2003).

Following the suggestion by Eichelberger et al. (1986) and Jaupart and Allegre (1991), we have assumed that the escape of gas from magma is controlled by the permeable gas flow. With the same understanding the effect of the magma permeability on eruptions has been extensively studied so far (Klug and Cashman, 1996; Klug et al., 2002; Melnik et al., 2005). The present paper has shown that gas can be transported by lateral permeable flow in magma, but the transported gas must be removed quickly near the boundary walls to maintain the gas flow. Near the boundary walls gas may be absorbed by the country

rock or migrate into the cracks that have been formed by the brittle failure of magma in glass transition (Gonnerman and Manga, 2003; Neuberg et al., 2006). If these processes do not work effectively enough, the gas pressure would be enhanced there so that further transportation and escape of gas could be suppressed. In this case, the eruption style is no longer controlled by the horizontal gas flow in magma but by some processes in and near the country rock or vertical gas flow in magma.

Finally, we recall various simplifications made in the present treatment. We have neglected the non-linear solubility, over-saturation of dissolved volatiles and dependence of magma viscosity on various conditions. In particular, we have not explicitly considered the gassy flow region that arises in the upper part of the conduit in the case of explosive eruptions. These limitations should be overcome for more quantitative simulations and more detailed discussions.

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References

- Alidibirov, M., Dingwell, D.B., 1996. Magma fragmentation by rapid decompression. *Nature* 380, 146–148.
- Blower, J.D., 2001. Factors controlling permeability–porosity relationships in magma. *Bull. Volcanol.* 63, 497–504.
- Burgisser, A., Gardner, J.E., 2005. Experimental constraints on degassing and permeability in volcanic conduit flow. *Bull. Volcanol.* 67, 42–56.
- Burnham, C.W., 1975. Water and magma: a mixing model. *Geochim. Cosmochim. Acta* 39, 1077–1084.
- Eichelberger, J.C., Carrigan, C.R., Wesrich, H.R., Price, R.H., 1986. Non-explosive silicic volcanism. *Nature* 323, 598–602.
- Gardner, J.E., Thomas, R.M.E., Jaupart, C., Tait, S., 1996. Fragmentation of magma during Plinian volcanic eruptions. *Bull. Volcanol.* 58, 144–162.
- Gonnerman, H., Manga, M., 2003. Explosive volcanism may not be an inevitable consequence of magma fragmentation. *Nature* 426, 432–435.
- Gurenko, A.A., Belousov, A.B., Trumbull, R.B., Sobolev, A.V., 2005. Explosive basaltic volcanism of the Chikurachki Volcano (Kurile arc, Russia): insights on pre-eruptive magmatic conditions and volatile budget revealed from phenocryst-hosted melt inclusions and groundmass glasses. *J. Volcanol. Geotherm. Res.* 147, 203–232.
- Ichihara, M., Rittel, D., Sturtevant, B., 2002. Fragmentation of a porous viscoelastic material: implications to magma fragmentation. *J. Geophys. Res.* 107. doi:10.1029/2001JB000591.
- Jaupart, C., Allegre, C.J., 1991. Gas content, eruption rate and instabilities of eruption regime in silicic volcanoes. *Earth Planet. Sci. Lett.* 102, 413–429.

- Klug, C., Cashman, K.V., 1996. Permeability development in vesiculating magmas: implication for fragmentation. *Bull. Volcanol.* 58, 87–100.
- Klug, C., Cashman, K.V., Bacon, C.R., 2002. Structure and physical characteristics of pumice from the climactic eruption of Mount Mazama (Crater Lake), Oregon. *Bull. Volcanol.* 64, 486–501.
- Macedonio, G., Neri, A., Marti, J., Folch, A., 2005. Temporal evolution of flow conditions in sustained magmatic explosive eruptions. *J. Volcanol. Geotherm. Res.* 143, 153–172.
- Marti, J., Soriano, C., Dingwell, D.B., 1999. Tube pumices as strain markers of the ductile–brittle transition during magma fragmentation. *Nature* 402, 650–653.
- Massol, H., Jaupart, C., 1999. The generation of gas overpressure in volcanic eruptions. *Earth Planet. Sci. Lett.* 166, 57–70.
- Mastin, L.G., 2005. The controlling effect of viscous dissipation on magma flow in silicic conduits. *J. Volcanol. Geotherm. Res.* 143, 17–28.
- McKenzie, D., 1984. The generation and compaction of partially molten rock. *J. Petrol.* 25 (3), 713–765.
- McMillan, P.F., 1994. Water solubility and specification models. In: Carroll, M.R., Holloway, J.R. (Eds.), *Volatiles in Magmas*. Mineralogical Society of America, pp. 131–156.
- Melnik, O., 2000. Dynamics of two-phase conduit flow of high-viscosity gas-saturated magma: large variations of sustained explosive eruption intensity. *Bull. Volcanol.* 62, 153–170.
- Melnik, O.E., Sparks, R.S.J., 1999. Non-linear dynamics of lava dome extrusion. *Nature* 402, 37–41.
- Melnik, O.E., Sparks, R.S.J., 2002. Modeling of conduit flow dynamics during explosive activity at Soufrier Hills Volcano, Montserrat. In: Druit, T.H., Kokelaar, B.P. (Eds.), *The Eruption of Soufrier Hills Volcano, Montserrat. from 1995 to 1999*. Memoirs. Geological Society, London, pp. 307–317.
- Melnik, O., Barmin, A.A., Sparks, R.S.J., 2005. Dynamics of magma flow inside volcanic conduits with bubble overpressure buildup and gas loss through permeable magma. *J. Volcanol. Geotherm. Res.* 143, 53–68.
- Mueller, S., Melnik, O., Spieler, O., Scheu, B., Dingwell, D.B., 2005. Permeability and degassing of dome lavas undergoing rapid decompression: an experimental determination. *Bull. Volcanol.* 67, 526–538.
- Murase, T., McBirney, A.R., 1973. Properties of some common igneous rocks and their melts at high temperatures. *Geol. Soc. Amer. Bull.* 84, 3563–3592.
- Neuberg, J.W., Tuffen, H., Collier, L., Green, D., Powell, T., Dingwell, D., 2006. The trigger mechanism of low-frequency earthquakes on Montserrat. *J. Volcanol. Geotherm. Res.* 153, 37–50.
- Papale, P., 1999. Strain-induced magma fragmentation in explosive eruptions. *Nature* 397, 425–428.
- Papale, P., 2001. Dynamics of magma flow in volcanic conduits with variable fragmentation efficiency and nonequilibrium pumice degassing. *J. Geophys. Res.* 106, 11043–11065.
- Proussevitch, A., Sahagian, D., 2005. Bubbledrive-1: a numerical model of volcanic eruption mechanisms driven by disequilibrium magma degassing. *J. Volcanol. Geotherm. Res.* 143, 89–111.
- Rust, A.C., Cashman, K.V., 2004. Permeability of vesicular silicic magma: inertial and hysteresis effects. *Earth Planet. Sci. Lett.* 228, 93–107.
- Saar, M.O., Manga, M., 1999. Permeability–porosity relationship in vesicular basalts. *Geophys. Res. Lett.* 26, 111–114.
- Sahagian, D., 1999. Magma fragmentation in eruptions. *Nature* 402, 589–591.
- Sahagian, D., 2005. Volcanic eruption mechanisms: insights from intercomparison of models of conduit processes. *J. Volcanol. Geotherm. Res.* 143, 1–15.
- Slezin, Y.B., 2003. The mechanism of volcanic eruption (a steady state approach). *J. Volcanol. Geotherm. Res.* 122, 7–30.
- Sparks, R.S.J., 1978. The dynamics of bubble formation and growth in magmas: a review and analysis. *J. Volcanol. Geotherm. Res.* 3, 1–37.
- Sparks, R.S.J., 2003. Forecasting volcanic eruptions. *Earth Planet. Sci. Lett.* 210, 1–15.
- Takeuchi, S., Nakashima, S., Tomiya, A., Shinohara, H., 2005. Experimental constraints on the low gas permeability of vesicular magma during decompression. *Geophys. Res. Lett.* 32, L10312. doi:10.1029/2005GL022491.
- Tuffen, H., Dingwell, D.B., 2005. Fault textures in volcanic conduits: evidence for seismic trigger mechanisms during silicic eruptions. *Bull. Volcanol.* 67, 370–387.
- Tuffen, H., Dingwell, D., Pinkerton, H., 2003. Repeated fracture and heating of silicic magma generates flow banding and earthquakes? *Geology* 31, 1089–1092.
- Turcotte, D.L., Schubert, G., 1982. *Flow in porous media. Geodynamics: Applications of Continuum Physics to Geological Problems*. John Wiley and Sons, New York, pp. 381–422.
- Woods, A.W., 1995. The dynamics of explosive volcanic eruptions. *Rev. Geophys.* 33, 495–530.
- Woods, A.W., Koyaguchi, T., 1994. Transitions between explosive and effusive eruptions of silicic magma. *Nature* 370, 641–644.
- Yoshida, S., Koyaguchi, T., 1999. A new regime of volcanic eruption due to the relative motion between liquid and gas. *J. Volcanol. Geotherm. Res.* 89, 303–315.
- Zhang, Y., 1999. A criterion for the fragmentation of bubbly magma based on brittle failure theory. *Nature* 402, 648–650.