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Measuring multifractality in seismic sequences

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Abstract

We investigated the multifractal structure of the interevent times between successive earthquakes that occurred in Umbria-Marche, which is one of the most seismically active areas of central Italy. We used the Multifractal Detrended Fluctuation Analysis (MF-DFA), which permits detection of multifractality in nonstationary series. Analyzing the time evolution of the multifractal behaviour of the seismic sequence, a loss of multifractality during the aftershocks is revealed.

Keywords: Multifractal Detrended Fluctuation Analysis; Earthquakes

1. Introduction

Marked point processes describe events that occur at random locations in space and time and are characterized by an intensity value. Earthquakes can be regarded as spatio-temporal point processes, marked by the magnitude. Their spatial (latitude, longitude, depth), temporal (occurrence instant) and energy (magnitude) parameters are featured by power-law behaviour, which implies self-similarity and absence of characteristic length-scale. The Gutenberg-Richter law states that the probability distribution of the released energy is a power-law (Gutenberg and Richter, 1944). The epicentres occur on a fractal-like distribution of faults (Kagan, 1992). The Omori's law states that the number of aftershocks, which follow a main event, decays as a power-law with exponent close to minus one (Utsu et al., 1995). The fractal behaviour revealed in these statistics could be considered as the end-product of a self-organized critical state of the Earth's crust, analogous to the state of a sandpile, which evolves naturally to a critical repose angle in response to the steady supply of new grains at the summit (Bak et al., 1988).

Recently, much work has been focused on the characterizing the temporal distribution of a seismic sequence. For completely random seismic sequences, the probability density function (pdf) of the interevent times follows an exponential decreasing form; this is typical of Poissonian processes, which are memoryless processes with all the events independent of each other. On the contrary, the interevent times are generally power-law distributed for time-clusterized sequences (Telesca et al., 1999, 2000 a,b), which are featured by time-correlation properties among the events. But the pdf of the interevent intervals is only one window into a point process, because it yields only first-order information and it reveals none about the correlation properties. Therefore, time-fractal second-order methods are necessary to investigate the temporal fluctuations of seismic sequences more deeply. The use of statistics like the Allan Factor (Allan, 1966), the Fano Factor (Lowen and Teich, 1995), the Detrended Fluctuation Analysis (DFA)

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(Peng et al., 1995), has allowed gaining more insight into the time dynamics of seismicity in terms of (i) discrimination between Poissonian and clusterized sequences (Telesca et al., 2001a,b), (ii) spatial variability of time-clustering behaviour (Telesca et al., 2001c, 2003a, 2004a,b; Telesca and Lapenna, 2004), and (iii) magnitude-variability of the property of time-clusterization (Telesca et al., 2002a; Telesca and Macchiato, 2004). All these measures are consistent with each other, so that we can define one scaling exponent that is sufficient to capture the time dynamics of a seismic process.

But one scaling exponent is sufficient to completely describe a seismic process under the hypothesis that this is monofractal. Monofractals are homogeneous objects, in the sense that they have the same scaling properties, characterized by a single singularity exponent (Stanley et al., 1996). The need for more than one scaling exponent can derive from the existence of a crossover time scale, which separates regimes with different scaling behaviours, suggesting e.g. different types of correlations at small and large time scales, thus leading to different types of time dynamics intrinsic in the same seismic sequence (Telesca et al., 2002a). Different values of the same scaling exponent could be required for different segments of the same seismic sequence, indicating a time variation of the scaling behaviour, relying to a time variation of the underlying dynamics (Telesca et al., 2001a). Typically an enhancement of the time-clustering is detected in correspondence to aftershocks (Telesca and Macchiato, 2004). Furthermore, different scaling exponents can be revealed for many interwoven fractal subsets of the sequence; in this case the process is not a monofractal but multifractal. A multifractal object requires many indices to characterize its scaling properties. Multifractals can be decomposed into many–possibly infinitely many–subsets characterized by different scaling exponents. Thus multifractals are intrinsically more complex and inhomogeneous than monofractals and characterize systems featured by a spiky dynamics, with sudden and intense bursts of high frequency fluctuations (Meneveau and Sreenivasan, 1991).

A seismic process can be considered as characterized by a fluctuating behaviour, with temporal phases of low activity interspersed between those where the density of the events is relatively large. This “sparseness” can be well described by means of the concept of multifractal.

Multifractality in earthquakes has been investigated in Enescu et al. (2005) concerning Vrancea seismicity. This study revealed two distinct scaling regimes: non homogeneous and multifractal at small scales, monofractal and close to Poissonian at large scales. Furthermore it was found that the multifractal behaviour at small scales (minutes–hours) is clearly an effect of the “short” aftershock sequences that occurred after some major Vrancea earthquakes. Scaling analysis of seismicity in the space–time–magnitude domain has been performed in Molchan and Kronrod (2005), pointing out to some evidence in favour of multifractality being present in seismicity.

The simplest type of multifractal analysis is given by the standard partition function multifractal formalism. By using this method aftershock-induced intermittent-type temporal fluctuations, interpreted in terms of multifractality, have been found in Irpinia (southern Italy) seismicity (Telesca et al., 2001d). But this method could suffer misleading results due to the presence of nonstationarity in the data. Another method based on the generalization of the Detrended Fluctuation Analysis (DFA) has been developed by Kantelhardt et al. (2002). This method, called Multifractal Detrended Fluctuation Analysis (MF-DFA) is able to reliably determine the multifractal scaling behaviour of nonstationary series.

2. Seismicity data

We study the sequence of 7521 earthquakes in a very seismically active area of central Italy, which was struck by a violent earthquake (M_D=5.8) on September 26, 1997. The epicentre distribution of the events occurred from 1983 to 2003 is shown in Fig. 1 (data extracted from the instrumental catalogue of National Institute of Geophysics and Volcanology—INGV, http://www.ingv.it). The earthquakes are located in a circular area centered on the epicentre of the strongest M5.8 earthquake, with a radius of 100 km. The completeness magnitude, estimated after performing the Gutenberg-Richter analysis is 2.4.

3. Methods and data analysis

The main features of multifractals are to be characterized by high variability on a wide range of temporal or spatial scales, associated to intermittent fluctuations and long-range power-law correlations.

The interevent intervals examined in this paper present clear irregular dynamics (Fig. 2), characterized by sudden bursts of high frequency fluctuations, which suggest to perform a multifractal analysis, in order to evidence different scaling behaviours for different intensities of fluctuations. We applied the Multifractal Detrended Fluctuation Analysis (MF-DFA), which operates on the series $x(i)$, where i=1,2,…,N and N is the length of the series. With x_{ave} we indicate the mean value. We assume that $x(i)$ are increments of a random walk

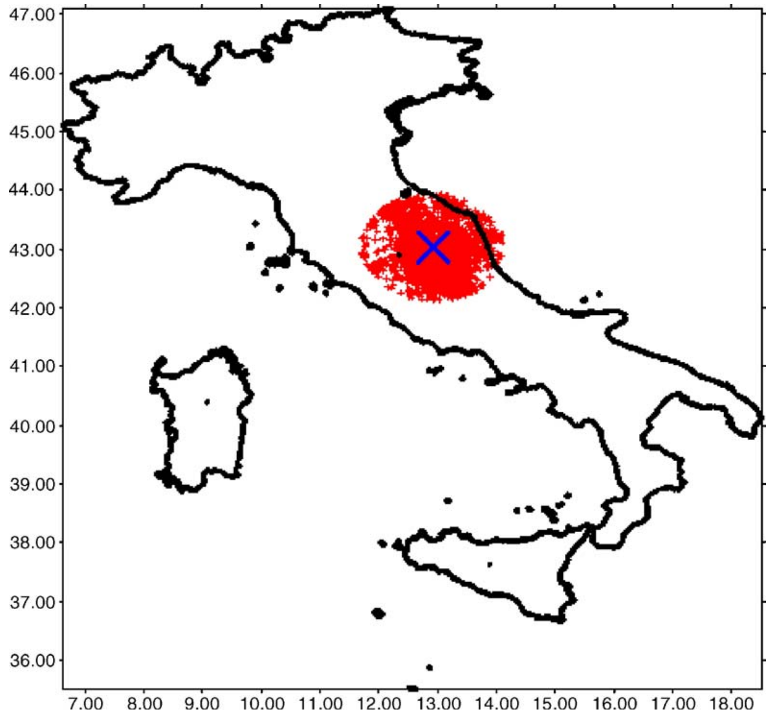


Fig. 1. Epicentral distribution of the seismicity of central Italy during the period 1983–2003.

process around the average x_{ave}, thus the "trajectory" or "profile" is given by the integration of the signal

$$y(i) = \sum_{k=1}^{i} [x(k) - x_{ave}]. \quad (1)$$

In our case $x(i)$ are the interevent times. Next, the integrated series is divided into $N_S = \text{int}(N/s)$ nonoverlapping segments of equal length s. Since the length N of the series is often not a multiple of the considered time scale s, a short part at the end of the profile $y(i)$ may remain. In order not to disregard this part of the series, the same procedure is repeated starting from the opposite end. Thereby, $2N_S$ segments are obtained altogether. Then we calculate the linear local trend for

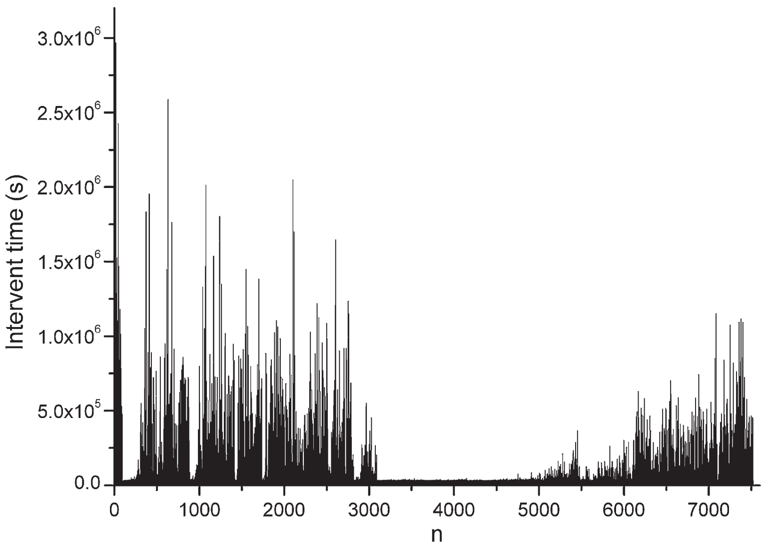


Fig. 2. Interevent interval time series of the seismicity shown in Fig. 1.

each of the $2N_S$ segments by a least square fit of the series. Then we determine the variance

$$F^2(s,v) = \frac{1}{s}\sum_{i=1}^{s}\{y[(v-1)s+i]-y_v(i)\}^2 \quad (2)$$

for each segment v, $v=1,\ldots, N_S$ and

$$F^2(s,v) = \frac{1}{s}\sum_{i=1}^{s}\{y[N-(v-N_S)s+i]-y_v(i)\}^2 \quad (3)$$

for $v=N_S+1,\ldots, 2N_S$. Here, $y_v(i)$ is the fitting line in segment v. Then, we average over all segments to obtain the q-th order fluctuation function

$$F_q(s) = \left\{\frac{1}{2N_S}\sum_{v=1}^{2N_S}[F^2(s,v)]^{\frac{q}{2}}\right\}^{\frac{1}{q}} \quad (4)$$

where, in general, the index variable q can take any real value except zero. Repeating the procedure described above, for several time scales s, $F_q(s)$ will increase with increasing s. Then analyzing log–log plots $F_q(s)$ versus s for each value of q, we determine the scaling behaviour of the fluctuation functions. If the series x_i is long-range power-law correlated, $F_q(s)$ increases for large values of s as a power-law

$$F_q(s) \approx s^{h_q}. \quad (5)$$

The value h_0 corresponds to the limit h_q for $q \to 0$, and cannot be determined directly using the averaging procedure of Eq. (4) because of the diverging exponent. Instead, a logarithmic averaging procedure has to be employed,

$$F_0(s) \equiv \exp\left\{\frac{1}{4N_S}\sum_{v=1}^{2N_S}\ln[F^2(s,v)]\right\} \approx s^{h_0}. \quad (6)$$

In general the exponent h_q will depend on q. For stationary series, h_2 is the well-defined Hurst exponent H (Feder, 1988). Thus, we call h_q the generalized Hurst exponent. h_q independent of q characterizes monofractal series. The different scaling of small and large fluctuations will yield a significant dependence of h_q on q. For positive q, the segments v with large variance (i.e. large deviation from the corresponding fit) will dominate the average $F_q(s)$. Therefore, if q is positive, h_q describes the scaling behaviour of the segments with large fluctuations; and generally, large fluctuations are characterized by a smaller scaling exponent h_q for multifractal time series. For negative q, the segments v with small variance will dominate the average $F_q(s)$. Thus, for negative q values, the scaling exponent h_q describes the scaling behaviour of segments with small fluctuations, usually characterized by larger scaling exponents.

We calculated the fluctuation functions $F_q(s)$ for scales s ranging from 10 events to $N/4$, where N is the total length of the series. The length of the series ($N=7520$) allows us to consider the estimated exponents reliable. Fig. 3 shows the fluctuation functions $F_q(s)$ for $q=-10$ and $q=+10$. The different slopes h_q of the fluctuation curves indicate that small and large interevent fluctuations scale differently. We calculated the fluctuation functions $F_q(s)$ for $q\in[-10,10]$,

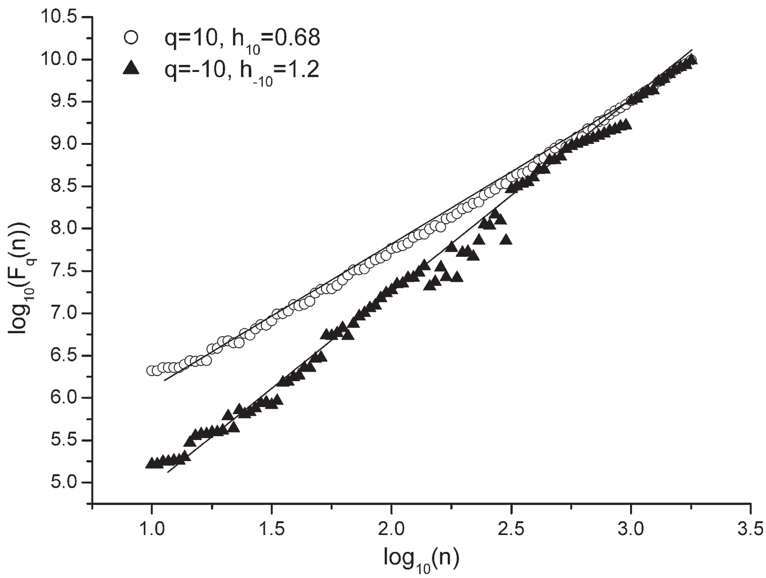


Fig. 3. Fluctuation functions for $q=-10$ and $q=10$. The different slopes suggest the presence of multifractality in the series.

with 0.5 step. Fig. 4 shows the q-dependence of the generalized Hurst exponent h_q determined by fits in the regime $10<s<N/4$. The $h_q \sim q$ relation is characterized by the typical multifractal form, monotonically decreasing with the increase of q.

Another manner to quantify multifractality in a series is by using the multifractal spectrum. The multifractal spectrum can be obtained using the relationship

$$\tau(q) = qh(q) - 1, \quad (7)$$

and then the Legendre transform (Parisi and Frisch, 1985),

$$\alpha = \frac{d\tau}{dq} \quad (8)$$

$$f(\alpha) = q\alpha - \tau(q), \quad (9)$$

where α is the Hölder exponent and $f(\alpha)$ indicates the dimension of the subset of the series that is characterized by α. The singularity spectrum quantifies in details the long-range correlation properties of a time series (Ashkenazy et al., 2003a,b). The multifractal spectrum gives information about the relative importance of various fractal exponents present in the series. In particular the width of the spectrum indicates the range of present exponents. To be able to make quantitative characterization of multifractal spectra, we fitted the spectrum to a quadratic function (Shimizu et al., 2002) around the position of its maximum at α_0, i.e. $f(\alpha)=A(\alpha-\alpha_0)^2+B(\alpha-\alpha_0)+C$: the coefficients are obtained by an ordinary least square procedure. Parameter B serves as an asymmetry parameter, which is zero for symmetric shapes, positive or negative for a left- or right-skewed (centered) shape, respectively. To obtain an estimate of the range of possible fractal exponents, we measured the width of the spectrum, extrapolating the fitted curve to zero. The width of the spectrum was then defined as $W=\alpha_1-\alpha_2$, with $f(\alpha_1)=f(\alpha_2)=0$. The measure of the width of multifractal spectra as a measure of degree of multifractality has also been suggested in relation to the investigation of past climate variations (Ashkenazy et al., 2003a,b). The maximum α_0, which is the value of α where $f(\alpha)$ assumes its maximum value, gives indication if the underlying process is regular in appearance; furthermore larger α_0 for more regular process. The width of the spectrum W is a measure of how wide the range of fractal exponents found in the signal is; and, thus, it measures the degree of multifractality of the series. The wider the range of possible fractal exponents, the "richer" the process in structure. Finally, the asymmetry parameter B captures the dominance of low or high fractal exponents with respect to the other. A right-skewed spectrum indicates relatively strongly weighted high fractal exponents, and low ones for left-skewed shapes (Shimizu et al., 2002). Fig. 5 shows the multifractal spectrum of the interevent times of the investigated seismicity and the quadratic fit. The position of its maximum is at $\alpha_0=0$. The best fit, obtained by a least square method, furnished the following values of the parameter $A=-7.51$, $B=-0.05$ and $C=1.02$. The width of the multifractal spectrum is $W=0.74$. From these parameter estimations we can deduce that the seismic process under study is rather multifractal, with more weighted low fractal exponents.

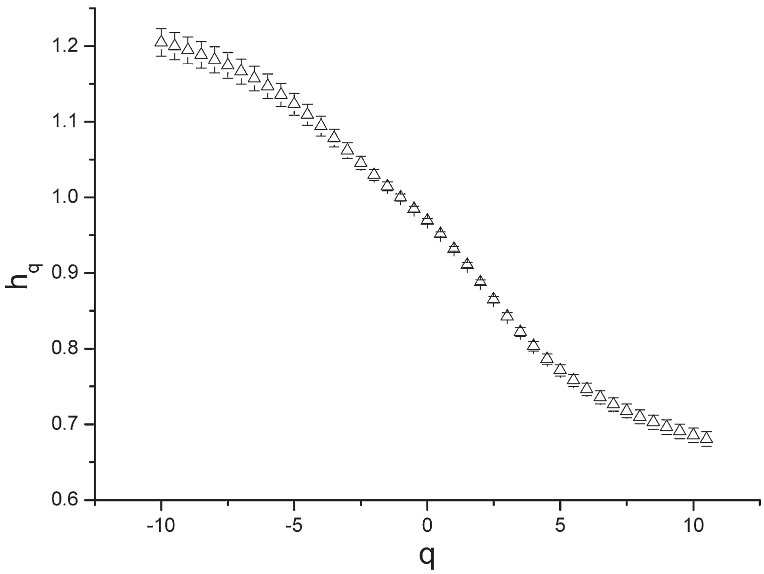


Fig. 4. Generalized Hurst exponents h_q versus q ranging between −10 and 10 (step of 0.5).

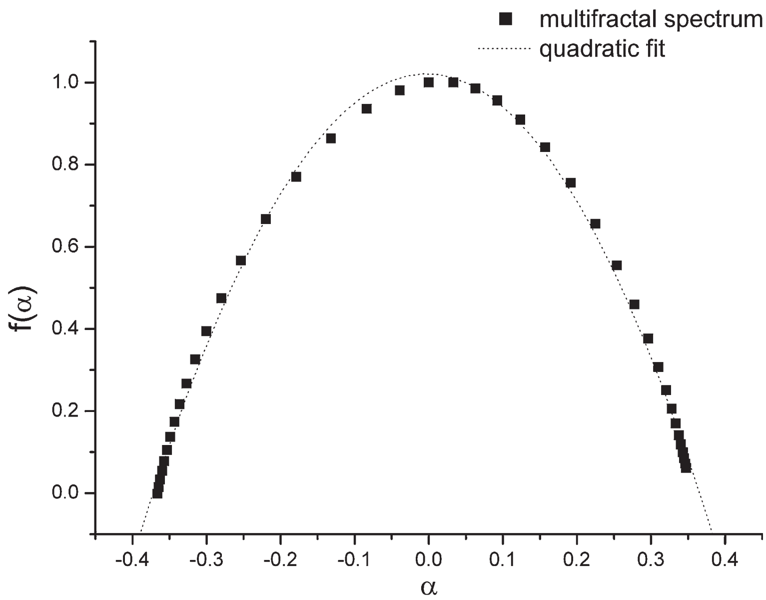
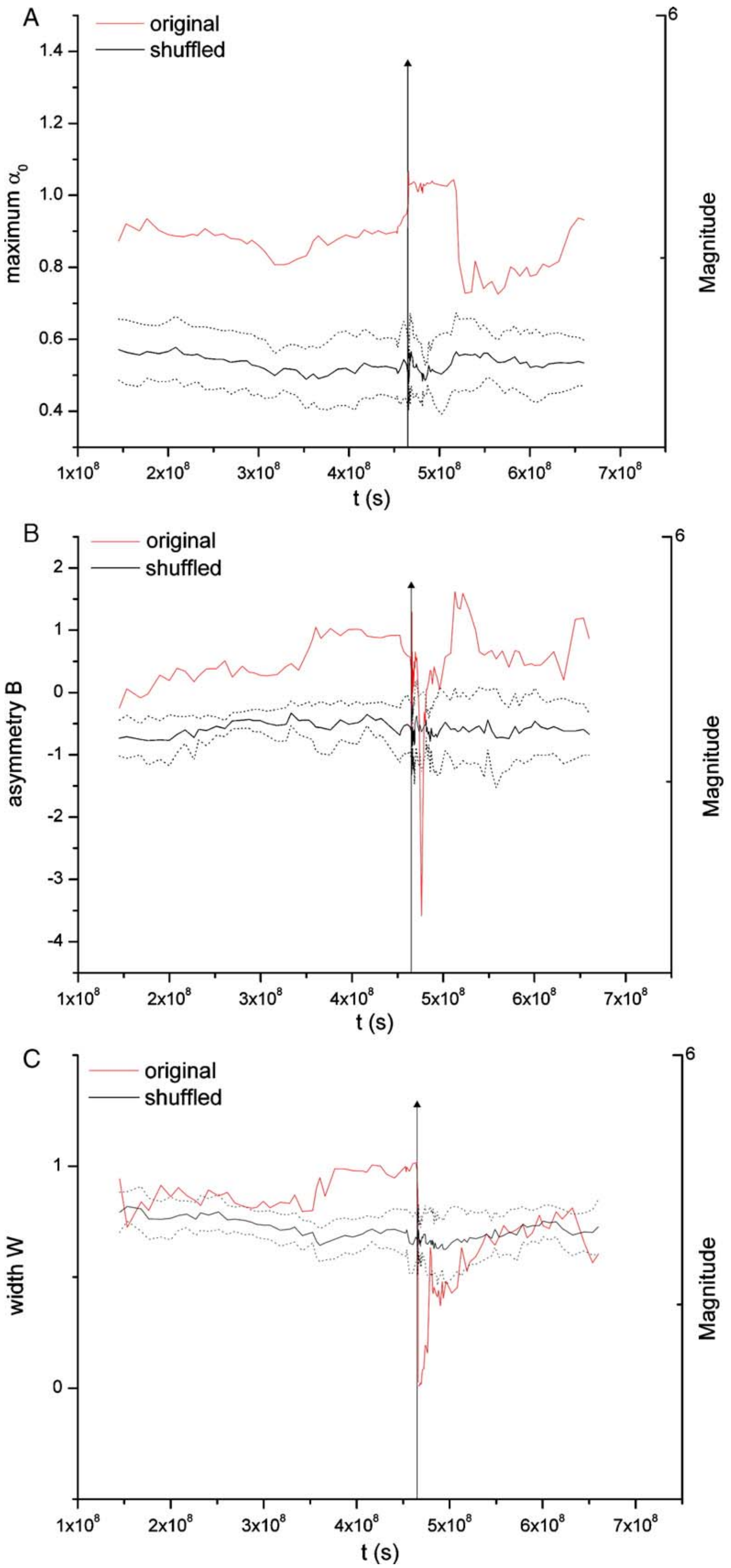


Fig. 5. Multifractal spectrum of the interevent time series plotted in Fig. 2. The dotted curve represents the quadratic fit.

The multifractal parameters calculated over the whole series are not informative about the dynamical changes in the behaviour of the seismic sequence. Thus, what we need is a time-dependent analysis of multifractality. By using the concept of sliding window (Gamero et al., 1997; Martin et al., 2000) one can calculate the temporal evolution of the multifractality. In our case we considered a sliding window of 10^3 events; we calculated the set of the generalized Hurst exponents $\{h_q(t): -10 \le q \le +10\}$ and then the three multifractal parameters (maximum α_0, asymmetry B and width W) in each window by fitting the local multifractal spectrum by a quadratic function. We also calculated the minimum and maximum values of α, which are also important to characterize multifractality in a series (Goltz, 1998). The number of events in each window was chosen in order to have sufficient amount of points to perform the estimates. The shift between two successive windows was set to 50 events, in order to smooth the results and evaluate the variation of the multifractality with a good time resolution. Keeping the number of events constant ensures comparable results, although it seems physically desirable to keep the time interval constant to include the temporal dynamics of the seismicity pattern, but then the number of earthquakes would change considerably because of the large temporal coupling as shown by, e. g., Omori's law (Goltz, 1998). Each value was associated to the occurrence time of the last event in the sliding window. Fig. 6 shows the time variation of the maximum α_0, asymmetry B and width W. All the three parameters are clearly characterized by a strong variability, and this suggests a strong variability of the multifractal character of the series. The most striking feature in all the plots is the significant change of the time pattern during the aftershocks, following the largest earthquake of the series (M=5.8). In particular, the maximum α_0 shows a sudden increase, indicating that most clusters possess a higher local fractal dimension than before the mainshock. This means that areas of high values of the measure increase while almost void areas expand, which in turn implies a rising energy concentration. The asymmetry B shows a decrease, assuming negative values during aftershock activation; more particularly, the asymmetry changes from positive values to negative values, indicating a change from left-skewed shapes to right-skewed ones, and a relative dominance of high fractal exponents. The behaviour of the width W, which can be more properly considered as a measure of the degree of the multifractality, shows a sudden decrease, indicating a change from a heterogeneous to a more homogeneous dynamics. This lowering of the multifractal width is due to the contemporary increase of the minimum α-value (α_{min}) and decrease of the maximum α-value (α_{max}) (Fig. 7). This behaviour indicates that the clustering within the most clustered areas becomes more intense (increase of α_{min}), while the clustering within the sparse areas decreases (decrease of α_{max}).

Fig. 6. Time variation of the three multifractal parameters: maximum (a), asymmetry (b) and width (c). In the figures the average of parameters obtained by 10 randomly shuffled versions of the original series (black curves) within their 1–σ range (dotted lines) are plotted.


A
original
shuffled
maximum α_0
Magnitude
t (s)
1.4
1.2
1.0
0.8
0.6
0.4
6
1x10^8
2x10^8
3x10^8
4x10^8
5x10^8
6x10^8
7x10^8
B
original
shuffled
asymmetry B
Magnitude
t (s)
2
1
0
-1
-2
-3
-4
C
original
shuffled
width W
Magnitude
t (s)
1
0

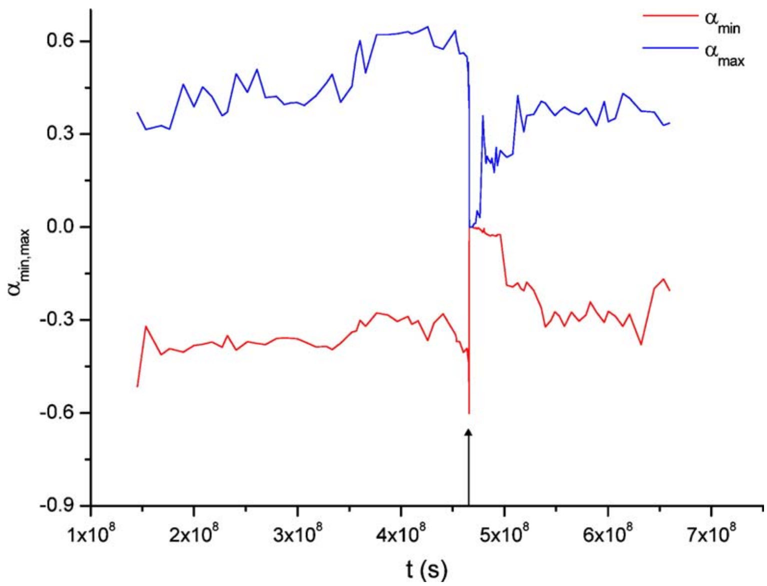


Fig. 7. Time variation of the minimum α-value and maximum α-value.

In order to verify that the variability of the parameters is not random but significant, we performed the same analysis on 10 randomly shuffled versions of the original series. We calculated for each shuffled series the maximum, the asymmetry and the width varying with time, and then we averaged them. In Fig. 6 we also plotted the averages within their $1-\sigma$ range. We can clearly observe that the parameters calculated for the original series differ significantly from those calculated for the shuffled series, especially during the occurrence of the largest shock of the sequence.

4. Conclusions

The potential of multifractal analysis is far from being fully exploited, since it was only very recently that attention has been drawn to the need for a thorough testing of the multifractal tools, and, in particular, a much deeper understanding of the nature of the seismic phenomena and their interactions with the multifractal methods used for their analysis (Telesca et al., 2001b, 2002b, 2003b).

The geophysical phenomenon underlying earthquakes is complex. The multifractal analysis, performed in the present study, has led to a better understanding of such complexity. The multifractality of a seismic phenomenon relies on the different scaling for long and short interevent intervals. The analysis of the time evolution of the multifractality, characterized by the five parameters (maximum, asymmetry, width, maximum α-value and minimum α-value), suggests that the seismic phenomenon under study is characterized by a dynamical change from heterogeneity toward homogeneity during the aftershock activation, revealed by a loss of multifractality after a main event.

References

Allan, D.W., 1966. Statistics of atomic frequency standards. Proc. IEEE 54, 221–230.

Ashkenazy, Y., Havlin, S., Ivanov, P.Ch., Peng, C.-K., Schulte-Frohlinde, V., Stanley, H.E., 2003a. Magnitude and sign scaling in power-law correlated time series. Physica, A 323, 19–41.

Ashkenazy, Y., Baker, D.R., Gildor, H., Havlin, S., 2003b. Nonlinearity and multifractality of climate change in the past 420,000 years. Geophys. Res. Lett. 30, 2146–2149.

Bak, P., Tang, C., Wiensenfeld, K., 1988. Self-organizing criticality. Phys. Rev., A 38, 364–374.

Enescu, B., Ito, K., Radulian, M., Popescu, E., Bazacliu, O., 2005. Multifractal and chaotic analysis of Vrancea (Romania) intermediate-depth earthquakes: investigation of the temporal distribution of events. PAGEOPH 162, 249–271.

Feder, J., 1988. Fractals. Plenum Press, New York.

Gamero, L., Plastino, A., Torres, M.E., 1997. Wavelet analysis and nonlinear dynamics in a nonextensive setting. Physica, A 246, 487–509.

Goltz, B., 1998. Fractals and chaotic properties of earthquakes. New York, Springer.

Gutenberg, B., Richter, C.F., 1944. Frequency of earthquakes in California. Bull. Seismol. Soc. Am. 34, 185–188.

Kagan, Y.Y., 1992. Correlations of earthquake focal mechanism. Geophys. J. Int. 110, 305–320.

Kantelhardt, J.W., Zschiegner, S.A., Konscienly-Bunde, E., Havlin, S., Bunde, A., Stanley, H.E., 2002. Multifractal detrended fluctuation analysis of nonstationary time series. Physica, A 316, 87–114.

Lowen, S.B., Teich, M.C., 1995. Estimation and simulation of fractal stochastic point processes. Fractals 3, 183–210.

Martin, M.T., Plastino, A.R., Plastino, A., 2000. Tsallis-like information measures and the analysis of complex signals. Physica, A 275, 262–271.

Meneveau, C., Sreenivasan, K.R., 1991. The multifractal nature of turbulent energy dissipation. J. Fluid Mech. 224, 429–484.

Molchan, G., Kronrod, T., 2005. On the spatial scaling of seismicity rate. Geophys. J. Int. 162, 899.

Parisi, G., Frisch, U., 1985. In: Ghil, M., Benzi, R., Parisi, G. (Eds.), In Turbulence and Predictability in Geophysical Fluid Dynamics and Climate Dynamics. Amsterdam, North Holland.

Peng, C.-K., Havlin, S., Stanley, H.E., Goldberger, A.L., 1995. Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series. CHAOS 5, 82–87.

Shimizu, Y., Thurner, S., Ehrenberger, K., 2002. Multifractal spectra as a measure of complexity in human posture. Fractals 10, 103–116.

Stanley, H.E., Afanasyev, V., Amaral, L.A.N., Buldyrev, S.V., Goldberger, A.L., Havlin, S., Leschhorn, H., et al., 1996. Anomalous fluctuations in the dynamics of complex systems: from DNA and physiology to econophysics. Physica, A 224, 302–321.

Telesca, L., Lapenna, V., 2004. Investigating the spatial variability of the time-scaling properties in Italian seismicity. Nonlinear Process. Geophys. 11, 545–552.

Telesca, L., Macchiato, M., 2004. Time-scaling properties of the Umbria-Marche 1997–1998 seismic crisis, investigated by the Detrended Fluctuation Analysis. Chaos, Solitons Fractals 19, 377–385.

Telesca, L., Cuomo, V., Lanfredi, M., Lapenna, V., Macchiato, M., 1999. Investigating clustering structures in time-occurrence sequences of seismic events observed in the Irpinia-Basilicata Region (Southern Italy). Fractals 7, 221–234.

Telesca, L., Lapenna, V., Vallianatos, F., 2000a. Self-similarity properties of seismicity in the Southern Aegean Area. Tectonophysics 321, 179–188.

Telesca, L., Cuomo, V., Lapenna, V., Macchiato, M., 2000b. Analysis of the time-scaling behaviour in the sequence of the aftershocks of the Bovec (Slovenia) April 12, 1998 earthquake. Phys. Earth Planet. Inter. 120, 315–326.

Telesca, L., Cuomo, V., Lapenna, V., Macchiato, M., 2001a. Identifying space-time clustering properties of the 1983–1997 Irpinia-Basilicata (southern Italy) seismicity. Tectonophysics 330, 93–102.

Telesca, L., Cuomo, V., Lapenna, V., Macchiato, M., 2001b. Statistical analysis of fractal properties of point processes modelling seismic sequences. Phys. Earth Planet. Inter. 125, 65–83.

Telesca, L., Cuomo, V., Lapenna, V., Macchiato, M., 2001c. Depth-dependent time-clustering behavior in seismicity of southern California. Geophys. Res. Lett. 28, 4323–4326.

Telesca, L., Cuomo, V., Lapenna, V., Macchiato, M., 2001d. Intermittent-type temporal fluctuations in seismicity of the Irpinia (southern Italy) region. Geophys. Res. Lett. 28, 3765–3768.

Telesca, L., Cuomo, V., Lapenna, V., Macchiato, M., 2002a. $1/f^{\alpha}$ fluctuations of seismic sequences. Fluctuation Noise Lett. 2, L357–L367.

Telesca, L., Lapenna, V., Vallianatos, F., 2002b. Monofractal and multifractal approaches in investigating scaling properties in temporal patterns of the 1983–2000 seismicity in the Western Corinth Graben (Greece). Phys. Earth Planet. Inter. 131, 63–79.

Telesca, L., Lapenna, V., Macchiato, M., 2003a. Spatial variability of time-correlated behaviour in Italian seismicity. Earth Planet. Sci. Lett. 212, 279–290.

Telesca, L., Lapenna, V., Macchiato, M., 2003b. Investigating the time-clustering properties in seismicity of Umbria-Marche region (central Italy). Chaos, Solitons Fractals 18, 203–217.

Telesca, L., Cuomo, V., Lapenna, V., Macchiato, M., 2004a. Detrended Fluctuation Analysis of the spatial variability of the temporal distribution of southern California seismicity. Chaos, Solitons Fractals 21, 335–342.

Telesca, L., Lapenna, V., Lovallo, M., Macchiato, M., 2004b. Long-range time-correlation properties of seismic sequences. Chaos, Solitons Fractals 21, 387–393.

Utsu, T., Ogata, Y., Matsu'ura, R.S., 1995. The centenary of the Omori formula for a decay law of aftershock activity. J. Phys. Earth 43, 1–33.