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Comparison of forecasting performance of nonlinear models of hydrological time series

Jozef Komorník [a], Magda Komorníková [b,c,*], Radko Mesiar [b,c], Danuša Szökeová [b], Ján Szolgay [b]

[a] *Faculty of Management, Comenius University, Bratislava, Slovakia*
[b] *Faculty of Civil Engineering, Slovak University of Technology, Slovakia*
[c] *UTIA AV CR Prague, Czech Republic*

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Abstract

Mean monthly flows of the Tatry alpine mountain region in Slovakia are predominantly fed by snowmelt in the spring and convective precipitation in the summer. Therefore their regime properties exhibit clear seasonal patterns. Positive deviations from these trends have substantially different features than the negative ones. This provides intuitive justification for the application of nonlinear two-regime models for modelling and forecasting of these time series. Nonlinear time series structures often have lead to good fitting performances, however these do not guarantee an equally good forecasting performance. In this paper therefore the forecasting performance of several nonlinear time series models is compared with respect to their capabilities of forecasting monthly and seasonal flows in the Tatry region. A new type of regime-switching models is also proposed and tested. The best predictive performance was achieved for a new model subclass involving aggregation operators.

Keywords: Time series; Trend; Regime-switching model; Aggregation operator; Mean monthly flow

1. Introduction

Reliable forecasts of monthly inflows to hydropower reservoirs are of particular interest for optimization of operation scheduling. Therefore (and also for many other reasons) river flow forecasting has always been an important issue in operational hydrology. Despite of the fact, that many approaches have contributed to the advancement of reliability of forecasting, due to the complexity of the phenomenon involving a wide range of spatial and temporal scales, many aspects of the problem of river flow forecasting still remain to be refined and solved.

Successful prediction of river flow series requires the understanding and adequate modelling of the underlying physical mechanism responsible of their generation. Outputs from many hydrological systems exhibit two distinct characteristics: nonlinearity and nonstationarity (Coulibaly and Baldwin, 2005), meaning that their statistical characteristics change over time due to the nonlinear dynamics of either system internal or external driving force mechanism.

Approaches accounting for nonlinearity and nonstationarity used for river flow forecasting cover a wide range of different methods from deterministic (from completely black-box models to very detailed distributed models) to stochastic approaches, not to forget the so-called hybrid (combined deterministic–stochastic) modelling approach

* Corresponding author. Address: Faculty of Civil Engineering, Slovak University of Technology, Slovakia. Tel.: +421 2 59274428; fax: +421 2 52961477.

E-mail addresses: Jozef.Komornik@fm.uniba.sk (J. Komorník), magda@math.sk (M. Komorníková), mesiar@math.sk (R. Mesiar), szoke@math.sk (D. Szökeová), szolgay@svf.stuba.sk (J. Szolgay).

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that tries to combine the good points of both methods (see, e.g., Singh, 1995; Reyes-Aldasoro et al., 1999; Ganguly and Bras, 2001).

Moreover, in the last years there has been an increasing interest in probabilistic forecast of hydrologic variables. Such forecasts aim at quantifying the prediction reliability through a probability distribution function (Krzysztofowicz, 2001) or a prediction interval (Chatfield, 2001) for the unknown future value. The evaluation of the uncertainty associated to the forecast can be seen as important information, not only to assess the quality of the prediction, but also as a basis for comparison of forecasts from different methods as input to operational actions and decisions conditioned on the distribution of the forecasted values (Todini, 2004).

Owing to the complexity and nonlinearity of the involved meteorological and hydrological processes, reliable site-specific predictions remain a difficult task for physically based watershed models driven by outputs from numerical weather forecasting models, especially when monthly and/or seasonal forecasts are needed which include quantitative information of the uncertainty of predictions as it is often the case in hydropower system operation.

Due to these difficulties, recent attempts have resorted also to a number of data driven approaches. In this group, in addition to deterministic data driven methods, such as artificial neural networks with various time steps ranging from hourly to annual (e.g., Saad et al., 1996; Jain et al., 1999; Coulibaly and Baldwin, 2005), various nonlinear prediction methods (Porporato and Ridolfi, 2001; Laio et al., 2003), have also found their applications. However these models do not always provide insight into the probabilistic structure of the data and their capabilities of giving information on the uncertainties associated with the forecasts are limited.

Probabilistic nonlinear predictors and nonlinear time series models, which have been investigated in both the statistical and the hydrological literature (e.g., Tsai and Chan, 2000; Tamea et al., 2005; Amendola, 2003; Wang et al., 2005a,b), offer the potential of explaining both nonlinear behaviour and giving information on forecast uncertainty. As pointed out by Amendola (2003), nonlinear time series structures often have lead to good fitting performances, however the nonlinear time series literature highlights that the good fitting results of nonlinear models do not guarantee an equally good forecasting performance (Chatfield, 2001). Further computing forecasts with nonlinear time series models is not so straightforward as in the case of linear models especially when the forecast lead is longer than one time step.

In this paper therefore the forecasting performance of several nonlinear time series models is compared, since there is limited knowledge of their predictive capabilities concerning the particular problem of forecasting monthly and seasonal flows needed for hydropower generation scheduling. A special type of aggregation operator based self-exciting threshold autoregressive model is suggested and included in the comparison.

The paper is organized as follows. After a general introduction of the use of nonlinear forecasting methods, the particular nonlinear time series methods suggested to be used for forecasting of monthly and seasonal runoff are described. The paper continues with their application to a near real case situation in the High Core Mountain region of Slovakia, where several reservoirs are in operation and further are planned to be built. The case study allowed for testing of some technical aspects of the proposed and other methods through their predictive performance. Final comments and recommendations conclude the paper; some ideas for future investigations are given.

2. Nonlinear time series modelling

Porporato and Ridolfi (2001) emphasized, that when proposing forecasting methods, one should

(1) take advantage of the deterministic components of the forecasted phenomenon and describe its main nonlinearities;
(2) admit, that there are some mechanisms of river flow dynamics for which an adequate physical description is not yet available;
(3) keep the number of parameters in proportion to the sample size of the observed available data in order to obtain robust methods and to avoid somewhat arbitrary calibrations;
(4) use flexible methods, which allow one to update the forecast so as to take advantage of the information of each event as it occurs.

The necessity to give information on the uncertainty of the forecasts could be added to this list (Todini, 2004). Using nonlinear system analysis enables to take many of these aspects into consideration, moreover certain class of nonlinear models allows for the option of issuing probabilistic forecasts, which makes them even more attractive. Linear time series models offer information on predictive uncertainty in general, however their use for forecasting of complex nonlinear phenomena is rather limited, since they accept the validity of the stationarity principle, which is not appropriate for operational forecasting and uncertainty assessment of nonlinear and nonstationary heteroskedastic processes in general. Recently, e.g., Pekárová and Pekár (2006) reported predictions of annual runoff of the Danube with autoregressive models which can be regarded as linear (Wang et al., 2005b). Tesfaye et al. (2006) used periodic stationary PARMA models for monthly flows. Linear ARMA family models (Box and Jenkins, 1970) became more popular in hydrology especially for the generation of monthly and annual flows for reservoir design and optimization (extensive reviews of the several classes of such models proposed for the modelling of water resources time series can be found, e.g., in

Lawrance and Kottegoda (1977), Salas and Obeysekera (1992) and Shrikanthan and McMahon (2002)).

Since modelled processes in hydrology are nonlinear and nonstationary in general, models originating from the classical Box–Jenkins methodology have to be reconsidered and substituted by new approaches, which allow for considering these properties of hydrologic time series. Modern nonlinear modelling techniques, which have been extensively applied in systems theory and econometrics, offer such a potential. The growing number of nonlinear structures proposed in literature has lead Tong (1990) to classify these into the first and the second generation models. The former include all the nonlinear structures for the conditional mean or the conditional variance. The latter are derived through the combination of the first generation models leading to a variety of nonlinear time series models. In the time series domain, several new types of models were proposed recently in financial mathematics, such as Threshold Autoregressive (TAR) and Self-Exciting Threshold Autoregressive (SETAR) or Smooth Transition Autoregressive (STAR) models (e.g., Amendola and Niglio, 2001; Clements and Smith, 1997a; Hansen, 1997; Granger and Teräsvirta, 1993; Lin and Granger, 1994; Tjøstheim, 1994; Tong, 1990), enabling to model nonlinearities and breaks in the development of the corresponding time series and the Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models (e.g., Bollerslev, 1986; Engle, 1982; Enders, 1997; Gourieroux, 1997; Hamilton, 1994), which enable to deal also with heteroskedastic data.

The number of attempts to conduct nonlinear analysis in the time series domain in hydrology and to apply these types of models is growing. Several papers were devoted to testing nonlinearity in particular time series at different time scales, e.g., Wang et al. (2005b) focused at general nonlinearity and low dimensional chaos, Tsai and Chan (2000) developed a test to detect SETAR bilinear and nonlinear continuous-time nonlinearity, Chen and Rao (2003) attempted to detect nonlinearity composed of linear stationary segments in streamflow, temperature, precipitation and Palmer drought index series, Wong et al. (2005) proposed a method for detecting change points in hydrological time series. Nonlinear time series models were also presented in a number of papers, e.g., Livina et al. (2003) proposed a model to reproduce the asymmetric periodic behaviour with large fluctuations around large streamflow and small fluctuations around small streamflow, Fortina et al. (2004) continued to develop the shifting level approach introduced in the 1970s, Akıntug and Rasmussen (2005) presented a Markov switching model for annual hydrologic time series.

Tol (1996) fitted a GARCH model for the conditional variance and the conditional standard deviation, in conjunction with an AR(2) model for the mean, to model daily mean temperature, Amendola and Storti (1999) attempted to use a Threshold AutoRegressive (TAR) model where the switch from the different regimes was due to the antecedent precipitation index. Also the use of some more complex models has been proposed, in particular the so called second generation nonlinear models like TAR-ARCH and TAR-BL in Amendola and Vitale (2000), Amendola and Storti (2001) and Amendola (2003) compared different approaches to compute forecasts from regime switching models. Wang et al. (2005a) proposed an ARMA-GARCH error model to capture the ARCH effect present in daily streamflow series, as well as to preserve seasonality in variance of the residuals,

Even if attempts to use nonlinear time series models lead to good fitting performances, as already pointed out before, good fitting results of nonlinear models do not guarantee an equally good forecasting performance (Chatfield, 2001). This is often due to the current position in the state space and to some factors which are beyond the variable specified in the prediction (Yao and Tong, 1994; Amendola, 2003). Furthermore, computing forecasts from nonlinear models is not such an easy task and when the forecast horizon is longer than one time step, this becomes even more complicated.

Their applicability for a number of particular problems in hydrology in general has to be proved and thoroughly tested case by case. Therefore in this paper, several regime switching nonlinear structures belonging to the TAR class, namely the SETAR, LSTAR and a new class of models, the so called aggregation operator based SETAR models (ATAR) introduced in Komorník and Komorníková (2005), are examined with respect to their abilities to forecast monthly and seasonal runoff. Various theoretical aspects of forecasting with the TAR class of models have been studied in the literature (see, e.g., Brown and Mariano, 1989; De Gooijer and De Bruin, 1998; Clements and Smith, 1997a,b, 1999; Clements et al., 2003). No such attempts were undertaken here, in the paper a practice oriented, case study based approach will be followed.

The paper focuses on application of above mentioned models to a near real case situation in the High Core Mountain region of Slovakia, which is an alpine region with mountainous runoff generation regime. In the area, which is usually considered as homogeneous with respect to seasonal runoff generation patterns (Kohnová, 1998), several reservoirs are in operation and further are planned to be built. The case study is constructed in such a way, that testing of some technical aspects of the proposed models and other methods through their fitting and comparison of predictive performance would allow both for deciding on their practical applicability and further development.

3. Theoretical background for time series modelling

In general, the time series consists from the following components:

1. *Trend*: The long-term component that represents the growth or decline in a time series over an extended period of time.

2. *Seasonal component*: A pattern of change in data that repeats itself from year to year.
3. *Cyclical component*: The wavelike fluctuation around the trend.
4. *Irregular component*: The residuals of the values of time series after the other components have been removed.

We can eliminate the first three ("systematic") components in general (for example by regression). There are no proven "automatic" techniques to identify trend components in the time series data; however, as long as the trend is monotonous (consistently increasing or decreasing) that part of data analysis is typically not very difficult. Seasonal dependence (seasonality) is formally defined as correlational dependence of order k between each ith element of the series and the $(i-k)$th element (Box and Jenkins, 1970) and measured by autocorrelation. If the unexplained noise component is not too large, seasonality can be visually identified in the series as a pattern that repeats every k elements. Seasonal patterns of time series can be examined, e.g., via correlograms. The correlogram (autocorrelogram) displays graphically and numerically the autocorrelation function (ACF), that is, serial correlation coefficients for consecutive lags in a specified range of lags. The frequencies for the cyclical component we identify here using spectral analysis (namely, the sample periodogram is computed). Then we can analyze the irregular component by the Box–Jenkins methodology (Box and Jenkins, 1970) or using nonlinear regime-switching models, as it will be shown in the following.

We denote the univariate time series of interest as y_t, where y_t can be residuals from the systematic components. The variable y_t is observed for $t = 1,2,\ldots,n$, while we assume that initial conditions or pre-sample values $y_0, y_{-1},\ldots,y_{1-p}$ are available whenever necessary. We denote by Ω_{t-1} the history or information set at time $t-1$, which contains all available information that can be exploited for forecasting future values $y_t, y_{t+1},\ldots$ When Ω_{t-1} does not contain any information that can be used in a linear forecasting model for y_t, the corresponding time series is usually called a *white noise* time series ε_t. Usually, it is required that ε_t has a constant (unconditional) mean equal zero and a constant (unconditional) variance as well.

In general any time series y_t can be thought of as being the sum of two parts: what can and what cannot be predicted using the knowledge from the past as gathered in Ω_{t-1}. That is, y_t can be decomposed as

$$y_t = E[y_t|\Omega_{t-1}] + v_t,$$

where v_t is called the *unpredictable* part with $E[v_t|\Omega_{t-1}] = 0$.

A classical *linear ARMA model* for predictable component of y_t assumes that it is a combination of p of its lagged values and of the q realizations of lagged values of zero-mean independent residuals ε_t with equal variance σ^2:

$$\begin{aligned} y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2} - \cdots - \phi_p y_{t-p} \\ = \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \cdots + \theta_q\varepsilon_{t-q}, \end{aligned} \tag{1}$$

where $\phi_1,\ldots,\phi_p$ (autoregressive AR coefficients) and $\theta_1,\ldots,\theta_q$ (moving average MA coefficients) are unknown parameters. For details on linear ARMA models, see e.g., Box and Jenkins (1970).

3.1. Regime-switching models

A natural approach to modelling time series (hydrological, financial, etc.) with nonlinear models seems to be to consider *different states of the world* or *regimes*, and to allow the possibility that the dynamic behavior of hydrologic variable depends on the regime that occurs at any given point at time. By *state-dependent dynamic behaviour* of a time series it is meant that certain properties of the time series, such as its mean, variance and autocorrelation, are different in different regimes.

In recent years several time series models have been proposed which formalize the idea of the existence of different regimes generated by a stochastic process (mainly in economy and finance). We discuss here some of these models. We restrict our attention here to models assuming that in each of the regimes a linear AR model can describe the dynamic behaviour of the time series adequately.

In the following sections, we discuss representations of the different regime-switching models and estimation and testing methods for the presence of regime-switching effects in the time series.

3.1.1. Threshold Autoregressive (TAR) models

Threshold Autoregressive (TAR) models are quite popular in the nonlinear time-series literature. This popularity is due to the fact that they are relatively simple to specify, estimate and interpret, at least in comparison with many other nonlinear time series models.

The idea of multi-regime forecasting models is not new (see e.g., Bacon and Watts, 1971). Tong (1983) initially proposed the *Threshold Autoregressive* (TAR) model (see also Tong and Lim, 1980; Tsay, 1989; Tong, 1990), which assumes that the regime that occurs at a time t can be determined by an observable *threshold variable* q_t relative to a *threshold value*, which we denote as c.

Suppose that the observed data are $(y_1,\ldots,y_n)$, where n is the total number of observations in a time series and it is called the *length* of the time series. Let

- $x_{t,i} = (1, y_{t-1},\ldots,y_{t-p_i})'$ and $\phi_i = (\phi_{0i},\phi_{1i},\ldots,\phi_{p_i i})'$ for $i = 1, 2$;
- $I[A]$ is an indicator function with $I[A] = 1$ if the event A occurs and $I[A] = 0$ otherwise;
- q_t is the transition variable;
- ε_t is i.i.d. $(0,\sigma^2)$.

TAR model is linear within a regime, but liable to move between regimes as the process crosses the threshold. The two-regime Threshold Autoregressive (TAR) model with regimes AR(p_1) and AR(p_2) takes the form:

$$y_t = x'_{t,1}\phi_1 I[q_t \leqslant c] + x'_{t,2}\phi_2 I[q_t > c] + \varepsilon_t.$$

Identification of the appropriate model orders p_1, p_2 and estimation of the threshold c and of the AR coefficients in the two regimes of the TAR model can be done with help of information criteria, e.g., the

– Akaike information criterion

$$\mathrm{AIC}(p_1, p_2) = \ln \hat{\sigma}^2 + \frac{2(p_1 + p_2)}{n}.$$

– Schwarz information criterion

$$\mathrm{SIC}(p_1, p_2) = \ln \hat{\sigma}^2 + \frac{(p_1 + p_2)\ln n}{n}.$$

– Hannan–Quinn information criterion

$$\mathrm{HQC}(p_1, p_2) = \ln \hat{\sigma}^2 + \frac{2(p_1 + p_2)\ln(\ln n)}{n}$$

(for details, see Liew and Chong, 2003), which are minimized.

3.1.2. Self-exciting TAR (SETAR) models

SETAR is a special case of TAR models, when the threshold variable q_t is taken to be a lagged value of the time series itself: $q_t = y_{t-d}$ for a certain integer $d > 0$ (see Petruccelli and Woolford, 1984; Chen and Tsay, 1991). For example, in the two-regime case with AR(p_1) and AR(p_2), the model is

$$y_t = (\phi_{0.1} + \phi_{1.1}y_{t-1} + \cdots + \phi_{p_1.1}y_{t-p_1})I[y_{t-d} \leqslant c] + (\phi_{0.2} + \phi_{1.2}y_{t-1} + \cdots + \phi_{p_2.2}y_{t-p_2})I[y_{t-d} > c] + \varepsilon_t. \quad (2)$$

The least squares estimate of c can be obtained by minimizing this residual variance:

$$\hat{c} = \arg\min_{c \in C} \hat{\sigma}^2(c),$$

where $\hat{\sigma}^2(c) = \frac{1}{n}\sum_{t=1}^{n}\hat{\varepsilon}_t(c)^2$ is a residual variance and C denotes the set of all allowed threshold values. A popular choice for C is $C = \{c | y_{[\pi_0(n-1)]} \leqslant c \leqslant y_{[(1-\pi_0)(n-1)]}\}$ where $y_{(0)}, \ldots, y_{(n-1)}$ denote the order statistics of threshold variable $y_{t-d}, y_{(0)} \leqslant \cdots \leqslant y_{(n-1)}$, and $[\cdot]$ denotes integer part. A safe choice for π_0 appears to be 0.15 (Franses and van Dijk, 2000).

Testing linearity against the alternative of a SETAR model is discussed in Chan (1990, 1991), Chan and Tong (1990), Hansen (1997, 2000). In the test used in this paper the estimates of the SETAR model are used to define a likelihood ratio or F-statistics, which tests the restrictions as given by the null hypothesis, that is

$$F(\hat{c}) = n\left(\frac{\tilde{\sigma}^2 - \hat{\sigma}^2}{\tilde{\sigma}^2}\right),$$

where $\tilde{\sigma}^2$s is an estimate of the residual variance under the null hypothesis of linearity. As minimizes the residual variance over the set C, $F(\hat{c})$ is equivalent to the supremum over this set C of the pointwise test-statistics $F(c)$ with an asymptotic χ^2 distribution with $p + 1$ degrees of freedom. The distribution of F is then nonstandard. Because the exact form of the dependence between the different $F(c)$'s is difficult to analyze, critical values are most easily determined by means of simulation (see Hansen, 1997, 2000 for more details).

3.1.3. Smooth transition AR (STAR) models

A more gradual transition between the different regimes can be obtained by replacing the indicator function $I[y_{t-d} > c]$ in (2) by a continuous function $G(q_t; \gamma, c)$, which changes smoothly from 0 to 1 as q_t increases (so-called *transition function*).

The idea of smooth transition between regimes dates back to Bacon and Watts (1971). It was introduced into the nonlinear time series literature by Chan and Tong (1986) and popularized by Granger and Teräsvirta (1993) and Teräsvirta (1994). A comprehensive review of the STAR model and extensions that allow for exogenous variables as regressors as well, is given in Teräsvirta (1998).

A formal representation of a 2-regimes STAR model is

$$y_t = \Phi_1(B)y_t[1 - G(q_t; \gamma, c)] + \Phi_2(B)y_t G(q_t; \gamma, c) + \varepsilon_t, \quad (3)$$

where

- ε_t is a white noise sequence with variance σ^2,
- the autoregressive polynomials

  $$\Phi_i(B) = \phi_{i,0} + \phi_{i,1}B + \phi_{i,2}B^2 + \cdots, \quad i = 1, 2$$

  in the shift operator B (defined by $By_t = y_{t-1}$) are related to regimes that are determined by values of a threshold variable q_t and its threshold level value c.

Logistic STAR (LSTAR) model is a STAR model with the *logistic* transition function:

$$G(q_t; \gamma, c) = \frac{1}{1 + \exp(-\gamma[q_t - c])},$$

where γ is the smoothness parameter. The parameter c in $G(q_t; \gamma, c)$ can be interpreted as the *threshold* between the two regimes corresponding to $G(-\infty; \gamma, c) = 0$ and $G(+\infty; \gamma, c) = 1$, in the sense that the logistic function changes monotonically from 0 to 1 as q_t increases, while $G(c; \gamma, c) = 0.5$. The parameter γ determines the *smoothness* of the change in the value of the logistic function, and thus the transition from one regime to the other. For more details, see Franses and van Dijk (2000).

Estimation of the parameters $\hat{\theta} = (\phi'_1, \phi'_2, \gamma, c)$ in the STAR model (3) can be made by a relatively straightforward application of nonlinear least squares method

$$\hat{\theta} = \arg\min_{\theta} \sum_{t=1}^{n}[y_t - F(x_t; \theta)]^2,$$

where $F(x_t; \theta) = x'_{t,1}\phi_1[1 - G(q_t; \gamma, c)] + x'_{t,2}\phi_2 G(q_t; \gamma, c)$.

Testing linearity against STAR constitutes the first step towards building STAR models. The problem of testing the linearity against STAR alternatives was addressed in (Luukkonen et al., 1988). This problem is complicated by the presence of unidentified nuisance parameters under the null hypothesis. We denote $p = \max(p_1, p_2)$. The null hypothesis of linearity can be expressed as equality of the autoregressive parameters in the two regimes of the STAR model in (3). Thus, H_0: $\phi_1 = \phi_2$, whereas the alternative hypothesis is H_1: $\phi_{j,1} \neq \phi_{j,2}$ for at least one $j \in \{0, \ldots, p\}$.

We replace the transition function $G(q_t; \gamma; c)$ by a suitable Taylor series approximation. In the reparametrized equation, the identification problem with the presence of unidentified nuisance parameters is no longer present, and linearity can be tested by means of a HCC (heteroscedasticity-consistent) variant of the LM-type test statistic with a standard asymptotic χ^2-distribution under the null hypothesis (see also Granger and Teräsvirta, 1993).

3.1.4. TAR model with aggregation operators

A new method of construction of regime-switching models, based on combinations of shape functions with aggregation operators (for details on aggregation operators, see, e.g., Calvo et al., 2002) has been indicated in Bognár (2005) and Komorník and Komorníková (2005) and is further developed here.

We rewrite a usual form of a regime-switching model for a time series y_t as

$$y_t = \Phi_1(B)y_t(1 - F(q_t)) + \Phi_2(B)y_tF(q_t) + \varepsilon_t, \qquad (4)$$

where

- $\Phi_1(B)$, $\Phi_2(B)$ are autoregressive polynomials in the shift operator B,
- ε_t's are assumed to be a martingale difference sequence with respect to the history of the time series up to time $t-1$, which is denoted as $\Omega_{t-1} = \{y_{t-1}, y_{t-2}, \ldots, y_{t-(p-1)}, y_{t-p}\}$, $p = \max(p_1, p_2)$, that is, $E[\varepsilon_t|\Omega_{t-1}] = 0$. We also assume that the conditional variance of ε_t is constant, $E[\varepsilon_t^2|\Omega_{t-1}] = \sigma^2$,
- F is a so-called transition function, i.e., a nondecreasing surjective map of the values of a threshold variable q_t to the interval $[0, 1]$:

$$F(q_t) = g(\gamma(q_t - c)) + \frac{1}{2}, \qquad (5)$$

where

$$g : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$$

is a nondecreasing shape function (an odd bijection), $c = F^{-1}(1/2)$ is the threshold constant and γ is the smoothness parameter.

We will use the standard logistic shape function (shifted to $g(0) = 0$)

$$g(x) = \frac{1}{1 + e^{-x}} - \frac{1}{2}. \qquad (6)$$

The idea followed here is to find an optimal form of transition between two possible regimes based on information contained in variables $y_{t-1}, \ldots, y_{t-k}$. Two possible suggested approaches are

Case 1. To apply first some continuous aggregation operator $\boldsymbol{A}: R^k \to R$ to the observed values y_{t-i}, $i = 1, \ldots, k$ and transform the resulting output by means of a transition function $F: R \to [0, 1]$, i.e.,

$$q_t = \boldsymbol{A}(y_{t-1}, \ldots, y_{t-k}). \qquad (7)$$

Case 2. Transform values $y_{t-1}, \ldots, y_{t-k}$ by a fixed transition transformation $f: R \to [0, 1]$ into values $u_i = f(y_{t-i})$, $i = 1, \ldots, k$ and then apply a continuous k-dimensional aggregation operator $\boldsymbol{B}: [0, 1]^k \to [0, 1]$, i.e.,

$$F(q_t) = \boldsymbol{B}(u_1, \ldots, u_k) = \boldsymbol{B}(f(y_{t-1}), \ldots, f(y_{t-k})). \qquad (8)$$

Typical continuous aggregation operators on the real line (they also map $[0, 1]^k$ onto $[0, 1]$) are

- ✓ Arithmetic mean $M(x_1, \ldots, x_k) = \frac{1}{k}\sum_{i=1}^k x_i$;
- ✓ Weighted means $W(x_1, \ldots, x_k) = \sum_{i=1}^k w_i x_i$, where $w_i \in [0, 1]$, $\sum_{i=1}^k w_i = 1$;
- ✓ OWA operators $W'(x_1, \ldots, x_k) = \sum_{i=1}^k w_i x_i'$, where x_i' is the ith order statistics from the sample $(x_1, \ldots, x_k)$.

In the class of OWA operators we can find the MIN and MAX operators, corresponding to extremal cases $w_1 = 1$ and $w_i = 0$ otherwise, resp. $w_k = 1$ and $w_i = 0$ otherwise. In our contribution we use weight triangles Δ:

1. weights Δ_2 generated by $q(x) = x^2$, i.e., $w_i = \left(\frac{i}{k}\right)^2 - \left(\frac{i-1}{k}\right)^2 = \left(\frac{2i-1}{k^2}\right)$
2. weights Δ_3 generated by $q(x) = x^3$, i.e., $w_i = \left(\frac{i}{k}\right)^3 - \left(\frac{i-1}{k}\right)^3 = \left(\frac{3i^2-3i+1}{k^3}\right)$
3. Sierpinski carpet weights

$$\Delta_S : w_i = \begin{cases} p(1-p)^{i-1}, & i < k \\ (1-p)^{k-1}, & i = k \end{cases}$$

4. Fibonacci weights $\Delta_F : w_i = \frac{F(i)}{F(k+2)-1}$, with $F(i) = 1, 1, 2, 3, 5, 8, \ldots$ for $i = 1, 2, 3, 4, 5, 6, \ldots$

We denote weighted means with Δ_2 as W_2, Δ_3 as W_3, Δ_S as W_S and with Δ_F as W_F. The number of variables that enter any applied aggregation operator (for individual time series models) is $k = k_0 - 1$, where k_0 is the first value of delay for which the value of the autocorrelation function is not significantly different from 0.

For testing the linearity of (4) (or H_0: $\gamma = 0$) in Case 1 we can proceed similarly as in Franses and van Dijk (2000) and utilizing the difference

$$F^*(q_t) = F(q_t) - 1/2 = g(\gamma(q_t - c)).$$

In the reparametrized model equation the linearity can be tested by means of a Lagrange Multiplier (LM) statistics with a standard asymptotic χ^2-distribution under the null hypothesis.

For testing the linearity of (4) in Case 2 we will use a modified distribution derived by Hansen (1997).

For more details about TAR model with aggregation operators, see Szökeová et al. (2006) and Mesiar et al. (2006). These new models will be briefly called ATAR models (Case 2) and LSTAR models with aggregation operators (Case 1 with logistic transition function).

3.2. Point forecasts

Computing point forecasts from nonlinear models is much more complicated than from linear model (Box and Jenkins, 1970; Franses, 1998). Consider the case where y_t is described by the general nonlinear auto-regressive model

$$y_t = F(q_{t-1};\theta) + \varepsilon_t \tag{9}$$

for some nonlinear function $F(q_{t-1};\theta)$.

The optimal h-step-ahead forecast of y_{t+h} at time t is given by

$$\hat{y}_{t+h|t} = E[y_{t+h}|\Omega_t], \tag{10}$$

where Ω_t denotes the history of the time series up to and including the observation at time t. Using (9) and the fact that $E[\varepsilon_{t+1}|\Omega_t] = 0$, the optimal 1-step-ahead forecast is

$$\hat{y}_{t+1|t} = E[y_{t+1}|\Omega_t] = F(q_t;\theta).$$

When the forecast horizon is longer than 1 period, things become more complicated, because in general the linear conditional expectation operator E cannot be interchanged with the nonlinear operator F, that is

$$E[F(\cdot)] \neq F(E[\cdot]).$$

Thus, the expected value of a nonlinear function is not equal to the function evaluated at the expected value of its arguments.

Several methods have been developed to obtain more adequate multiple-step-ahead forecasts. Brown and Mariano (1989) attempt to obtain the conditional expectation (10) directly by computing

$$\hat{y}_{t+h|t} = \int_{-\infty}^{\infty} F(\hat{y}_{t+h-1|t} + \varepsilon;\theta) f(\varepsilon)\,\mathrm{d}\varepsilon,$$

where f denotes the density of ε_t.

An alternative approach to computing multiple-step-ahead forecasts is to use Monte Carlo or bootstrap methods to approximate the conditional expectation (10). The h-step-ahead Monte Carlo forecast is given by

$$\hat{y}_{t+h|t} = \frac{1}{L}\sum_{i=1}^{L} F(\hat{y}_{t+h-1|t} + \varepsilon_i;\theta), \tag{11}$$

where L is some large number and the ε_i are drawn from the presumed distribution of ε_{t+1}. The bootstrap forecast is very similar, the only difference being that the residuals from the estimated model $\hat{\varepsilon}_t$, $t = 1,\ldots,n$ are used,

$$\hat{y}_{t+h|t} = \frac{1}{L}\sum_{i=1}^{L} F(\hat{y}_{t+h-1|t} + \hat{\varepsilon}_i;\theta).$$

Lin and Granger (1994) and Clements and Smith (1997a) compare various methods to obtain multiple-step-ahead forecasts for SETAR and STAR models, respectively. Their main findings are that the Monte Carlo and bootstrap methods compare favourably to the other methods. In this paper the Monte Carlo method has been applied.

3.3. Comparison of forecasting performance

For comparison of the forecasting performance of alternative models the mean square error (MSE), root mean square error (RMSE) and mean absolute error (MAE) were applied:

$$\mathrm{MSE} = \frac{1}{P}\sum_{T=1}^{P}(\hat{y}_T - y_T)^2,$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{P}\sum_{T=1}^{P}(\hat{y}_T - y_T)^2}, \quad \mathrm{MAE} = \frac{1}{P}\sum_{T=1}^{P}|\hat{y}_T - y_T|,$$

where P is the number of forecast periods and $\hat{y}_T$ is the predicted value for y_T.

In Diebold and Mariano (1995) a test statistics of the null hypothesis of equal accuracy of two competing h-step ahead forecasts of a time series variable y_t, (denoted as $\hat{y}_{1,t|t-h}$ and $\hat{y}_{2,t|t-h}$, respectively, which have been produced for $t = N+h,\ldots,N+P+h-1$, rendering P forecasts in total, where N is the number of observations in the estimation sample) is developed. This test is also used later in the paper. Specifically, they propose a test of the null of equal forecast accuracy for an arbitrary loss function $\imath(e_{i,t|t-h})$, where $e_{i,t|t-h}$ is the corresponding h-step ahead forecast error, that is, $e_{i,t|t-h} = y_t - \hat{y}_{i,t|t-h}$, $i = 1$, 2. The loss difference is defined as $\Delta_t \equiv \imath(e_{1,t|t-h}) - \imath(e_{2,t|t-h})$ so that equal forecast accuracy entails $E[\Delta_t] = 0$.

Assuming covariance stationarity of the loss difference series, Diebold and Mariano (1995) show that the asymptotic distribution of the sample mean loss differential

$$\overline{\Delta} \equiv \frac{1}{P}\sum_{t=N+h}^{N+P+h-1}\Delta_t \tag{12}$$

is given by

$$\sqrt{N}(\overline{\Delta} - \mu) \to N(0, V(\overline{\Delta})),$$

where

$$V(\overline{\Delta}) = \frac{1}{P}\left(\gamma_0 + 2\sum_{i=1}^{h-1}\gamma_i\right), \quad \gamma_i = \mathrm{cov}(\Delta_t, \Delta_{t-i})$$

for $i = 1,\ldots,h-1$

assuming that h-step ahead forecasts exhibit dependence up to order $h-1$. Thus, an asymptotically standard normal statistics for testing the null hypothesis of equal forecast accuracy can be obtained as

$$DM = \frac{\overline{\Delta}}{\sqrt{\widehat{V}(\overline{\Delta})}}, \tag{13}$$

where $\widehat{V}(\overline{\Delta})$ is a consistent estimate of $V(\overline{\Delta})$, based on the sample autocovariances $\hat{\gamma}_i$ given by $\hat{\gamma}_i = \frac{1}{P}\sum_{t=N+h+i}^{N+P+h-1}(\Delta_t - \overline{\Delta})(\Delta_{t-i} - \overline{\Delta})$.

4. Application to real data modelling

The Tatry mountains region (Fig. 1) were selected as the pilot area for testing the approaches, which is usually considered as homogeneous with respect to the runoff regime in general (Hlavčová et al., 1998). Cluster analysis of Slovak catchments based on physiographic and climatic variables also resulted in a homogeneous pooling group located in this region (Kohnová, 1998). It is therefore of interest to investigate the structural properties of mean monthly discharge time series in this area with respect to their nonlinear properties, which possibly could be also similar. Moreover the Liptovská Mara, Bešeňová and Čierny Váh hydropower plants are in operation in this region and the Garajky reservoir for drinking water supply is planned to be build. Water is also withdrawn from rivers for snowing of ski tracks. Forecasting of mean monthly and seasonal reservoir inflows and mean monthly river flows is therefore of practical interest.

The area is also called the region of the high core mountains and it is building the inside of the West Carpathian belt. The high core mountains comprise the Východné, Západné Tatry and Nízke Tatry. The Tatry mountains belong to the highest mountains in Slovakia, with elevation rising from 800 to 2600 m above sea level. The long-term mean annual precipitation amounts vary from 900 mm in the lowest parts of the mountains to 2000 mm and more in the highest elevations. The average snow cover duration is approximately 200 days a year. The mean monthly temperature in January ranges between −10 and −6 °C and in July between 11 and 15 °C. The Východné and Západné Tatry are prolonged from the west to the east. The Nízke Tatry are elongated in an east–west direction.

Geologically, the Východné and Západné Tatry form a relatively highly elevated block, almost completely lined by the overlying sediments of the Central Carpathian Palaeogene basin. The southern and central parts of this block are formed by crystalline rock outcroppings of pre-Mesozoic sediments. The basement consists prevailingly of granitoids and metamorphic rocks with several tectonic slices. The Nízke Tatry ridge as a whole has the character of an arc-horst, the central and southern parts consist predominantly of pre-Alpine crystalline schists and granitoids, with the northern slopes being mainly composed of Late Paleozoic and Mesozoic lithostratigraphic units.

Pleistocene glaciation formed the alpine character of this region. The quarternary sediments and foothills consist of accumulated glacial, glaciofluvial, fluvial and proluvial sediments. These storages influence the runoff regime, which is considered to be less variable in general than that in adjacent mountainous areas of Slovakia.

A total of seven small and middle sized catchments in this region were selected for the case study. Their physiographic characteristics are presented in Tables 1 and 2 contains the basic hydrologic characteristics.

The length of observations in all individual catchments was 36 years (1966–2001). 32 years were used for model identification and parameter estimation, the rest of the data

Fig. 1. The location of the test region in Slovakia.

was used for verification and model forecasting performance comparison.

The flow regime of these rivers has alpine character with highest scores of occurrence of annual maximum mean monthly flows in April (Boca, Biely Váh, L'ubochnianka, Revúca) and May (Belá, Váh, Poprad,) and of the annual minimum monthly flows in January (Boca, Biely Váh, Váh, Revúca), February (L'ubochnianka, Poprad) or March (Belá). These flows are predominantly fed by snowmelt. Relatively high mean monthly flows occur also in June, July and August, these are mainly caused by convective precipitation. The low flow period starts in September and lasts till spring in general. Descriptive statistics of the test data and of its seasonality is listed in Table 3. The periods of 6, 4 and 3 months can be hydrologically explained by the grouping of high and low runoff periods in the series caused by the mixed snowmelt and convective precipitation fed runoff regime.

The nonlinear time series models described above were applied to data describing the mean monthly flows on the Belá, Biely Váh, Boca, L'ubochnianka, Poprad, Revúca and Váh Rivers. Though all these rivers were analyzed exchauslively, details about Poprad River are given here only. In the other six cases, we will include only the best fitting models.

In all cases, we have first determined the systematic components of the time series and we have modelled the remaining residua only. Harmonic regression was used instead of the more standard lag 12 differencing operator. We have preferred this approach, since it preserves the complete information and allows for physical interpretation (what is not always the case, when applying differentiation).

Concerning the systematic components, we got these results:

- In all cases, we have verified no trend to be present.
- By each river, the significant period for the seasonal component is $L = 12$ as verified by means of the correlogram.

Table 1
Physiographic catchments characteristics of the test basins

River	Site	Area (km^2)	Forestation (% of basin area)	Elevation (m a.s.l.)	Slope (deg)
Biely Váh	Východná	98	39	1065	9.6
Boca	Král'ova Lehota	115	84	1107	23.2
Belá	Podbanské	91	58	1544	26.4
Váh	Liptovský Mikuláš	1101	59	1090	17.4
Revúca	Podsuchá	218	60	993	19.2
Poprad	Matejovce	316	40	1018	9.2
L'ubochnianka	L'ubochňa	118	95	926	25.1

Table 2
Basic hydrologic characteristics of the test basins estimated from the period 1931 to 1980

River	Site	Mean annual temperature (°C)	Mean annual specific runoff (l/s km^2)	Mean annual precipitation (mm)
Biely Váh	Východná	4.2	15.9	926
Boca	Král'ova Lehota	4.2	19.0	1174
Belá	Podbanské	1.7	38.5	1647
Váh	Liptovský Mikuláš	4.2	19.0	1046
Revúca	Podsuchá	4.9	22.5	1195
Poprad	Matejovce	4.6	13.5	864
L'ubochnianka	L'ubochňa	5.0	20.1	1132

Table 3
Descriptive statistics of test data and of its seasonality

River	Site	Mean flow (m^3/s)	St. deviation (m^3/s)	Significant periods (month)	St. deviation of residuals (m^3/s)
Biely Váh	Východná	1.51	0.95	12, 6, 14.4	0.77
Boca	Král'ova Lehota	1.86	1.52	12, 6	1.24
Belá	Podbanské	3.52	2.99	12, 6, 4, 3	1.79
Váh	Liptovský Mikuláš	20.01	12.91	12, 6	9.25
Revúca	Podsuchá	4.67	3.02	12, 6, 4, 14.4	2.28
Poprad	Matejovce	3.95	2.24	12, 6, 14.4	1.52
L'ubochnianka	L'ubochňa	2.33	1.21	12, 6, 4	0.96

- For the cyclical component, by means of the spectral analysis we have determined as a significant frequency that one corresponding to 6 months (significance was tested by Fisher test).

The major part of our modelling work was devoted to the residua modelling.

We have started with the SETAR model. Applying AIC and BIC information criteria for linear models (see, e.g., Franses and van Dijk, 2000), we have determined an appropriate order for AR models. In all cases the order $p = 1$ was shown to be the best fitting. After the estimation of parameters of SETAR (p_1, p_2, d) models for $p_1, p_2, d \leqslant 2$ (and with step $\frac{c_{\max} - c_{\min}}{100}$ when estimating the best fitting threshold parameter c), we have tested linearity (besides $p = 1$ we have considered also $p = 2$) against the alternative of a SETAR model with heteroscedasticity-robust variant of LM test, applying the distribution derived by Hansen (1997). The null hypothesis can be rejected at conventional significance levels (for all 7 rivers) for several combinations of p_1, p_2 and d. Best SETAR models were chosen based on the minimal p-values and on minimal values of AIC, SIC and HQC information criteria for TAR models (we have taken into account also the threshold value c – if possible to be in the middle of the data). Concerning the diagnostic checking, after modelling of residuals, the remainder was again tested for serial correlation and remaining nonlinearity. In all cases our models were accepted (observe that we have proceed similarly also in the other types of models described below). For two best SETAR models we have computed 1-, 3-, 6- and 12-step ahead forecasts, and in each case we computed the prediction errors MSE, RMSE and MAE for the original data (i.e., summing residual forecasts with systematic components).

As the next step, we have tested the linearity against the LSTAR model with the threshold variable y_{t-1}, y_{t-2} and in the AO (aggregation operator) form. The number of entries into AO was determined by means of the autocorrelation function. In the case of all seven discussed rivers, LSTAR models with the threshold variables y_{t-d}, $d = 1$, 2 were rejected. However, in case of LSTAR models with AO (we applied W_F, W_S, W_2, W_3, M, MAX and MIN aggregation operators) the null hypothesis can be rejected at conventional significance levels for several combination of p_1,p_2,c and γ. We have chosen for each river 3 models with the smallest p-value. For these models we have estimated the parameters for $p_1, p_2 \leqslant 2$, $\gamma \in [0.5; 10]$ and $c \in [c_{0.1}; c_{0.9}]$. The best model for each type of the threshold variable was chosen based on the AIC, BIC and HQC information criteria, c and γ (note that for large γ the model behaves like SETAR). For these models, 1-, 3-, 6- and 12-step ahead forecasts were computed.

The last group of models considered was the class of ATAR models. We have estimated the parameters in the case of aggregation operators MAX, MIN, M, W_2 and W_3. We have applied the logistic shape function shifted into $f(0) = 0$. In all cases, after estimation of parameters for $p_1, p_2 \leqslant 2$, we have tested the linearity against the actual ATAR model, exploiting the modified distribution derived by Hansen (1997). As before, the best model for each type of aggregation operator was chosen based on AIC, SIC and HQC information criteria and values c, γ. For the best model, we have computed 1-, 3-, 6- and 12-step ahead forecasts.

Finally, for the 10 (11 for L'ubochnianka) best models we have applied the Diebold–Mariano test for 1-, 3-, 6- and 12-step-ahead forecasts. Based on this test and on the prediction errors, we have chosen for each river one model with the best predictive ability. From 7 observed rivers, in 4 cases (Biely Váh, Boca, L'ubochnianka, Poprad) the best model was from the new ATAR class, in 2 cases (Revúca, Váh) the SETAR model was the winner and only in 1 case (Belá) the LSTAR model with aggregation operator was the best fitting.

The best models are described below for the 6 rivers except the Poprad. For the Poprad River, we have included the complete description of discussed models in the respective classes (SETAR, LSTAR with AO and ATAR). Note that in the case of 12-step-ahead forecasts we have also included the "naïve" model, based on seasonality $L = 12$, $y_{t+12} = y_t$. However, this model is evidently worse than models chosen from the SETAR, LSTAR with AO as well as from ATAR classes.

5. Numerical results

Next we present an overview of the best models in all considered classes. The data from 32 years (1966–1997) have been used for model building. Subsequent data from 4 years (1998–2001) served for testing the quality of prediction. The best models in the classes mentioned above have been fitted.

5.1. Systematic components

$$Sc_{\text{Poprad}} = \underset{(0.08)}{3.95} - \underset{(0.11)}{2.14}\cos\frac{\pi t}{6} - \underset{(0.104)}{0.44}\cos\frac{5\pi t}{36} + \underset{(0.103)}{0.55}\cos\frac{\pi t}{3} - \underset{(0.10)}{0.52}\sin\frac{\pi t}{3},$$

$$Sc_{\text{Bela}} = \underset{(0.11)}{3.52} - \underset{(0.15)}{2.83}\cos\frac{\pi t}{6} + \underset{(0.11)}{1.19}\cos\frac{\pi t}{3} - \underset{(0.12)}{0.9}\sin\frac{\pi t}{3} + \underset{(0.12)}{0.85}\sin\frac{\pi t}{2} - \underset{(0.11)}{0.37}\cos\frac{2\pi t}{3} - \underset{(0.12)}{0.42}\sin\frac{2\pi t}{3},$$

$$Sc_{\text{Boca}} = \underset{(0.06)}{1.86} - \underset{(0.09)}{0.84}\cos\frac{\pi t}{6} + \underset{(0.09)}{0.45}\sin\frac{\pi t}{6} - \underset{(0.08)}{0.82}\sin\frac{\pi t}{3},$$

$$Sc_{\text{Biely Vah}} = \underset{(0.04)}{1.51} + \underset{(0.06)}{0.29}\sin\frac{\pi t}{6} - \underset{(0.06)}{0.61}\cos\frac{\pi t}{6} - \underset{(0.05)}{0.29}\sin\frac{\pi t}{3} - \underset{(0.05)}{0.22}\cos\frac{5\pi t}{36},$$

$$Sc_{\text{Lubochnianka}} = \underset{(0.05)}{2.33} + \underset{(0.07)}{0.54}\sin\frac{\pi t}{6} - \underset{(0.07)}{0.57}\cos\frac{\pi t}{6} - \underset{(0.06)}{0.53}\sin\frac{\pi t}{3} - \underset{(0.06)}{0.11}\cos\frac{\pi t}{3} + \underset{(0.06)}{0.32}\cos\frac{\pi t}{2},$$

$$Sc_{\text{Revuca}} = \underset{(0.13)}{4.67} + \underset{(0.18)}{1.30}\sin\frac{\pi t}{6} - \underset{(0.18)}{1.40}\cos\frac{\pi t}{6} - \underset{(0.16)}{1.71}\sin\frac{\pi t}{3} + \underset{(0.15)}{0.80}\cos\frac{\pi t}{2} - \underset{(0.15)}{0.75}\cos\frac{5\pi t}{36},$$

$$Sc_{\text{Vah}} = \underset{(0.50)}{20.01} + \underset{(0.71)}{2.65}\sin\frac{\pi t}{6} - \underset{(0.71)}{10.47}\cos\frac{\pi t}{6} - \underset{(0.63)}{6.21}\sin\frac{\pi t}{3} + \underset{(0.63)}{2.56}\cos\frac{\pi t}{3}.$$

5.2. Self-exciting TAR (SETAR) models

Table 4 contains p-values for the test of linearity against a 2-regime SETAR alternative applied to the residua y_t after systematic components for mean monthly river flows of the Poprad, the threshold value c and AIC, SIC, HQC information criteria.

Based on minimal p-value and information criteria of the type AIC, SIC and HQC for SETAR models, optimal models of the type SETAR(1,2) and SETAR(2,1) have been selected for $d = 1$. In both models, the lower regime ($y_{t-1} < -0.67$) contains 134 observations, whereas the upper regime contains the remaining 250 observations. These models will be denoted as M_1 and M_2.

Model M_1(σ = 1.341)

$$x_t = Sc_{\text{Poprad}} + \Big(\underset{(0.01)}{-0.50} + \underset{(0.007)}{0.17}\, y_{t-1}\Big)(1 - I(c > -0.67))$$
$$+ \Big(\underset{(0.004)}{0.14} + \underset{(0.003)}{0.17}\, y_{t-1} - \underset{(0.002)}{0.02}\, y_{t-2}\Big) I(c > -0.67) + \varepsilon_t.$$

Model M_2(σ = 1.337)

$$x_t = Sc_{\text{Poprad}} + \Big(\underset{(0.01)}{-0.53} + \underset{(0.007)}{0.21}\, y_{t-1} - \underset{(0.003)}{0.10}\, y_{t-2}\Big)(1 - I(c > -0.67))$$
$$+ \Big(\underset{(0.004)}{0.14} + \underset{(0.003)}{0.17}\, y_{t-1}\Big)(c > -0.67) + \varepsilon_t.$$

The prediction errors for 1-, 3-, 6- and 12-step-ahead forecasts are in Table 5.

Based on Diebold–Mariano test and on the prediction errors, we have chosen model M_2 as that with the better predictive ability. Original data and 1-, 3-, 6- and 12-step-ahead forecasts for 48 months with model M_2 are presented in Fig. 2. Original data x_t (mean monthly river flows) are drawn by full line and predictions by dashed line.

5.3. Two-regimes LSTAR models with threshold variable of aggregated type (case 1)

The first step was testing the linearity against the LSTAR model with the threshold variable y_{t-1}, y_{t-2} and in the AO (aggregation operator) form. Table 6 contains p-values of the heteroscedasticity-consistent variant of LM-type test against LSTAR nonlinearity, based on AR(1) and AR(2) models.

In the case of all 7 discussed rivers, LSTAR models with the threshold variables y_{t-d}, $d = 1, 2$ were rejected. However, in case LSTAR models with AO the null hypothesis can be rejected at conventional significance levels for several combination of p_1, p_2, c and γ. We have chosen for each river 3 models with the smallest p-value. For these models we have estimated the parameters for $p_1, p_2 \leqslant 2$, $\gamma \in [0.5; 10]$ and $c \in [c_{0.1}; c_{0.9}]$. Table 7 contains values of c, γ, σ^2, AIC, BIC and HQC information criteria for Poprad River (aggregation operators are W_S with $p = 0.5$, W_3 and M). The number of variables that enter each applied AO (for Poprad River) is $k = 5$.

The best model for each type of the threshold variable was chosen based on the AIC, BIC and HQC information criteria, c and γ. These models will be denoted as M_3 (with MAX), M_4 (with W_S) and M_5 (with W_3). Table 8 contains the parameter estimates for all three models for residuals after systematic components for Poprad River. The prediction errors for 1-, 3-, 6- and 12-step-ahead forecasts are in Table 9.

Table 4
p-Values for the test of linearity against a 2-regime SETAR alternative, threshold value c and AIC, SIC, HQC for Poprad River

384	p_1	p_2	p-Value	σ^2		c	p_1+p_2	AIC		SIC		HQC		Sum
	$d=1$													
Poprad M.	1	1	0.0224	1.8330	8	−0.43	2	0.616	8	0.637	4	0.625	5	17
	1	2	0.0265	1.7990	3	−0.67	3	0.603	2	0.634	3	0.615	2	7
	2	1	0.0206	1.7900	1	−0.67	3	0.598	1	0.629	1	0.610	1	3
	2	2	0.0190	1.7930	2	−0.67	4	0.605	3	0.646	7	0.621	4	14
	$d=2$													
Poprad M.	1	1	0.0800	1.8200	6	−0.97	2	0.609	4	0.630	2	0.617	3	9
	1	2	0.0640	1.8180	5	−0.82	3	0.613	5	0.644	5	0.626	6	16
	2	1	0.0580	1.8200	6	−0.97	3	0.614	6	0.645	6	0.627	7	19
	2	2	0.0508	1.8130	4	−0.97	4	0.616	7	0.657	8	0.632	8	23

Table 5
The prediction errors for 1-, 3-, 6- and 12-step-ahead forecasts

d	p_1, p_2	1-Step forecasts			3-Step forecasts			6-Step forecasts			12-Step forecasts		
		MSE	RMSE	MAE	MSE	RMSE	MAE	MSE	RMSE	MAE	MSE	RMSE	MAE
1	2, 1	2.91	1.71	1.15	3.93	1.98	1.18	4.17	2.04	1.21	4.54	2.13	1.23
1	1, 2	2.92	1.71	1.12	3.96	1.99	1.18	4.14	2.03	1.20	4.53	2.13	1.22

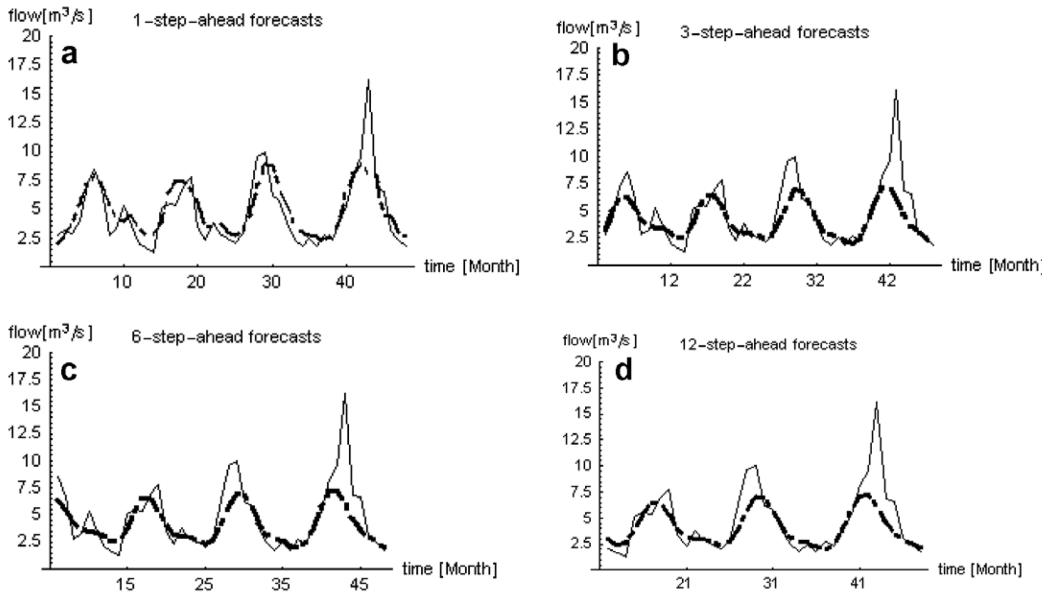

Fig. 2. Original data and (a) 1-, (b) 3-, (c) 6- and (d) 12-step-ahead forecasts for 48 months with model SETAR(2,1).

Table 6
p-Values of the HCC variant of LM-type test against LSTAR nonlinearity, based on AR(1) and AR(2) models

River	k	p	For LSTAR with		For LSTAR with aggregation operator						
			$d=1$	$d=2$	MAX	MIN	M	W_S	W_F	W_2	W_3
Belá	14	2	0.5257	0.6841	0.7784	0.4971	0.5245	0.0278	0.0396	0.0636	0.1201
Biely Váh	15	1	0.3276	0.4492	0.0652	0.2533	0.0307	0.0289	0.0289	0.0096	0.0094
Boca	4	2	0.5679	0.6211	0.0210	0.0215	0.0116	0.0223	0.0183	0.0208	0.0266
L'ubochianka	3	1	0.0884	0.1105	0.00011	0.00059	0.00039	0.00012	0.00012	0.00011	0.00007
Poprad M.	5	2	0.3652	0.4732	0.0229	0.0790	0.0708	0.0389	0.0465	0.0551	0.0438
Revúca	3	1	0.1336	0.2811	0.00006	0.0009	0.0004	0.0002	0.0002	0.00010	0.00006
Váh	7	1	0.8263	0.9214	0.0154	0.0249	0.0310	0.0360	0.0326	0.0273	0.0333

Table 7
Values of c, γ, σ^2, AIC, BIC and HQC information criteria for Poprad River

384	p_1	p_2	σ^2		γ	c	p_1+p_2	AIC		SIC		HQC		Sum
Poprad Matejovce	*W_S with p = 0.5*													
	1	1	1.8319	4	8.5	0.49	2	0.616	4	0.636	4	0.624	4	12
	1	2	1.7924	3	8.5	0.49	3	0.599	3	0.630	2	0.611	3	8
	2	1	1.7698	2	8.5	0.41	3	0.586	1	0.617	1	0.599	1	3
	2	2	1.7651	1	8.5	0.44	4	0.589	2	0.630	3	0.605	2	7
	W_3													
	1	1	1.8505	4	4.5	1.12	2	0.626	4	0.646	4	0.634	4	12
	1	2	1.8093	3	8.5	0.14	3	0.609	3	0.639	3	0.621	3	9
	2	1	1.7803	2	5.5	0.14	3	0.592	1	0.623	1	0.605	1	3
	2	2	1.7770	1	8.5	0.11	4	0.596	2	0.637	2	0.612	2	6
	MAX													
	1	1	1.8556	4	4.5	0.24	2	0.629	4	0.649	3	0.637	4	11
	1	2	1.8066	2	4.5	0.21	3	0.607	1	0.638	1	0.619	1	3
	2	1	1.8091	3	0.5	0.64	3	0.608	2	0.639	2	0.621	2	6
	2	2	1.8064	1	4.5	0.16	4	0.612	3	0.653	4	0.628	3	10

Based on Diebold–Mariano test and on the prediction errors, we have chosen model M_5 with the better predictive ability. Original data and 1-, 3-, 6- and 12-step-ahead forecasts for 48 months with model M_5 are presented in Fig. 3.

Table 8
Parameter estimates for LSTAR models with AO

Type	c	γ	$\varphi_{0.1}$	$\varphi_{1.1}$	$\varphi_{2.1}$	$\varphi_{0.2}$	$\varphi_{1.2}$	$\varphi_{2.2}$
MAX	0.21 (0.008)	4.5	−0.40 (0.009)	0.33 (0.007)	0	0.06 (0.003)	0.27 (0.002)	−0.08 (0.002)
W_S	0.41 (0.011)	8.5	−0.14 (0.005)	0.51 (0.003)	−0.24 (0.002)	0.14 (0.006)	0.15 (0.003)	0
W_3	0.14 (0.009)	5.5	−0.06 (0.004)	0.51 (0.003)	−0.19 (0.002)	0.35 (0.008)	0.07 (0.003)	0

Table 9
The prediction errors for 1-, 3-, 6- and 12-step-ahead forecasts

AO	p_1, p_2	1-Step forecasts			3-Step forecasts			6-Step forecasts			12-Step forecasts		
		MSE	RMSE	MAE	MSE	RMSE	MAE	MSE	RMSE	MAE	MSE	RMSE	MAE
MAX	1, 2	2.88	1.70	1.13	4.16	2.04	1.20	4.29	2.07	1.20	5.21	2.28	1.29
W_S	2, 1	3.24	1.80	1.17	3.78	1.94	1.20	4.11	2.03	1.24	4.11	2.03	1.24
W_3	2, 1	2.88	1.70	1.18	3.93	1.98	1.18	4.36	2.09	1.22	4.89	2.21	1.24

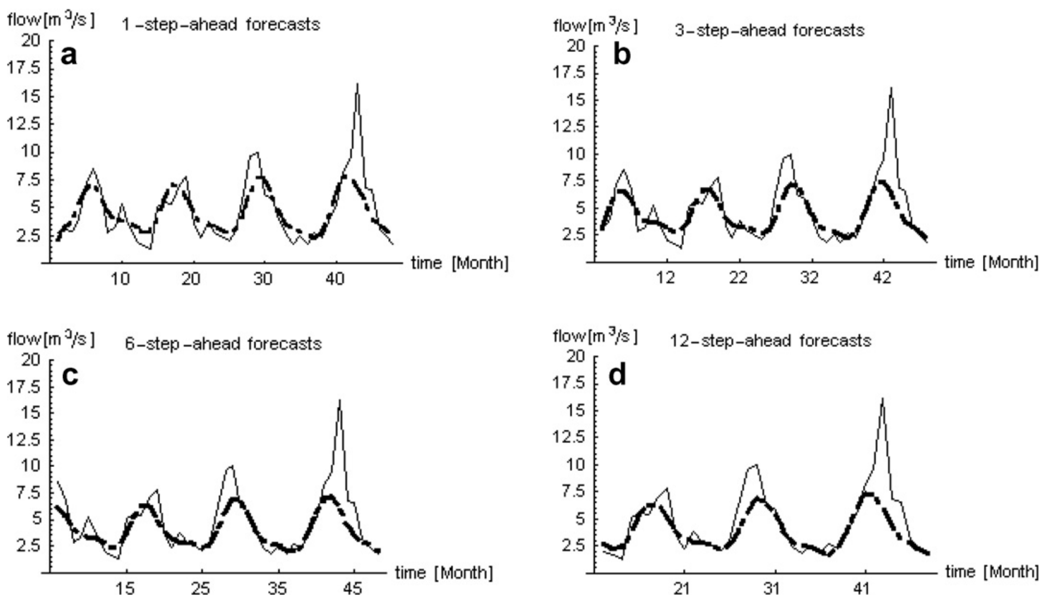

Fig. 3. Original data and (a) 1-, (b) 3-, (c) 6- and (d) 12-step-ahead forecasts for 48 months with model M_5.

Model $M_5(\sigma = 1.330)$

$$q_t = 0.5y_{t-1} + 0.25y_{t-2} + 0.125y_{t-3} + 0.0625y_{t-4} + 0.0625y_{t-5},$$

$$F(q_t; 8.5; 0.41) = \frac{1}{1 + e^{-8.5(q_t - 0.41)}},$$

$$x_t = Sc_{\text{Poprad}} + \left(\underset{(0.004)}{-0.06} + \underset{(0.003)}{0.51}\, y_{t-1} - \underset{(0.002)}{0.19}\, y_{t-2}\right)(1 - F(q_t; 8.5; 0.41)) + \left(\underset{(0.008)}{0.35} + \underset{(0.003)}{0.07}\, y_{t-1}\right) F(q_t; 8.5; 0.41) + \varepsilon_t.$$

5.4. ATAR models (case 2) with the logistic type function

Table 10 contains p-values for the test of linearity against a 2-regime ATAR alternative applied to the residua y_t after systematic components for monthly average Poprad River flows, the threshold value c, smoothness parameter γ and AIC, SIC, HQC information criteria. As aggregation operators were used M, W_2, W_3, MAX and MIN.

The best model for each type of aggregation operators was chosen based on the minimal p-value and AIC, BIC, HQC information criteria. These models will be denoted as M_6 (with W_2), M_7 (with W_3), M_8 (with MAX), M_9 (with MIN) and M_{10} (with M). Table 11 contains the parameter estimates for all five ATAR models for residuals after systematic components for Poprad River. The prediction errors for 1-, 3-, 6- and 12-step-ahead forecasts are in Table 12. In last row of these table is also prediction errors for "naïve" 12-step-ahead forecasts.

Based on Diebold–Mariano test and on the prediction errors, we have chosen model M_{10} with the better predictive ability for 3- and 12-step-ahead forecasts, M_6 for 1-step-ahead forecasts and M_7 for 6-step-ahead forecasts. Original data and 1-, 3-, 6- and 12-step-ahead forecasts for 48 months with these models are presented in Fig. 4.

Table 10
p-Values for the test of linearity against a 2-regime ATAT alternative, σ^2, c, γ and AIC, SIC, HQC information criteria

384	p_1	p_2	p-Value	σ^2		γ	c	p_1+p_2	AIC		SIC		HQC		Sum
Poprad Matejovce	W_2														
	1	1	0.0150	1.8166	4	8.5	0.01	2	0.607	3	0.628	1	0.616	2	6
	1	2	0.0678	1.8098	3	8.5	0.04	3	0.609	4	0.640	3	0.621	4	11
	2	1	0.0291	1.7909	1	4.5	−0.16	3	0.598	1	0.629	2	0.611	1	4
	2	2	0.0291	1.7909	1	4.5	−0.16	4	0.604	2	0.645	4	0.620	3	9
	W_3														
	1	1	0.0155	1.8175	4	6.5	−0.13	2	0.608	3	0.628	2	0.616	2	7
	1	2	0.0815	1.8136	3	6.5	−0.06	3	0.611	4	0.642	3	0.623	4	11
	2	1	0.0263	1.7885	2	2.5	0.29	3	0.597	1	0.628	1	0.609	1	3
	2	2	0.0262	1.7884	1	2.5	0.32	4	0.602	2	0.643	4	0.618	3	9
	MAX														
	1	1	0.0117	1.8101	4	4.5	0.29	2	0.604	2	0.624	1	0.612	1	4
	1	2	0.0464	1.8016	3	4.5	0.26	3	0.604	3	0.635	3	0.617	3	9
	2	1	0.0438	1.8003	1	0.5	0.46	3	0.604	1	0.634	2	0.616	2	5
	2	2	0.0438	1.8003	1	0.5	0.44	4	0.609	4	0.650	4	0.625	4	12
	MIN														
	1	1	0.0190	1.8227	4	8.5	−0.24	2	0.611	1	0.631	1	0.619	1	3
	1	2	0.1883	1.8207	3	5.5	1.12	3	0.615	4	0.646	3	0.627	3	10
	2	1	0.1035	1.8182	2	8.5	−0.27	3	0.613	2	0.644	2	0.626	2	6
	2	2	0.0699	1.8105	1	8.5	−1.00	4	0.614	3	0.656	4	0.631	4	11
	M														
	1	1	0.0150	1.8167	4	8.5	−0.01	2	0.607	2	0.628	1	0.616	1	4
	1	2	0.0682	1.8122	3	8.5	−0.01	3	0.610	4	0.641	3	0.622	3	10
	2	1	0.0488	1.8027	1	8.5	0.01	3	0.605	1	0.636	2	0.617	2	5
	2	2	0.0488	1.8027	1	8.5	0.01	4	0.610	3	0.651	4	0.626	4	11

Table 11
Parameter estimates for ATAR models

Type	c	γ	$\varphi_{0.1}$	$\varphi_{1.1}$	$\varphi_{2.1}$	$\varphi_{0.2}$	$\varphi_{1.2}$
MAX	0.29 (0.012)	4.5	−0.22 (0.009)	0.45 (0.007)	0	0.04 (0.003)	0.26 (0.002)
MIN	−0.24 (0.014)	8.5	−0.12 (0.003)	0.30 (0.003)	0	0.38 (0.011)	0.22 (0.007)
M	0.01 (0.002)	8.5	−0.42 (0.006)	0.34 (0.004)	−0.17 (0.003)	0.41 (0.008)	0.19 (0.004)
W_2	−0.16 (0.009)	4.5	−0.65 (0.009)	0.35 (0.005)	−0.30 (0.005)	0.51 (0.008)	0.11 (0.004)
W_3	−0.29 (0.015)	2.5	−0.83 (0.012)	0.36 (0.006)	−0.38 (0.005)	0.58 (0.005)	0.06 (0.004)

Table 12
The prediction errors for 1-, 3-, 6- and 12-step-ahead forecasts

Type	AO	p_1, p_2	1-Step forecasts			3-Step forecasts			6-Step forecasts			12-Step forecasts		
			MSE	RMSE	MAE	MSE	RMSE	MAE	MSE	RMSE	MAE	MSE	RMSE	MAE
ATAR	M	2, 1	2.99	1.83	1.25	3.62	1.90	1.24	3.88	1.97	1.25	4.19	2.05	1.25
	MAX	1, 1	2.84	1.68	1.19	3.94	1.98	1.16	4.32	2.08	1.20	5.17	2.27	1.28
	MIN	1, 1	2.98	1.73	1.25	3.65	1.91	1.25	3.87	1.97	1.25	4.22	2.05	1.26
	W_2	2, 1	3.09	1.76	1.22	3.66	1.91	1.26	3.88	1.97	1.27	4.22	2.05	1.28
	W_3	2, 1	3.23	1.79	1.23	3.69	1.92	1.23	3.98	1.99	1.24	4.35	2.08	1.24
Naive												6.95	2.63	1.83

Model M_7 ($\sigma = 1.337$; improving 37%)

$$f(z) = \frac{1}{1+e^{-2.5(z+0.29)}};$$
$$F(q_t) = 0.5f(y_{t-1}) + 0.25f(y_{t-2}) + 0.125f(y_{t-3}) + 0.0625f(y_{t-4}) + 0.0625f(y_{t-5}),$$
$$x_t = Sc_{\text{Poprad}} + (-0.83 + 0.36y_{t-1} - 0.38y_{t-2})(1 - F(q_t)) + (0.58 + 0.06y_{t-1})F(q_t).$$

5.5. Diebold–Mariano test of equal forecast accuracy

Finally, we compared the quality of out-of-sample predictions for the above mentioned selected 10 models M_1, M_2, M_3, M_4, M_5, M_6, M_7, M_8, M_9, M_{10}. The quality comparison between all pairs of these models was based on the Diebold–Mariano (DM) test of equal forecast accuracy. The results for 1-, 3-, 6- and 12-step-ahead forecasts are

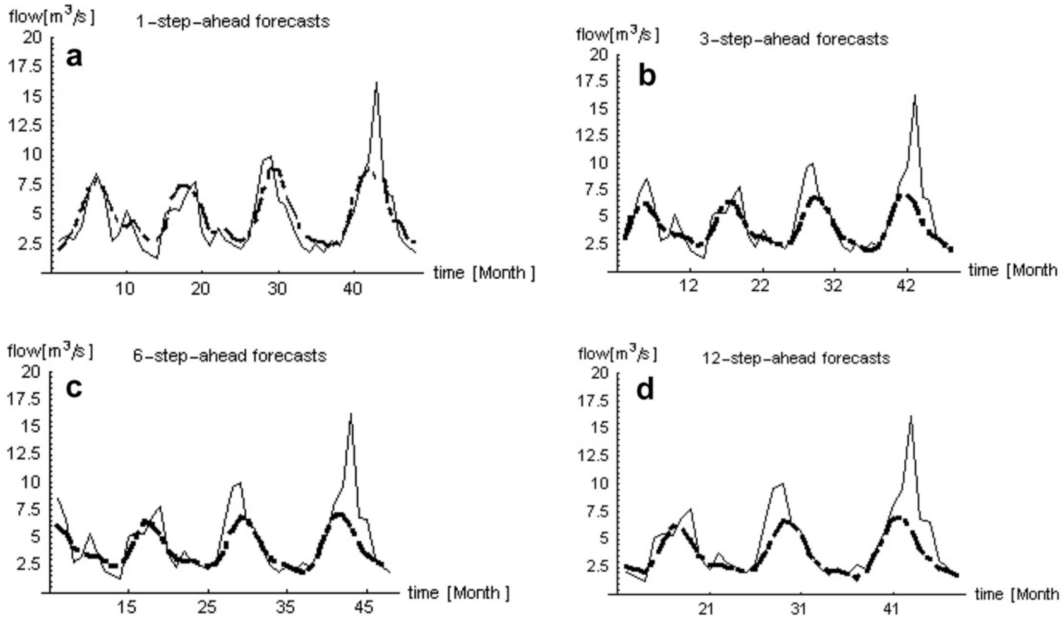

Fig. 4. Original data and (a) 1-, (b) 3-, (c) 6- and (d) 12-step-ahead forecasts for 48 months with models ATAR(2,1).

presented in Tables 13a–13d. The element in the intersection of the ith row and jth column of this table is equal to

(a) 1
(b) −1
(c) 0

if the prediction quality of the model in the row i in comparison with the model in the column j is

(a) significantly better,
(b) significantly worse,
(c) is not significantly different.

Results of Tables 13 demonstrate that the models M_{10} for 3- and 12-step-ahead forecasts, M_6 for 1-step-ahead forecasts and M_7 for 6-step-ahead forecasts have a significantly better prediction quality for residuals from systematic components for Poprad River than all competing models.

5.6. Best models for the remaining 6 river

5.6.1. River Belá

Best models is LSTAR(1,2) with W_S (for $k = 14$). Standard deviation of residuals is $\sigma = 1.43$ (improving 52%)

$$q_t = \sum_{j=1}^{14} \Delta_{S,j} y_{t-j}, \quad F(q_t; 8.5; 0.41) = \frac{1}{1 + e^{-8.5(q_t - 0.45)}},$$

$$x_t = Sc_{\text{Bela}} + \left(\underset{(0.004)}{-0.11} + \underset{(0.003)}{0.17} y_{t-1} \right)(1 - F(q_t; 8.5; 0.45)) + \left(\underset{(0.008)}{0.53} + \underset{(0.003)}{0.14} y_{t-1} - \underset{(0.002)}{0.17} y_{t-2} \right) F(q_t; 8.5; 0.45) + \varepsilon_t.$$

Table 13a
Results of Diebold–Mariano test of equal forecast accuracy for 1-step-ahead forecasts

		ATAR					LSTAR with AO			SETAR		Sum
		M	MAX	MIN	W_2	W_3	W_S	W_2	W_3	1.2	2.2	
ATAR	M	X	1	0	−1	−1	−1	−1	0	−1	0	−4
	MAX	−1	X	1	−1	−1	−1	−1	1	−1	−1	−5
	MIN	0	−1	X	−1	0	−1	−1	0	−1	−1	−6
	W_2	1	1	1	X	1	0	1	0	0	0	5
	W_3	1	1	0	−1	X	−1	−1	0	0	0	−1
LSTAR with AO	MAX	1	1	1	0	1	X	0	1	0	0	5
	W_S	1	1	1	−1	1	0	X	1	0	0	4
	W_3	0	−1	0	0	0	−1	−1	X	−1	−1	−5
SETAR	2, 1	1	1	1	0	0	0	0	1	X	0	4
	2, 2	0	1	1	0	0	0	0	1	0	X	3

Table 13b
Results of Diebold–Mariano test of equal forecast accuracy for 3-step-ahead forecasts

		ATAR					LSTAR with AO			SETAR		Sum
		M	MAX	MIN	W_2	W_3	W_S	W_2	W_3	1, 2	2, 2	
AOTAR	M	X	1	1	1	1	1	1	1	1	1	9
	MAX	−1	X	1	1	−1	1	1	1	1	1	5
	MIN	−1	−1	X	1	−1	1	1	1	1	1	3
	W_2	−1	−1	−1	X	−1	1	−1	−1	−1	1	−5
	W_3	−1	1	1	1	X	1	1	1	1	1	7
LSTAR with AO	MAX	−1	−1	−1	1	−1	X	−1	−1	−1	−1	−9
	W_S	−1	−1	−1	1	−1	−1	X	−1	1	1	−1
	W_3	−1	−1	−1	1	−1	1	1	X	1	1	1
SETAR	2, 1	−1	−1	−1	1	−1	1	−1	−1	X	1	−3
	2, 2	−1	−1	−1	−1	−1	1	−1	−1	−1	X	−7

Table 13c
Results of Diebold–Mariano test of equal forecast accuracy for 6-step-ahead forecasts

		ATAR					LSTAR with AO			SETAR		Sum
		M	MAX	MIN	W_2	W_3	W_S	W_2	W_3	1, 2	2, 2	
AOTAR	M	X	0	1	1	−1	1	1	1	1	1	6
	MAX	0	X	1	1	−1	1	1	1	1	1	6
	MIN	−1	−1	X	1	−1	1	1	1	1	1	3
	W_2	−1	−1	−1	X	−1	−1	1	−1	−1	−1	−7
	W_3	1	1	1	1	X	1	1	1	1	1	9
LSTAR with AO	MAX	−1	−1	−1	1	−1	X	1	−1	−1	−1	−5
	W_S	−1	−1	−1	1	−1	−1	X	−1	−1	−1	−9
	W_3	−1	−1	−1	1	−1	1	1	X	1	1	1
SETAR	2, 1	−1	−1	−1	1	−1	1	1	−1	X	−1	−3
	2, 2	−1	−1	−1	1	−1	1	1	−1	1	X	−1

Table 13d
Results of Diebold–Mariano test of equal forecast accuracy for 12-step-ahead forecasts

		ATAR					LSTAR with AO			SETAR		Sum
		M	MAX	MIN	W_2	W_3	W_S	W_2	W_3	1, 2	2, 2	
AOTAR	M	X	1	1	1	1	1	1	1	1	1	9
	MAX	−1	X	1	1	0	1	1	1	1	1	6
	MIN	−1	−1	X	1	−1	1	1	1	1	1	3
	W_2	−1	−1	−1	X	−1	1	−1	−1	−1	−1	−7
	W_3	−1	0	1	1	X	1	1	1	1	1	6
LSTAR with AO	MAX	−1	−1	−1	−1	−1	X	−1	−1	−1	−1	−9
	W_S	−1	−1	−1	1	−1	1	X	−1	−1	−1	−5
	W_3	−1	−1	−1	1	−1	1	1	X	−1	−1	−3
SETAR	2, 1	−1	−1	−1	1	−1	1	1	1	X	−1	−1
	2, 2	−1	−1	−1	1	−1	1	1	1	1	X	1

5.6.2. River Biely Váh

Best models is ATAR(2,1) with W_2 (for $k = 3$). Standard deviation of residuals is $\sigma = 0.58$ (improving 39%).

$$f(z) = \frac{1}{1 + e^{-3.5(z+0.19)}};$$
$$F(q_t) = 0.111f(y_{t-1}) + 0.333f(y_{t-2}) + 0.555f(y_{t-3}),$$
$$x_t = Sc_{\text{Biely Vah}} + \left(\underset{(0.016)}{-0.57} + \underset{(0.006)}{0.07}\, y_{t-1} - \underset{(0.015)}{0.40}\, y_{t-2}\right)(1 - F(q_t)) + \left(\underset{(0.01)}{0.30} + \underset{(0.008)}{0.22}\, y_{t-1}\right)F(q_t) + \varepsilon_t.$$

5.6.3. River Boca

Best models is ATAR(2,2) with W_2 (for $k = 4$). Standard deviation of residuals is $\sigma = 0.98$ (improving 36%):

$$f(z) = \frac{1}{1 + e^{-8.5(z-0.07)}};$$
$$F(q_t) = 0.6255f(y_{t-1}) + 0.1875f(y_{t-2}) + 0.3125f(y_{t-3}) + 0.43755f(y_{t-4}),$$
$$x_t = Sc_{\text{Boca}} + \left(\underset{(0.007)}{-039} + \underset{(0.006)}{0.37}\, y_{t-1} - \underset{(0.005)}{0.40}\, y_{t-2}\right)(1 - F(q_t)) + \left(\underset{(0.008)}{0.55} + \underset{(0.004)}{0.11}\, y_{t-1} + \underset{(0.003)}{0.04}\, y_{t-2}\right)F(q_t) + \varepsilon_t.$$

Table 14
Comparison of the naïve model and the best nonlinear regime-switching model

River	Type	1-Step forecast			3-Step forecast			6-Step forecast			12-Step forecast		
		MSE	RMSE	MAE	MSE	RMSE	MAE	MSE	RMSE	MAE	MSE	RMSE	MAE
Belá	LSTAR	3.24	1.80	1.29	3.25	1.80	1.21	3.22	1.79	1.19	3.27	1.81	1.13
	Naïve	6.88	2.62	1.80	18.51	4.30	3.34	17.73	4.21	3.41	6.06	2.46	1.67
Biely Váh	ATAR	0.97	0.99	0.70	1.13	1.06	0.67	1.24	1.14	0.69	1.42	1.19	0.73
	Naïve	1.82	1.35	0.86	3.91	1.98	1.50	4.00	2.00	1.47	2.25	1.50	1.09
Boca	ATAR	1.69	1.30	1.02	1.40	1.18	0.93	1.25	1.12	0.84	1.26	1.12	0.83
	Naïve	2.54	1.59	1.05	3.58	1.89	1.37	3.63	1.91	1.32	2.10	1.45	1.11
L'ubochnianka	ATAR	1.35	1.16	0.97	1.13	1.06	0.79	1.16	1.07	0.78	1.17	1.08	0.76
	Naïve	2.35	1.53	1.02	4.54	2.13	1.53	4.63	2.15	1.53	2.36	1.54	1.11
Poprad	ATAR	2.84	1.68	1.19	3.62	1.90	1.24	3.88	1.97	1.25	4.19	2.05	1.25
	Naïve	6.19	2.45	1.72	18.90	4.35	3.43	22.77	4.77	3.96	6.95	2.64	1.83
Revúca	SETAR	6.46	2.54	2.11	5.95	2.44	1.83	5.98	2.45	1.85	6.56	2.56	1.93
	Naïve	12.87	3.55	2.47	24.29	4.93	3.66	24.82	4.98	3.30	12.67	3.56	2.76
Váh	SETAR	97.38	9.87	8.01	86.27	9.29	6.41	88.91	9.43	6.13	92.74	9.63	6.88
	Naïve	153.30	12.38	9.26	385.73	19.64	15.42	409.48	20.24	16.48	172.55	13.14	10.21

5.6.4. River Lubochnianka

Best models is ATAR(2,1) with W_2 (for $k = 3$). Standard deviation of residuals is $\sigma = 0.71$ (improving 41%):

$$f(z) = \frac{1}{1 + e^{-5.5(z+0.22)}};$$

$$F(q_t) = 0.111f(y_{t-1}) + 0.333f(y_{t-2}) + 0.555f(y_{t-3}),$$

$$x_t = Sc_{\text{Lubochnianka}} + \left(\underset{(0.011)}{-0.59} + \underset{(0.009)}{0.17}\, y_{t-1} - \underset{(0.008)}{0.25}\, y_{t-2}\right)(1 - F(q_t)) + \left(\underset{(0.007)}{0.40} + \underset{(0.005)}{0.15}\, y_{t-1}\right)F(q_t) + \varepsilon_t.$$

5.6.5. River Revúca

Best models is SETAR(1,2) (for $d = 1$). Standard deviation of residuals is $\sigma = 2.03$ (improving 33%)

$$x_t = Sc_{\text{Revuca}} + \left(\underset{(0.005)}{-0.23} + \underset{(0.003)}{0.29}\, y_{t-1}\right)(1 - I(c > 0.42)) + \left(\underset{(0.007)}{0.70} + \underset{(0.002)}{0.16}\, y_{t-1} - \underset{(0.002)}{0.09}\, y_{t-2}\right)I(c > 0.42) + \varepsilon_t.$$

5.6.6. River Váh

Best models is SETAR(2,2) (for $d = 1$). Standard deviation of residuals is $\sigma = 7.59$ (improving 41%)

$$x_t = Sc_{\text{Vah}} + \left(\underset{(0.005)}{-0.72} + \underset{(0.0009)}{0.39}\, y_{t-1} - \underset{(0.0004)}{0.16}\, y_{t-2}\right)(1 - I(c > -0.37)) + \left(\underset{(0.005)}{1.38} + \underset{(0.0004)}{0.20}\, y_{t-1} + \underset{(0.0004)}{0.06}\, y_{t-2}\right)I(c > -0.37) + \varepsilon_t.$$

Though several prediction characteristics are numerically described in the preceding tables, to stress the quality of obtained predictions, Table 14 brings the comparison of the naïve model and our best model.

6. Conclusions

We have discussed the prediction ability of several nonlinear time series models and proposed a new class regime switching models based on aggregation operators for the forecasting of flows on selected 7 Slovak rivers, which use a transition function obtained as the outputs of an aggregation operator with inputs given by the images (in a common shape function) of a fixed number of delayed values of the modelled time series.

Mean monthly flows of the test region used in this study, the Tatry alpine mountain region, are predominantly fed by snowmelt in the spring and convective precipitation in the summer. Therefore their hydrological regime exhibits clear seasonal patterns. Positive deviations from these trends have substantially different features than the negative ones. This provided intuitive justification for the application of nonlinear two-regime models for modelling and forecasting of these time series. In all cases, the possible linearity of models was rejected.

From seven rivers used in the case study, in four cases (Biely Váh, Boca, Lubochnianka, Poprad) the best model was from the new ATAR class, in two cases (Revúca, Váh) the SETAR model was the winner and only in one case (Belá) the LSTAR model with aggregation operator performed best. Despite of the declared homogeneity of the driving forces of runoff generation in the Tatry mountains region, the structural properties of mean monthly discharge time series in this area with respect to their nonlinear properties are obviously not similar. Reasons for this fact may be many, however it is believed, that differences in underground storages formed by the quarternary sediments and of accumulated glacial, glaciofluvial, fluvial and proluvial sediments strongly influence the runoff regime in the particular rivers. These storages clearly influence the variability of the series and their memory properties; however the extent and effect of this influence will have to be further investigated.

Concerning the further development of the mathematical models proposed in the paper, we intend to develop the approach in two directions. In practical forecasting in this case study the model structure and parameters were fixed, with no influence on these by newly observed values. We intend to modify the model with each new piece of data in the future. Another direction of improving the results could be the focus on external observations (for example the rainfall), which would be included into the switching mechanism.

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