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Temperature reconstruction for Arctic glaciers

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Abstract

Numerous data indicate discernible warming of the Northern Hemisphere in the 20th century especially at high latitudes. Summer melting at the surface of Arctic glaciers is likely to increase this warming. To verify this phenomenon, we reconstruct the past temperatures of the Arctic ice caps at their surface and at the 10 m depth. The reconstructions are made for two Arctic ice caps (Austfonna and Akademii Nauk) by means of the inversion of the measured temperature-depth profiles. In addition, we use the measured oxygen–isotope ratio and the melt feature index as input data. We find that the present ice temperature is the highest over the last 1000 years, while the lowest temperature occurred in the 1700s. The surface temperature variations of Akademii Nauk (Severnaya Zemlya) and Austfonna (Svalbard) Ice Caps exceed the average Arctic temperature anomalies over the last 150 years by 6 to 7 °C.

Keywords: Arctic; Past temperature reconstruction; Inverse problem

1. Introduction

The analysis of the borehole temperature and the ice-core isotopic composition extracted by GISP II revealed that on the temperature scale the warming is at least three times the coincident temperature change at the tropics and mid-latitudes (Cuffey et al., 1995). It was also noted that for a smaller period of time covering the last four centuries the climate change in the Arctic had been augmented by several positive feedbacks, including ice and snow melting (Overpeck et al., 1997). Deep-ground temperature analysis in the Canadian Arctic (Taylor, 1991) revealed the enhanced surface temperature warming (by 5 °C over the last 70–100 years). Additionally, according to Mareschal and Beltrami (1992), eastern Canada experienced warming of 1 to 2 °C over the past 100–200 years, and temperature reconstructions based on seventeen borehole temperature profiles (Huang et al., 1996) indicate that in the 20th century the warming has become the predominant phenomenon in the northeast USA and southeast Canada. The reconstructions of surface temperature in both Hemispheres show that warming in the 20th century is unprecedented for at least the last two millennia (Mann and Jones, 2003), with the magnitude of the ground surface warming over the last five centuries larger in the Northern Hemisphere than in the Southern Hemisphere (Huang et al., 2000).

Many Eurasian Arctic glaciers are subjected to melting due to high summer air temperature. Melt water percolates into snow-firn layers. The intensity of melting during the summer months is proportional to the third power of the mean air temperature (Krenke and Khodakov, 1966). The refreezing of melt water results in the sub-surface production of heat. Hence, small

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changes of summer air temperature can produce significant changes in snow-firn temperature. It is expected therefore that warming should be more pronounced in Arctic glaciers. However, the filtration of melt water complicates both the dating of ice cores and the application of the well-known procedure for the calibration of the ice-core isotopic record by the measured temperature-depth profile (Cuffey et al., 1994).

In the present paper, we develop two methods to estimate the past temperatures of the Arctic glaciers subjected to intense summer melting. The first method is based on the temperature estimation at the so-called "10 m depth" at which the seasonal climatic changes are attenuated (Blatter, 1987; Nagornov et al., 2001). Using Tikhonov's regularization method (Tikhonov and Arsenin, 1977) we invert the measured temperature-depth profiles for Akademii Nauk and Austfonna ice caps. The second method, which allows reconstruction of past surface temperatures, is based on the measured temperature-depth profiles, the oxygen–isotope ratio, and the melt feature index for the above-mentioned ice caps. We show that the temperatures reconstructed by using the first and the second method are in a good agreement.

2. The input data

In this paper, we use the temperature-depth profiles derived from the borehole drillings on several ice caps (Arkhipov, 1999). The borehole temperature for the Akademii Nauk ice cap is shown in Fig. 1 (measured with the accuracy of 0.1 °C). As a result of the contemporary warming of the Arctic glaciers, the temperature gradient changes its sign at the depth of 100 to 200 m (Kotlyakov et al., 2004). The estimated ranges of the accumulation rate are 0.3 to 0.5 m/year for the Akademii Nauk and 0.2 to 0.8 m/year for the Austfonna (Arkhipov, 1999). As the thermal regime of the bottom part of the glaciers is at steady state, the heat flow from the Earth's interior can be directly calculated by using the measured temperature profiles.

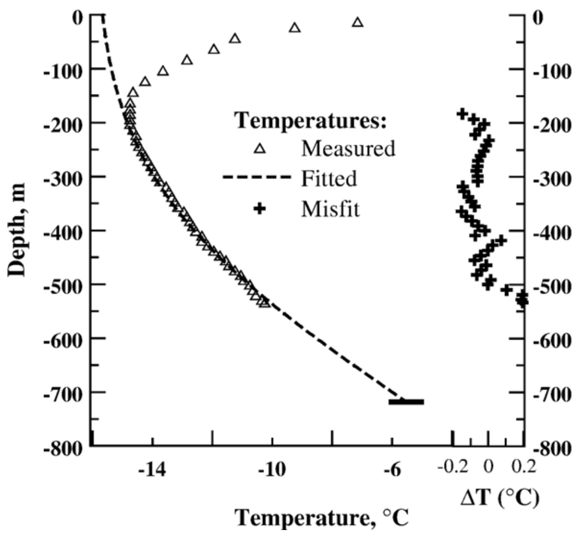

Fig. 1. Temperatures in the Akademii Nauk ice cap (measured, fitted steady state, and misfit).

3. Regularization method for reconstruction of the glacier surface temperature

A mathematical statement of the forward problem involves the one-dimensional heat conduction equation together with initial and boundary conditions. The principal factors affecting the deep ice temperature have been considered elsewere (Paterson and Clarke, 1978; Robin, 1983). In the present paper, we adopt the statement of the forward problem similar to that used by MacAyeal et al. (1991). The equation and the additional conditions are:

$$\begin{aligned} &\rho C\left(\frac{\partial T}{\partial t}+w\frac{\partial T}{\partial z}\right)=\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right),0<t<t_f,0<z<H,\\ &T(z,0)=T_0(z),0<z<H,\\ &T(0,t)=T_s(t),0<t\le t_f,\\ &-k\cdot\frac{\partial T(H,t)}{\partial z}Q(t),0<t\le t_f, \end{aligned} \quad (1)$$

where z is the vertical coordinate, t is time, ρ is the density of a glacier, C is its heat capacity, k is the thermal conductivity, $Q(t)$ is the heat flux, $T_0(z)$ is the initial temperature associated with the temperature in the past, w is the vertical velocity of the ice, H is the depth of the borehole, and t_f is the terminal time corresponding to the borehole temperature measurements. The frictional heating term is neglected in Eq. (1) since we only consider glaciers whose thickness is approximately 500 m, whereas the layer heating becomes substantial at 800 m depth (Radok et al., 1970). The borehole temperature-depth profile $T(z,t_f)$ is affected by the climatic surface changes and other glacier parameters according to a well-known heat-transfer problem approach. The 10 m temperature that corresponds to the coordinate $z=0$ (Blatter, 1987) is denoted hereafter as $T_s(t)$. The annual surface temperature variations can penetrate only to a specific depth, which depends on the site location and time. Generally, this depth is about 10 m.

The borehole temperature-depth profile $T(z,t_f)$ is assumed to be a response to changes in the surface climate. It is expected that the temperature $T_s(t)$ can be

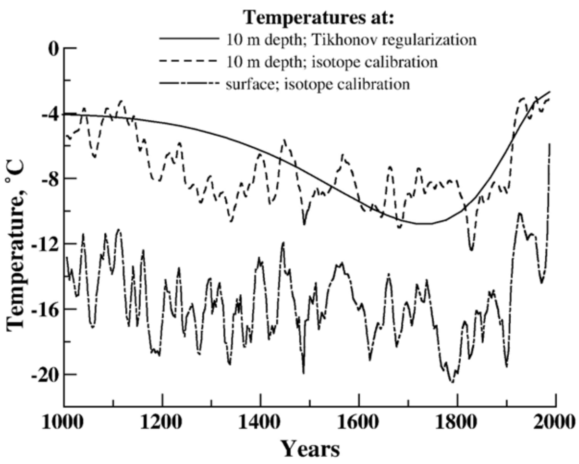

Fig. 2. Reconstructed temperatures for the Austfonna ice cap.

found if $\theta(z) \equiv T(z,t_f)$ is known as a function of z. This is the so-called inverse problem (MacAyeal et al., 1991; Dahl-Jensen et al., 1999). To resolve this problem we apply the Tikhonov's regularization method (Tikhonov and Arsenin, 1977). This method is valid for inverse problems with given (and non-exact) input data. The solution of this problem at the terminal time can be re-written as the operator relationship $\theta(z) \equiv A\{T_s\}$. For the statement under consideration we can introduce a quasi-solution $\overline{T}_s(t)$ as a function that minimizes the functional

$$\int_0^H \left(A\{\overline{T}_s(t)\} - \theta(z)\right)^2 dz = \min\left[\int_0^H \left(A\{T_s(t)\} - \theta(z)\right)^2 dz\right] \equiv \alpha, \qquad (2)$$

where the minimum is searched in the set of continuous functions $T_s(t)$. To obtain a stable solution, the Tikhonov's method assumes adding a stabilizing term to the functional:

$$\Psi(T_s) = \int_0^H \left(A\{T_s(t)\} - \theta(z)\right)^2 dz + \beta_1 \cdot \Omega[T_s(t)], \qquad (3)$$

where $\beta_1 > 0$ is the regularization parameter which must be chosen according to the accuracy of the input data. The additional functional

$$\Omega(T_s(t)) = \int_0^{t_f} \left[T_s^2 + \beta_2 \left(\frac{dT_s}{dt}\right)^2\right] dt, \qquad (4)$$

is related to the smoothness of the quasi-solution ($\beta_2 > 0$). It has been proved (Tikhonov and Arsenin, 1977) that the procedure of minimization of Ψ is stable with respect to small pertrubations of the input data, whereas the minimization of the functional Eq. (2) does not possess the same property. Therefore, Ω plays a role of a stabilizer.

The procedure of finding the minimum of Ψ is based on the iterative gradient method. At the beginning, we define the zero iteration $(T_s(t_0), T_s(t_1), \dots, T_s(t_k))^0 \equiv \vec{\mu}^0$, where $t_1, t_2, \dots, t_k$ are given nodes. The n-th iteration is then determined by $\vec{\mu}^{n+1} = \vec{\mu}^n - \gamma^n \cdot \mathrm{grad}\,\Psi(\vec{\mu}^n)$, where $\gamma^n > 0$ is the gradient step.

4. Algorithm of finite-difference inversion

The zero approximation to the unknown temperature is chosen as a linear function of $T_s(0)$ and $T_0(0)$. In order to calculate μ^n we use the implicit finite-difference scheme for the heat conduction equation symbolically presented as $A\{\vec{\mu}^n\} = \theta^n(z)$. The main functional can be written as $\Psi^n = \int_0^H (A\{\vec{\mu}^n\} - \theta_k(z))^2 dz + \beta \cdot \Omega[\vec{\mu}^n]$,. The iteration μ_j^{n+1} is found as $\vec{\mu}_j^{n+1} = \mu_j^n - \gamma_j^n \cdot \frac{\partial \Psi^n(\mu_j^n)}{\partial \mu_j^n}$. The derivative $\frac{\partial \Psi^n(\mu_j^n)}{\partial \mu_j^n}$ can be calculated provided that $\frac{\partial A}{\partial \mu_j^n}$ is known. In turn, the derivative $\frac{\partial A}{\partial \mu_j^n}$ depends on $W^j = \frac{\partial \theta^n}{\partial \mu_j^n}$, which is the solution of the problem obtained by differentiating all of the Eq. (1) with respect to μ_j^n. The iterations are repeated until the minimum of Ψ is achieved.

5. Application of the model to the reconstruction of the 10 m temperatures

The inverted temperatures for Austfonna and Akademii Nauk ice caps are shown in Figs. 2 and 3, respectively (solid curves). Note that over the last two hundred years the 10 m temperature variations in these glaciers were much larger than the corresponding changes of the air temperature estimated by isotope analysis (Overpeck et al., 1997). This phenomenon is likely to be associated with the complex interaction between melt water and snow-firn pack.

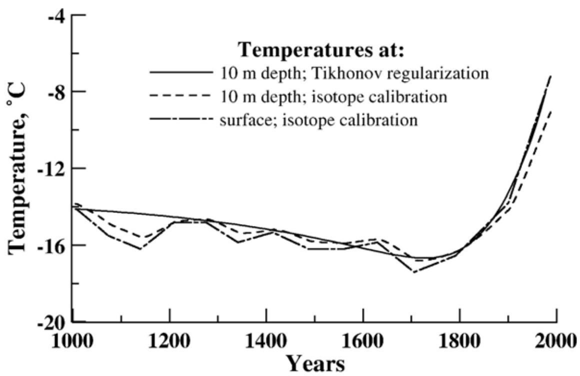

Fig. 3. Reconstructed temperatures for the Akademii Nauk ice cap.

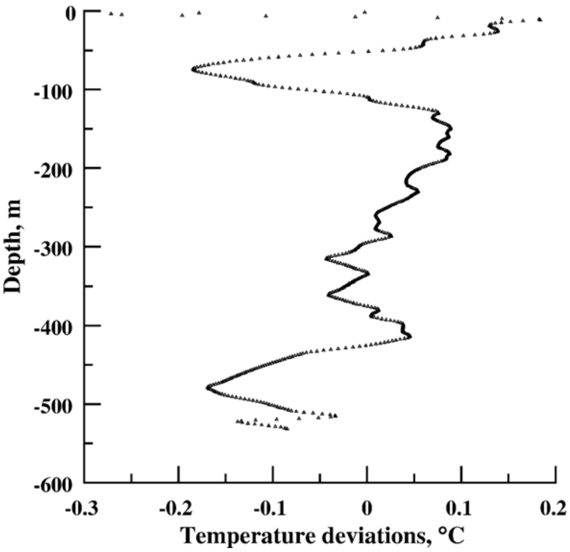

Fig. 4. Deviation between the observed and calculated temperatures for the Akademii Nauk ice cap.

The values of the heat flow (0.035, 0.05, and 0.06 W/m^2) were assumed for Austfonna. Our calculations show that the uncertainty in the heat flow at the Austfonna base introduces certain differences in the reconstructed profiles prior to the 1800s. The borehole drilled to the bottom of Austfonna has shown that the water table reached up to 50 m (Kotlyakov et al., 2004), indicating that the glacier was melting at its base. If heat flow of 0.029 W/m^2 is assumed, the calculated temperature-depth profile is close to the measured temperature-depth profile and it becomes possible to estimate the rate of melting at the bed. The difference between the heat flows above and below the bed is expended on melting; its value is equal to $\rho L\dot{a}$, where ρ is the density of ice, L is the heat latent, and $\dot{a}$ is the rate of melting. For heat flow from the rock of 0.05 W/m^2, the rate of melting is $\dot{a}$=2.1 mm/year, whereas for heat flow of 0.1 W/m^2 the value of $\dot{a}$ is 7.3 mm/year (Langseth et al., 1990). For accumulation rates between 0.2 and 0.8 m/year the reconstructed surface temperatures are close to each other from 0 to 200 years ago. Their deviations become noticeable at longer time intervals. The deviation between the observed and the calculated temperatures on the Akademii Nauk ice cap is shown in Fig. 4. Note that this deviation is two times larger than the accuracy of the temperature measurements.

6. The sensitivity study

The sensitivity of the inversion method is estimated by re-solving the test problem. We generate a synthetic temperature-depth profile by reiterating the procedure and using the data presented by MacAyeal et al. (1991). The initial temperature profile is assumed to be a steady state:

$$T_0(z) = T_{01} + \frac{Q}{k}\sqrt{\pi H\chi/2A}\left\{erf\left[(z-h)\sqrt{A/2H\chi}\right] + erf\left[H\sqrt{A/2H\chi}\right]\right\},$$

where $T_{01}=-25$ °C, H=2000 m, $\chi=1.4\cdot10^{-6}$ m^2/s, k=2 W/m/°C, A=0.25 m/year, and Q=0.05 W/m^2. The advection velocity profile $w(z)$ is chosen according to the relationship of Dansgaard and Johnsen (1969). The surface-temperature history is given by the function $T_s(t)=T_{01}+\delta T\cdot\sin(\omega t)$, where $\delta T=5$ °C and $\omega=7.96\cdot10^{-11}$ s^{-1} represents a 2500 year climatic cycle. The coordinate and the time steps for the solution of the forward problem, as well as the synthetic temperature depth profile definition are the same as those used by MacAyeal et al. (1991).

Fig. 5 shows the reconstructed surface temperature. One can see that the developed method retrieves the glacier surface temperature quite satisfactorily. Moreover, the method is stable with respect to input data. Indeed, let random errors be added to the test borehole profile:

$$T_{si} = T_{sti} + \delta T_{\rm err}\cdot\xi_i,$$

where i is the number of the coordinate node, $\delta T_{\rm err}$ is the amplitude of the temperature error, and ξ_i is a random

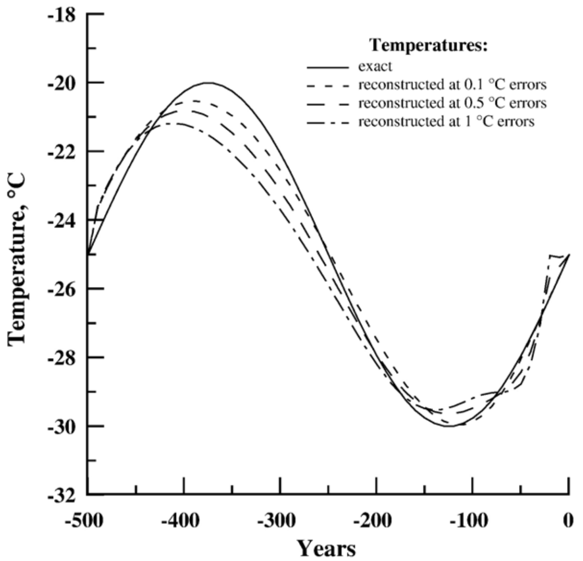

Fig. 5. Influence of the input data errors on temperatures reconstructed using the Tikhonov's method.

variable uniformly distributed on the interval $[-1,1]$. The most likely error of the real temperature measurements in the borehole is about 0.05–0.1 °C. Thus, the developed method is valid for the errors of such magnitude in the input data. For comparison, Fig. 6 shows the temperatures reconstructed by the control method, which is very sensitive to small variations of the input data.

7. Reconstruction of the surface temperature

The conventional method of the past temperature inversion for the glaciers is based on the stable-isotope ratios, for example $\delta^{18}O$. The glacier surface temperature $T_s(t)$ depends linearly on $\delta^{18}O$: $T_s(t)=\alpha+\beta\cdot\delta^{18}O$ (Paterson, 1994). The parameters α and β depend on the location of a glacier. Cuffey et al. (1994) calibrated the $\delta^{18}O$ paleothermometer for Central Greenland. In the present paper, the coefficients of the linear dependence between $\delta^{18}O$ and near-surface temperature are found by solving the inverse problem for the heat transfer equation.

The calibration of the $\delta^{18}O$ isotope paleothermometer method is applied to the Austfonna and Akademii Nauk ice caps. Together with the near-surface temperature changes the melt water refreezing significantly affects the temperature field of the Arctic ice caps (Paterson and Clarke, 1978). This phenomenon is taken

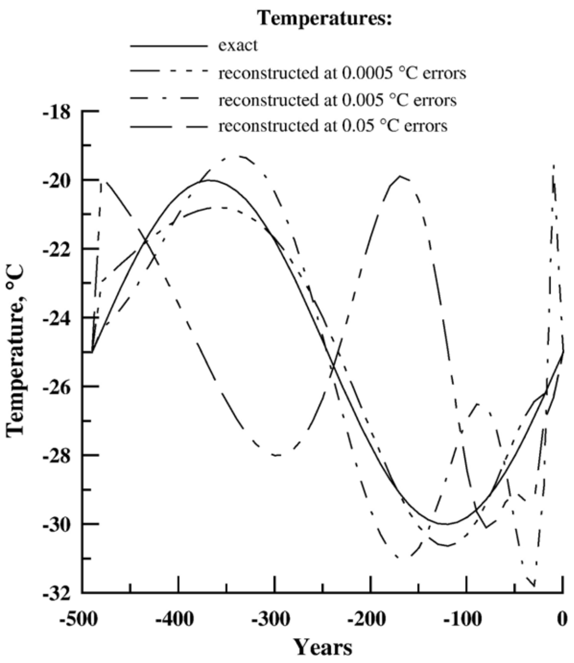

Fig. 6. Influence of the input data errors on temperatures reconstructed using the control method.

into account by introducing the effective heat source in the heat transfer equation and by using the measured melt feature index in the ice cores.

The mathematical statement of the forward problem for the glacier temperature is similar to Eq. (1). The differences between the two statements are the following: First, we add a heat source f to the right-hand side of the first equation in Eq. (1). Second, we rewrite the boundary condition in the form $T(0,t)=\alpha\psi(t)+b$, where $\psi(t)=\delta^{18}O(t)$ is the measured oxygen–isotope ratio (Arkhipov, 1999). Last, we replace the initial condition in Eq. (1) by the equation $T(z,0)=T_0+T_{st}(z)$, where the surface temperature T_0 at $t=0$ is the unknown parameter of the inverse problem.

For the firn density–depth dependence we adopt the analytical approximation $\rho=\rho_{\rm ice}(1-c_0\ \exp(-\gamma z))$, where $\rho_{\rm ice}$ is ice density. Based on the firn density measurements of Arctic glaciers (Arkhipov, 1999), we determined the values $\gamma\approx 0.1\ {\rm m}^{-1}$, $c_0\approx 0.58$ for Austfonna and $\gamma\approx 0.28\ {\rm m}^{-1}$, $c_0\approx 0.61$ for Akademii Nauk ice caps.

The thermal conductivity of ice $k(T,\rho)$ depends on temperature and firn density according to the relationship

$$k(T,\rho)=\frac{k_f(\rho)}{k_{\rm ice}}\cdot 9.828\cdot\exp(-0.0057(273.15+T)),\quad (T<0°C) \tag{5}$$

where $k_f(\rho)=0.021+0.00042\rho+2.2\cdot 10^{-9}\rho^3\ {\rm W\ m^{-1}\ K^{-1}}$ (Paterson and Clarke, 1978). For the purpose of linearization of the heat-transfer problem we use a steady-state temperature in Eq. (5). The thickness of the Eurasian Arctic does not exceed ~ 500–700 m. If the temperature deviations from the steady state do not exceed 10 °C, the effect of non-linear properties on the temperature profiles is less than 0.05 °C, which is comparable with the accuracy of the measurements.

The density of the heat sources f in the heat-transfer equation is the sum of the viscous heat dissipation and the latent heat released during the refreezing of the percolated melt water. The heat resulting from the melt water refreezing is introduced in the sub-surface layer. Its density is given by the equation

$$f(z,t)=(L-CT(z,t))w_0\rho_0\rangle P(t)\langle g(z), \tag{6}$$

where L is the latent heat, w_0 is the advection rate near the ice surface equal to the accumulation rate a_0; ρ_0 is the firn density of sub-surface layers, $T(z,t)$ is the firn temperature, $\langle P(t)\rangle$ is the measured melt feature index, and $g(z)$ is the distribution function of melt water (Paterson and Clarke, 1978). The melt feature index is equal to the relative concentration of the refrozen ice per unit volume (Fig. 7).

The melt water distribution function $g(z)$ is ill-determined (Paterson and Clarke, 1978). Paterson and Clarke (1978) used the following formula for the Devon Island ice cap:

$$g(z) = \begin{cases} (2/l_0^2)\cdot(l_0-2|z-d|), |z-d|<l_0/2; \\ 0, |z-d|<l_0/2; \end{cases} \tag{7}$$

where $l_0 = 0.2$ m. The value ρ_0 in Eq. (6) follows from the relationship $\int_{z_1}^{z_2} \rho(z)P(t,z)g(z)dz \approx \rho_0(\tilde{z})\cdot\langle P(t)\rangle, \tilde{z}\in [z_1, z_2]$, where the interval $[z_1, z_2]$ corresponds to the area of non-zero values of $g(z)$. Note that the melt feature index is a non-smooth function of time since $P\approx 0$ in cold months and $P\approx P_{max}$ in summer months. Due to the intense summer melting, water penetrates into several annual layers. Therefore, we smooth the melt feature index data as follows

$$\bar{P}_j = (P_{j-1} + P_j + P_{j+1})/3 \approx \frac{1}{t_{j+\frac{3}{2}} - t_{j-\frac{3}{2}}} \int_{t_{j-\frac{3}{2}}}^{t_{j+\frac{3}{2}}} P(t)dt,$$

where j is the index of the annual layer.

Given that the value of the specific heat capacity C in Eq. (6) is $2.04\cdot 10^3$ J kg^{-1}K^{-1} and the value of the latent heat L is $3.35\cdot 10^5$ J kg^{-1}, the ratio $C/L \sim 10^{-2}$ °C^{-1}. The average near-surface temperature of the Arctic ice caps is about −10 °C. From this, $\frac{C}{L}(-T(z,t)) \sim 10^{-1}$ and the heat source f takes the form $f = La_0\rho_0\langle P(t)\rangle g(z)$. Due to the possible discrepancy between the real melt water and the measured melt feature index (Paterson and Clarke, 1978), it is necessary to introduce additional parameter P_0 in Eq. (6). This parameter is unknown and must be defined as a result of the solution of the inverse problem. Thus, the heat source term in the heat-transfer equation takes the form

$$f(z,t) = La_0\rho_0\langle P(t)\rangle g(z)P_0, \tag{8}$$

where P_0 must be determined (together with the paleothermometer coefficients a, b and T_0) as the result of the inversion.

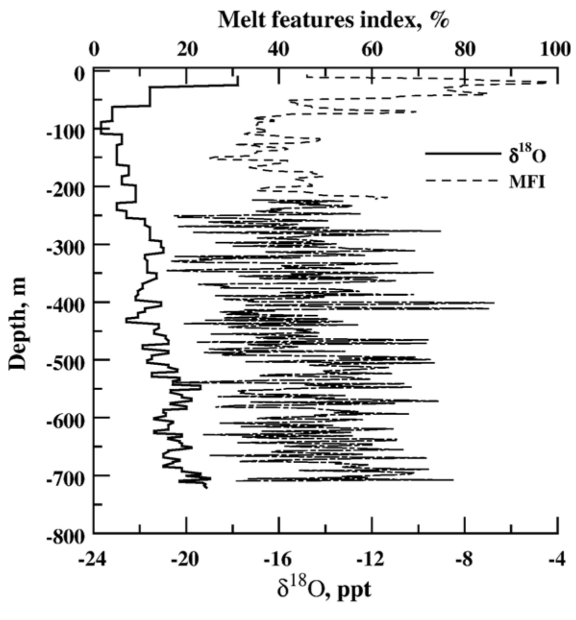

Fig. 7. The measured melt feature index and the oxygen isotopic ratio for the Akademii Nauk ice cap.

8. Calibration of the isotope paleothermometer

The inverse problem reduces to the minimization of the deviation between the observed and the calculated temperature profiles. The solution of the forward problem Eq. (1) can be written as follows: $T(z,t_f) = T_{st}(z) + aT_1(z,t_f) + bT_2(z,t_f) + T_0T_3(z,t_f) + P_0T_4(z,t)$, where the functions $T_1(z,t_f)$, $T_2(z,t_f)$, $T_3(z,t_f)$ and $T_4(z,t_f)$ are solutions of the problems:

$$\rho C\frac{\partial T_1}{\partial t} = \frac{\partial}{\partial z}\left(k(z)\frac{\partial T_1}{\partial z}\right) - \rho c w(z)\frac{\partial T_1}{\partial z}; 0<t<t_f; 0<z<H;$$

$$T_1(0,t) = \psi(t); \frac{\partial T_1}{\partial z}(H,t) = 0;$$

$$T_1(z,0) = 0; \tag{9}$$

$$\rho C\frac{\partial T_2}{\partial t} = \frac{\partial}{\partial z}\left(k(z)\frac{\partial T_2}{\partial z}\right) - \rho c w(z)\frac{\partial T_2}{\partial z}; 0<t<t_f; 0<z<H;$$

$$T_2(0,t) = 1; \frac{\partial T_2}{\partial z}(H,t) = 0;$$

$$T_2(z,0) = 0; \tag{10}$$

$$\rho C\frac{\partial T_3}{\partial t} = \frac{\partial}{\partial z}\left(k(z)\frac{\partial T_3}{\partial z}\right) - \rho c w(z)\frac{\partial T_3}{\partial z}; 0<t<t_f; 0<z<H;$$

$$T_3(0,t) = 0; \frac{\partial T_3}{\partial z}(H,t) = 0;$$

$$T_3(z,0) = 1; \tag{11}$$

$$\rho C\frac{\partial T_4}{\partial t}=\frac{\partial}{\partial z}\left(k(z)\frac{\partial T_4}{\partial z}\right)-\rho cw(z)\frac{\partial T_4}{\partial z}+f;\,0<t<t_f;\,0<z<H;$$

$$T_4(0,t)=0;\frac{\partial T_4}{\partial z}(H,t)=0;$$

$$T_4(z,0)=0; \qquad (12)$$

In Eq. (12), f is given by Eq. (8) with $P_0=1$.

The deviation between the observed and the calculated temperature-depth profiles, or the so-called discrepancy functional,

$$S(a,b,T_0,P_0)=\int_0^h\{\theta(z)-T_{st}(z)-aT_1-bT_2-T_0T_3-P_0T_4\}^2dz$$

should be minimized. This leads to the system of linear equations for unknown parameters a, b, T_0 and P_0:

$$\begin{cases} a\int_0^H T_1^2dz+b\int_0^H T_1T_2dz+T_0\int_0^H T_1T_3dz+P_0\int_0^H T_1T_4dz=\int_0^H\tilde{\theta}(z)T_1dz\\ a\int_0^H T_1T_2dz+b\int_0^H T_2^2dz+T_0\int_0^H T_2T_3dz+P_0\int_0^H T_2T_4dz=\int_0^H\tilde{\theta}(z)T_2dz\\ a\int_0^H T_1T_3dz+b\int_0^H T_2T_3dz+T_0\int_0^H T_3^2dz+P_0\int_0^H T_3T_4dz=\int_0^H\tilde{\theta}(z)T_3dz\\ a\int_0^H T_1T_4dz+b\int_0^H T_2T_4dz+T_0\int_0^H T_3T_4dz+P_0\int_0^H T_4^2dz=\int_0^H\tilde{\theta}(z)T_4dz \end{cases} \qquad (13)$$

where $\tilde{\theta}(z)=\theta(z)-T_{st}(z)$. Note that similar decomposition of the solution can be used to find the past surface temperature by the inversion of the measured temperature-depth profiles in analogous problems for glaciers (Cuffey et al., 1995) and for rocks (Huang et al., 2000). In the present paper, the solution of the system Eq. (13) is found in explicit form. Other approaches to solve paleo-reconstruction problems are based on numerical minimization of the discrepancy functional (Cuffey et al., 1995; Huang et al., 2000), which can introduce additional errors in the reconstructions.

The results of the paleothermometer calibration for Austfonna ice cap are shown in Figs. 2 and 8. The accumulation rate for Austfonna ice cap varies from 0.2 to 0.8 m/year (Arkhipov, 1999). At higher accumulation rates, the temperature signal penetrates faster from the surface of a glacier to its interior. Calculations show that the minimum deviation between the computed and the measured profiles is achieved for accumulation rates $a_0\approx0.5$–0.6 m/year. For the accumulation rate varying from 0.2 to 0.8 m/year, the variation of the paleothermometer parameter a does not exceed 12% (provided that the thickness of a glacier is fixed). The corresponding value of the reverse dependence of $\delta^{18}O$ on the near-surface temperature varies in the range 0.32–0.37 °C/ppt. Such a low sensitivity in the variation of the paleothermometer coefficients is due to the decreasing of the advection rate with depth. The heat transfer in the glacier is related both to diffusion and to advection. However, diffusion is predominant in most of the glacier thickness, meaning that the integral terms of Eq. (13) vary insignificantly with changes in accumulation rate.

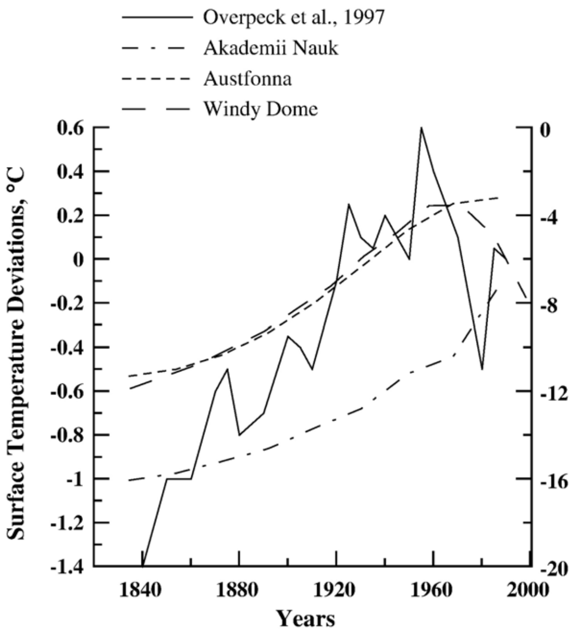

Fig. 8. The standardized proxy Arctic-wide summer-weighted annual temperature deviations (Overpeck et al., 1997) and the reconstructed 10 m depth temperatures of the Eurasian Arctic ice caps.

The heat source parameter P_0 depends on the heat location depth d. Though the value of d is not well known, we fit it in the process of the solution, taking the d that corresponds to $P_0=1$. For d varying from 0.5 to 2 m, the heat source parameter varies from 0.54 to 3.3, and $P_0=1$ for $d=1.3$ m ($a_0=0.5$ m/year). This agrees with the assumption that melt water in the Arctic glaciers penetrates into several annual layers. As noted, the smoothed data for the melt feature index are used as the input. Given the original measured melt feature index, the paleothermometer parameters a and b differ from the ones for the smoothed data by less than 4% and 5%, respectively.

We also investigate the influence of the heat flow on the paleothermometer parameters. For the heat flux varying from 0.01 to 0.05 W/m^2, the paleothermometer parameter a varies from 2.6 to 2.9 °C/ppt, and the parameter b varies from 30 to 33 °C. For the value $Q \approx 0.029$ W/m^2 corresponding to the phase transition at the bottom, the variation of the parameter a does not exceed 7%.

The temperature changes at the 10 m depth correspond well enough to the surface temperature changes inverted by using δ^{18}O data (Fig. 2). The temperature at the depth of 10 m is higher than the temperature at the surface due to the presence of the additional heat source related to the release of heat by the melt water crystallization in the sub-surface layers. Similar conclusions hold true for the Akademii Nauk ice cap (Fig. 3); however, in that case the temperature difference between the surface and the 10 m depth is much smaller. We found that the melt water penetrates into one to two annual layers for the Akademii Nauk ice cap and into three to four annual layers for the Austfonna ice cap. The discrepancy in the number of layers can be attributed to the lower mean annual temperatures and shorter ablation period for the Akademii Nauk ice cap. The present ice temperature is the highest over the last 1000 years, while the lowest temperature occurred in the 1700s.

Within the considered range of the accumulation rate ($0.3 \leq a_0 \leq 0.5$ m/year), the relative error of the paleothermometer coefficient a related to uncertainty of the dating of the δ^{18}O does not exceed 8% (given the initial accumulation rate $a_0 = 0.2$ m/year). The corresponding values of the coefficient α of the inverse dependence of δ^{18}O on temperature are 0.60 ± 0.03 °C/ppt. This is in good agreement with the results of the paleothermometer calibration for Greenland (Cuffey et al., 1994).

Fig. 8 shows the trends of the mean global temperature for the Arctic region and the retrieved temperatures of the glaciers under consideration. For the Austfonna and Akademii Nauk ice caps, the temperature trends are similar, while the temperature on the Vetreniy (Windy Dome) glacier (Kotlyakov et al., 2004) decreased by 4 °C over the last 30 years (Fig. 6). Similar behavior of temperature was found earlier in the Canadian Arctic (Fig. 9) (Taylor, 1991). The temperature scale of the warming we have calculated for the Arctic glaciers for the last 150 years (about 8 °C) is 6 to 7 °C larger than that averaged over the Arctic region (about 2 °C). Such temperature changes on Arctic glaciers are likely related to the increase of the mean summer air temperature caused by surface melting (Zagorodnov et al., 1989; Zagorodnov and Arkhipov, 1990), which causes abrupt albedo changes and decreases the stability of the atmosphere (Overpeck et al., 1997). However, other factors such as the configuration of atmospheric waves and the distribution of land and ocean in the Arctic should also contribute to non-uniformity in the spatial distribution of Arctic warming, meaning that strong summer melting is not the only factor determining the enhanced annual average Arctic temperatures. Here we determine just the surface temperature changes for the region where the temperature-depth profile has been measured; understanding of the specific reasons for warming in the whole Arctic is beyond the scope of this analysis.

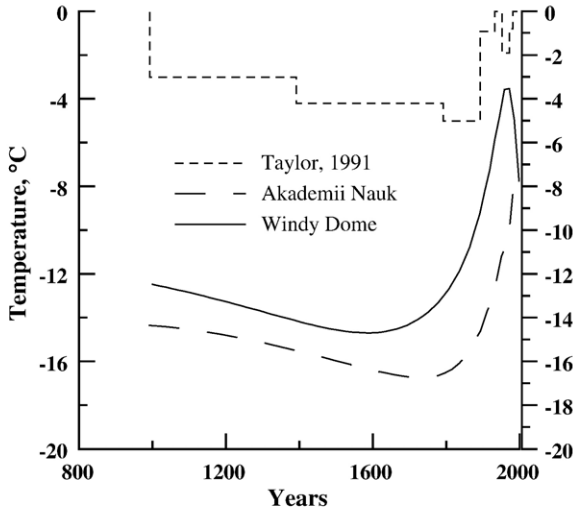

Fig. 9. The surface temperature deviations in the Canadian Arctic (Taylor, 1991) and the reconstructed 10 m temperatures of the Eurasian Arctic ice caps.

For the heat source location depth d varying from 0.1 to 0.8 m, the heat source parameter P_0 varies from 1.43 to 0.1; $P_0 \approx 1$ for $d \leq 0.3$ m. In this case, in contrast with that of Austfonna, the melt water is obviously located in one annual layer. The value of d indicates relatively small influence of melt water refreezing on the temperature field of the Akademiya Nayk ice cap. The 10 m temperature changes inverted due to the regularization method (solid curve in Fig. 3) and the near-surface temperature changes inverted due to δ^{18}O data (dashed curve in Fig 3) are in good agreement for the Akademii Nauk ice cap.

Hence, despite the presence of the melting processes and the melt water penetration, the oxygen–isotope paleothermometer approach is justified for the Arctic glaciers subjected to intense summer melting. The parameters a and b of the oxygen–isotope paleothermometer ($T = a\delta^{18}O + b$) are determined for the glaciers Austfonna ($a = 2.6$–2.9 °C/ppt, $b = 30$–33 °C) and Akademii Nauk ($a = 1.6$–1.73 °C/ppt, $b = 19.8$–

23.6 °C). The variations of a and b are related to the errors of the temperature measurements in the boreholes and to the uncertainty in the values of the accumulation rates and heat flows.

9. Conclusions

Two methods have been developed and applied to reconstruct past temperature at the surface and 10 m depth of Arctic ice caps with intense summer melting. The results obtained by using both methods are in a good agreement. Additionally, the paleothermometer parameters for the Akademii Nauk glacier are close to those previously determined for Greenland (Cuffey et al., 1994). Values of the parameters of the isotope paleothermometer ($T = a\delta^{18}O + b$) are $a = 2.6$–2.9 °C/ppt, $b = 30$–33 °C for Austfonna and $a = 1.6$–1.73 °C/ppt, $b = 19.8$–23.6 °C for Akademii Nauk. The bottom part of the Akademii Nauk ice is close to its steady state while the bottom part of Austfonna glacier is in the interchange condition. The estimated rate of melting at the bottom of Austfonna is 2–7 mm/year.

Over the last 150–200 years, the increase of temperature in the sub-surface layers of the Austfonna and Akademii Nauk glaciers was indeed synchronous and is estimated to be about 8 °C. This magnitude of warming is significantly larger than that averaged over the whole Arctic region (≈2 °C). The intense summer melting and the consequent heat release during recrystallization of the percolated melt water is a likely cause of the large observed warming.

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