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Correlations in aftershock and seismicity patterns

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Abstract

Correlations in space and time play a fundamental role in earthquake processes. One direct manifestation of the effects of correlations is the occurrence of aftershocks due to the stress transfer in the vicinity of a main shock. Less obvious and more speculative changes in correlations may occur in the background seismicity before large earthquakes. Using statistical physics it is possible to introduce a measure of spatial correlations through a *correlation length*. This quantity characterizes how local fluctuations can influence the occurrence of earthquakes over distances comparable with the correlation length. In this work, the physical basis of spatial correlations of earthquakes is discussed in the context of critical phenomena and the percolation problem. The method of two-point correlation function is applied to the seismicity of California. Well defined variations in time of the correlation length are found for aftershock sequences and background seismicity. The scaling properties of our obtained distributions are analyzed with respect to changes in several scaling parameters such as lower magnitude cutoff of earthquakes, the maximum time interval between earthquakes, and the spatial size of the area considered. This scaling behavior can be described in a unified manner by utilizing the multifractal fit. Utilizing the percolation approach the time evolution of clusters of earthquakes is studied with the correlation length defined in terms of the radius of gyration of clusters. This method is applied to the seismicity of California.

Keywords: Earthquakes; Two-point correlation function; Correlation length; Percolation; Radius of gyration; Multifractal scaling

1. Introduction

Earthquake catalogs contain information concerning the history and mechanisms of nonlinear processes in the upper brittle layer of the Earth's crust. These catalogs are also a source of information for various earthquake prediction algorithms. Some of these algorithms are based on studies of possible repetitive precursory patterns of seismicity. The existence of these patterns requires that earthquakes are correlated in space and time. Therefore, studies of earthquake correlations represent an important part of the general problem of understanding the physics of earthquakes and are also crucial for the development of any prediction methods. It is also believed that correlations are inherently present in the Earth's crust and are signatures of the nonlinear dynamics of rheological, frictional, chemical, and other processes. From this point of view, the Earth's crust can be considered as a strongly-correlated nonlinear dissipative system. To study different aspects of the correlations of this system we will utilize concepts developed in statistical physics.

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Spatial and temporal correlations are inherent in the theory of static and dynamic critical phenomena where they play a central role (Ma, 1976; Hohenberg and Halperin, 1977). In the last two decades the ideas of criticality have been applied to earthquake physics (Rundle et al., 1997, 2000, 2003; Harris, 1998; Bowman et al., 1998; Jaumé and Sykes, 1999). Recent studies suggest that many dynamical dissipative systems fluctuate around a steady state, i.e., system dynamics builds up long-range correlations in the system and produce power-law distributions of event sizes such as the Gutenberg–Richter scaling for earthquakes (Gutenberg and Richter, 1954) before a large earthquake destroys the correlated state. The system retreats from the steady state only to start rebuilding the long-range correlations (Sornette and Helmstetter, 2002). This idea is appealing from the geophysical point of view and could be associated with the presence of characteristic earthquakes.

It is natural to hypothesize that a seismogenic zone treated as a strongly-correlated, highly nonlinear dissipative dynamical system can be characterized by suitably defined correlation lengths. For any given point in space and time, this system has one or more characteristic lengths over which it is correlated. The occurrence of an earthquake at a point can trigger earthquakes over distances comparable to a correlation length. The probability of such an event occurring is quantified using a correlation function. To fully define a problem we also assume that the correlation properties of the system are changing over time. This measure of correlations can be used to monitor the seismic activity in a region.

The first quantitative studies of correlations in seismogenic zones were carried out by Kagan and Knopoff (1976, 1980) and Kagan (1981a,b) who considered the two-, three-, and four-point correlation functions for the spatial distribution of earthquake epicenters and hypocenters. The power-law behavior of the correlation functions they obtained indicated a self-similar spatial structure of earthquake clusters and was related to the fractal geometry of fault networks. For shallow earthquakes, no characteristic length scale was found for distances between hypocenters ranging from a few kilometers up to 1000 km utilizing several regional and worldwide catalogs available at that time. These results can be used in constructing geometrical models for earthquake fault systems. The second-order moment (the two-point correlation function) for central California (1969–1982) was also analyzed by Reasenberg (1985) who found the presence of clustering related to the aftershock process. This analysis was aimed at finding precursory behavior in California seismicity by analyzing the pairwise statistical interdependence of earthquakes.

The fractal properties of the spatial distribution of earthquake hypocenters were examined by Robertson et al. (1995) for four seismic data sets in California. They found that the fractal capacity dimension of the distribution of faults $d_f \approx 1.9$, is nearly equal to the fractal dimension of the backbone of a 3d percolating cluster. Kosobokov and Mazhenkov (1994) introduced an additional term ($C\log L$, where L is a linear size of an area) into the Gutenberg–Richter scaling which quantifies the spatial distribution of earthquakes. Their analysis shows that the coefficient C is a fractal dimension of a set of epicenters, vary from 1.0 to 1.5, and correlate with the geometry of tectonic features. Guo and Ogata (1997) estimated the fractal dimension of the hypocenter distribution for 34 aftershock sequences in Japan from 1971 to 1995 based on the two-point correlation function. The values of fractal dimensions found for different regions correspond to various degrees of heterogeneity of fault networks. Nanjo and Nagahama (2004) studied 14 aftershock sequences following large main shocks in Japan. They argue that aftershock distributions become less clustered with increasing fractal dimensions of active fault systems.

The concept of a growing correlation length prior to large earthquakes was examined by Zöller et al. (2001), Zöller and Hainzl (2002). They suggested that the growth of correlations reflects the changes in the state of the regional stress field prior to large earthquakes. They used the single-link cluster algorithm (Frohlich and Davis, 1990) to construct clusters of earthquakes. The correlation length was defined as a median of a cumulative distribution function of cluster links.

Keilis-Borok and Soloviev (2003) and Zaliapin et al. (2003a,b) have used the notion of Premonitory Seismicity Patterns (PSPs) to analyze the sequence of earthquake events in their retrospective analysis of seismicity and also in several prediction algorithms. One of these PSPs is an increase in correlation range. They used several patterns of increased seismic activity to monitor an increase in correlations among earthquakes.

An attempt to introduce a universal scaling law for earthquakes has been proposed by Christensen et al. (2002) and Bak et al. (2002). These authors estimated the probability distribution of the interoccurrence time intervals τ between earthquakes, $P_{L,m}(\tau)$, within an area of linear size L and cutoff magnitude m_c as scaling

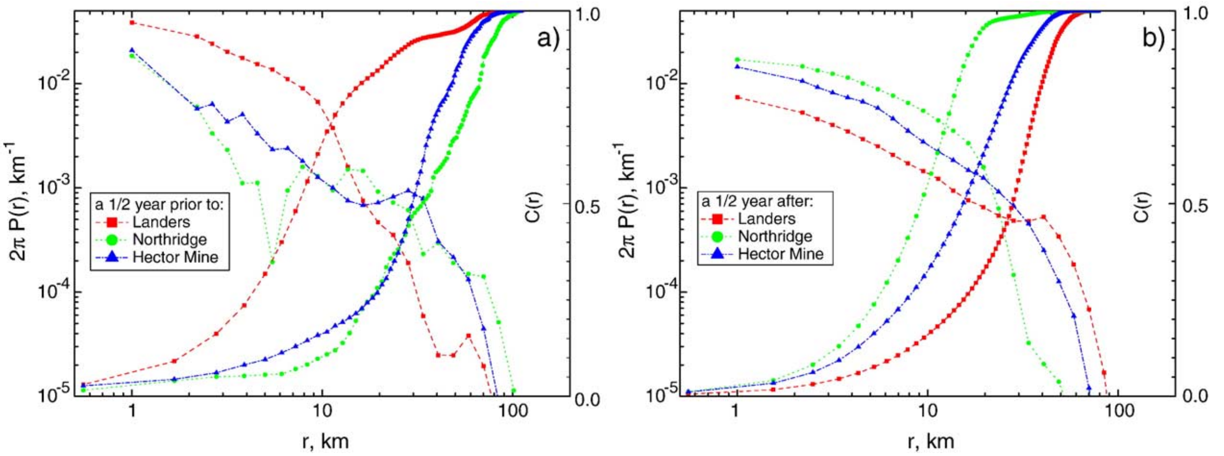

Fig. 1. Dependence of the probability density function $P(r)$ and cumulative correlation function $C(r)$ on the separation distance r prior to (a) and after (b) several large earthquakes in California. For each curve a 1/2 year time interval has been chosen with square areas of size $1^\circ \times 1^\circ$ centered on the epicenter of each main shock and with all earthquakes greater than $m_c=2.0$, included. The values of $P(r)$ decrease with increasing r and the values of $C(r)$ increase.

parameters. For Southern California they found a universal scaling with

$$P_{L,m_c}(\tau) = \tau^{-p} g\left(\tau L^{d_f} 10^{-bm_c}\right). \tag{1.1}$$

They associated τ^{-p} with Omori's law ($p \approx 1.0$), 10^{-bm} with GR scaling ($b \approx 1.0$), and L^{df} with fractal spatial scaling ($d_f \approx 1.2$) for the $2d$ location of earthquake epicenters. Two distinct scaling regions were found, for short times, these results correspond to a generalized Omori's law for aftershocks (Shcherbakov et al., 2004) and for long times they are associated with the uncorrelated regime of main shocks. To take into account the spatial heterogeneity and nonstationarity of earthquake occurrence rates, Corral (2003, 2004) has found that the fast decay for long times is not exponential, but another power law.

In the work reported here we have employed two approaches to study the correlations between earthquakes. In the next two sections, we apply the two-point correlation function method to aftershock sequences of major California earthquakes. In this context, we have studied the temporal evolution of the correlation length defined as a mean of the two-point distribution function prior to and after the main shocks. To study the scaling properties of the two-point correlation functions, we have used a multifractal analysis. In Section 4, we have mapped the seismic activity onto the framework of the percolation problem and have studied the temporal behavior of the correlation length prior to and after major earthquakes in California. In our analyses we have used the catalogs provided by the Southern California Earthquake Center (SCSN catalog, http://www.data.scec.org/) and the Northern California Earthquake Data Center (NCSN catalog, http://quake.geo.berkeley.edu/ncedc/).

2. Two-point correlation function

Earthquake occurrences can be treated as a stochastic point process. To study spatial correlations in such a process it is useful to define a two-point correlation integral (Grassberger and Procaccia, 1983)

$$C(r) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{i,j=1}^{N} \theta\left(r - |\mathbf{x}_i - \mathbf{x}_j|\right) \equiv 2\pi \int_o^r P(r')r'dr', \tag{2.1}$$

where r is the Euclidian distance between pairs of point events (earthquake epicenters), $\mathbf{x}_i$ are their vector coordinates, $\theta(x)$ is the Heaviside step function. The summation is over all pairs of N events, and $P(\mathbf{r})$ is the probability density function. The two-point correlation integral (2.1) was introduced as a measure of the strangeness of attractors of dissipative dynamical systems that exhibit chaotic behavior. It was shown (Grassberger and Procaccia, 1983) that for these systems the integral behaves as a power-law for small r

$$C(r) \propto r^{\nu} . \tag{2.2}$$

The correlation length ξ is defined as the first moment of the density correlation function $P(r)$

$$\xi \equiv E[r] = 2\pi \int_0^\infty r^2 P(r) dr . \tag{2.3}$$

Another possibility is to use a median value of the correlation integral as a correlation length.

In this section, we have applied the two-point correlation function analysis to foreshock and aftershock sequences of large earthquakes in California. We have also studied the temporal evolution of the correlation length prior to and after these earthquakes. In Fig. 1 the correlation integral C (2.1) and the probability density function $P(r)$ are given for the Landers, Northridge, and Hector Mine earthquakes. Time intervals of 180 days before and 180 days after the main shocks have been taken. All earthquakes with magnitudes greater than $m_c = 2.0$ in square areas of $1^\circ \times 1^\circ$ size centered on the main shock epicenters for the prescribed time periods have been considered. The visual inspection of the figure shows a change in the behavior of the correlation function before and after main shocks. The occurrence of aftershocks creates a broader scaling regime in the correlation function that we attribute to the self-similar nature of the aftershock process. This has a geometric component reflected in the fractal structure of a fault network upon which aftershocks occur.

In Fig. 2 the time evolution of the correlation length prior to and after the main shocks under consideration is given. We have employed a running window with the fixed number of events $W_N = 200$ and a constant event shift to calculate the values of the correlation length from (2.3) in each event window. Time intervals of 2 years prior to and after the main shocks have been considered. Further analysis have showed that the behavior of the mean, median, and the second moment are basically the same. The drastic change in the behavior of the correlation length at the time of the main shocks is attributed to the occurrence of aftershocks which reduce the mean value of the distance between pairs of events, whereas, foreshocks are more uniformly spread and contribute to larger values of the correlation length.

3. Scaling properties of the two-point correlation function

The observed features of the two-point correlation function suggest that the system exhibits self-similar behavior over some range of scales associated with its nonlinear dynamics and/or the fractal geometry of the fault network. Usually, this type of behavior depends on several scaling parameters such as the linear size of the region under consideration, the lower magnitude cutoff of earthquakes used, the maximum time interval between earthquakes specified, etc. These parameters introduce finite-size effects due to spatial limits of the region considered and the availability of earthquake catalog data in it. Varying these parameters, one can observe functional changes in the scaling behavior of a given distribution. Moreover, in some instances it is possible to introduce a scaling law that incorporates in a unified manner the size effects of the distribution function. In statistical physics this approach is known as a

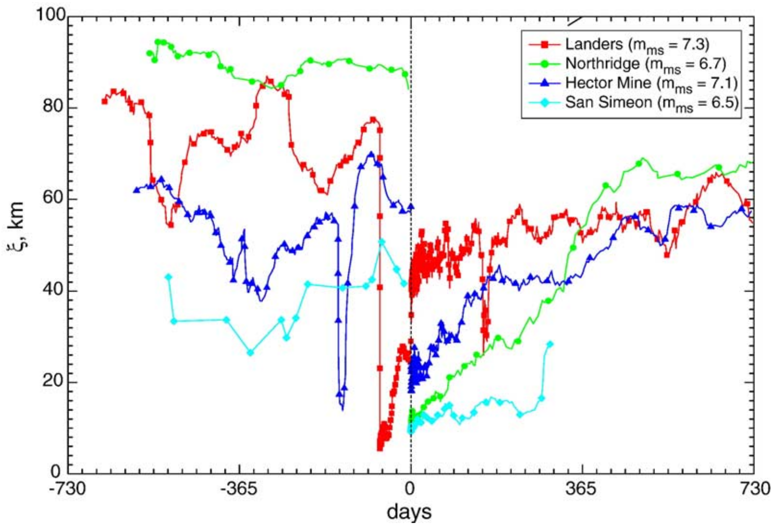

Fig. 2. Time evolution of the correlation length ξ defined in (2.3) prior to and after several large earthquakes in California. A moving event window with 200 events and shifts by 10 events has been used for the time interval of 2 years prior to and after the main shocks to compute the time evolution of the correlation length for each window.

finite-size scaling analysis (Barber, 1983). In this section, we have analyzed the scaling properties of the two-point correlation function introduced previously by varying the lower magnitude cutoff m_c of earthquakes, the maximum time interval between earthquakes T, and the linear size of the region under consideration L or the maximum distance between pairs of earthquakes R. We have performed this analysis for aftershock sequences of large main shocks in California and for the seismicity for all of Southern California.

In Fig. 3, we have plotted the density correlation function for Southern California earthquakes over a period of 10 years starting from January 1, 1995 till September 30, 2004. A region with size $5.0° \times 5.0°$ has been chosen. To construct the distributions, we have considered all pairs of earthquakes in the catalog with magnitudes greater than m_c which occurred within a time interval T and a spatial distance R. For this analysis we have used $m_c = 1.6$, 2.0, 2.4, 2.8, 3.2, 3.6 and $T = 90$ days. The visual inspection shows that the magnitude cutoff m_c does not significantly change the functional form of the distributions. This can be explained if the geometrical component of the fault network plays a significant role in the scaling properties of the distributions. Therefore, for the further analysis we have used only the spatial scaling parameters L and R.

The scaling analysis of the two-point correlation function for Southern California earthquakes is given in Fig. 4a where we have varied the parameters L and m_c and have kept $T = 90$ days fixed. The region of size $5° \times 5°$ has been considered during the time interval of 10 years as in Fig. 3 with earthquakes greater than $m_c = 1.6$, 2.0, 2.4, 2.8, 3.2, 3.6, 4.0 and pairs of events occurring within the boxes of linear sizes 50, 100, 150, 200, and 250 km centered at 34.5° and 116.5°W.

Aftershock sequences of large earthquakes contribute significantly to the scaling properties of the distributions and can alter them dramatically. To analyze separately the behavior of the two-point correlation function for an aftershock sequence, we have shown in Fig. 5a the scaling for an aftershock sequence following the Hector Mine earthquake during the time interval of two years. We have considered all pairs of earthquakes with magnitudes greater than $m_c = 1.6$, 2.0, 2.4, 2.8, 3.2, 3.6 which occurred within a time distance of $T = 120$ days. Spatial boxes of increasing linear sizes of $L = 25$, 50, 75, 100, 125, and 150 km around the epicenter have been used as a scaling parameter. The constructed curves show rather complex scaling.

To describe the observed scaling behavior of the two-point correlation function, we utilize the multifractal scaling analysis (Halsey et al., 1986; Kadanoff et al., 1989)

$$\frac{\log_{10} P_{m_c}(r, L)}{\log_{10}(L/L_0)} = f\left(\frac{\log_{10}(r/r_0)}{\log_{10}(L/L_0)}\right), \tag{3.1}$$

where f is a scaling function. The quantity $\log_{10}(r/r_0)/\log_{10}(L/L_0)$ is called α and the fit (3.1) is called an f–α representation in the standard multifractal terminology (Halsey et al., 1986). The constants r_0 and L_0 give an appropriate units of length. The values $\mathrm{d}f/\mathrm{d}\alpha$ define a

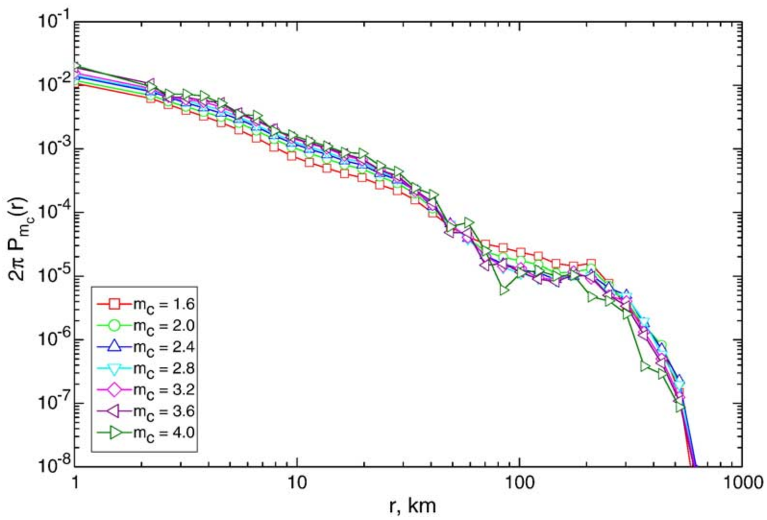

Fig. 3. The two-point correlation function for Southern California over a period of 10 years starting from January 1, 1995 till September 30, 2004. The region of size $5.0° \times 5.0°$ has been chosen. For this plot we have used $m_c = 1.6$, 2.0, 2.4, 2.8, 3.2, 3.6, 4.0 and $T = 90$ days.

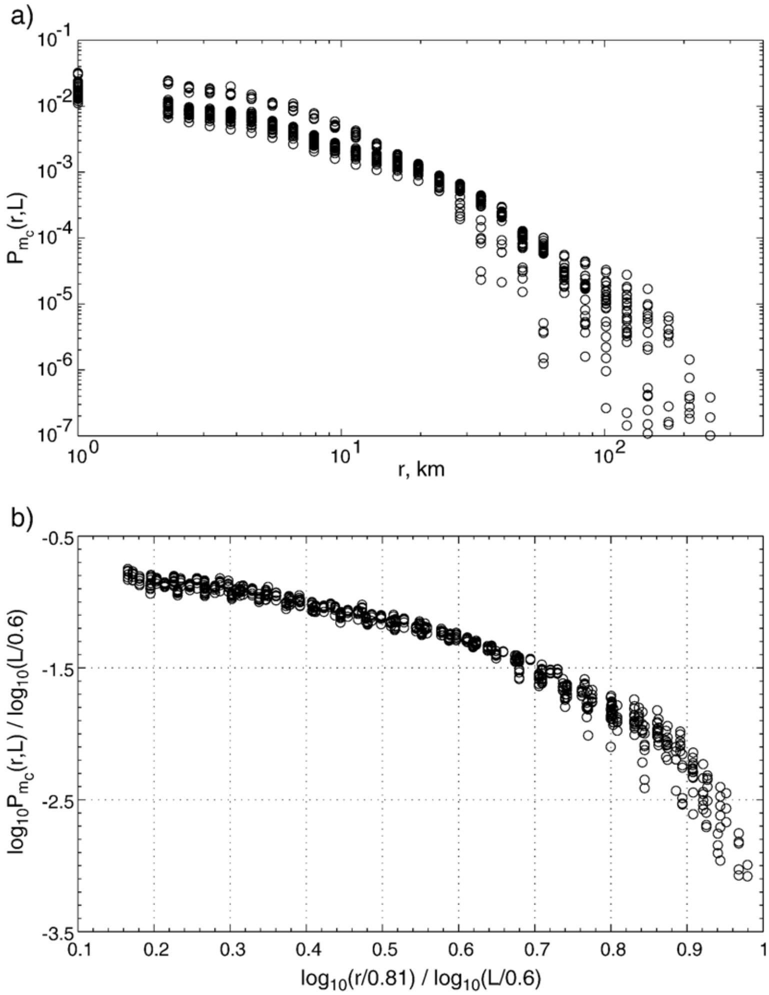

Fig. 4. The multifractal scaling analysis of the two-point correlation function $P_m(r,L)$. (a) Nonscaled plots of the distributions $P_m(r,L)$ for a range of values of magnitude cutoffs m_c=1.6, 2.0, 2.4, 2.8, 3.2, 3.6, 4.0 and linear sizes L=50, 100, 150, 200, 250 km of boxes centered at 34.5°N and 116.5°W. The time interval of approximately 10 years has been used starting from January 1, 1995 and all pairs of events within T=90 days have been considered. (b) The distributions have been re-scaled according to the multifractal fit (3.1) with scaling parameters r_0=0.81 km and L_0=0.6 km.

spectrum of scaling exponents in the problem. In the simple case when f is a linear function the problem could be described by only two exponents.

The re-scaled two-point correlation functions according to the multifractal fit (3.1) of the data given in Fig. 4a are given in Fig. 4b. In the same manner in Fig. 5b, we have plotted the re-scaled distribution for the aftershock sequence of the Hector Mine earthquake. For each curve we have re-scaled the x-axis with $\log_{10}(r/r_0)/\log_{10}(L/L_0)$ and the y-axis with $\log_{10}P_{m_c}(r,L)/\log_{10}(L/L_0)$. The values of r_0=0.81 km and L_0=0.6 km for Southern California (Fig. 4b) and r_0=0.85 km and L_0=0.16 km for the aftershocks of the Hector Mine earthquake (Fig. 5b) have been used to obtain the best collapse of all curves into a single one. Our scaling results show that the two-point correlation functions exhibit multifractal scaling rather than simple scaling.

4. Earthquake clustering as a percolation problem

The concept of percolation was introduced to study various aspects of disordered systems and their properties. It answers questions concerning the connectivity properties in such systems. For example, what is the condition under which fluid flows through a porous media or when is electrical connectivity possible in a complex network of resistors? Formally, the percolation problem is stated in the context of studies of statistical and thermodynamical properties of clusters of sites or bonds defined on a graph. The clusters are formed by

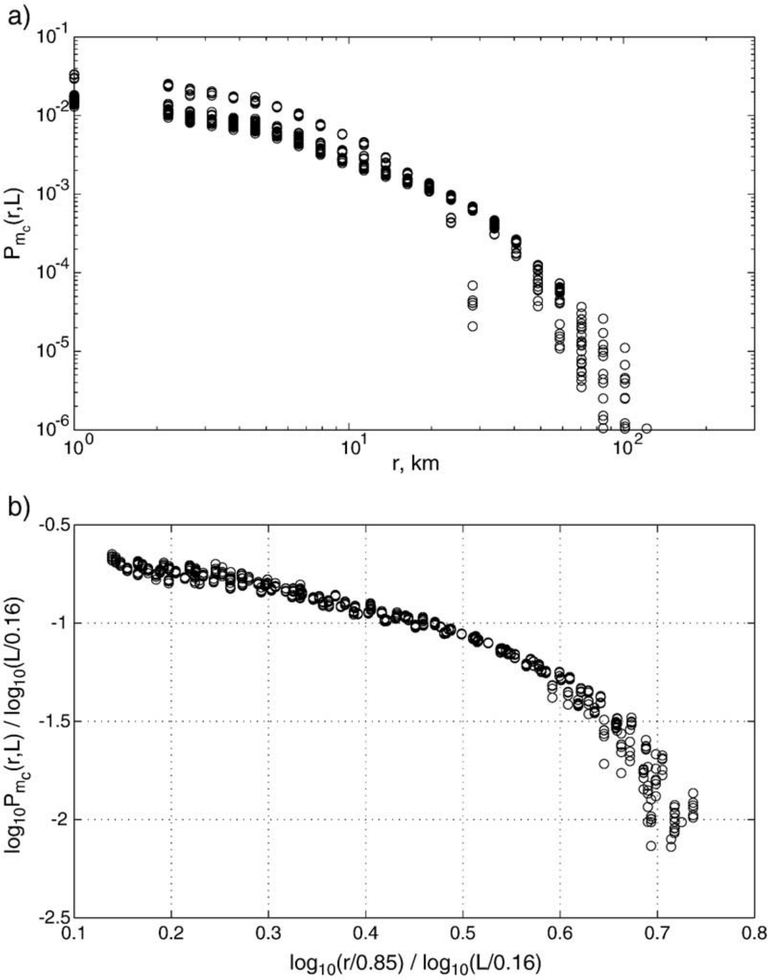

Fig. 5. The same analysis as in Fig. 4 for the aftershocks of the Hector Mine earthquake. (a) Nonscaled plots of the distributions $P_m(r,L)$ for magnitude cutoffs m_c=1.6, 2.0, 2.4, 2.8, 3.2, 3.6, and linear sizes L=25, 50, 75, 100, 125, 150 km of the box centered on the epicenter. The time interval of 2 years has been used following the main shock and all pairs of events within T=120 days have been considered. (b) The distributions have been re-scaled according to the multifractal fit (3.1) with scaling parameters r_0=0.85 and L_0=0.16.

connecting nearest-neighbor sites or bonds of a graph with a suitably defined probability. Solutions to this problem have found many applications in applied and theoretical physics. An introduction to percolation theory and its applications can be found in Stauffer and Aharony (1992) and Sahimi (1994).

Clusters exhibit critical behavior near the percolating threshold that defines a transition in topological structure of the disordered media. This phase transition is characterized by divergence of the correlation length and a characteristic time that is a trade mark of any critical phenomena. In the context of earthquakes we hypothesize that a seismogenic zone can be considered as a strongly correlated nonlinear dynamical system where earthquakes are outcomes of this dynamics.

To study percolating properties of earthquake clusters we have divided the seismogenic zone into a grid of square boxes N_{box} of a linear size l. We have considered earthquakes greater than a specified magnitude m_c that occurred within a specified time window W_T or within an event window W_n. A spatial box is occupied if one or more earthquakes occur within it. Adjacent occupied boxes define a cluster. For each time or event window one can study the temporal evolution of such clusters. The geometry of clusters in the percolation problem is an important subject of research. There are several characteristics that quantify it. One of them is the radius of gyration R_s

$$R_s^2 = \sum_{i=1}^{s} \frac{|\mathbf{r}_i - \mathbf{r}_0|^2}{s}, \tag{4.1}$$

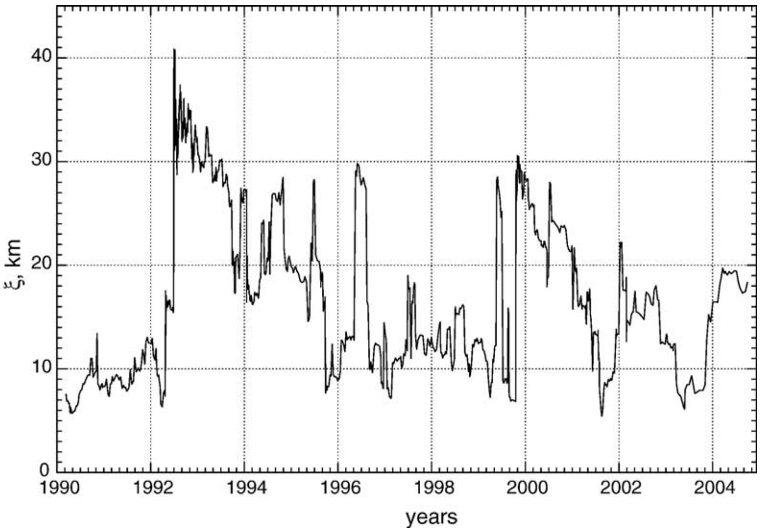

Fig. 6. Time evolution of the correlation length ξ defined in the context of the percolation approach (4.3) for Southern California starting from January 1, 1990 till September 30, 2004. A region of size $5.0^\circ \times 5.0^\circ$ and a moving window with events and shifts by 50 events have been used. Earthquakes greater than $m_c = 2.0$ have been considered.

with s is the cluster size, $\mathbf{r}_i$ is the position of the ith site belonging to the cluster, and $\mathbf{r}_0$ is the position of the center of mass of the cluster

$$\mathbf{r}_0 = \sum_{i=1}^{s} \frac{\mathbf{r}_i}{s}. \quad (4.2)$$

The correlation or connectivity length ξ is defined as a weighted average distance of two sites belonging to the same cluster (Stauffer and Aharony, 1992)

$$\xi^2 = 2\frac{\sum_s R_s^2 s^2 n_s}{\sum_s s^2 n_s} \quad (4.3)$$

where n_s is the number of clusters of size s. The correlation length defines a typical radius of the connected clusters over which the system is macroscopically homogeneous.

In Fig. 6, we have shown the temporal evolution of the correlation length (4.3) in Southern California for 10 years starting from January 1, 1990 till September 30, 2004. A region of size $5^\circ \times 5^\circ$ divided into $N_{\text{box}} = 100 \times 100$ boxes of an average size of $l \approx 5.07$ km has been analyzed with a moving event window of size 500 and a shift size of 50 events. All earthquakes

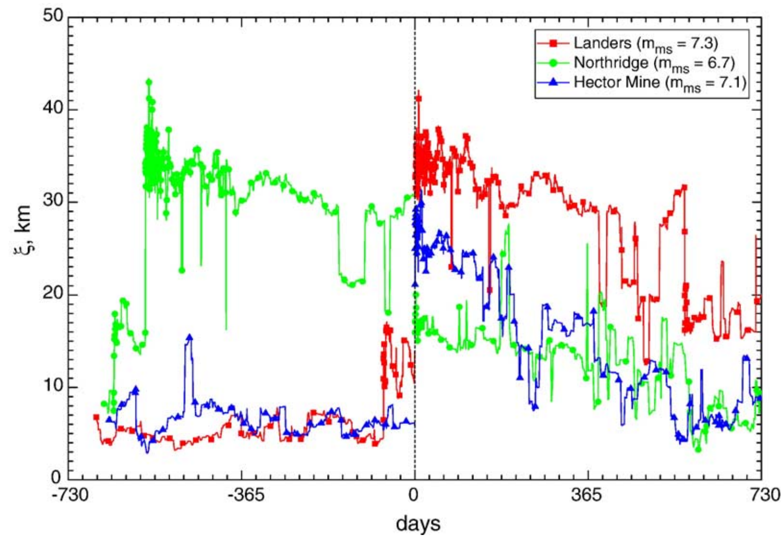

Fig. 7. Time evolution of the correlation length ξ defined in (4.3) 2 years prior to and 2 years after large earthquakes in California. A moving event window with 300 events and shifts by 10 events has been used with earthquakes greater than $m_c = 2.0$.

greater than $m_c = 2.0$ has been considered. We have also plotted, in a manner similar to the analysis performed in the section on the two-point correlation function, the time evolution of the correlation length 2 years prior to and 2 years after the major earthquakes in California. This is shown in Fig. 7.

5. Concluding remarks

Clustering of earthquakes in space and time is a well established observational fact. This effect is directly related to the space-time correlations between events that play a critical role in earthquake processes. One of the main aims of this work is to study such effects through the time evolution of a correlation length. This quantity is used as a measure of correlations in the system and plays a fundamental role in studies of critical phenomena. Changes in correlation length can be used as a precursor to large earthquakes, although this effect is not well observed and remains controversial.

In this work, we have discussed the physical basis of spatial correlations of earthquakes in the context of critical phenomena. We have studied the temporal evolution of the correlation length defined in the context of the two-point correlation function and the percolation problem. We have not found any pronounced temporal changes in the correlation length prior to large earthquakes in California. We have also studied the scaling properties of the two-point correlation function using the multifractal analysis. The results obtained suggest that on the global scale the two-point correlation function exhibits multifractal scaling with a spectrum of correlation dimensions. Whereas, on small scales of order of 25–50 km simple scaling is observed. This suggests that the highly heterogeneous distribution of seismicity occurring on the fractal network of faults exhibits rather complex behavior.

Partial information on correlations is also present in the Gutenberg–Richter frequency–magnitude statistics (Gutenberg and Richter, 1954) which is reflected in the change of the shape of the distribution constructed for different time periods. Time correlations are well documented for aftershock sequences and described by the modified Omori's law. This law specifies a temporal decay of aftershock rates and possesses a power-law scaling. To describe the observed scaling of aftershocks, Shcherbakov et al. (2004) introduced a generalized formula for the scaling rate which incorporates the three empirical laws, i.e., the Gutenberg–Richter relation, the modified Omori's law, and Båth's law (Shcherbakov and Turcotte, 2004).

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