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Cumulative Benioff strain-release, modified Omori's law and transient behaviour of rocks

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Abstract

Irreversible thermodynamic theories with internal state variables can be used to derive a general constitutive law for both transient and steady-state behaviours of rocks. This constitutive law can represent the concepts of damage and damage evolution in either the fibre-bundle model or continuum damage mechanics. We have previously proposed an empirically based constitutive law for both the transient and steady-state behaviours of rocks ultimately derived from laboratory experimental data. We show here that this law is concordant with the general constitutive law derived from irreversible thermodynamic theories, and that the relaxation modulus has a temporal power–law that depends on a structural fractal property of rocks. Our constitutive law predicts forms for the cumulative Benioff strain-release for precursory seismic activations and the modified Omori's laws of aftershocks, both aspects of the temporal fractal properties of seismicity. These seismic properties can also be derived by the fibre-bundle model or continuum damage mechanics. Our model suggests that these time-scale invariant processes of seismicity may be regulated by the fractal structures of crustal rocks.

Keywords: Damage mechanics; Fibre-bundle model; Irreversible thermodynamics; Omori's law; Fractal; Seismicity

1. Introduction

When an earthquake happens, the sudden stress changes induce a transient response of rocks in the crust and upper mantle, which is recorded as time series of surface displacement measured by creep meters (e.g., Wesson, 1987), the global positioning system (GPS) (e.g., Freed and Bürgmann, 2004), or on estimating the data of repeating earthquakes (e.g., Nadeau and McEvilly, 1999; Uchida et al., 2003). The time series data symbolizes a temporal seismicity pattern as a summation of transient responses of the surface on various time-scales. The transient responses over a time scale of a few years to major large earthquakes which repeat over many centuries or millennia are often analyzed by empirical constitutive laws based on experimental observation of steady-state creep curve of rocks, i.e., the Newtonian or non-Newtonian curve (e.g., Wesson, 1987; Teisseyre, 1995; Freed and Bürgmann, 2004). However, the constitutive law of rock behaviours has not been derived for the responses in the shorter time-scale to smaller events. We need to consider the temporal seismicity pattern consisting of many earthquakes in various time-scales.

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From this point of view, we need to derive a constitutive law for rock behaviour with the purpose of a new recognition of the temporal seismicity pattern. Then, the law requires two aspects. One is to express not only the viscoelastic but also brittle behaviour such as microcracking and failure inside the rock body as well as at the crustal scale. Fibre-bundle models (e.g., Krajcinovic, 1989; Kachanov, 1999; Turcotte et al., 2003; Turcotte and Glasscoe, 2004; Nanjo and Turcotte, 2005) and continuum damage mechanics (e.g., Lemaitre, 1985; Krajcinovic, 1989; Lyakhovsky et al., 1993, 1997; Main, 2000; Abdel-Tawab and Weitsman, 2001; Turcotte et al., 2003; Nanjo et al., 2005) have been used in explaining the brittle behaviour and applied to the rheology of continental lithosphere (e.g., Nanjo and Turcotte, 2005) and the earthquake problems (e.g., Turcotte et al., 2003; Shcherbakov et al., 2005; Nanjo et al., 2005).

Another requirement is to describe the transient response of rocks. The transient behaviour of rocks (e.g., transient creep or stress relaxation) is denoted as the response to the sudden changes of the stress and strain-rate such as earthquakes, that is, a kind of relaxation behaviour (e.g., Findley et al., 1976; Shimamoto, 1987) shown in Fig. 1. In contrast, the rock behaviour is generally represented by the Dorn's non-linear equation for the steady-state creep of rocks (e.g., Dorn, 1954; Poirier, 1985):

$$\dot{\varepsilon} = A\sigma^m \exp\left(-\frac{Q}{RT}\right), \tag{1}$$

where $\dot{\varepsilon}$ is the strain-rate, σ is the applied stress, m is the stress exponent reflecting the deformation mechanism, Q is the activated energy, R is the universal gas constant, T is the absolute temperature, and A is a constant. The steady-state flow law (Eq. (1)) does not express the transient response of the rocks (Fig. 1) (e.g., Shimamoto, 1987; Nagahama, 1994; Kawada and Nagahama, 2004). To fill this gap, a constitutive law for the transient behaviour, as well as steady-state behaviour of rocks, was derived based on irreversible thermodynamics (e.g., Schapery, 1964, 1966, 1969) and nonlinear viscoelastic theory (Schapery and Riggins, 1982; Shimamoto, 1987; Nagahama, 1994; Kawada and Nagahama, 2004; Kawada et al., 2005). The resulting behaviour shows temporal power–law relaxation, exhibited as a long time tail (Takayasu, 1990).

The cumulative Benioff 'strain' (the sum of the square root of the energy release for sequential earthquakes) has been suggested as a precursory phenomenon of large earthquakes, increasing as an inverse power–law of time before a mainshock. It is important to note here that the Benioff strain is not the actual Kostrov strain that might be measured by satellite data (Main, 1999). Such accelerations of Benioff strain have been observed in many case studies (e.g., Varnes, 1989; Bufe and Varnes, 1993; Newman et al., 1995; Bowman et al., 1998; Rundle et al., 2000; Rabinovitch et al., 2002; Turcotte et al., 2003; Nanjo and Nagahama, 2004), and interpreted as analogous to tertiary creep (e.g., Saito, 1969; Varnes, 1989) and crack propagation (e.g., Das and Scholz, 1981; Main, 1988; Varnes, 1989) of rocks. On the other hand, temporal patterns of aftershocks show temporal time-scale invariance of the aftershock decay, characterized by the modified Omori's law (Utsu, 1961). Recently, it has been shown that the modified Omori's law can be related to the steady-state flow law of rocks (Eq. (1)) (e.g., Shcherbakov et al.,

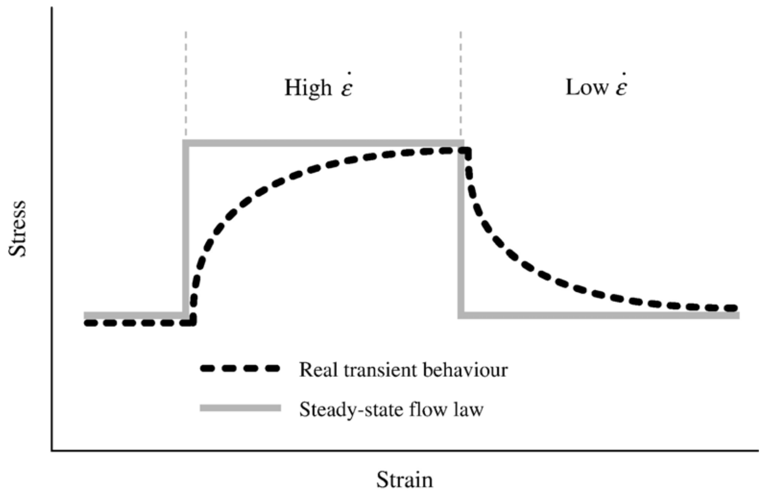

Fig. 1. A schematic figure on the responses of rocks to the sudden change in stress or strain-rate. This figure is modified from Shimamoto (1987).

2005; Nanjo et al., 2005). These temporal scale-invariant processes observed prior or subsequent to mainshock are the transient responses of rocks to the earthquakes. However, the processes have not so far been linked to the constitutive law for the transient behaviour of rocks.

In this paper, we derive a unified constitutive law for both the transient and steady-state behaviours of rocks from the experimental data. We then show that the model is consistent with some seismic patterns, such as the cumulative Benioff strain-release for precursory seismic activations and the modified Omori's laws of aftershocks. We show that these seismic patterns have some temporal fractal properties which can be explained equally well by the fibre-bundle model or by continuum damage mechanics. Then, in terms of the irreversible thermodynamic theories with the internal state variables, we show a general constitutive law which can be linked to the fibre-bundle model or continuum damage mechanics, and discuss the relation between this general constitutive law and our constitutive law. Finally, we point out that these temporal fractal properties of seismicities may be regulated by the fractal structures of crustal rocks.

2. Constitutive law for transient behaviour of rocks

The transient behaviour of viscoelastic materials is usually formulated based on the Boltzmann superposition principle for stress relaxation (e.g., Boltzmann, 1876; Findley et al., 1976). The output stress σ is represented by the convolutional integral of the input of the strain rate $\dot{\varepsilon}$ and the response (Green's) function or relaxation modulus $E(t)$ (e.g., Findley et al., 1976) at the deformation time t. When the input $\dot{\varepsilon}$ is constant, $E(t)$ represents $\mathrm{d}\sigma/\mathrm{d}\varepsilon$ which means the change in the stress of the rheological material against an increment of the strain. Then, the constitutive law for the viscoelastic behaviour of rocks is given by $E(t)$ (e.g., Shimamoto, 1987; Nagahama, 1994; Kawada and Nagahama, 2004; Kawada et al., 2005).

Some constitutive laws for the transient behaviour are briefly introduced. For analytical reason, the secant modulus $E_S(t)=\sigma/\varepsilon$ is utilized instead of $E(t)$. Analyses of experimental data on the transient behaviours of halite, marble and lherzolite yield the formulations:

$$\frac{\sigma}{\varepsilon}\equiv E_S(\xi)=\frac{E'}{g(\varepsilon)}\xi^{-\beta},\quad \xi=\frac{t}{C}\exp\left(-\frac{Q}{RT}\right),\tag{2}$$

where β is a positive exponent, ε is the strain, $g(\varepsilon)$ is the strain-dependent function ($g(\varepsilon)=1+a\varepsilon$), E' and a are the material constants, and C is a constant (Shimamoto, 1987; Nagahama, 1994; Kawada and Nagahama, 2004). Moreover, ξ stands for the temperature reduced time obtained by normalizing the various temperature behaviours, and is defined in the condition that T is convertible to the deformation time t (e.g., Schwarzl and Staverman, 1952; Findley et al., 1976). The power–law of ξ (or t) in Eq. (2) is indicative of temporal scale-invariance of rock behaviours (sometimes described as a 'temporal fractal property' or 'long time tail behaviour', e.g., Takayasu, 1990; Nagahama, 1994; Kawada and Nagahama, 2004; Kawada et al., 2005). Eq. (2) leads to $E(\xi)$ as the constitutive law for the transient behaviour of rocks as follows

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon}\equiv E(\xi)=\frac{(1-\beta)E'}{g(\varepsilon)}\xi^{-\beta},\tag{3}$$

based on the relation between $E(t)$ and $E_S(t)$: $E_S(t)=\frac{1}{t}\int_0^t E(t')\mathrm{d}t'$, where t' is a dummy variable (Shimamoto, 1987; Nagahama, 1994; Kawada and Nagahama, 2004). Note that $0<\beta<1$.

The temporal power–law decay function is simply considered as the summation of a suite of exponential decay functions with different lifetimes λ. Then, $E(t)$ is derived also by the Laplace transform of the distribution function of λ as follows;

$$E(t)=\int_0^\infty D(\lambda)\exp\left(-\frac{t}{\lambda}\right)\mathrm{d}\lambda,\tag{4}$$

where $D(\lambda)$ is the distribution function of lifetimes λ (e.g., Gross, 1953; Findley et al., 1976; Saito and Murayama, 1987). When $D(\lambda)\propto\lambda^{-\beta-1}$, $E(t)$ shows temporal power–law relaxation (Nagahama, 1994; Kawada and Nagahama, 2004; Kawada et al., 2005). If the lifetime λ depends on the scale length L of the underlying process, then $D(\lambda)$ depends to some extent on rock heterogeneity, and hence may be termed a 'structural' distribution. If $\lambda\sim L$, then the spatial heterogeneity is also fractal.

When $\dot{\varepsilon}=\varepsilon/t$, Eq. (2) generates a relation (e.g., Shimamoto, 1987; Nagahama, 1994; Kawada and Nagahama, 2004):

$$\dot{\varepsilon}=\left[\left(\frac{g(\varepsilon)}{\varepsilon E'}\right)^{\frac{1}{\beta}}\left(\frac{\varepsilon}{C}\right)\right]\sigma^{\frac{1}{\beta}}\exp\left(-\frac{Q}{RT}\right).\tag{5}$$

Eq. (5) is a general expression for a power–law rheology for a thermally activated process. It reduces to the Dorn's equation (Eq. (1)) for the special case of a steady-state flow law, when the term in square brackets reduces to a constant (Shimamoto, 1987; Nagahama, 1994; Kawada and Nagahama, 2004; Kawada et al., 2005). Comparing Eq. (5) with Eq. (1) implies that, the empirical stress exponent m in Eq. (1) corresponds to $1/\beta$ in Eq. (5), and the constant A can be further decomposed into terms representing the strain, deformation mechanism and material constants. In this sense, the power–law relation between $E(\xi)$ and ξ (or $E(t)$ and t) yields the relation of the power–law form $\dot{\varepsilon} \propto \sigma^{1/\beta}$ and represents the transient behaviour as well as the steady-state behaviour (Shimamoto, 1987; Nagahama, 1994; Kawada and Nagahama, 2004; Kawada et al., 2005). This is supported also by the analyzing experimental results of samples of halite (Shimamoto, 1987; Nagahama, 1994), marble and lherzolite (Kawada and Nagahama, 2004). For halite, these authors found $1/\beta$ of 15 ($\xi < 10^5$ s) and 6.7 ($\xi > 10^5$ s). Those values reflect the low-temperature shear deformation or primary creep (Shimamoto, 1986, 1987; Nagahama, 1994) and the steady-state behaviour (Heard, 1972; Shimamoto, 1987; Nagahama, 1994), respectively.

3. Fibre-bundle model for brittle behaviour of rocks

Seismic activation prior to a seismic event and the decay of aftershock activity are different aspects of the evolution of damage in the brittle processes of rocks. This brittle process is irreversible, and has been studied based on the continuum damage mechanics (e.g., Lyakhovsky et al., 1993, 1997; Turcotte et al., 2003; Nanjo et al., 2005) and fibre-bundle model (e.g., Turcotte et al., 2003; Turcotte and Glasscoe, 2004; Nanjo and Turcotte, 2005). For comparison we briefly derive the law for the brittle behaviour in terms of the 1-D fibre bundle model (e.g., Newman and Phoenix, 2001; Turcotte et al., 2003; Turcotte and Glasscoe, 2004; Nanjo and Turcotte, 2005). Let us consider a bundle having $n(t)$ fibres and $n(0) = n_0$ (n_0 is a constant). A fibre is subjected to the stress σ_f following the Hooke's law:

$$\sigma_f(t) = K\varepsilon_f, \qquad \dot{\varepsilon} = \frac{\varepsilon_f}{t}, \tag{6}$$

where K is the elastic constant, and ε_f is the strain of one fibre. When the fibre is broken down, the failed fibre is replaced by a new fibre, which is hypothesized as an analogy to a repetitive earthquake rupture or a migration of a dislocation. Then, $n(t)$ requires standing for the numbers of the unbroken and unreplaced remaining fibres. Then, the breakdown rule of the fibres is expressed by

$$\frac{dn(t)}{dt} = -v(\sigma_f)n(t), \tag{7}$$

where σ_f is the stress for the respective bundles and v is defined as the hazard rate (e.g., Weibull, 1951; Newman and Phoenix, 2001). This hazard rate is given by

$$v(\sigma_f) = v_f\left(\frac{\sigma_f}{\sigma_0}\right)^{\rho}, \tag{8}$$

where ρ is a positive exponent, and v_f is a constant (e.g., Newman and Phoenix, 2001; Turcotte et al., 2003; Turcotte and Glasscoe, 2004; Nanjo and Turcotte, 2005). From Eqs. (6)–(8) with the discussion of lifetimes of the respective original fibres (Eqs. (25)–(30) in Turcotte and Glasscoe (2004)), the constitutive relation between the total stress σ and strain-rate $\dot{\varepsilon}$ is derived as the power–law $\dot{\varepsilon} \propto \sigma^{\rho+1}$. When the hazard rate (Eq. (8)) follows a thermally activated process, the constitutive relation reduces to Dorn's equation (Eq. (1)) (e.g., Nanjo and Turcotte, 2005), which corresponds to Eqs. (2) and (5) if $\rho + 1 = m = 1/\beta$. Moreover, the analytical results of the brittle failure of rocks also yield the Newtonian or non-Newtonian law, and the exponent m or $1/\beta$ for the brittle and viscoelastic behaviours ranges from 1 to 60 (Nakamura and Nagahama, 1999).

Here, the brittle behaviour of rocks is formulated as the stress power–law $\dot{\varepsilon} \propto \sigma^{1/\beta}$ from the fibre-bundle model. Therefore, we point out that $E(t) \propto t^{-\beta}$ inclusively expresses the failure or brittle behaviour as well as viscoelastic (transient and steady-state) behaviour. In the next sections, based on these laws, we consider the temporal seismicity patterns in surface displacement and cumulative Benioff strain release.

4. Surface displacement and the constitutive law of rock behaviours

Freed and Bürgmann (2004) analyzed the time series of surface displacement due to the 1999 Hector Mine earthquake (moment magnitude 7.1) by GPS data, and described the transient response for about three years after the earthquake by the experimental flow law (Eq. (1)) with $m = 3.5$ (in Fig. 2). This means that the response in the longer time-scale (on the order of a year) to the earthquake can be represented by the simpler flow law of the steady-state behaviour (Eq. (1)). Here we fitted the constitutive law of Eq. (5) to the time series data measured by a creep meter, as shown in Fig. 3 (Wesson, 1987) for a magnitude 4.8 earthquake near San Juan Batista in California. Here, ε is regarded as the displacement divided by a characteristic length, and stress rate is considered as an approximate constant. Then, Eq. (5) is

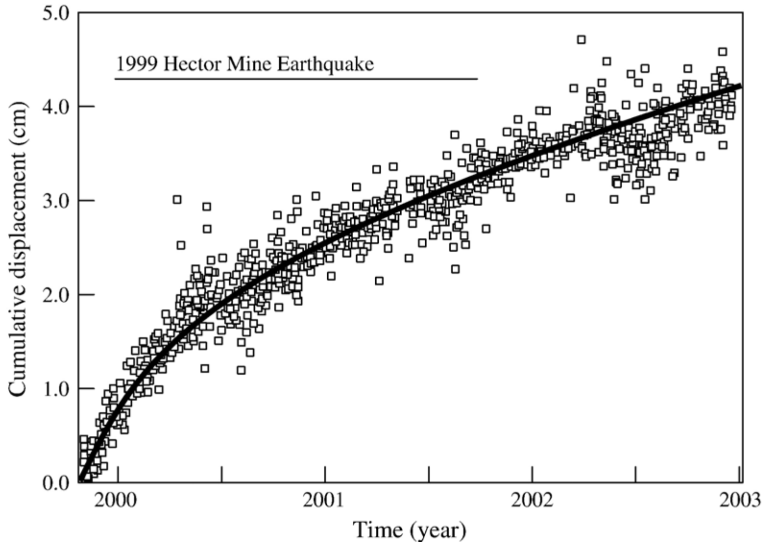

Fig. 2. Horizontal displacement time series subsequent to the 1999 Hector Mine earthquake as observed at a GPS station in a Mojave desert of South California. The fitted curve is the power–law for the steady-state behaviour in Eq. (1) of wet olivine, where $m=3.5$, $A=3.6\times10^5$ ($\mathrm{MPa}^{-m}\ \mathrm{s}^{-1}$), and $Q=480$ ($\mathrm{kJ\ mol}^{-1}$). This figure is modified from Freed and Bürgmann (2004).

transformed into $\log d=\beta\log t+C_\mathrm{f}$, where d is cumulative displacement and C_f is a constant depending on the material constant E', characteristic length and stress rate.

In this data the temporal power–law behaviour of the laboratory-derived constitutive law of (Eq. (5)) describes the transient response after the earthquake in this field example (Fig. 3). In our model (Eq. (5)) β characterizes the transient behaviour of rocks, derived above from Eq. (2) in the form of Eq. (3), interpreted as a structural fractal property of rocks $D(\lambda)$, yielding the long time tail behaviour described by Eq. (4). Therefore, observed time series of surface displacement due to a mainshock with many associated small events can be described by Eq. (5). In our model this arises from a temporal fractal property in Eq. (4), regulated by the fractal structure of crustal rocks represented by $D(\lambda)$.

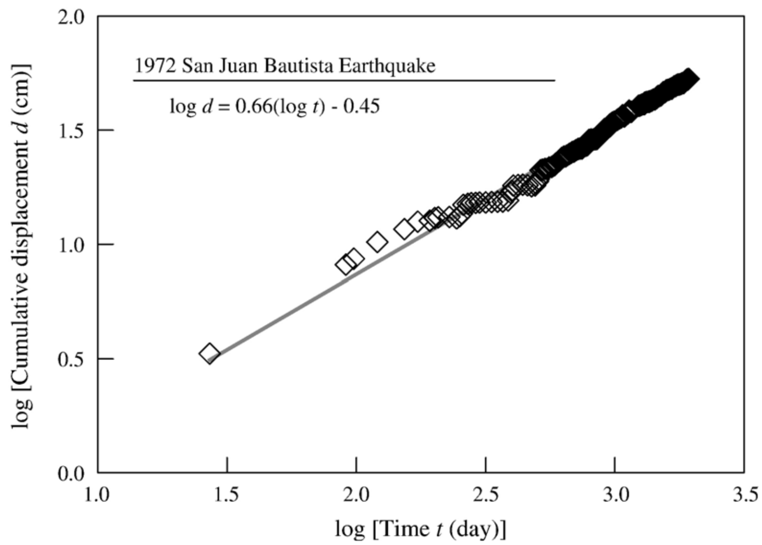

Fig. 3. Right-lateral displacement time series subsequent to the 1972 San Juan Bautista earthquake (local magnitude $M_\mathrm{L}=4.8$) as observed at a creep meter setting along the San Andreas Fault in the San Juan Bautista area. The line is fitted based on Eq. (2), and β-value is evaluated from the slope of the regressive line. The original data of Fig. 7 in Wesson (1987) is analyzed.

5. Cumulative Benioff strain-release and the constitutive law of rock behaviours

Seismic activation has been observed prior to a lot of major earthquakes, and been quantified as accelerated releases of the cumulative (square root sum) of seismic energy in the time series (e.g., Varnes, 1989; Sykes and Jaumé, 1990; Bufe and Varnes, 1993; Newman et al., 1995; Bowman et al., 1998; Rundle et al., 2000; Turcotte et al., 2003). This precursory phenomenon is analogous to the tertiary creep behaviour, and is formulated by generalizing the Saito's (1969) equation as follows:

$$\dot{\varepsilon} = \varphi(t_c - t)^{-\gamma}, \tag{9}$$

where t_c is the occurrence time of the earthquake, γ is a positive exponent, and φ is a constant (e.g., Varnes, 1989). This equation is linked to that of crack propagation (e.g., Das and Scholz, 1981; Main, 1988; Varnes, 1989; Bufe and Varnes, 1993) and rock failure (e.g., Voight, 1989; Varnes, 1989; Bufe and Varnes, 1993). These authors use the Benioff cumulative 'strain'-release Ω, i.e., the summation of the square root of the energy release at earthquake sequence (e.g., Benioff, 1951; Varnes, 1989; Bufe and Varnes, 1993; Newman et al., 1995; Bowman et al., 1998; Rundle et al., 2000; Turcotte et al., 2003; Nanjo and Nagahama, 2004). Here we note the caveat that the exponents γ from the Benioff strain cannot be compared directly with similar exponents derived from the tectonic (Kostrov) strain (Main, 1999). Integration of Eq. (9) after replacing $\dot{\varepsilon}$ by $\mathrm{d}\Omega/\mathrm{d}t$, in the condition $\Omega = \Omega_c$ at $t = t_c$, results in

$$\Omega = \Omega_c - \frac{\varphi}{s}(t_c - t)^s, \qquad s = 1 - \gamma. \tag{10}$$

Based on Eq. (10), we analyze the accelerating precursory seismic activation on two major earthquakes occurred in Himachal Himalaya, India (10 December 1975, local magnitude $M_L = 5.3$) and in north-eastern Caribbean Sea (14 February 1980, $M_L = 4.8$). Ω is calculated as $\Sigma[\sqrt{e_{seis}}]$, and the released energy at an event e_{seis} is $\log e_{seis} = 1.5M_L + 4.8$ (e.g., Varnes, 1989; Bufe and Varnes, 1993; Kanamori and Anderson, 1975; Bowman et al., 1998). In Fig. 4, the temporal power–law relation of Eq. (10) is realized in the plot of $\log(\Omega_c - \Omega)$ vs. $\log(t_c - t)$. s-values are approximately concordant with the estimation by Bowman et al. (1998) when $\log(t_c - t) > 6.4$. Moreover, electromagnetic radiation induced by uniaxial and triaxial rock fracture also shows the power–law relation between t and Ω in Eqs. (9) or (10) (Rabinovitch et al., 2002).

The power–law increase in the cumulative Benioff strain-release can be equally well described by the fibre-bundle model (e.g., Newman et al., 1995; Turcotte et al., 2003) used in Eqs. (6)–(8) in Section 3. Let us consider a bundle composed of n_0 fibres with n_c failed fibres in which the respective fibres are subjected to σ_0 at initial

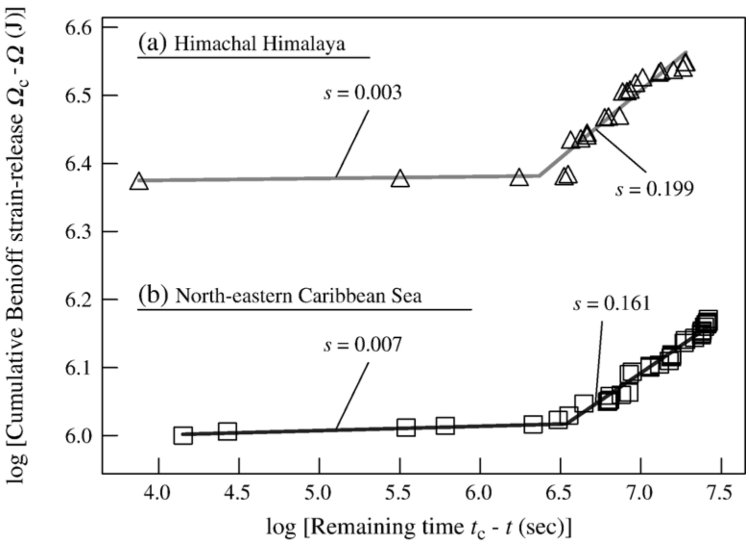

Fig. 4. The power–law relation between the cumulative Benioff strain-release vs. the remaining time until the earthquake occurred in (a) Himachal Himalaya (32.80°N, 76.70°E) on 10 December 1975 (local magnitude M_L is 5.3) and in (b) north-eastern Caribbean Sea (18.43°N, 64.39°E) on 14 February 1980 (M_L is 4.8). The regressive lines represent the power–law relation in Eq. (10), and s-values are evaluated from the slope of the lines. The data sets for (a) and (b) are from Das Gupta (1984) and Frankel (1982), respectively.

condition. Then, the stress of the surviving fibres σ_f is prescribed by

$$\sigma_f = \frac{n_0}{n(t)}\sigma_0. \tag{11}$$

The breakdown rule and hazard rate follow Eqs. (7) and (8), respectively. Substituting Eqs. (8) and (11) into Eq. (7), and integrating with the initial condition $n(t_c)=0$, we obtain

$$n(t) = n_0[\rho v_0(t_c - t)]^{1/\rho}. \tag{12}$$

Based on the Eqs. (6), (11) and (12), σ_f and ε_f become the power–law function depending on $(t_c - t)$. Then, the stored elastic energy in a fibre per unit volume $e_f(t)$ is given by

$$e_f(t) = \frac{1}{2}K\varepsilon_f^2. \tag{13}$$

Here, η is introduced as a fraction of e_f released in the acoustic-emission events at the failure of a fibre and is analogous to the seismic efficient (e.g., Turcotte et al., 2003). The energy e_a released in the acoustic-emission events is formulated by

$$e_a(t) = -\eta e_f(t)\frac{d}{dt}n(t). \tag{14}$$

Eqs. (6) and (11)–(14) yields a relation:

$$e_a(t) \propto (t_c - t)^{-\frac{1+\rho}{\rho}}. \tag{15}$$

Therefore, the Benioff strain-release can be formulated as

$$\frac{d\Omega}{dt} \propto \sqrt{e_a(t)} \propto (t_c - t)^{-\frac{1+\rho}{2\rho}}. \tag{16}$$

Eq. (16) is different from the equation proposed by Turcotte et al. (2003), and is identical to Eq. (9) after replacing $\dot{\varepsilon}$ by $d\Omega/dt$, and the exponent ρ is linked to those of γ in Eq. (9), s in Eq. (10), m in Eq. (1) and β in Eqs. (2), (3) and (5): $1+\rho = 1+1/[2\gamma-1] = 1+1/[1-2s] = m = 1/\beta$. This linkage of the various exponents yields a unifying relationship between the temporal power–law increase in cumulative Benioff strain-release, the long time tail behaviour of rocks, the structural fractal property of the rocks, and the ρ-value in the fibre-bundle model. Therefore, we can recognize the temporal pattern of the small events prior to major earthquakes from parameters in the long time tail of the transient behaviour of rocks. Moreover, we should point out that, in our model, the precursory seismic activation is prescribed by the fractal structure parameter $D(\lambda)$ of the crustal rocks.

6. Modified Omori's law and the constitutive law of rock behaviours

The modified Omori's law (Utsu, 1961) is given in the form:

$$\frac{dN}{d\tau} = B(\tau + c)^{-p}, \tag{17}$$

where τ is the time elapsed after the main shock, $dN/d\tau$ is the rate of occurrence of aftershocks greater than a magnitude, p is the positive exponent, and B and c are constants. Shcherbakov et al. (2005) combined the temporal statistical laws for aftershocks as the (modified) Omori's law (Omori, 1894; Utsu, 1961), the Gutenberg–Richter law (Gutenberg and Richter, 1954) and the Båth's law (Båth, 1965), and derived the generalized Omori's law:

$$\frac{1}{N_T}\frac{dN}{d\tau} = \frac{p-1}{c}\left(1 + \frac{\tau}{c}\right)^{-p}, \tag{18}$$

where N_T is the total number of aftershocks with magnitude greater than a value. Nanjo et al. (2005) pointed out the relation between the modified or generalized Omori's law and the Dorn's equation (Eq. (1)) (or constitutive Eq. (5)). By the continuum damage model (e.g., Nanjo et al., 2005) which is equivalent to the fibre-bundle model (Krajcinovic, 1989; Turcotte et al., 2003), the brittle behaviour of rocks is described as the power–law relation $\dot{\varepsilon} \propto \sigma^m$ in the similar way to the fibre-bundle model in Eqs. (6)–(8) of Section 3 (e.g., Turcotte et al., 2003; Shcherbakov et al., 2005; Nanjo et al., 2005).

Here, let us consider the following situation; when a large earthquake happens, the strain is suddenly applied and the elastic energy is stored. A large part of the energy is immediately released by the earthquake, while the other is released by the aftershocks. The release rate of aftershock energy $e_r(\tau)$ can be calculated by the continuum damage model (Nanjo et al., 2005) as

$$\frac{1}{e_{rT}}\frac{de_r(\tau)}{d\tau} \propto \left(1 + \frac{\tau}{\zeta}\right)^{-\frac{m}{m-1}}, \tag{19}$$

where e_{rT} is the total energy of the aftershock sequence, and ζ is a constant. Eq. (19) is equivalent to the generalized Omori's law in Eq. (18) (e.g., Shcherbakov et al., 2005; Nanjo et al., 2005), and the exponent p is linked to m and β by $p/(p-1) = m = 1/\beta$. Therefore, the temporal fractal decay of aftershocks is affected by β in our constitutive law (Eq. (5)). This means that the decay of aftershocks is regulated by the structural fractal property of rocks. Moreover, p-value correlates to the fractal dimension of pre-existing fault systems in the aftershock region (e.g., Nanjo et al., 1998; Nanjo and Nagahama, 2004), and the temporal power–law behaviour of the rocks is also related to the fractal structure of the pre-existing active fault systems.

7. Discussion

The constitutive law for the transient behaviour of rocks similar to Eq. (3) or Eq. (5) is derived from the irreversible thermodynamic theories (Biot, 1954; Schapery, 1964, 1966). Especially, from the Lagrange equation (Eq. (19) in Schapery, 1966), Schapery (1966) derived a nonlinear constitutive formulation for viscoelastic response with microstructural change:

$$\sigma = a_e h_e \frac{dq}{d\varepsilon} \int_0^{\xi} \tilde{E}(\xi - \xi') \frac{d\varepsilon}{d\xi'} d\xi', \tag{20}$$

where q is the generalized coordinate, $\tilde{E}$ is the relaxation modulus, ξ' is the arbitrary temperature reduced time, and a_e and h_e are constants. When q is regarded as an internal state variable (a hidden variable) indicating the internal state on the microstructures such as molecular configuration in polymer, defect in crystal or microcrack (e.g., Fung, 1965; Schapery, 1966), q is identical to the damage parameter in the continuum damage mechanics (e.g., Lemaitre, 1985; Krajcinovic, 1989; Lyakhovsky et al., 1993, 1997; Abdel-Tawab and Weitsman, 2001; Turcotte et al., 2003; Shcherbakov et al., 2005; Nanjo et al., 2005). Then, Eq. (3) is linked to Eq. (20) under $a_e h_e (dq/d\varepsilon) = 1/g(\varepsilon)$ and $\tilde{E}(\xi) = (1-\beta)\xi^{-\beta}/E'$. Therefore, $g(\varepsilon)$ depends on a damage parameter, and Eq. (20) represents the damage evolution covering the irreversible behaviour associated with both brittle and ductile processes. Moreover, Schapery (1969) pointed out that $a_e h_e (dq/d\varepsilon)$ in Eq. (20) expresses the effect of the hysteresis in the multiple step stress relaxation.

The damage evolution like the constitutive law (Eq. (20)) is equivalent to the following equation under non-equilibrium condition:

$$\frac{dq}{dt} = -\Gamma \frac{\partial F}{\partial q}, \tag{21}$$

where F is the free energy for materials with damage, and Γ is a positive constant reflecting the temporal scale of an irreversible process (e.g., Myasnikov et al., 1990; Lyakhovsky et al., 1993). This equation corresponds to J-integral (e.g., Rice, 1968; Schapery, 1984; Lyakhovsky et al., 1993) and the Ginzburg-Landau equation (e.g., Honenberg and Halperin, 1977; Lyakhovsky et al., 1993). Moreover, Eq. (21) regulates the continuum damage mechanics (e.g., Myasnikov et al., 1990; Lyakhovsky et al., 1993), and q for the continuum damage mechanics ($0 \leq q \leq 1$) is regarded as $1-(n/n_0)$ for the fibre-bundle model (e.g., Turcotte et al., 2003). Hence, Eqs. (16) and (19) are also prescribed by Eq. (21). Therefore, the framework of our constitutive law for the transient behaviour of rocks (Eqs. (3) and (5)) includes the concept of damage and/or damage evolution considered in the framework of the fibre-bundle model and continuum damage model. So, Eq. (5) can be applied to the brittle processes such as the seismic activation prior to an event and the decay of aftershock activity.

There are many alternative damage models to predict power–law rheology or modified Omori's law. Main (1999, 2000) presented the damage model based on the Charles' (1958a,b) law for subcritical crack growth and Voight's (1989) equation for terminal stage of failure. In our model, Eqs. (9) and (10) are solutions of the Voight's equation (e.g., Bufe and Varnes, 1993; Main, 1999, 2000) and Eq. (8) is identical to Charles' (1958a, b) law. Hence, Main's damage model is also concordant with our model. In the future, we need also to investigate the relation of our model to the other damage models.

8. Conclusions

The cumulative Benioff strain-release for preseismic activations and modified Omori's law of aftershocks are analyzed by a general constitutive law derived from the irreversible thermodynamic theories with the internal state variables. This constitutive law is related to the empirically based constitutive law for transient behaviour of rocks and can be linked to the damage models such as the fibre-bundle model or continuum damage mechanics. We pointed out that the temporal seismicity patterns are recognized as the summation of transient behaviours of crustal rocks and have the temporal fractal properties, and that these temporal fractal properties of seismicity are constrained by the fractal structures of crustal rocks.

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