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GEOCHEMISTRY

Effective Coefficients of Quasi-Stationary Maxwell Equations with Multiscale Random Inhomogeneous Electroconductivity

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Large-scale details of a medium structure, for example, large blocks and easily distinguished layers, can be described in a model designed for mathematical modeling in the theory of electromagnetic sounding. Small-scale details of the medium can hardly be described, but they should be taken into account within the statistical approach using effective coefficients. Fractal theory is one of the most rapidly developing theories for investigating natural objects and phenomena. There is experience in applying the theory of fractals to geoelectric problems [1]. Such an approach requires the application of a complex geometrical language and, as a rule, large computational resources. In our work, we use the approach suggested by Kolmogorov [2], which allows us not to decline the hypothesis of a continuous medium. The difference between the approaches is as follows: in the first case, the geometry of the medium is specified explicitly; in the second case, we calculate the coefficients of equations that describe the properties of the medium. Infinite multiplicative Kolmogorov cascades also lead to extremely inhomogeneous fractal sets. Such an approach was used in filtration problems [3–5]. In our work, we obtained effective coefficients in quasi-stationary Maxwell equations for the fields in isotropic random inhomogeneous media under the condition that only statistical information is available about the fluctuations of parameters. The peculiarities of the problems suggested here are multiscale and fractal properties of the coefficients, which have logarithmic statistics. The theoretical results are confirmed by numerical modeling.

FORMULATION OF THE PROBLEM

According to [6], the quasi-stationary approximation of Maxwell equations for monochromatic fields

$$\tilde{\mathbf{E}}(\mathbf{x}, t) = \mathrm{Re}(\mathbf{E}(\mathbf{x})e^{-i\omega t}), \quad \tilde{\mathbf{H}}(\mathbf{x}, t) = \mathrm{Re}(\mathbf{H}(\mathbf{x})e^{-i\omega t})$$

in the absence of extraneous currents is written as

$$\begin{aligned} \mathrm{rot}\mathbf{H}(\mathbf{x}) &= \sigma(\mathbf{x})\mathbf{E}(\mathbf{x}), \\ \mathrm{rot}\mathbf{E} &= i\omega\mu\mathbf{H}, \end{aligned} \qquad (1)$$

where $\mathbf{E}$ and $\mathbf{H}$ are vectors of the electric and magnetic field intensity; μ is magnetic penetrability; $\sigma(\mathbf{x})$ is specific electroconductivity; ω is cyclic frequency; and $\mathbf{x}$ is the vector of spatial coordinates. The magnetic penetrability μ for rocks is equal to the magnetic penetrability of a vacuum. We shall assume that specific electroconductivity is constant beyond the finite volume V with a sufficiently smooth surface S. At the boundary of S, tangent components of the electric and magnetic field intensity are continuous.

We assume that the field of electroconductivity is known. This means that the field is measured at each point $\mathbf{x}$ in a small sample with size l_0. The random function of spatial coordinates $\sigma(\mathbf{x})$ is considered as a limit $\sigma(\mathbf{x})_{l_0} \to \sigma(\mathbf{x})$ at $l_0 \to 0$. The dependence of $\sigma(\mathbf{x})_l$ on scale l can be considered as a factor that allows us to develop new approaches to the investigation of a randomly inhomogeneous medium. Let field $\sigma(\mathbf{x})_{l_1}$ be specified. How can we make a transition to a rougher grid l_2? It is possible to smooth field $\sigma(\mathbf{x})_{l_1}$ over scale $l_2 > l_1$, but will the obtained field be the real electroconductivity for scale l_2? Generally speaking, this is not true. A new measurement of the electroconductivity will be needed in samples with size l_2. The necessity of this procedure is caused by the fact that fluctuations of electroconductivity from the scale range (l_1, l_2) correlate with fluctuations of the induced electric field. Following Kolmogorov [2], we shall consider the dimensionless field $\psi(\mathbf{x}, l_1, l_2) = \dfrac{\sigma(\mathbf{x})_{l_2}}{\sigma(\mathbf{x})_{l_1}}$, where $\sigma(\mathbf{x})_{l_1}$ and

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$\sigma(\mathbf{x})_{l_2}$ are field $\sigma(\mathbf{x})_{l_0}$ smoothed over scales l_1 and l_2. Let us extend function $\psi(\mathbf{x}, l_1, l_3)$ and $\psi(\mathbf{x}, l_2, l_3)$ into Taylor series at point l_2. Neglecting the terms of the second order of smallness and using the evident property $\psi(\mathbf{x}, l_1, l_3) = \psi(\mathbf{x}, l_1, l_2)\psi(\mathbf{x}, l_2, l_3)$, we get

$$\frac{\partial \ln \sigma(\mathbf{x})_l}{\partial \ln l} = \varphi(\mathbf{x}, l), \qquad (2)$$

where $\varphi(\mathbf{x}, l) = \left.\frac{d\psi(\mathbf{x}, l, l\lambda)}{d\lambda}\right|_{\lambda = 1}$, $\lambda = \frac{l_1}{l}$. The solution of Eq. (2) describes electroconductivity as a function of field φ, which determines all statistical properties of the electroconductivity

$$\sigma(\mathbf{x})_{l_0} = \sigma_0 \exp\left[-\int_{l_0}^{l} \varphi(\mathbf{x}, l_1)\frac{dl_1}{l_1}\right], \qquad (3)$$

where σ_0 is constant. It is assumed that the conductivity has inhomogeneities of scale l from range (l_0, L), where l_0 and L are the minimal and maximal scales of measurements $\sigma(\mathbf{x}) = \sigma(\mathbf{x})_{l_0}$ and field φ is isotropic and statistically homogeneous. Fields with different scales for any x and y are statistically independent:

$$\Phi((\mathbf{x}-\mathbf{y})^2, l, l_1) = \Phi((\mathbf{x}-\mathbf{y})^2, l)\delta(\ln l - \ln l_1). \quad (4)$$

This hypothesis is usually assumed to be true in different models [2]. It reflects the fact that statistical dependence decreases for different scales. In the scale invariant system, this means that the correlation function does not depend on the scale at $\mathbf{x} = \mathbf{y}$. If dispersion $\varphi(\mathbf{x}, l)$ is finite, integral (3) tends to a normal field for large values of $\frac{L}{l_0}$. From hereon, it is assumed for simplicity that field $\varphi(\mathbf{x}, l)$ has a normal distribution.

SUBGRID MODELING

We divide conductivity function $\sigma(\mathbf{x}) = \sigma(\mathbf{x})_{l_0}$ into two components with respect to scale l. The large-scale component $\sigma(\mathbf{x}, l)$ was obtained by statistical averaging over all $\varphi(\mathbf{x}, l_1)$ with $l_0 < l_1 < l$, $l - l_0 = dl$, where dl is small. The small-scale (subgrid) component is equal to $\sigma'(\mathbf{x}) = \sigma(\mathbf{x}) - \sigma(\mathbf{x}, l)$:

$$\sigma(\mathbf{x}, l) = \sigma_0 \exp\left[-\int_{l}^{L} \varphi(\mathbf{x}, l_1)\frac{dl_1}{l_1}\right] \times \left\langle \exp\left[-\int_{l_0}^{l} \varphi(\mathbf{x}, l_1)\frac{dl_1}{l_1}\right]\right\rangle, \qquad (5)$$

$$\sigma'(\mathbf{x}) = \sigma(\mathbf{x}, l)\left[\frac{\exp\left(-\int_{l_0}^{l} \varphi(\mathbf{x}, l_1)\frac{dl_1}{l_1}\right)}{\left\langle \exp\left[-\int_{l_0}^{l} \varphi(\mathbf{x}, l_1)\frac{dl_1}{l_1}\right]\right\rangle} - 1\right],$$

$$\langle \sigma'(\mathbf{x})\rangle = 0,$$

$$\sigma(\mathbf{x}, l) = \left[1 - \bar{\varphi}(l)\frac{dl}{l} + \frac{1}{2}\Phi_0(l)\frac{dl}{l}\right]\sigma_l(\mathbf{x}), \qquad (6)$$

where brackets (<>) indicate statistical averaging, $\bar{\varphi}(l) = \langle \varphi(\mathbf{x}, l)\rangle$ and $\Phi_0(l) = \Phi(0, l)$. Large-scale (ongrid) components of electric and magnetic field intensity $\mathbf{E}(\mathbf{x}, l)$ and $\mathbf{H}(\mathbf{x}, l)$ are obtained as average solutions of equation system (1), in which large-scale component $\sigma(\mathbf{x}, l)$ is fixed, while small-scale component $\sigma'(\mathbf{x})$ is a random value. Subgrid components of the electric and magnetic fields are equal to $\mathbf{E}'(\mathbf{x}) = \mathbf{E}(\mathbf{x}) - \mathbf{E}(\mathbf{x}, l)$, $\mathbf{H}'(\mathbf{x}) = \mathbf{H}(\mathbf{x}) - \mathbf{H}(\mathbf{x}, l)$. Let us substitute relations for $\mathbf{H}(\mathbf{x})$ and $\sigma(\mathbf{x})$ into equation system (1) and average over small-scale component

$$\begin{aligned} \mathrm{rot}\mathbf{H}(\mathbf{x}, l) &= \sigma(\mathbf{x}, l)\mathbf{E}(\mathbf{x}, l) + \langle \sigma'\mathbf{E}'\rangle_{\sigma(\mathbf{x}, l)}, \\ \mathrm{rot}\mathbf{E}(\mathbf{x}, l) &= i\omega\mu\mathbf{H}(\mathbf{x}, l), \end{aligned} \qquad (7)$$

where $\langle\cdot\rangle_{\sigma(\mathbf{x}, l)}$ denotes averaging over all $\varphi(\mathbf{x}, l_1)$ at l_1 in the interval $l_0 < l_1 < l$ if component $\sigma(\mathbf{x}, l)$ is fixed. Let us subtract equation system (7) from (1), and, leaving only the terms of the first order of magnitude, we get subgrid equations

$$\begin{aligned} \mathrm{rot}\mathbf{H}' &= \sigma(\mathbf{x}, l)\mathbf{E}' + \sigma'\mathbf{E}(\mathbf{x}, l), \\ \mathrm{rot}\mathbf{E} &= i\omega\mu\mathbf{H}'. \end{aligned} \qquad (8)$$

We consider that $\mathbf{E}(\mathbf{x}, l)$ and $\mathbf{H}(\mathbf{x}, l)$ are known and find a solution for $\mathbf{E}'$ and $\mathbf{H}'$. For the fields in which a small variation in the scale causes significant fluctuations of the field itself (this is characteristic of fractal fields), it is possible to consider that $\sigma(\mathbf{x}, l)$, $\mathbf{E}(\mathbf{x}, l)$, $\mathbf{H}(\mathbf{x}, l)$, and their derivatives change more slowly than σ', $\mathbf{H}'$, and their derivatives. Let us introduce the notation $k = (1 + i)\sqrt{\omega\mu\sigma(\mathbf{x}, l)/2}$. For definiteness, we select the value of the root at which $\mathrm{Re}k > 0$, $\mathrm{Im}k > 0$, $r = |\mathbf{x} - \mathbf{x}'|$. The solution of equation system (8) is written as

$$\mathbf{E}'(\mathbf{x}) \simeq \frac{1}{4\pi} i\omega\mu \int_V \frac{e^{ikr}}{r}\sigma'(\mathbf{x}')d\mathbf{x}'\mathbf{E}(\mathbf{x}, l), \qquad (9)$$

$$\mathbf{H}' \simeq \frac{1}{4\pi}\int_V \frac{e^{ikr}}{r}\nabla\sigma' d\mathbf{x}'\frac{1}{\sigma(\mathbf{x}, l)}\mathrm{rot}\mathbf{H}(\mathbf{x}, l). \qquad (10)$$

It follows from (5) and (6) that, if difference $l - l_0$ is sufficiently small, then $\langle\sigma'(\mathbf{x})\sigma'(\mathbf{x}')\rangle \simeq \Phi(r, l)\sigma^2(\mathbf{x}, l)\frac{dl}{l}$. Using (9), we get

$$\langle\sigma'(\mathbf{x})\mathbf{E}'(\mathbf{x})\rangle_{\sigma(\mathbf{x},\, l)} \simeq k^2\int_0^\infty re^{ikr}\Phi(r, l)dr\frac{dl}{l}\sigma(\mathbf{x}, l)\mathbf{E}(\mathbf{x}, l). \tag{11}$$

Correlation function $\Phi(|\mathbf{x} - \mathbf{x}'|, l)$ is small beyond the region with radius $L \ll L_0$ with the center at point $\mathbf{x}$, where L_0 is the scale of the entire region. Thus, integration over a finite radius in (11) is substituted by integration with an infinite limit. At internal points, such substitution is valid. In a narrow band near the boundary of the order of L, estimate $\langle\sigma'(\mathbf{x})\mathbf{E}'(\mathbf{x})\rangle_{\sigma(\mathbf{x},\, l)}$ can have a poor accuracy. Using (6), (7) and taking into account (11), we get an estimate for the average value of the density of electric field $\langle\mathbf{j}\rangle_{\sigma_l(\mathbf{x})}$ at internal points of the region accurate to the second order of smallness with respect to $\frac{dl}{l}$:

$$\langle\mathbf{j}\rangle_{\sigma_l(\mathbf{x})} \simeq \sigma_l(\mathbf{x})\mathbf{E}(\mathbf{x}, l) + k^2\int_0^\infty re^{ikr}\Phi(r, l)dr\frac{dl}{l}\sigma_l(\mathbf{x})\mathbf{E}(\mathbf{x}, l). \tag{12}$$

If $\omega\mu L^2\sigma(\mathbf{x}, l) \ll 1$, we can neglect the integral term in (12). This condition is true in a wide range of frequencies for the problems of skin layer in an inhomogeneous medium with scales of inhomogeneities $L \ll L_0$. Thus, if we replace the specific electroconductivity $\sigma(\mathbf{x})$ with the smoother $\sigma_l(\mathbf{x})$ in Eq. (1), the average value of the current density changes insignificantly at small scales of inhomogeneities. Using (10), integrating by parts, and using the fact that integration over the complete spatial angle gives $\int\frac{x_jx_m}{r^2}d\vartheta = \frac{4\pi\delta_{jm}}{3}$, we obtain

$$\langle\sigma'\mathrm{rot}\mathbf{H}'\rangle_{\sigma(\mathbf{x},\, l)} \simeq \frac{2}{3}\left(\Phi_0(l)\frac{dl}{l} + k^2\int_0^\infty re^{ikr}\Phi(r, l)dr\frac{dl}{l}\right) \times \sigma(\mathbf{x}, l)\mathrm{rot}\mathbf{H}(\mathbf{x}, l). \tag{13}$$

Taking into account (13), we get from (6), (7) the mean value of the electric field accurate to the second order of smallness with respect to $\frac{dl}{l}$

$$\langle\mathbf{E}(\mathbf{x})\rangle_{\sigma_l(\mathbf{x})} = \left(\left(1 - \frac{1}{6}\Phi_0(l)\frac{dl}{l} + \bar{\varphi}(l)\frac{dl}{l}\right)^{-1} - \frac{2}{3}k^2\int r\frac{e^{ikr}}{r}\Phi(r, l)d\mathbf{x}'\frac{dl}{l}\right)\frac{1}{\sigma_l(\mathbf{x})}\mathrm{rot}\mathbf{H}_l(\mathbf{x}).$$

If $\omega\mu L^2\sigma(\mathbf{x}, l) \ll 1$, then in order to solve system (1) using the more smooth coefficient $\sigma(\mathbf{x})_l = \sigma_{0l}\exp\left[-\int_l^L\varphi(\mathbf{x}, l_1)\frac{dl_1}{l_1}\right]$ instead of $\sigma(\mathbf{x})_{l_0}$ and to obtain the correct mean value of the electric field strength, one should use σ_{0l}, whose limit value at $l \to l_0$ satisfies equation

$$\frac{d\ln\sigma_{0l}}{d\ln l} = -\frac{1}{6}\Phi_0(l) + \bar{\varphi}(l). \tag{14}$$

In a scale invariant medium, the mean values Φ_0, $\bar{\varphi}$ do not depend on scale l. In this case, solution of Eq. (14) is written as

$$\sigma_{0l} = \sigma_{0L}\left(\frac{l}{L}\right)^{-\frac{1}{6}\Phi_0 + \bar{\varphi}}, \tag{15}$$

where constant σ_{0L} describes the current in the medium at the largest scale at $l = L$ and determines the effective electroconductivity in the ongrid region.

NUMERICAL MODELING

The following numerical problem is solved in order to verify the formulas given above. Alternating the magnetic field with cyclic frequency ω influences the conducting medium. We consider that the strength of the external magnetic field is $\mathbf{H} = (0, H_y(z), 0)$, $H_y(z) = H_0$ at $z = 0$. The following dimensionless variables are used in the calculation: $\mathbf{x} = \frac{\hat{\mathbf{x}}}{L_0}$, $\sigma = \frac{\hat{\sigma}}{\sigma_0}$, $\sigma_0 = \langle\hat{\sigma}\rangle$, $\mathbf{H} = \frac{\hat{\mathbf{H}}}{H_0}$, $\mathbf{E} = \frac{L_0\sigma_0}{k_1H_0}\hat{\mathbf{E}}$, $k_1 = L_0\sqrt{\mu\omega\sigma_0}$. Thus, the problem is solved in a unit cube with $\sigma_0 = 1$, $H_0 = 1$. Equations (1) in dimensionless form are written as

$$\begin{aligned}\mathrm{rot}\mathbf{H}(\mathbf{x}) &= k_1\sigma(\mathbf{x})\mathbf{E}(\mathbf{x}),\\ \mathrm{rot}\mathbf{E}(\mathbf{x}) &= ik_1\mathbf{H}(\mathbf{x}).\end{aligned} \tag{16}$$

The magnetic and electric fields beyond the cube and at its boundaries are as follows:

$$H_x = H_z = 0,\ H_y = \exp\left(-\frac{k_1z}{\sqrt{2}}\right)\exp\left(\frac{ik_1z}{\sqrt{2}}\right),$$

$$E_x = \exp\left(-\frac{k_1z}{\sqrt{2}}\right)\exp\left(\frac{ik_1z}{\sqrt{2}} - \frac{i\pi}{4}\right),\ E_y = E_z = 0.$$

The conductivity field is modeled in the cube on the grid 256 × 256 × 256 with respect to spatial variables. Substitution of variables $l = 2^\tau$ is introduced for the calculations. The integral in Eq. (3) is replaced by the finite-difference formula, in which it is convenient to make a transition to logarithms with base 2:

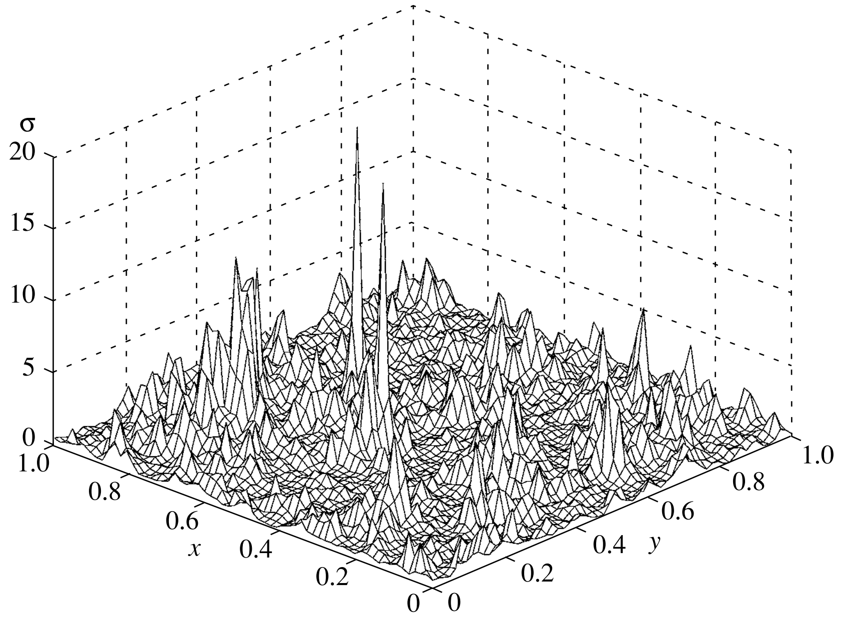

Fig. 1. Field of specific electroconductivity over middle section $z = 1/2$ for three scales $\sigma(\mathbf{x})_{l_0} = 2^s$, where $s = \left(-\sum_{i=-6}^{-4} \varphi(\mathbf{x}, \tau_i)\Delta\tau\right)$.

$$\sigma(\mathbf{x})_l = \exp\left[-\ln 2 \int\limits_{\log_2 l_0}^{\log_2 L} \varphi(\mathbf{x}, \tau)d\tau\right] \approx 2^{-\sum\limits_{i=-6}^{-4} \varphi(\mathrm{x}, \tau_i)\Delta\tau}, \tag{17}$$

where $\Delta\tau$ is a step by τ. In the calculations, $\Delta\tau$ was assumed to be equal to unity. The correlation of random field $\varphi(\mathbf{x}, \tau)$ was taken as

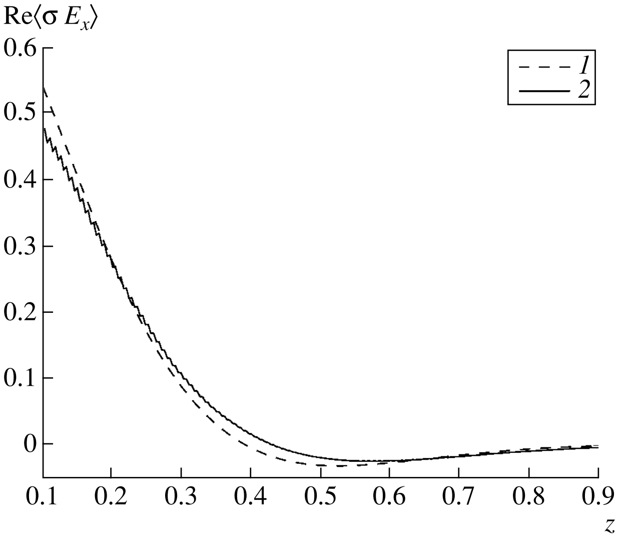

Fig. 2. (*1*) Real component of electric current density over x-axis for constant electroconductivity $\sigma = 1$. (*2*) Average real component of electric field density over x-axis obtained numerically for $\bar{\varphi} = 0$ with specific electric conductivity (17).

$$\langle\varphi(\mathbf{x}, \tau_i)\varphi(\mathbf{y}, \tau_i)\rangle - \langle\varphi(\mathbf{x}, \tau_i)\rangle\langle\varphi(\mathbf{y}, \tau_i)\rangle = \frac{\Phi_0}{\ln 2}\exp\left[-\frac{(\mathbf{x}-\mathbf{y})^2}{2^{2\tau_i}}\right]. \tag{18}$$

Field $\varphi(\mathbf{x}, \tau_i)$ is generated independently for each τ_i because statistical independence of fields with different scales is assumed. The number of additives in (17) and scales were chosen so that the scale of the strongest pulsations of conductivity would allow us to approximately substitute the probabilistic mean values by spatially average values, while the scale of the smallest pulsations would provide that the finite difference problem would approximate Eq. (1) quite well. Generally, three layers were used in the calculations $i = -6, -5, -4$. The minimal scale is $l_0 = \frac{1}{64}$, while the maximal scale is $L = \frac{1}{16}$. Algorithms with respect to rows and columns [7] were applied for numerical modeling of the random field. Constant Φ_0 in (18) should be taken from experimental data for inhomogeneous media. The author of [8] gives estimates of correlation functions for some parameters of natural media. In our work, we used $\Phi_0 = 0.3$. The method used in the calculations was based on a finite-difference scheme suggested in [9] and the decomposition method described in [10]. According to the procedure for deducing subgrid formulas for verification, repeated solution of the complete problem is required with probability averaging over small-scale pulsations. As a result, we get a sub-

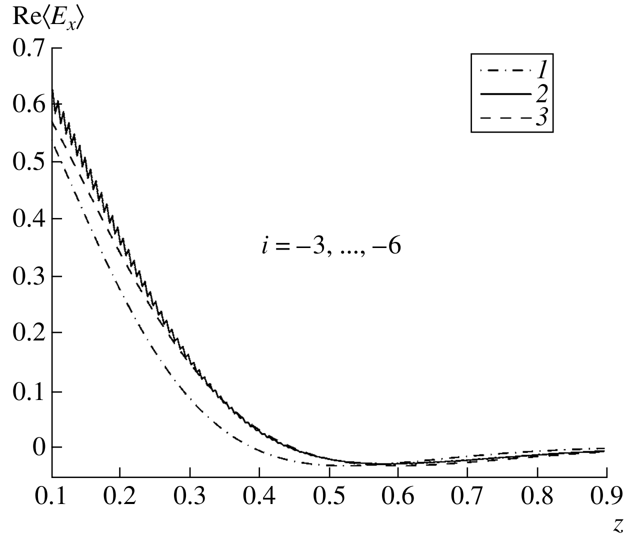

Fig. 3. (*1*) $\mathrm{Re}E_x$ for constant electroconductivity $\sigma = 1$ (*1*). (*2*) Mean values of $\mathrm{Re}E_x$ obtained numerically for (17) with $i = -6, \ldots, -3$. (*3*) Effective mean value of $\mathrm{Re}E_x$, Eq. (15), $\bar{\varphi} = \Phi_0/2$.

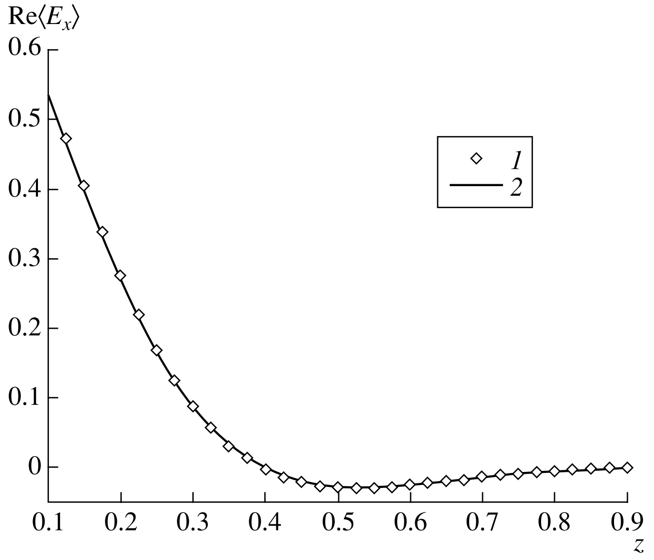

Fig. 4. (*1*) Component $\mathrm{Re}E_x$ at constant electroconductivity $\sigma = 1$. (*2*) Mean values of component $\mathrm{Re}E_x$ obtained numerically with σ calculated from Eq. (17), $i = -4, \ldots, -6$ at $\bar{\varphi} = \Phi_0/6$.

grid additive, which can be compared with a theoretical formula. In our work, we numerically calculate characteristics of current density and strength of the electric field in scale range (L, l). Their spatial averaging over planes (x, y) is performed for each value of z. Next, they are compared to the same characteristics obtained from theoretical formulas for the same scale range (L, l) and $l \to l_0$. The comparison starts from the problem with constant conductivity equal to $\sigma = 1$. Numerical calculation showed that one realization is sufficient to calculate the mean strength of the electric field. The size of the cube is not large enough to calculate the density of the electric field. Therefore, we used additional averaging over the Gibbs ensemble (the solution of the problem was performed twelve times with subsequent additional spatial averaging). In the calculations, parameter $k_1 = 6\sqrt{2}$, i.e., $L_0 = 6h_{skin}$, where h_{skin} is the thickness of skin layer. Figure 1 shows the field of specific electroconductivity for three scales calculated from Eq. (17) at the middle section $z = 1/2$. Figure 2 shows the results of numerical modeling of the real part of the electric field density average over internal points of the region and the real part of the electric field density obtained for the problem with the coefficient of electroconductivity $\sigma = 1$. Figure 3 shows the real part of the component of electric field strength along the x-axis obtained for constant specific electroconductivity $\sigma = 1$ (Fig. 3, no. 1). It also shows the real part of the electric field strength average over the x-axis obtained from numerical calculations based on Eq. (17), in which four additives $i = -6, \ldots, -3$ were used in the exponent of electroconductivity (Fig. 3, no. 2). The effective mean field (no. 3) was calculated using σ_{0l} from Eq. (15). Figure 4 shows the average numerical component of the real part of the electric field strength along the x-axis with the specific electroconductivity based on Eq. (17) and $\bar{\varphi} = \Phi_0/6$. In this case, the exponent in Eq. (15) is equal to zero and the average component of E_x coincides with the component of the electric field strength along the x-axis obtained for constant specific electroconductivity $\sigma = 1$.

Thus, the analysis in our approach does not go beyond differential equations and the theory of random functions. The main objects are parameters, mean values, and correlation functions that could be measured at least in principle. Theoretical estimates indicate that the mean density of electric current in the skin layer problem depends only slightly on small-scale inhomogeneities, unlike the average electric field strength, for which power dependence on the scale of inhomogeneities was revealed in the scale invariant medium. Direct numerical testing yielded a good agreement between the results based on theoretical formulas and the numerical experiment.

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