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Estimation of Block Sizes for Rock Masses with Non-persistent Joints

By

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Summary

Discontinuities or joints in the rock mass have various shapes and sizes. Along with the joint orientation and spacing, the joint persistence, or the relative size of the joint, is one of the most important factors in determining the block sizes of jointed rock masses. Although the importance of joint persistence on the overall rock mass strength has long been identified, the impact of persistence on rock strength is in most current rock mass classification systems underrepresented. If joints are assumed to be persistent, as is the case in most designs, the sizes of the rock blocks tend to be underestimated. This can lead to more removable blocks than actually exist in-situ. In addition, a poor understanding of the rock bridge strength may lead to lower rock mass strengths, and consequently, to excessive expenditure on rock support.

In this study, we suggest and verify a method for the determination of the block sizes considering joint persistence. The idea emerges from a quantitative approach to apply the GSI system for rock mass classification, in which the accurate block size is required. There is a need to statistically analyze how the distribution of rock bridges according to the combination of joint orientation, spacing, and persistence will affect the actual size of each individual block. For this purpose, we generate various combinations of joints with different geometric conditions by the orthogonal arrays using the distinct element analysis tools of UDEC and 3DEC. Equivalent block sizes (areas in 2D and volumes in 3D) and their distributions are obtained from the numerical simulation. Correlation analysis is then performed to relate the block sizes predicted by the empirical equation to those obtained from the numerical model simulation. The results support the concept of equivalent block size proposed by Cai et al. (2004, Int. J. Rock Mech. Min. Sci., 41(1), 3–19).

Keywords: Block size, jointed rock mass, GSI system, joint persistence, simulation.

Symbols

A_0 Block area determined by persistent joint sets,
A_b Block area delineated by discontinuous joint sets,
L Characteristic length of the rock mass under consideration,
$\bar{l}_i$ Accumulated joint length of set i,
p_f Joint persistence factor ($\sqrt{p_1 p_2}$ for 2D and $\sqrt[3]{p_1 p_2 p_3}$ for 3D),
p_i Joint persistence factor of set i,
s_i Average joint spacing of set i,
V_0 Block volume determined by persistent joint sets,
V_b Block volume delineated by discontinuous joint sets,
γ_i Angle between two joint sets.

1. Introduction

The rock mass classification systems, such as RMR (Bieniawski, 1973, 1976), Q (Barton et al., 1974), and GSI (Hoek et al., 1995) systems, are suggested based on accumulated engineering experiences and are excellent tools to characterize the complicated mechanical properties of in-situ rock masses during the design of rock structures. Since the mechanical and hydraulic behaviors of jointed rock masses are dominated by the discontinuities, most rock mass classification systems focus on determining specific values which represent discontinuity characteristics for design purpose. Because there are so many elements or factors that influence the engineering properties of rock masses and the inherent variability of the value is very large, it is almost impossible to exactly reflect all factors into a rock mass classification scheme. Consequently, the classification items and scores in most widely used rock mass classification systems are based on the experiences and subjectivity of the originators. In some cases, certain assumptions are essential, which may lead to inevitable errors in rock mass characterization. For example, the Q and GSI rock mass classification systems do not consider joint persistence explicitly. Joint persistence is only indirectly referred as "block interlocking" in the GSI system, using descriptive terms. Even though the joint persistence has been considered in the RMR system, the weighted values are underestimated as pointed out by Kim (2002). In addition, joints are often assumed fully persistent for the stability analysis of tunnels or slopes. This oversimplification may lead to overestimation of the size of the joints and hence the numbers of removable blocks near the excavated faces, resulting in excessive expenditure on rock support system. On the contrary, if the joint persistence characteristics are properly considered, the block size can be estimated accurately, and safe and economic design of the rock structure can be achieved.

Cai et al. (2004) proposed a quantitative approach, using block volume and joint condition factor, to utilize the GSI system. The GSI system is a rock mass classification scheme which provides the Mohr-Coulomb and Hoek-Brown strength parameters as well as deformation modulus of jointed rock masses (Hoek et al., 1995, 2002). Figure 1 presents the quantified GSI system chart. Once the block volume (V_b) and the joint condition factor (J_c) are known, the GSI value can be determined from the chart or from the following equation (Cai and Kaiser, 2006)

$$\mathrm{GSI} = \frac{26.5 + 8.79 \ln J_c + 0.9 \ln V_b}{1 + 0.0151 \ln J_c - 0.0253 \ln V_b}, \tag{1}$$

where J_c is a dimensionless factor and V_b is in cm^3.

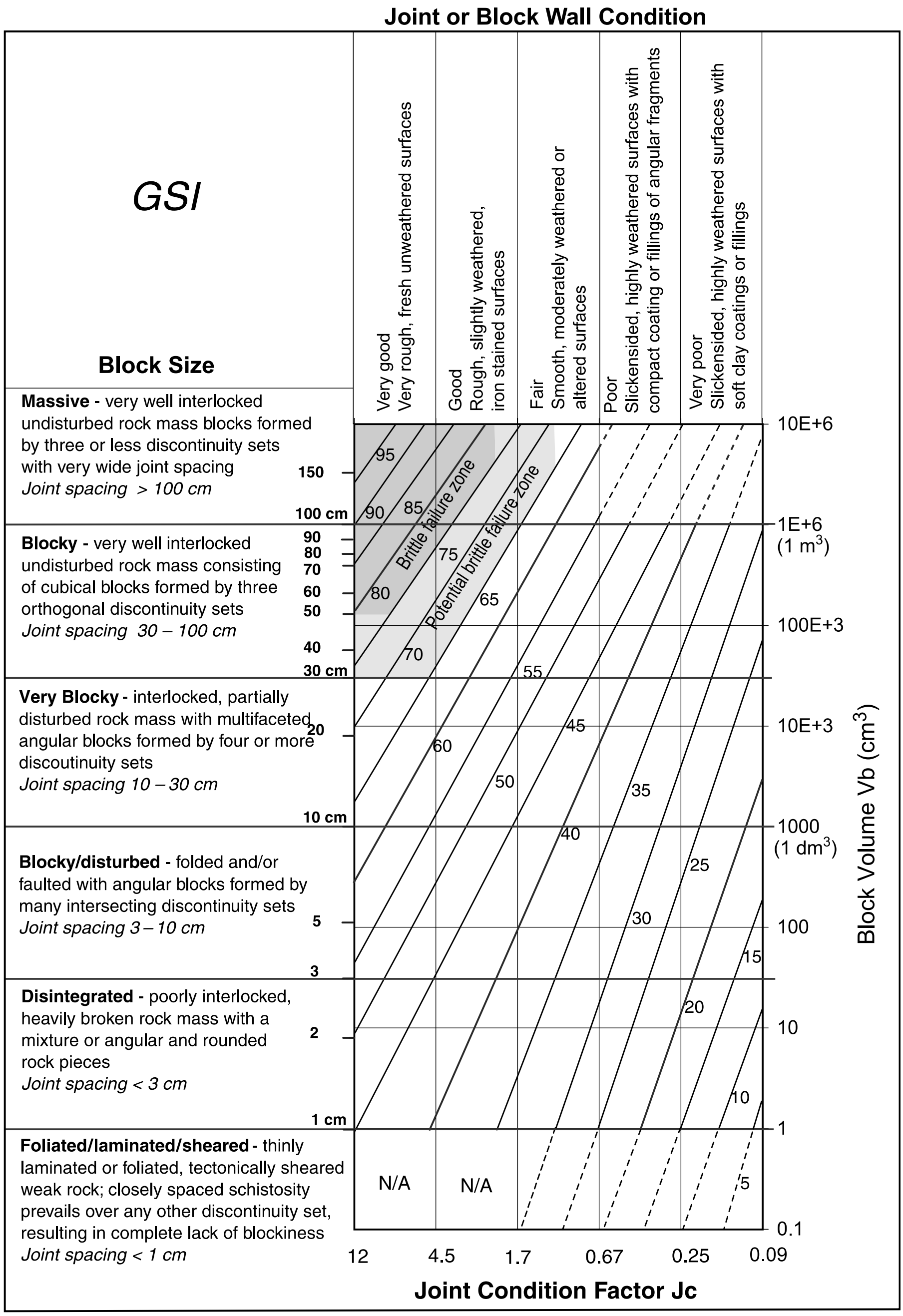


Fig. 1. Quantification of GSI chart (Cai et al., 2004)

One outstanding issue in this approach is the consideration of the joint persistence for the calculation of block volumes, i.e., to have a quantitative means to account the ‘‘block interlocking’’ verbal description in the original qualitative approach. A concept of equivalent block volume was proposed by Cai et al. (2004) but additional work was necessary to further verify the proposed idea.

To design the rock structures properly, a solid method is needed to estimate the block size as close to reality as possible, considering the joint persistence characteristics. There is a need to analyze statistically how the distribution of rock bridges according to the combination of joint orientation, spacing, and persistence will affect the actual size of each individual block. For this purpose, we generate various combinations of geometric conditions of discontinuities by the orthogonal arrays using UDEC (Itasca, 2004b) and 3DEC (Itasca, 2004a). Equivalent block area (2D) or volume (3D) and their distributions were obtained from the numerical simulation. Correlation analysis was then performed to relate the block sizes obtained from the numerical model simulation to those predicted by the empirical equation proposed by Cai et al. (2004).

Numerous tests or analyses on the level of each parameter are necessary to acquire a specific and statistically sound result through parametric analysis. Furthermore, because the number of tests increases exponentially with the increase of the number of parameters and levels, modification of the level of parameters to reduce test cases can result in false consequences as the weighted value of each parameter may be disregarded. Combinations of joint spacing, dip direction, dip angle, and length by the orthogonal arrays using the experimental design were generated to prevent this type of statistical errors from happening. Various combinations of parameters were used as input data in calculating block areas and volumes delineated by discontinuous joints.

In the next section, previous studies on rock bridges and joint persistence are briefly reviewed. Orthogonal arrays and the experimental design of the numerical simulation cases are described in Section 3. Simulation results and correlation analysis are presented in Section 4.

2. Joint Persistence and Rock Bridge

Joints usually occupy only a part of the surface extended by the joint plane to a given rock volume, a fact that is called persistence. Persistence is the term used to describe the areal extent or size of a discontinuity within a plane (Brady and Brown, 1992). There is no doubt that the stability of rock structures depends on the joint persistence. Instability is much more likely to occur if joints are fully persistent, given the usually much higher resistance of intact rock compared to that of joints (Einstein, 1993).

Persistence can be defined by relating the joint size to a reference size; either the sum of trace length l_i relative to the length of a collinear scan line L as $\Sigma l_i/L$ or the sum of individual joint surface areas a_i to the surface of a coplanar reference plane A as $\Sigma a_i/A$ (Dershowitz and Einstein, 1988). In practice, the true joint area can never be known accurately so that the joint length definition of joint persistence is often used. An illustration of persistent and non-persistent joints is presented in Figs. 2 and 3.

In jointed rock masses, rock bridges exist due to the non-persistent nature of the joints. A rock bridge is defined as a small bridge of rock separating coplanar or non-coplanar discontinuities. Rock bridges occur at a number of different scales and with a variety of geometries; only one example is presented in Fig. 3.

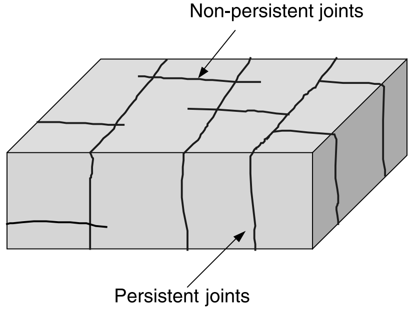


Fig. 2. Illustration of joint persistence

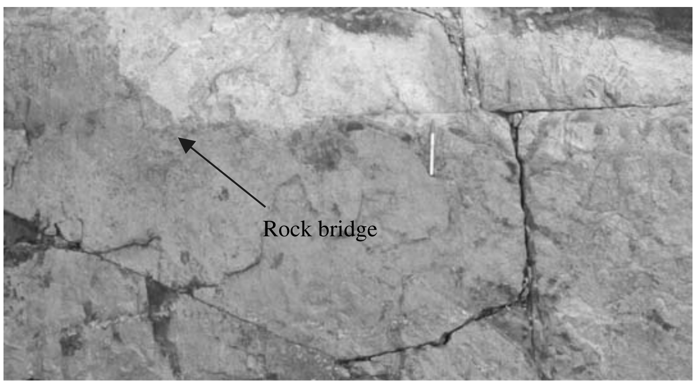


Fig. 3. Examples of rock bridge and non-persistent joints

The rock bridges play an important role in stabilizing the removable rock blocks. In particular, a rock block cannot fall or slide from an excavation or slope until the appropriate rock bridges have failed. The rock bridge failure involves the failure of the intact rock, which can be an order of magnitude stronger than the rock mass (Kemeny, 2005). The importance of rock bridges or non-persistence of joints on the stability of rock slopes has been studied by Einstein et al. (1983), Nichol et al. (2002), Sjöberg (1996) and others, and the effect of rock bridges on the strength or deformation properties of rock masses has been discussed by Kemeny and Cook (1986, 1991), Shen et al. (1995), Kemeny (2003, 2005), and amongst others.

If the joints are not persistent, i.e., with rock bridges, the rock mass strength is higher and the global rock stability is enhanced. Consequently, the apparent block volume should be larger for rock masses with non-persistent joints (Cai et al., 2004). The presence of discontinuous joints has a significant effect on the properties and behavior of rock masses and must be included in the engineering characterization of the rock masses. Diederichs and Kaiser (1999) demonstrated that the capacity of 1% rock bridge area equivalent to $10 \times 10\,\text{cm}$ or $100\,\text{cm}^2$ per $1\,\text{m}^2$ total area in a strong rock (UCS $> 200\,\text{MPa}$) is equivalent to the capacity of at least one cablebolt. Cai et al. (2004) proposed that if the joint length are only about 20% of the reference length, the equivalent block size is about five times larger than that with persistent joints. The issue of joint persistency is important but at the same time, it is one of the most difficult issues to be addressed in rock mechanics. In the present study, we use the distinct element method, in combination with the experimental design, to systematically investigate the effect of joint persistence on the block size distribution of jointed rock masses. The goal is to verify the concept of the equivalent block volume proposed by Cai et al. (2004) and to provide a method for the proper consideration of the joint persistence for use in the GSI system.

3. Estimation of Block Sizes Using Distinct Element Method

3.1 Orthogonal Arrays by the Experimental Design

Because the inherent variability of spatial characteristics of joints, such as joint orientation, length, and spacing, etc., is considerable and the number of influential parameters in determining a joint network is large, it is impractical to consider every simulation case by varying each parameter individually. Therefore, in order to save time and honor the statistical distributions of the parameters, it is necessary to develop an experimental design to collect sufficient sample data and develop a statistical analysis method to draw accurate conclusions from the collected sample data.

The statistical experimental design is widely used to assist data acquisition, decrease experiment error, and provide statistical analysis tools to describe the degree of errors. In statistical experimental design, two basic principles, i.e., replication and randomness, are required. Replication means that the same results can be obtained by the same experimental conditions. Randomness is required to ensure objectivity by putting the experiment objects into different conditions or by randomly arranging their experimental order. Furthermore, it is necessary to have a homogeneous experimental environment when applying the probability theory (Devore, 2000; Mendenhall and Sincich, 1995).

Orthogonal arrays are used to define the combination of joint spacing, dip direction, angle, and length of different joint sets for the estimation of the block area and volume using UDEC and 3DEC, respectively. The principle of orthogonal array design is to eliminate the inclination effect by other factors when certain factor is being investigated. When there are many parameters to be studied, the main effect of each parameter and some of the reciprocal actions are estimated, while other reciprocal actions are disregarded to reduce the number of tests. The benefit of orthogonal array design is that it calculates the parameter changes from experimental or field-mapping data, facilitates the easy preparation of input data for analysis of variance, and accommodates many parameters in experiment or simulation without increasing the test scale (Devore, 2000; Mendenhall and Sincich, 1995).

Assume that there is an orthogonal array presented as $L_9(3^4)$. This means that four factors or parameters could be used in the experiment and there are three stages in which the parameters can change their values. 81 experiments are required to obtain the results that are statistically significant and representative. If the orthogonal array is used, however, only nine experiments are needed.

In addition, we conducted ANOVA (Analysis of Variance) analysis using calculated results from the orthogonal arrays to analyze the effect of joint spacing, angle, and length on the decision of the block area or volume. ANOVA is used to uncover the main and interaction effect of categorical independent variables on an interval dependent variable. The key statistics in ANOVA is the *F*-test of the difference of group means. *F*-test is intended to find out if the means of the groups formed by values of the independent variable (or combinations of values for multiple independent variables) are different enough not to have occurred by chance. If the group means do not differ significantly, then it is inferred that the independent variables do not have an effect on the dependent variable. If the *F*-test shows that the independent variables are related to the dependent variable, then multiple comparison tests of significance are used to explore which value groups of the independent variables have the most influence to the relationship. Unlike regression, ANOVA does not assume linear relationships and it handles interaction effects automatically. It is not a test of difference in variances, but rather an assumption of relative homogeneity of variances. Thus, one of the key assumptions in ANOVA analysis is that the groups formed by the independent variables are relatively equal in size and have similar variances on the dependent variable. Like regression, ANOVA is a parametric procedure which assumes multivariate normality (Devore, 2000; Mendenhall and Sincich, 1995). The statistical analysis in this study has been achieved using the commercial software SPSS (SPSS Inc., 2004).

3.2 Calculation of Block Size Considering Joint Persistence

As stated above, block size, which is determined from the joint orientation, spacing, number of joint sets, and persistence, is an important indicator of rock mass quality. Block size is an areal (2D) or volumetric (3D) expression of joint density. Assume that the joint sets are persistent, the block area in 2D and the block volume in 3D can be calculated as

$$A_0 = \frac{s_1 s_2}{\sin \gamma_1}, \tag{2}$$

$$V_0 = \frac{s_1 s_2 s_3}{\sin \gamma_1 \sin \gamma_2 \sin \gamma_3}, \tag{3}$$

where s_i and γ_i are the joint spacing and the angle between joint sets, respectively (see Fig. 4). The subscript 0 indicates that the block size is calculated assuming persistent joints.

If s_i and $\bar{l}_i$ are the average joint spacing and the accumulated joint length of set *i* in the sampling plane, respectively, and *L* is the characteristic length of the rock mass under consideration, a joint persistence factor p_i is defined as

$$p_i = \begin{cases} \dfrac{\bar{l}_i}{L} & l_i < L \\ 1 & l_i \geq L. \end{cases} \tag{4}$$

An example that shows two joint sets with $p_i = 0.7$, according to the definition in Eq. (5), is presented in Fig. 5.

Because the joints are discontinuous, an equivalent spacing for continuous joint has to be found to use Eqs. (2) and (3) to calculate the block size. Based on the consideration that short joints are insignificant to the stability of the caverns with a

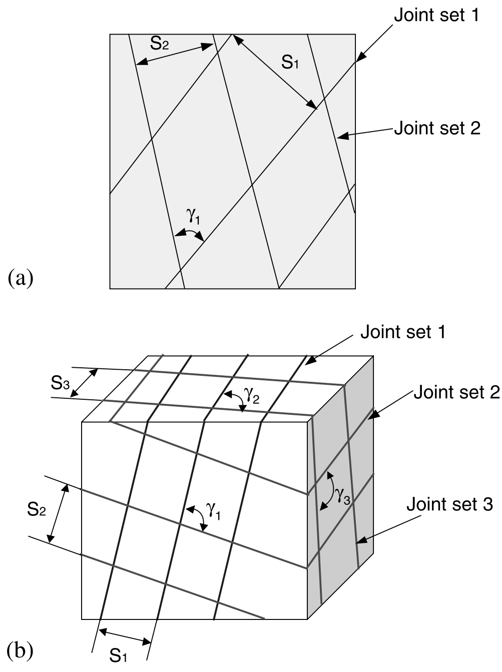


Fig. 4. Illustration of rock block areas **a** and volumes **b**

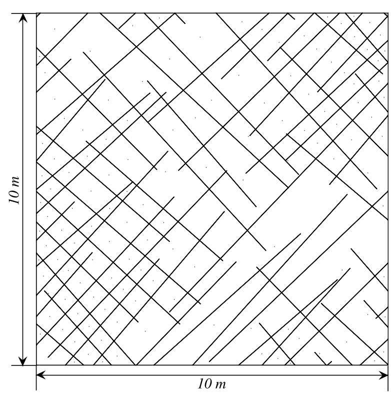


Fig. 5. An example showing joint persistence of 0.7

larger span, or are insignificant to the rock mass properties with a longer characteristic length (Cai and Horii, 1992), the equivalent block area and volume are expressed as

$$A_b = \frac{s_1 s_2}{\sin\gamma_1 \sqrt{p_1 p_2}}, \tag{5}$$

$$V_b = \frac{s_1 s_2 s_3}{\sin\gamma_1 \sin\gamma_2 \sin\gamma_3 \sqrt[3]{p_1 p_2 p_3}}, \tag{6}$$

where the subscript b indicates that the block size is calculated assuming non-persistent joints.

Equation (6) was proposed by Cai et al. (2004) to account the influence of joint persistence on the block volume and Eq. (5) is the equivalent version for two-dimensional block systems. The equations have been suggested based on the field experience but the validity has not been fully tested using real field mapping data. Given the difficulty in obtaining any real field data that truly represents the state of joint persistence, it is unlikely that this is achievable using current field mapping technology. Hence, an alternative approach, i.e., distinct element modeling method, is used to generate rock block systems that consider the joint persistence.

3.3 Simulation of the Block Systems Using UDEC and 3DEC

Block systems are generated by numerical simulation using the distinct element method. The generation of the blocks can be achieved using continuous and discontinuous joints. This feature is utilized to calculate the block sizes delineated by non-persistent joints.

3.3.1 Block Area (2D Simulation)

First, 2D block systems are simulated using UDEC. UDEC requires the following inputs to generate joint sets: average angle between the horizontal plane and the joint set, and its standard deviation (a_m, a_s); average spacing of each joint set and its standard deviation (s_m, s_s); average length and its standard deviation (t_m, t_s); average gap and its standard deviation (g_m, g_s) (see Fig. 6).

We consider two joint sets in a region of 10×10 m (width $\times$ length). The joint spacings are selected based on the spacing of "*very blocky*" to "*massive*" rock masses in the quantitative GSI chart shown in Fig. 1. This is because the role of rock bridges are more important in hard, massive rock masses than in soft, disintegrated rock masses. The average joint spacings thus selected are 10, 30, 50, 100, 150, and 200 cm for the orthogonal array.

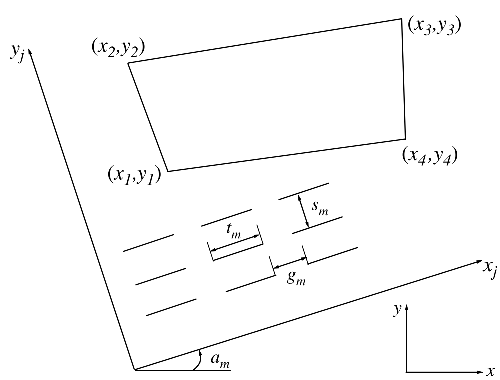


Fig. 6. Parameters for generating joint sets in UDEC (Itasca, 2004b)

The average length of joint set 1 is fixed at 1000 cm (10 m), and the average length of joint set 2 is varied at 100, 300, 500, 700, and 900 cm to consider the non-persistency of the joints. The average angles between the horizontal plane and joint set 1 are assumed to be 15°, 30°, 45°, and 60° for the orthogonal array. The existing two joint sets are first assumed perpendicular to each other, which is representative of most joint geometries observed in the field. At the same time, we also investigated joint geometries with two non-orthogonal joint sets. The angles between the two joint sets are 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, and 150°.

For each joint set, the position and orientation of individual joint are random because of the consideration of deviations in the input parameters. In this fashion, the average gaps of joint sets was found to have a negligible effect on the block area calculation, and this means that the persistence characteristics can be fully described by joint length alone. Consequently, the joint average gaps are not considered in the orthogonal array. In this way, we selected six, four, and five levels of factors for joint spacing, joint angle with respect to the horizontal plane, joint length, respectively, and the combination of the parameters resulted in 49 simulation cases in the orthogonal array as shown in the Appendix (Table 3).

In addition, the standard deviation of each parameter was considered separately. The standard deviations were assumed 5° and 10° for the joint angles, 10, 20, 30% of the means for joint spacings and lengths, respectively. Considering all the possible combination in the orthogonal array, 294 simulations, resulted from multiplying 49 cases by 6 conditions due to both the orthogonal arrays and the standard deviation, were required to generate two-dimensional block systems delineated by the two joint sets.

3.3.2 Block Volume (3D Simulation)

To generate three-dimensional block systems using 3DEC (Itasca, 2004a), the following input data are needed: average dip direction/dip angle of each joint set and its standard deviation (dd_m, dd_s, d_m, d_s), average spacing of each joint set and its standard deviation (s_m, s_s), and average persistence of each joint set.

A representative volume element (RVE), which is $10 \times 10 \times 10$ m in dimension and contains three joint sets, is considered in the present study. The average spacings considered for each joint sets in the orthogonal array are 20, 30, 50, 100, 150, and 200 cm. For simplicity, only mutually orthogonal joint sets are considered and the joint dip directions/dips for the three joint sets are 000/00°, 000/90°, and 090/90°, respectively.

Unlike UDEC, joint persistence can be directly inputted in 3DEC. The joint persistence of joint set 1 is set to 1.0, which means that this joint set is continuous or persistent. Joint sets 2 and 3 are discontinuous whose persistence varies (0.1, 0.3, 0.5, 0.7, and 0.9). Not all joint sets in 3DEC can be considered as non-persistent. At least one joint set has to be continuous to generate block systems in 3DEC. As will be seen in the simulation results presented later, this shortcoming of the code does not impede us from reaching a sound conclusion about the influence of joint persistence on the block sizes.

The joint spacing and persistence factors with six and five levels, respectively, were selected to generate an orthogonal array for the calculation of block volumes. This resulted in an orthogonal array which contained 25 numerical experiments, as shown in Appendix (Table 4).

In the mean time, although the orientations of joint sets were not directly used in the orthogonal array design, different values were assumed to consider their variability, i.e., standard deviations of the average dip directions/dip angles were assumed to be 0°, 5°, 10°, for different simulation cases. The standard deviations of the joint spacing were assumed to be 10, 20, 30% of the mean values in the analysis. This resulted in: three cases whose standard deviation of joint set spacing were 10, 20 and 30%, respectively, without considering the standard deviation of the dip directions/dips; three cases whose standard deviations of the joint set spacing were 10, 20 and 30%, respectively, while the standard deviations of dip direction/dip were all 5°; three cases whose standard deviation of the joint set spacing were 10, 20 and 30%, respectively, while the standard deviations of dip directions/dips were all 10°. In total, there were 225 cases, resulted from multiplying 25 cases by 9 conditions due to both the orthogonal arrays and the standard deviation, for the three-dimensional block system simulation using 3DEC. The orthogonal array design greatly reduced the effort of numerical model simulation while at the same time ensured the statistical significance and the validity of the simulation results.

4. Simulation Results

4.1 Block Area Size and Distribution

Figure 7 presents one example showing the blocks generated by two persistent joint sets in two-dimensions. The joint spacings for the two joint sets are 50 and 70 cm and the joint spacing deviations are considered in the model. Block shapes are regular and the block size distribution can be approximated by a lognormal distribution. Figure 8(a) presents the blocks delineated by two non-persistent joint sets whose average joint orientation angles and spacings are same as in Fig. 7. The joint persistences for the two joint sets are all 0.5. The standard deviations for the joint set orientations are 5°. It is seen that some joints terminate in the intact rock, which resembles the rock bridges observed in rock masses. In engineering practice, what

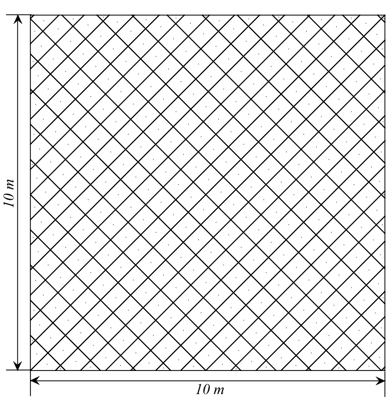


Fig. 7. Example of the block system generated by two continuous joint sets using UDEC

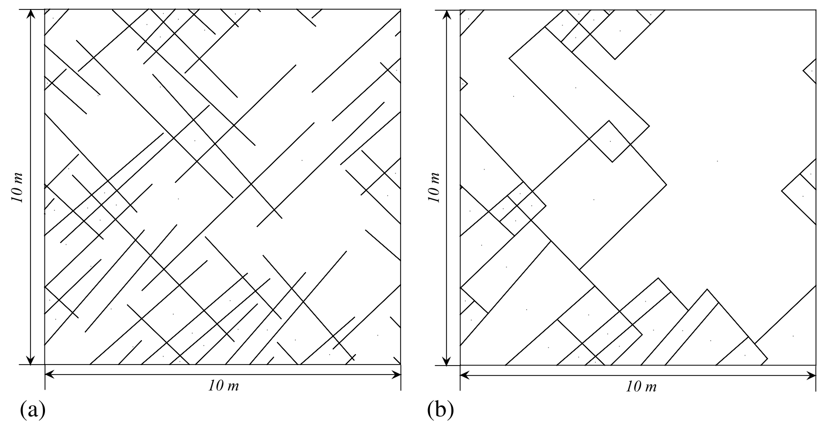


Fig. 8. Example of the block system generated by two discontinuous joint sets in UDEC: **a** before incomplete joint deletion; **b** after deletion of joints terminating inside blocks

controls the stability of a cavern or a slope is the distinct rock blocks, which are shown in Fig. 8(b). The block system shown in Fig. 8(b) is generated by eliminating the incomplete joints (joints terminating inside intact rocks). Rock blocks show different sizes and shapes, which follow certain distributions depending on the variability of the input parameters. In this study, we have generated the block systems with different joint lengths according to the joint persistence characteristics.

The important concept that needs further explanation is the adoption of the distinct block system in design. The deletion of the incomplete joints ignores fracture propagation near the joint tips. When there are a few joints in the rock, the effect of the rock bridge and the fracture propagation from the joint tips cannot be ignored. However, when there are many joints in the rock mass, as shown in Fig. 8(a), the distinct blocks shown in Fig. 8(b) will dominate the behavior of the rock masses when loaded because discontinuous joints have tensile strength. It is similar to the fact that groundwater will flow dominantly in the connected fracture networks. When the rock mass shown in Fig. 8(a) is loaded, it will behave predominantly the same way as if the rock mass shown in Fig. 8(b) is loaded. The contribution or influence from the isolated, incomplete joints, as they are called in UDEC, is negligible. Of course, certain degree of approximation is involved in the analogy. For rock engineering application, this is sufficient in terms of defining the macroscopic strength and deformation parameters of the joint rock masses.

Next, the block areas in each simulation case were calculated to identify the distribution characteristics of the block size of different joint persistence characteristics, using Bestfit in @RISK (Palisade Corporation, 2001). Bestfit is a statistical tool which automatically determines the probability density function of the input data through its test-of-goodness process. Figure 9(a) presents the probability density function of the average area for all simulated cases and Fig. 9(b) shows the individual block size distribution for simulation Case-9 (Table 3). Both distributions follow lognormal distributions, which is in agreement with the observation by Mahtab and Grasso (1992), Song and Lee (2001), Song et al. (2001) and others that the probability density function of the block size and the joint persistence show a lognormal distribution.

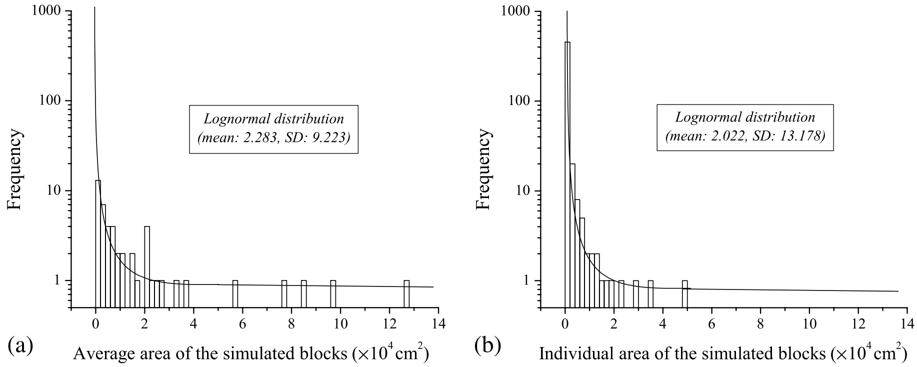


Fig. 9. Block size distribution functions: **a** average block areas of all simulation cases; **b** individual block size distribution for simulation Case-9 (2D)

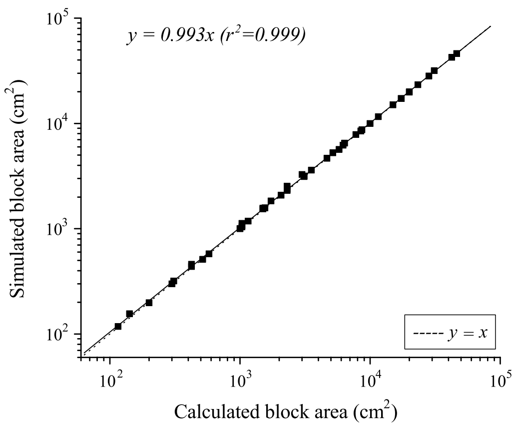


Fig. 10. Regression between calculated and simulated block areas delineated by two continuous joint sets

To validate Eq. (5), the equation for the calculation of the equivalent block size in 2D considering joint persistence, we compared the calculated areas by Eq. (5) to the average areas calculated from UDEC by the orthogonal array. Figure 10 shows the regression analysis result of the average areas calculated by both Eq. (2) (horizontal axis) and UDEC (vertical axis) for the case with continuous joints. The correlation between the two approaches is very good.

Figure 11 compares the average block sizes (areas) of the 294 cases simulated by UDEC, using the orthogonal array in Table 3 and considering the variability of each parameter, to those calculated by Eq. (5), for the case with non-persistent joints. The horizontal axis indicates the block size calculated by Eq. (5) and the vertical axis corresponds to the block size simulated by UDEC. In this figure, Groups 1 to 3 represent all the simulation cases that the standard deviation of the angle between the two joint sets is 5° and the standard deviations of joint spacing and length are 10, 20, and 30% of the mean values, respectively. Groups 4 to 6 represent all the remaining cases that the standard deviation of the angle between the two joint sets is 10° and the standard deviations of joint spacing and length are 10, 20, 30% of the mean values, respectively.

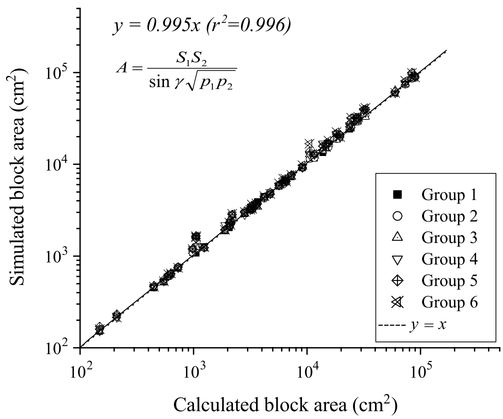


Fig. 11. Regression between calculated and simulated block areas delineated by two discontinuous joint sets

Using SPSS, regression analysis was performed and the correlation between the UDEC result (y) and analytical results (x) is found as

$$y = 0.995x \quad (r^2 = 0.996). \tag{7}$$

Consequently, Eq. (5) can be written as

$$A_b = 0.995\frac{s_1 s_2}{\sin\gamma_1\sqrt{p_1 p_2}} \approx \frac{s_1 s_2}{\sin\gamma_1\sqrt{p_1 p_2}}, \tag{8}$$

where A_b is the average equivalent block size calculated from the average joint spacing, orientation, and persistence in 2D. For practical purpose, a slope of 1 is sufficiently accurate.

The block sizes formed by discontinuous joint sets are normalized to the ones formed by continuous joint sets in Fig. 12. The combined joint persistence factor p_f is defined as $\sqrt{p_1 p_2}$ and plotted as the horizontal axis in Fig. 12. Although large variability is observed in the data set, there is a general trend that as the persistence factor approaches

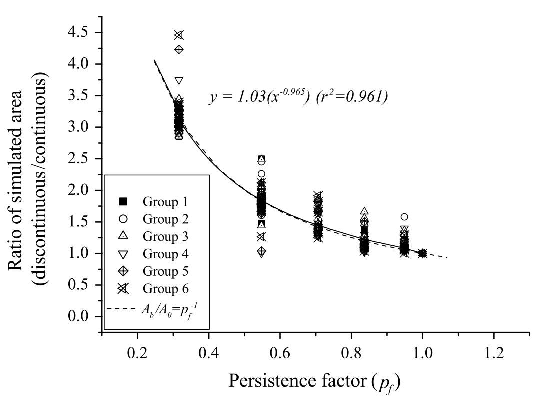


Fig. 12. The relationship between the normalized block size and the joint persistence factor

Table 1. Summary of the variance analysis results (2D)

Source of variation	Sum of square	Degree of freedom	Mean square	F	Significance
Spacing factor ($s_1 \times s_2$)	144.657	18	8.037	12.151	0.000
Angle factor ($\sin \gamma_1$)	6.100	4	1.525	2.306	0.044
Persistence factor ($\sqrt{p_1 p_2}$)	11.664	4	2.916	4.409	0.000
Error	13.228	20	0.661	–	–

unity, the normalized block size moves toward 1. The regression curve is obtained as

$$\frac{A_b}{A_0} = 1.03(p_f)^{-0.965} \quad (r^2 = 0.961), \tag{9}$$

where A_0 is the block area calculated from joint spacings and angles without the consideration of joint persistence.

An ANOVA analysis was conducted to evaluate the effect of geometric characteristics of joints, such as joint spacing, angle between joint sets, and joint persistence, on the decision of the block size in the simulation. Table 1 presents the result of the ANOVA analysis by SPSS. The sum of square is the sum of square errors for each treatment, and the degree of freedom is the number of populations being tested minus one. The sums of square divided by the degree of freedom is called mean square. F is the ratio of the mean square of treatment factor to the mean square of Error.

According to the statistical analysis, the F value, which represents the contribution of each factor to the statistical significance, is the highest for joint spacing (12.151), moderate for joint persistence (4.409), and lowest for the angle between joint sets (2.306). This means that joint spacing and persistence are more important than the angle between the joint sets for the decision of block sizes. Joint spacing is the single most important parameter that defines the block size. Since the contribution of joint persistence is greater than that of the angle between joint sets, the persistence characteristics should be considered separately in the rock mass classification.

The reason that we fixed the length of joint set 1 to 1000 cm while changing the lengths of joint set 2 is to limit the simulation cases and at the same time to ensure that realistic joint systems can be generated for all the combinations of joint geometric parameters. In fact, the length of joint set 1 can be shorter, but there is a limitation to

Table 2. Orthogonal array for the input parameter for block system generation using UDEC (short lengths for both joint sets)

No.	s_1 (cm)	s_2 (cm)	a_1 (°)	a_2 (°)	L_1 (cm)	L_2 (cm)
1	30	200	30	−15	300	500
2	30	150	15	−30	300	500
3	50	150	30	−15	500	500
4	30	200	15	−15	500	300
5	30	150	30	−30	500	300
6	50	200	30	−30	300	300
7	50	150	15	−15	300	300
8	50	200	15	−30	500	500

Note: s joint spacing, a_1, a_2 joint inclination angle relative to x-axis, positive anticlockwise, L joint length.

the joint spacing and angle to be included in the orthogonal array. One separate investigation considering shorter joint length for both joint sets is conducted and the simulation cases are listed in Table 2. In this case, the two joint sets are orthogonal and the joint lengths are 300 and 500 cm. The correlation between the UDEC result (y) and the equation result (x) is $y = 0.994x$, ($r^2 = 0.995$), showing approximately the same results for the case with longer joint length of joint set 1. Therefore, it can be said that the general conclusions reached based on simulation cases listed in Table 3 hold true for block systems with shorter lengths for both joint sets.

4.2 Block Volume Size and Distribution

Rock blocks are three-dimensional in nature. Based on the same approach adopted for two-dimensional block area investigation using UDEC, the block volumes in a three-dimensional setting are generated using 3DEC. Figure 13 presents one example of the three-dimensional block system generated by three continuous joint sets. The joint spacings for all three joint sets are 1 m and variabilities of joint orientations and

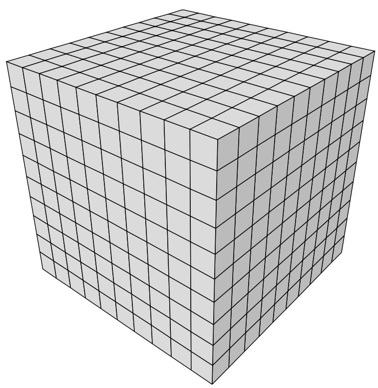
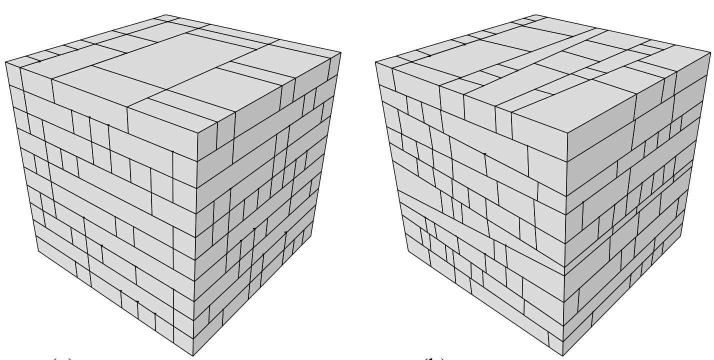


Fig. 13. Example of the blocks delineated by three continuous joint sets in 3DEC



(a) (b)

Fig. 14. Examples the blocks delineated by discontinuous joint sets in 3DEC: **a** without joint orientation and spacing deviations; **b** with joint orientation and spacing deviations

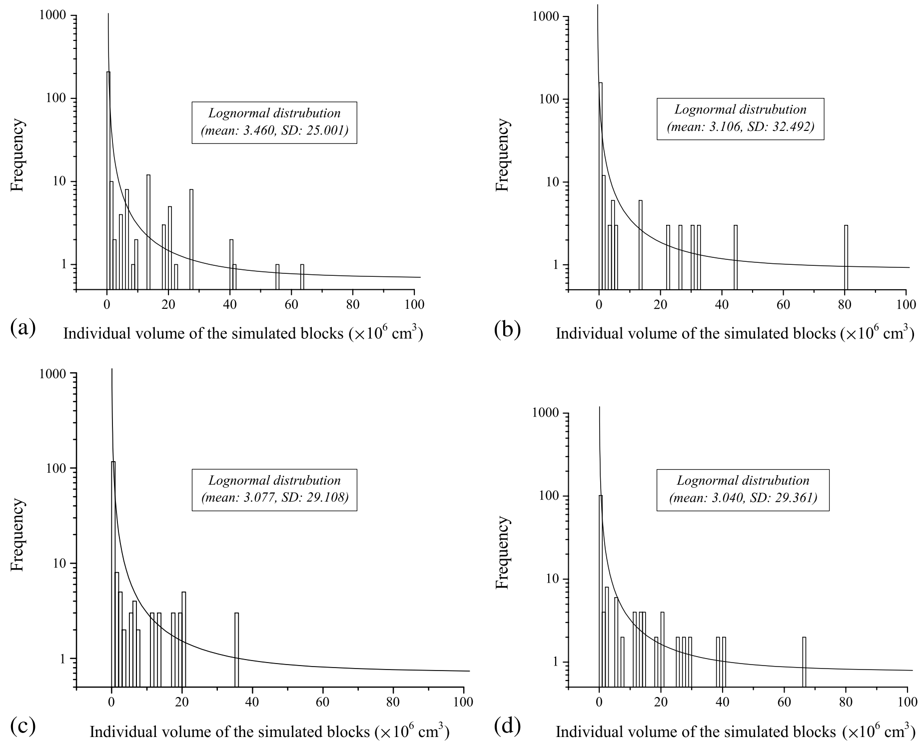


Fig. 15. Examples of distribution functions of the block volumes delineated by discontinuous joint sets

spacings are not considered. This results in a set of blocks of equal volumes. Figure 14(a) presents another example showing the blocks delineated by one continuous joint set and two discontinuous joint sets, whose average orientations and spacings are the same as in Fig. 13, while the variabilities of the joint orientation and spacing are not considered. The joint persistence for the two discontinuous joint sets is 0.5. When the deviations of joint orientation (5°) and spacing are considered, the resulting block system is shown in Fig. 14(b).

The distribution characteristics of the three-dimensional blocks generated with different joint persistence characteristics are analyzed using Bestfit. Figure 15(a) presents one example of the probability density distribution of each block without considering the deviations of joint orientation and spacing (Case-20 in Table 4). Figure 15(b) to (d) presents the probability density distributions of each calculated block volume of the same joint average spacings as in Fig. 15(a), but with average deviations for dip direction and dip of 5° in all cases, while the average deviations of joint set spacing are assumed to be 10, 20, and 30% of the mean values, respectively. In all cases simulated, the probability density distributions are best described by the lognormal distribution.

The block volumes obtained from 3DEC simulations of the cases listed in Table 4 are compared to the block volumes calculated using Eq. (6) and the result is presented in Fig. 16. Very good correlation is found between the 3DEC simulation and the

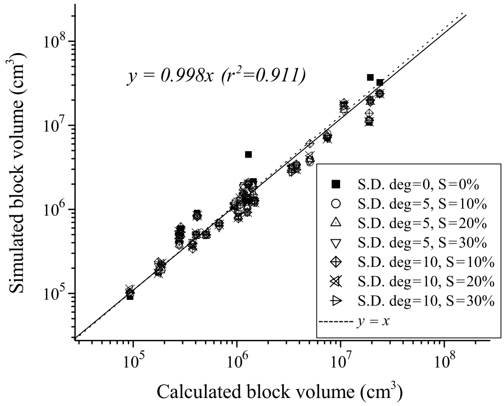


Fig. 16. Correlation between calculated (Eq. (7)) and simulated (3DEC) block volumes

proposed equation results. Since the average angles between the joint sets are 90° they are not included in the analysis as variables. Based on the regression analysis result, Eq. (6) can be rewritten as

$$V_b = 0.998\frac{s_1 s_2 s_3}{\sqrt[3]{p_1 p_2 p_3}} \approx \frac{s_1 s_2 s_3}{\sqrt[3]{p_1 p_2 p_3}} \quad (r^2 = 0.911,\ p_i < 1). \tag{10}$$

In this fashion, the method proposed by Cai et al. (2004) to calculate the equivalent block volume considering joint persistence has been validated by distinct element model simulation using 3DEC. The consideration of the joint persistence factor for each joint sets (p_i, $i = 1, 2, 3$) and their combination in the form $\sqrt[3]{p_1 p_2 p_3}$ is adequate to account the effect of joint persistence on the calculation of the equivalent block volume.

However, the role of the persistence is more important in the massive blocks than in the disintegrated blocks. This mean that the contribution of the rock bridges to the stability is more significant for blocky and massive rock masses. Analysis of the block system delineated by shorter joints and wider joint spacing is required to resolve this issue. A new orthogonal array is therefore created as shown in Appendix (Table 5).

Compared to Table 4, Table 5 presents the orthogonal array to generate massive blocks whose joint spacings are between 100 and 300 cm and the joint persistence factor is less than 0.5. The levels of joint spacing selected for the simulation are 100, 150, 200, and 300 cm and the levels of joint persistence factor are 0.1, 0.3, and 0.5, resulting in an orthogonal array which contains 16 simulation cases. A total of 144 simulations, resulted from multiplying 16 cases by 9 conditions due to both the orthogonal arrays and the standard deviation, are conducted. In all simulations, persistent joints are assumed for one joint set due to the limitation of the block generation routine in 3DEC.

Figure 17 presents the correlation between the average block volumes simulated by 3DEC and those calculated by Eq. (6), for wider joint spacing and lower joint persistence factor indicated in Table 5. Compared to Fig. 16, where the joint spacings are relatively small, Fig. 17 shows a slightly different correlation coefficient. The 3DEC simulated block volume is about 5% smaller than that calculated by Eq. (6). Figure 18 combines the simulation results for narrow and wide joint spacing. Based on

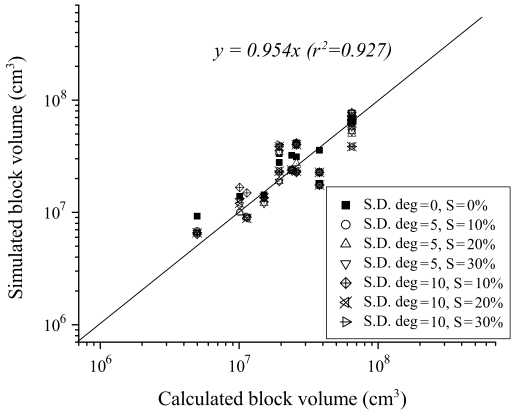


Fig. 17. Correlation between the calculated and simulated block volumes with wide joint spacing

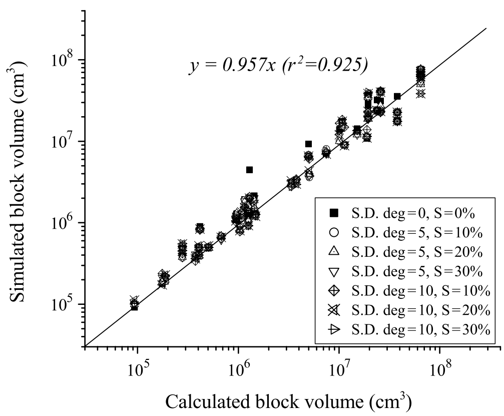


Fig. 18. Correlation between the calculated and simulated block volumes (all data)

the regression analysis results, Eq. (10) could be rewritten as

$$V_b = 0.957\frac{s_1 s_2 s_3}{\sqrt[3]{p_1 p_2 p_3}} \approx \frac{s_1 s_2 s_3}{\sqrt[3]{p_1 p_2 p_3}} \quad (r^2 = 0.925,\ p_i < 1). \tag{11}$$

Figure 19 presents the relationship between the normalized block volume and the combined joint persistence factor as well as the critical block size due to the persistence factor. The average block volume obtained from the 3DEC simulation is normalized to the average block volume calculated without the joint persistence consideration. The combined joint persistence factor p_f is defined as $p_f = \sqrt[3]{p_1 p_2 p_3}$. The regression equation obtained is

$$\frac{V_b}{V_0} = 1.03({p_f}^{-1.034}) \quad (r^2 = 0.829), \tag{12}$$

or simply as

$$\frac{V_b}{V_0} \approx {p_f}^{-1}, \tag{13}$$

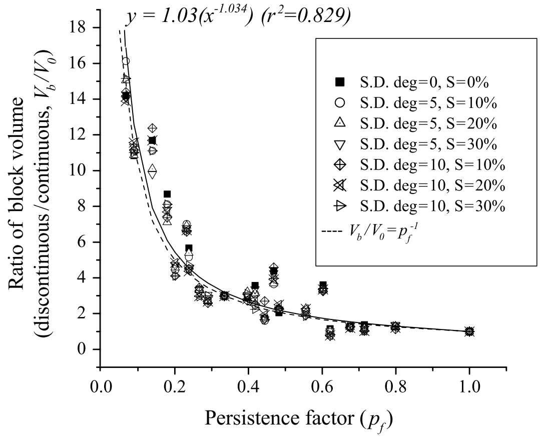


Fig. 19. Relationship between the normalized block volume and the joint persistence factor

where V_0 is the block volume calculated from joint spacings and angles without the consideration of the joint persistence. Equation (13) ensures consistent results be obtained when all joint sets are persistent. From the above analysis, the concept of the equivalent block volume and its calculation equation proposed by Cai et al. (2004) has been validated. When the joint persistence factor is greater than 0.5, the equivalent block volume delineated by discontinuous joint sets is roughly the same as the one determined by continuous joint sets. However, when the persistence factor is smaller than 0.5, the equivalent block volume delineated by discontinuous joint sets increase drastically as compared to the block volume without the joint persistence consideration. As can be seen in Fig. 1, when the block volume is one order of magnitude different, the difference in the GSI value could be as high as 10. This means that estimated rock mass strength, especially the tensile strength, with the consideration of joint persistence, will increase accordingly. With the proper consideration of the joint persistence, more accurate calculation of rock mass strength and hence rock mass characterization can be conducted and safe and economic design of structures in rock masses can be achieved.

5. Conclusions

In this study, distinct element analysis tools (UDEC and 3DEC) are used to generate block systems delineated by joint sets with different persistence. The objective is to statistically analyze how the joint geometric parameters, such as joint orientation, spacing, and persistence, will affect the size of each individual block and average size of the block system, with the focus on the study of the influence of joint persistence on the determination of the block sizes.

Based on the experimental design, blocks systems with various joint persistence factors, spacing, orientation, etc., are generated. It is found that the probability density functions of the block area (2D) and volume (3D) obey the lognormal distributions, which agree with the conclusions reached by other investigators. According to the result of the ANOVA analysis, the F value for joint persistence, which represents the factor's contribution to the statistical significance of the block size, is higher than that of the angle between joint sets, indicating that it is important to consider joint persistence in a rock mass classification program.

Correlation analysis was performed to relate the block size generated from the numerical tools to the ones calculated using the equation proposed by Cai et al. (2004). For two-dimensional case, the block sizes (A_b) delineated by two discontinuous joint sets are related to the blocks sizes (A_0) without the consideration of joint persistence as $A_b/A_0 \approx {p_f}^{-1}$, where p_f is the combined joint persistence factor defined as $p_f = \sqrt{p_1 p_2}$ for 2D models. For three-dimensional models, the block volumes (V_b) delineated by three discontinuous joint sets are related to the blocks volumes (V_0) without the consideration of joint persistence as $V_b/V_0 \approx {p_f}^{-1}$, where p_f is the combined joint persistence factor defined as $p_f = \sqrt[3]{p_1 p_2 p_3}$. Using the equations given above, the effects of joint persistence on the block area and volume delineated by discontinuous joint sets can be quantified, and approximate equivalent block sizes of jointed rock masses that contain discontinuous joints can be obtained. In this fashion, the interlocking due to the presence of discontinuous joints is indirectly considered by increased block size in the GSI system. This scheme is quantitative and it further expands the easy utilization of the GSI system for the determination of strength and deformation parameters of jointed rock masses.

Appendix: Orthogonal Arrays Used for the Simulation

Table 3. Orthogonal array for the input parameter for block system generation using UDEC

No.	s_1 (cm)	s_2 (cm)	a_1 (°)	a_2 (°)	L_1 (cm)	L_2 (cm)	No.	s_1 (cm)	s_2 (cm)	a_1 (°)	a_2 (°)	L_1 (cm)	L_2 (cm)
1	100	100	15	−15	1000	500	26	10	10	15	−15	1000	100
2	30	100	60	−15	1000	300	27	50	10	45	−30	1000	500
3	50	30	60	−30	1000	100	28	150	30	15	−30	1000	300
4	150	50	45	−60	1000	100	29	200	200	45	−15	1000	300
5	100	50	45	−30	1000	700	30	200	150	15	−30	1000	500
6	10	200	60	−60	1000	500	31	10	50	15	−45	1000	300
7	50	10	30	−60	1000	300	32	30	30	45	−15	1000	700
8	30	50	30	−45	1000	500	33	200	10	15	−45	1000	100
9	150	150	60	−15	1000	100	34	30	150	30	−45	1000	100
10	50	50	30	−15	1000	300	35	10	100	45	−45	1000	900
11	10	10	30	−15	1000	900	36	50	100	15	−45	1000	100
12	10	200	45	−30	1000	100	37	10	100	45	−30	1000	100
13	10	10	15	−30	1000	900	38	150	10	45	−45	1000	500
14	200	10	45	−15	1000	300	39	100	10	60	−45	1000	300
15	150	100	30	−30	1000	300	40	50	150	45	−15	1000	900
16	10	150	15	−60	1000	300	41	100	200	30	−15	1000	100
17	100	30	15	−60	1000	900	42	10	10	60	−45	1000	700
18	150	200	30	−45	1000	900	43	100	150	45	−45	1000	300
19	200	50	60	−30	1000	900	44	10	150	30	−30	1000	700
20	10	30	45	−45	1000	300	45	10	50	15	−15	1000	100
21	10	30	30	−15	1000	500	46	100	10	30	−30	1000	100
22	200	100	30	−60	1000	700	47	30	10	45	−60	1000	100
23	150	10	15	−15	1000	700	48	10	10	30	−30	1000	300
24	30	200	15	−30	1000	300	49	200	30	30	−45	1000	100
25	50	200	15	−45	1000	700							

Note: s joint spacing, a_1, a_2 joint inclination angle relative to x-axis, positive anticlockwise, L joint length.

Table 4. Orthogonal array for the input parameters of joints for block system generation using 3DEC

No.	Spacing 1 (cm)	Spacing 2 (cm)	Spacing 3 (cm)	Persistence 1	Persistence 2
1	50	150	50	0.1	0.7
2	20	30	150	0.1	0.1
3	100	50	50	0.3	0.9
4	20	150	30	0.5	0.9
5	30	30	50	0.9	0.5
6	20	100	50	0.7	0.3
7	50	200	30	0.9	0.3
8	100	150	200	0.9	0.1
9	100	200	150	0.7	0.7
10	30	20	100	0.1	0.9
11	150	20	50	0.5	0.1
12	150	150	100	0.7	0.5
13	50	30	20	0.7	0.9
14	30	50	30	0.7	0.1
15	20	200	20	0.3	0.5
16	100	100	30	0.1	0.5
17	150	50	20	0.1	0.3
18	30	100	20	0.5	0.7
19	100	30	100	0.5	0.3
20	30	150	150	0.3	0.3
21	50	50	150	0.5	0.5
22	150	100	150	0.9	0.9
23	50	100	100	0.3	0.1
24	20	50	100	0.9	0.7
25	150	30	30	0.3	0.7

Table 5. Orthogonal array for the analysis to generate the blocks in 3DEC with wider joint spacing and lower joint persistence

No.	Joint spacing 1 (cm)	Joint spacing 2 (cm)	Joint spacing 3 (cm)	Joint set 1 persistence	Joint set 2 persistence
1	200	100	300	0.5	0.5
2	300	200	100	0.1	0.5
3	300	100	150	0.1	0.5
4	100	200	150	0.5	0.1
5	100	150	300	0.1	0.5
6	100	300	200	0.1	0.5
7	150	300	100	0.5	0.5
8	300	300	300	0.3	0.1
9	200	150	100	0.1	0.1
10	200	200	200	0.3	0.5
11	150	150	150	0.3	0.5
12	200	300	150	0.1	0.3
13	150	200	300	0.1	0.3
14	150	100	200	0.1	0.1
15	100	100	100	0.3	0.3
16	300	150	200	0.5	0.3

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