
A comparative study of hydraulic conductivity estimations using geostatistics

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Abstract Three approaches for estimating the hydraulic conductivity (K) of the Trifa aquifer, Morocco were investigated: (1) kriging of the K values obtained from pumping tests, (2) cokriging of the pumping test data with electrical resistivity data as a secondary variable, and (3) cokriging of the pumping test data with the slope of the water table. Gauss-transformed values of the variables are used because they provide more robust variograms and transformed values of the primary and secondary variables show correlations higher than the raw values, which is beneficial in cokriging. In cokriging with electrical resistivity, two zones are considered since the geological deposits are different from the north to the south of the aquifer, which is reflected in different correlations between the variables. Comparison of the three approaches is based mainly on the estimation errors, and to a lesser degree on the cross-validations of the corresponding variogram models and general considerations, like the measurements' reliability and aquifer make-up. The best-estimated K is given by cokriging with the slope of the water table and is therefore preferred for further use in groundwater flow modeling. Thus, electrical resistivity or the slope of the water table can both be used as secondary variables to estimate K , especially in heterogeneous aquifers with lateral variations in lithology, as is the case of the Trifa aquifer.

Résumé Trois approches pour estimer la conductivité hydraulique (K) de l'aquifère de Trifa au Maroc, ont été étudiées : (1) le krigeage des valeurs de K obtenues à partir d'essais de pompages, (2) le cokrigeage des données d'essais de pompage et les données de résistivité électrique comme seconde variable, et (3) le cokrigeage des

données d'essais de pompage avec la pente de la nappe. Les valeurs de transformée de Gauss des variables sont utilisées car elles apportent des variogrammes plus robustes, et car les valeurs transformées des variables primaires et secondaires montrent de meilleures corrélations que les valeurs brutes, ce qui est bénéfique au cokrigeage. Avec le cokrigeage de la résistivité électrique, deux zones sont considérées dès lors que les dépôts géologiques sont différents du nord vers le sud de l'aquifère, ce qui se reflète dans différentes corrélations entre les variables. La comparaison des trois différentes approches est basée principalement sur les erreurs des estimations, et à un moindre degré sur les validations croisées des modèles de variogramme correspondants et des conditions générales, comme la reproductibilité des mesures et les limites de l'aquifère, qui se reflètent dans différentes corrélations entre les variables. Le K le mieux estimé provient du cokrigeage avec la pente de la nappe, et serait donc préféré dans une utilisation ultérieure en modélisation. Ainsi, la résistivité électrique ou la pente de la nappe peuvent toutes les deux être utilisées comme variables secondaires pour estimer K , et notamment dans les aquifères hétérogènes présentant des variations latérales de la lithologie comme c'est le cas dans l'aquifère de Trifa.

Resumen Se investigaron tres aproximaciones, para estimar la conductibilidad hidráulica (K) del acuífero de Trifa, Marruecos: (1) Krigeaje de los valores de K obtenidos de las pruebas de bombeo, (2) Cokrigeaje de los datos de prueba bombeo con los datos de resistividad eléctrica como variable secundaria, y (3) Cokrigeaje de los datos de prueba de bombeo con la pendiente del nivel freático. Se usan valores transformados-Gauss de las variables, porque ellos proporcionan unos variogramas más robustos y los valores transformados de las variables primarias y secundarias, muestran correlaciones más altas que los valores sin tratamiento, lo cual es beneficioso en el cokrigeaje. En el cokrigeaje con la resistividad eléctrica, dos zonas son consideradas, ya que los depósitos geológicos son diferentes de norte al sur del acuífero, lo cual se refleja en las correlaciones diferentes entre las variables. La comparación de las tres aproximaciones está basada principalmente en los errores de estimación, y en un grado menor en las validaciones cruzadas de los modelos del variograma correspondiente y en consideraciones generales, como la

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fiabilidad de las medidas y la constitución del acuífero. El K mejor estimado se da por el cokrigeaje con la pendiente del nivel freático y se prefiere por consiguiente para el uso extenso en el modelamiento del flujo de agua subterránea. Así, tanto la resistividad eléctrica, como la pendiente del nivel freático, pueden usarse como las variables secundarias para estimar K , sobre todo en los acuíferos heterogéneos con variaciones laterales en la litología, como es el caso del acuífero de Trifa.

Keywords Hydraulic conductivity · Kriging · Cokriging · Trifa aquifer

Introduction

The hydraulic conductivity is probably the most important hydrogeologic parameter for the construction of a groundwater model. It determines, to a large extent, the groundwater flow and the groundwater head distribution. However, pumping tests that produce hydraulic conductivity data are often limited in number and not well distributed over the study area. Consequently, obtaining reliable values for the hydraulic conductivity of an aquifer is often difficult due to lateral and vertical heterogeneities that are usually present in water-bearing geological strata. Nevertheless, as will be shown in this paper, good estimates can be obtained using correlated secondary variables that reflect the local variations in the hydraulic conductivity.

Several geostatistical methods have been proposed to improve the spatial estimation of regionalized variables. Matheron (1963, 1971) developed kriging and cokriging techniques. Delhomme (1976, 1978) introduced additional techniques as kriging combined with linear regression and kriging with external drift. The theory of geostatistics and its applications in earth sciences is well described in the books of Matheron (1965, 1971), Journel and Huijbregts (1978), Isaaks and Srivastava (1989), Goovaert (1997), and Kitanidis (1997). Applications of geostatistical methods in hydrology have been realized in several studies like Ahmed and de Marsily (1987); Ahmed et al. (1988) who used cokriging, kriging combined with linear regression, kriging with an external drift, and kriging with a guess field, to estimate aquifer transmissivity.

As measurements of the hydraulic conductivity are often rare due to their expensive cost and being not well distributed to suit monovariate geostatistics, the kriging method alone will not be adequate for accurate estimation of the spatial hydraulic conductivity distribution. Therefore, the search for well-correlated secondary variables remains an interesting issue. Authors like Archie (1942, 1950), Zohdy (1974), and Keller and Frischknecht (1970) have underlined the ability of electrical resistivity to determine the physical characteristics of rocks. A number of authors have used linear regression to estimate hydraulic conductivity from electrical resistivity and excellent papers have been published on this issue such as Kelly (1977), Kosinsky and Kelly (1981), Sri Niwas and Singhal (1981, 1985), Kwader (1985), Huntley (1986). Ahmed and de

Marsily (1988) introduced geostatistical methods as alternative and efficient tools that allow a flexible use of the relationship between electrical and hydraulic conductivity, such that a notable improvement in the spatial estimations of hydraulic conductivity is obtained. More recently, studies by Purvance and Andricevic (2000), De Lima and Sri Niwas (2000), Sri Niwas and de Lima (2003), and Troisi et al. (2000) used electrical resistivity for aquifer parameter characterization. In this study, electrical resistivity will also be used to estimate the hydraulic conductivity by geostatistical means, but the results show that caution is needed to accurately interpret the relationship between electrical resistivity and hydraulic conductivity.

In addition to using electrical resistivity, hydraulic head can be used to estimate the hydraulic conductivity as shown by Kitanidis and Vomvoris (1983), Hoeksema and Kitanidis (1984), Dagan (1985), and Sun and Yeh (1992). These studies solve the inverse problem by combining solution of the flow equation with linear estimation, and show that hydraulic head can be used with local transmissivity data to estimate the spatial distribution of the aquifer transmissivity. Wagner (1992) used hydraulic head and concentration measurements in estimating hydraulic conductivity within prescribed zones. Woodbury and Smith (1988) used both temperature and hydraulic-head measurements to estimate hydraulic conductivity distribution under steady-state conditions. Harvey and Gorelick (1995) combined hydraulic conductivity measurements, solute arrival time and steady-state hydraulic heads for mapping hydraulic conductivity. Yeh and Zhang (1996), and Li and Yeh (1999) applied cokriging to estimate hydraulic conductivity using pressure head, solute concentration, and solute arrival time in a hypothetical, heterogeneous vadose zone under steady-state infiltrations at different degrees of saturation. All these papers demonstrate the validity of using hydraulic head to estimate the hydraulic conductivity of an aquifer. The objective of this study is to estimate the hydraulic conductivity using cokriging with the gradient of the hydraulic head since, as can be inferred from Darcy's law, this is directly linked to the hydraulic conductivity.

Problem definition

Trifa is the most productive agricultural plain of eastern Morocco, bounded by the Kiss Wadi in the East, the Beni Znassen Mountains in the south, the Moulouya River in the west and the Ouled Mansour hills in the north (Fig. 1). Development of agricultural activities during the last few decades has increased the flux of human immigration and resulted in contamination and depletion of groundwater resources, causing damage to the environment of the whole region. In fact, the irrigation requirements for intensive agricultural practices have become so large that groundwater, being the main source for meeting agricultural and domestic water needs, is severely depleted, especially during dry seasons.

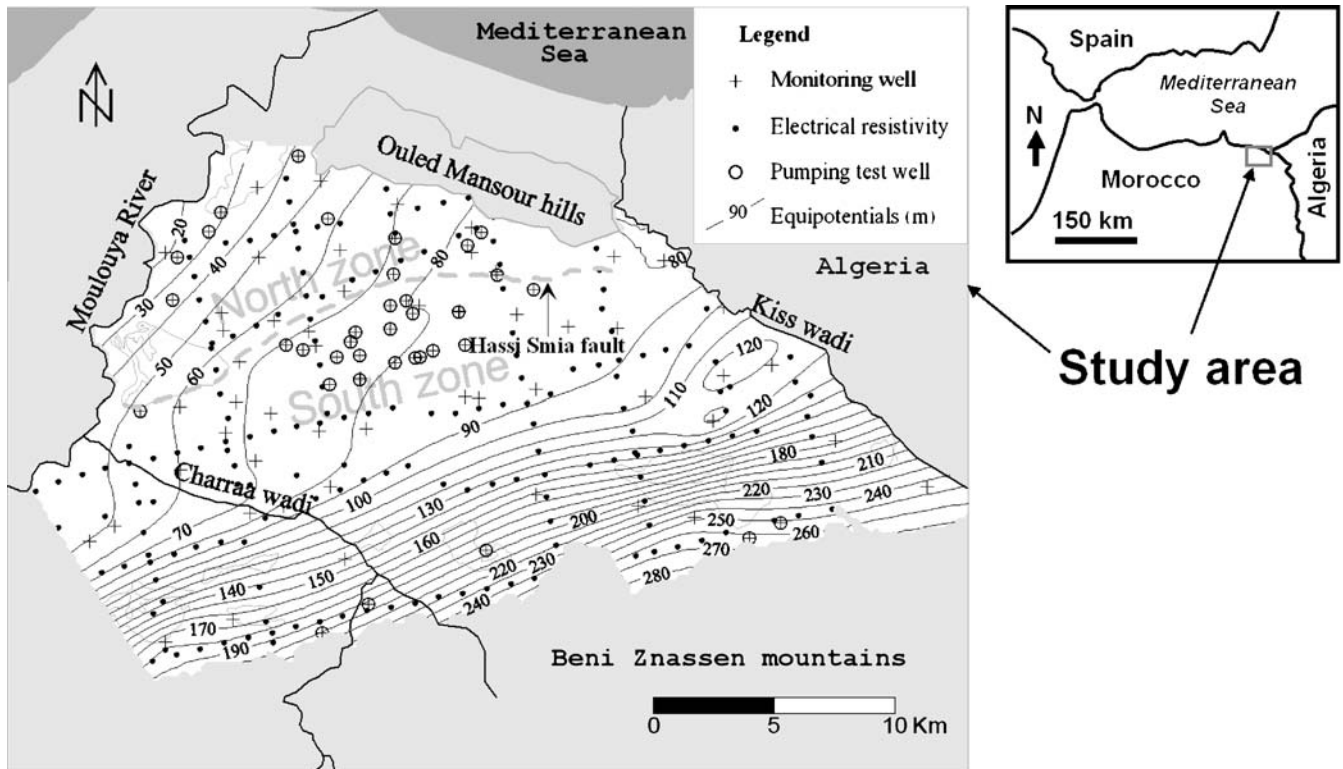


Fig. 1 The Trifa aquifer (white area) with locations of monitoring wells, pumping test wells, and electrical resistivity measurements; also shown are the groundwater-head equipotential lines (m above sea level)

The groundwater reservoir of Trifa consists of an unconfined aquifer, which in the north is essentially made up of Quaternary deposits underlain by impervious Tertiary deposits, while in the south the aquifer is made up of two layers, i.e. Quaternary deposits underlain by pervious Jurassic formations. The Trifa aquifer is characterized by the presence of the E–W oriented Hassi Smia fault (Fig. 1). From a hydrogeological point of view, the area can be sub-divided into two zones:

1. The zone north of the Hassi Smia fault, where the phreatic aquifer is shallow and mainly made up of marly-lacustrine limestone of early Quaternary age, underlain by impervious marls (Tertiary)
2. The zone south of the Hassi Smia fault, where the aquifer is deep and made up of a mixture of silt and sandy-silt of late Quaternary age, underlain by impervious marls (Tertiary) or pervious Jurassic limestone

Several pumping and observation wells are present in the area as shown in Fig. 1. Groundwater head measurements are available at 59 locations, which were obtained from the “Direction Générale de l’Hydraulique”, the central water-managing office of Morocco. The data consist of bi-monthly measurements for the period 1995–1996, distributed over the study area. Average values were calculated and used for constructing a groundwater-head contour map as shown in Fig. 1 (El Idrysy 2003). From the contour map, the direction of the groundwater flow can be deduced. It appears that in the south, the groundwater emerges from the Beni Znassen

Mountains and flows to the north. The Kiss River in the East drains part of the flow, but the majority arrives in the central plane, where most of the pumping wells are situated. Here, the groundwater flow seems to slow down and gradually change direction to the NW and NE, where a large amount drains to the Moulouya River in the NW and the remaining to the Charraa wadi, which crosses the Trifa plain in its southwestern part and flows to the Moulouya River.

When looking more closely at the intervals between the groundwater equipotential lines, one can notice clear differences between the aquifer sub-zones, i.e. in the south the equipotentials are very close together and the gradient is rather steep, while in the centre and north these are wider apart with almost no gradient. This pattern is likely due to the hydrogeological setting, in particular the conductive properties of the aquifer.

Pumping wells are mainly concentrated in the central part of the aquifer where groundwater is abstracted as an additional source for irrigation. The availability of many wells and the intense agricultural activity in the Trifa plain encouraged the water managers to conduct several pumping tests. Data obtained from the “Direction Générale de l’Hydraulique” consists of 33 pumping tests, of which 8 are situated in the zone north of the Hassi Smia fault. In addition, 5 other pumping tests have been conducted in the zone north of the Hassi Smia fault for another project (El Mandour 1998). Hence, in total, there are 13 pumping-test results available north of the Hassi Smia fault. Of the 25 pumping test south of Hassi Smia fault, 4 near the southern limit of the plain have been conducted in the Jurassic limestone deposits. The hydraulic conductivity

values obtained from the pumping tests vary from 0.1 to 1 m/d in the outer parts of the plain to large values of 10 to 180 m/d in the central part. Notice that the variation of the hydraulic conductivity across the aquifer shows an obvious similarity with the pattern of groundwater equipotential lines mentioned above. This correlation is significant as will be shown further on.

In 1995, electrical resistivity soundings were carried out across the Trifa plain by the “Laboratoire Public d’Essais et d’Etudes”, a geophysical laboratory based in Morocco. The measurements consisted of apparent resistivities with electrode spacings of 200 and 2,000 m, covering the whole area as shown in Fig. 1. With these measurements, the electrical resistivities of the different ground layers were estimated (EL Idrysy and De Smedt 2004). Orders of magnitude of the obtained electrical resistivities of the three geological units correspond well with those of the same type and age cited by Keller and Frischknecht (1970). The electrical resistivity of the Quaternary deposits, which are made up of terrestrial sediments, ranges from 20 to 150 Ωm ; while the underlying Tertiary deposits, consisting of marine sedimentary marl, have a smaller electrical resistivity in the order of 20 to 50 Ωm , and the Jurassic limestone unit in the south has very high resistivities ranging from 60 to 500 Ωm . Due to heterogeneity, especially of the Quaternary sediments, the electrical resistivity also varies locally and can be used as a substitute to mimic the variation of the hydraulic conductivity.

Methods

Three procedures are used to estimate the regional distribution of the hydraulic conductivity of the Quaternary layer: kriging, cokriging with the electrical resistivity, and cokriging with the piezometric gradient. A brief description of the geostatistical methods used in this study is presented in the Appendix. Each geostatistical technique proceeds as follows: after an exploratory data analysis, a variogram-model is inferred then cross-validated, and finally the estimation is carried out and displayed together with an error map.

The cross-validation consists of testing the estimation procedure at the locations of the existing data. The sample of a given location is removed from the data set and the value at that location is estimated using the remaining samples. This is repeated for all the samples. The validation criteria are based on the statistical properties of the differences or error, e , between the observed values and their estimates, and the standardized error, e/σ_0 . Quantitatively, the cross-validation procedure tests the robustness of a model by evaluating the mean and variance of the errors and standardized errors. Obviously, the mean values should be close to zero, the variance of the errors as small as possible, and the variance of the standardized error close to one. In addition one can verify whether all measurements are well reproduced by comparing the absolute value of the standardized error with

some confidence criterion value, in this study taken as 2.5. Estimates that do not fulfill this criterion are denoted as outliers. The UNIX based software ISATIS (Bleines et al. 2000) is used throughout this study in the exploratory data analysis and variable transformation as well as in the variography, cross-validation and estimation.

Results and discussion

Data transformation

In total, there are 34 pumping-test results available that yield values for the hydraulic conductivity in the Quaternary deposits. These values range between less than 0.1 m/d to about 200 m/d. Figure 2 shows the cumulative frequency distribution of these measurements, where the ordinate is a Gauss-transformed axis (i.e. a standardized normally distributed variable with the same cumulative frequency as the K -values; Journel and Huijbregts 1978; see pp. 478–479). The distribution of the hydraulic conductivity measurements is obviously not normal as can be noticed from the shape of the curve connecting the data points. Also, the distribution appears to be very skewed. For heavily skewed distributions as in the present case, the estimation of the variogram can be very sensitive to outliers, which can lead to less robust estimates with linear kriging techniques. In such cases, a classical approach is to transform the variable into a Gaussian one (Bleines et al. 2000). The transformation is obtained by Gaussian anamorphosis, which is a standard tool in geostatistics and available in ISATIS for transforming a variable with any distribution into a Gaussian one, and vice versa, by means of a mathematical function approximated by a polynomial expansion of Hermite polynomials (Bleines et al. 2000; Wackernagel 2003). Notice that the transformed K^* -values are more or less

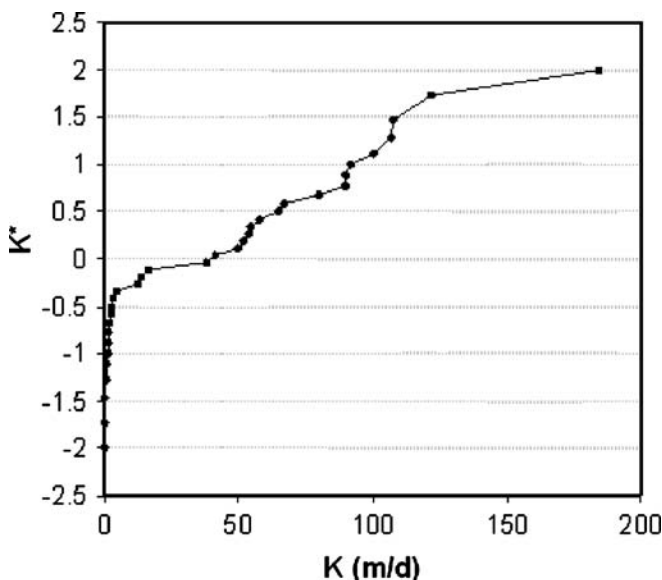


Fig. 2 Plot of Gauss-transformed values of the hydraulic conductivity K^* versus observed values of the hydraulic conductivity K obtained from pumping tests

uniformly varying between -2 and 2 . For geostatistical analysis, variables having a normal distribution are preferred; therefore, the Gauss-transformed K^* -values were used in the estimation methods and cross-validation. To return to the natural hydraulic conductivity values K , a back-transformation is performed using, again, the Gaussian anamorphosis tool of ISATIS.

Estimation by kriging

The Gauss-transformed hydraulic conductivity is estimated using the available values of the variable only. The experimental and model variograms of the Gauss-transformed hydraulic conductivity are presented in Fig. 8a. The fitted variogram is a nested model combining a Gaussian and a spherical structure.

Before kriging is performed, the fitted variogram is verified by the cross-validation procedure, the results of which are given in Table 1. The mean values of the errors and standardized errors are close to zero and the standardized error variance is close to 1, but the cross-validation still points out two outliers. Nevertheless, the fitted variogram can be considered as accurately representing the spatial variability and, consequently, can be used for estimation by kriging. The estimated maps of the Gauss-transformed hydraulic conductivity K^* and its corresponding standard deviation σ_0^* of the kriging errors are presented in Fig. 3. The map of the K^* -values reflects the situation encountered in the field, i.e. the corresponding highest K -values ($K^* > 0$) are found in the centre of the plain where most of the pumping wells are situated. Notice that the estimation error for the K^* -values is rather high, especially in locations far from the centre of the plain where there are fewer or no pumping tests available.

Estimation by cokriging with electrical resistivity

By cokriging with the electrical resistivity, it is intended that the estimation of the hydraulic conductivity be improved by exploiting the expected correlation between the hydraulic conductivity and the electrical resistivity, and by the fact that the secondary variable is known in more locations than the primary variable. Because measurements of both the hydraulic conductivity and electrical resistivity are not available at the same locations, electrical resistivity values at the locations of the pumping

tests are interpolated by means of kriging. The estimation errors associated with kriging are assumed to be small because of the dense set of electrical resistivity data. Any possible errors that might be present in the hydraulic conductivity values derived from the pumping tests are also neglected, and the differences of support scales at which the two variables are defined.

However, there is one more complication. Figure 4 shows the correlation between hydraulic conductivity K and electrical resistivity R . When considering all values together in a scatter-plot of observed K -values versus kriged R -values at the pumping test locations, there appears to be no correlation as the value of the correlation coefficient is 0.05, or almost nil. However, when the zones north and south of the Hassi Smia fault are considered separately, significant correlations between hydraulic conductivity and electrical resistivity become evident, which can either be positive or negative depending on the lithology. The scatter-plot shows a negative correlation for the north zone comprising limestone deposits, and a positive correlation for the south zone, where the deposits are mainly unconsolidated silt and sandy-silt. As demonstrated by other authors such as Kosinsky and Kelly (1981) and Heigold et al. (1979), consolidated rocks, as in the zone north of the Hassi Smia fault, are often electrically resistive but less hydraulically conductive. While in course-grained unconsolidated porous media, as in the zone south of the Hassi Smia fault, the hydraulic conductivity and electrical resistivity are both relatively high and positively correlated.

For the zone north of the Hassi Smia fault, there are 13 data pairs that show a negative correlation coefficient of -0.52 , and for the zone to the south there are 21 data pairs with a positive correlation coefficient of 0.49 (Fig. 4). Furthermore, when using Gauss-transformed hydraulic conductivity values K^* and Gauss-transformed electrical resistivity values, which are denoted by R^* , the correlations improve. For the north zone, the correlation between K^* and R^* becomes -0.66 , and for the south zone 0.51; both values are statistically significant (with 95% confidence, critical values for a directional test of the correlation coefficient are respectively 0.553 for 13 samples and 0.433 for 21 samples).

Therefore, geostatistical analyses is carried out on K^* and R^* for both zones separately. The observed and fitted variogram and cross-variogram for both zones are shown in Fig. 5. The fitted variograms are nested models combining Gaussian and spherical structures for the north zone, and spherical and exponential structures for the south zone, although some of the data points seem to suggest that a finite sill might not be appropriate for the site. This is also associated with the drawback of dividing the area into two zones (north and south) so that the data are reduced to 13 and 21 values respectively, which increases uncertainty with respect to the fitted variograms.

The cross-validation results of the variogram models are given in Table 1 where the results obtained for both zones are combined. The cokriging procedure reproduces

Table 1 Results of the cross-validation tests for north and south zones combined

Criterion	Kriging	Cokriging with electrical resistivity	Cokriging with slope of the water table
Mean error	0.089	0.007	0.048
Error variance	0.241	0.220	0.252
Mean standardized error	0.198	-0.018	0.174
Standardized error variance	1.14	1.07	1.28
Number of outliers	2	2	2

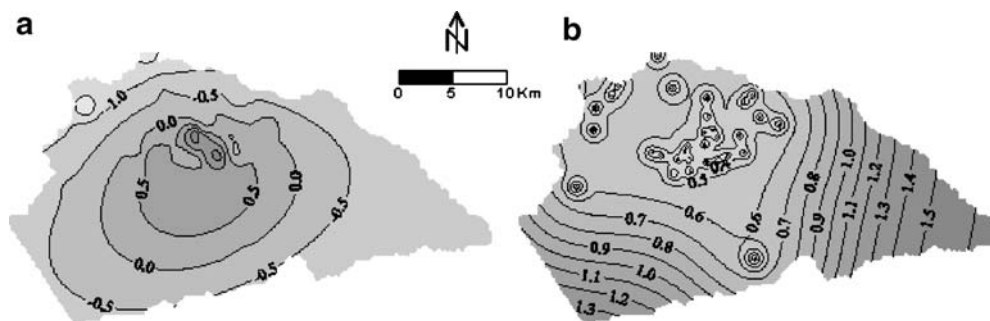


Fig. 3 Results of kriging: **a** estimated Gauss-transformed hydraulic conductivity K^* , and **b** its corresponding standard deviation σ_0^* of the estimation errors

two outliers, as in the kriging case. However, when examining the statistics of the estimation error and standardized error shown in Table 1, the cross-validation parameters of cokriging with electrical resistivity are much better than those of kriging.

Subsequently, using the variograms and cross-variograms in Fig. 5, the Gauss-transformed hydraulic conductivity is estimated by cokriging using the Gauss-transformed electrical resistivity as the secondary variable. This is done separately for the two zones and afterwards the results are joined together. The estimated Gauss-transformed hydraulic conductivity K^* and its corresponding standard deviation σ_0^* of the cokriging errors are given in Fig. 6, which combines the results of both the north and south zones that were estimated separately by cokriging.

The spatial distribution of the estimated Gauss transformed hydraulic conductivity shown in Fig. 6a is clearly different from the one shown in Fig. 3a. In Fig. 6a, the K^* -values show more local variation due to the effect of the more-sampled secondary variable R^* . The difference is also reflected in the map of the standard deviation of the estimation errors. The estimation error is much smaller in

this case, especially in the outer parts of the plain where the lack of pumping test data is compensated by the availability of the electrical resistivity values.

Estimation using the slope of the water table

The slope of the water table can be extracted from the estimated map of the hydraulic heads shown in Fig. 1. From a theoretical point of view, one expects an inverse relationship between the slope of the water table and the hydraulic conductivity, because Darcy's law implies that high-conductivity values correspond to small gradients and vice-versa. Of course, such a relationship is not straightforward due to variable groundwater fluxes in space and time. Nevertheless, some correlation can be present, which can be exploited in cokriging.

Slopes of the groundwater heads S in all 34 pumping test locations were calculated from the water table map in Fig. 1, using ArcView geographical information system (GIS). Any possible errors introduced by this are ignored in the subsequent analyses. A scatter plot of the Gauss-transformed hydraulic conductivities K^* and the corresponding values of the Gauss-transformed slope of the groundwater heads S^* is presented in Fig. 7. The correlation coefficient is -0.50 , which is superior to the critical correlation coefficient value of 0.339 at a significance level of 95% for 34 data pairs. Therefore, the slope is a useful secondary variable for cokriging of the hydraulic conductivity values. In the estimation of the hydraulic conductivity, only the water-table slope values at the piezometer locations are used because these are considered to be more reliable than other interpolated values.

The variograms and cross-variogram are shown in Fig. 8, and can be fitted with nested Gaussian and spherical components. The cross-validation of the model, shown in Table 1, yields again two outliers, which is the same as for kriging and cokriging with R^* . When comparing the statistics of the estimation errors and standardized errors, the mean error of cokriging with S^* is somewhat better than kriging, but not as good as cokriging with the electrical resistivity.

The results of cokriging with the slope of the water table are presented in Fig. 9. The smoothness of the estimated hydraulic conductivity map in Fig. 9a is somewhat intermediate between the one of kriging (Fig. 3a) and cokriging with electrical resistivity

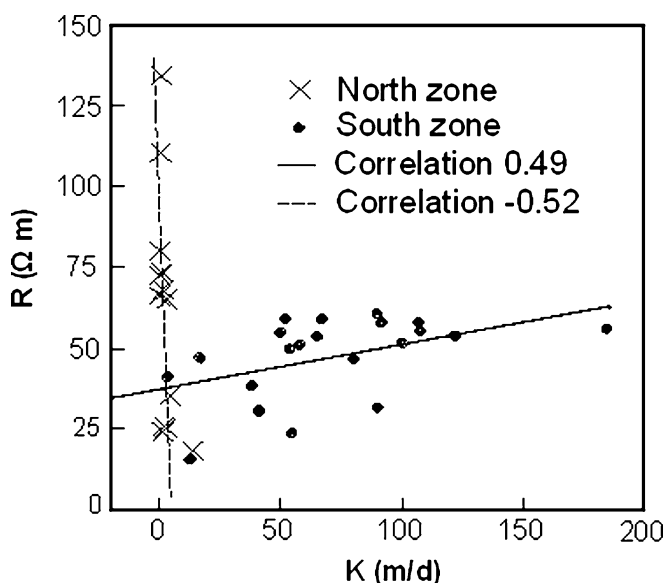


Fig. 4 Scatter-plot showing the correlation between hydraulic conductivity K and electrical resistivity R (the crosses represent samples located north of the Hassi Smia fault and the dots south of the fault)

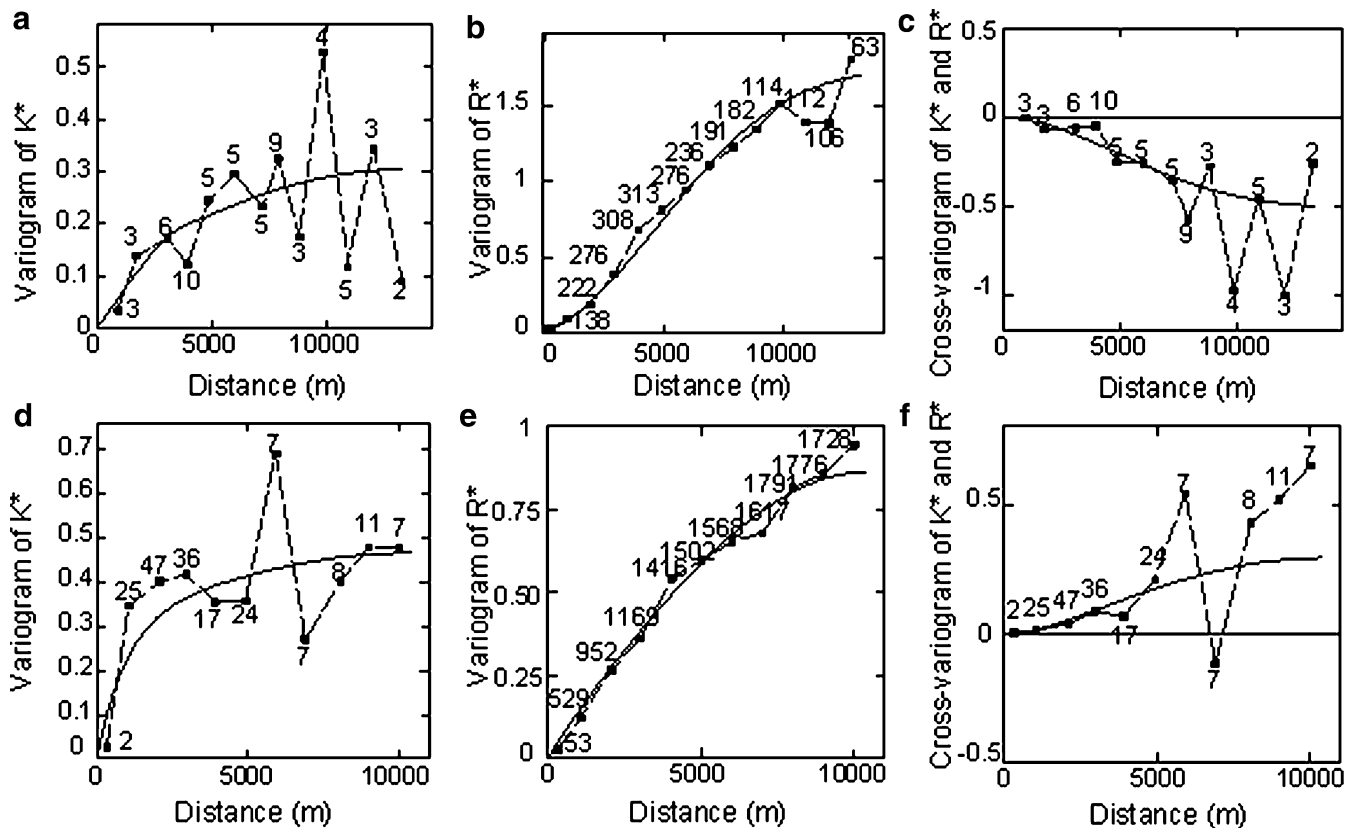


Fig. 5 a Variogram of the Gauss-transformed hydraulic conductivity K^* , b variogram of Gauss-transformed electrical resistivity R^* , and c cross-variogram of K^* and R^* in zone north of the Hassi Smia fault; and d variogram of the Gauss-transformed hydraulic conductivity K^* , e variogram of Gauss-transformed electrical resistivity R^* , and f cross-variogram of K^* and R^* in the zone south of the Hassi Smia fault. The calculated experimental variograms are shown in the figures by dashed lines, along with the number of data pairs used to calculate these. The fitted variogram and covariogram models are shown by solid lines

(Fig. 6a). This is due to the fact that the number of available samples of the slope of the water table is less than the electrical-resistivity values, but compared to kriging gives a better indication about the variations of the hydraulic conductivity in locations where there are no pumping tests. This is also confirmed in the map of the estimation errors (Fig. 9b). The errors are small near the samples of the primary variable K^* , but increase farther away from these locations. The increase of magnitude in the error is different from kriging or cokriging with R^* . Compared to kriging, the error increases more slowly due to the presence of the secondary variable whose influence is proportional to its

correlation with the primary variable. Compared to cokriging with electrical resistivity values, the increase is larger, due to the fact that there are lesser secondary values available that could reduce the error.

Comparison and final results

From the estimated hydraulic conductivity maps, one can notice the same type of variation over the study area: i.e. the hydraulic conductivity is high in the centre and decreases in a radial pattern towards the outer limits of the aquifer. This can be explained by the geological history of the region of Trifa that underwent a well-known subsidence that oc-

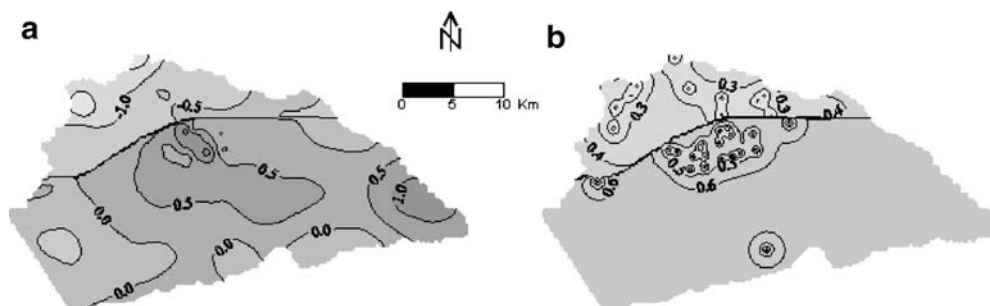


Fig. 6 Results of cokriging with electrical resistivity: a estimated Gauss-transformed hydraulic conductivity, K^* , and b its corresponding standard deviation σ_0^* of the estimation errors

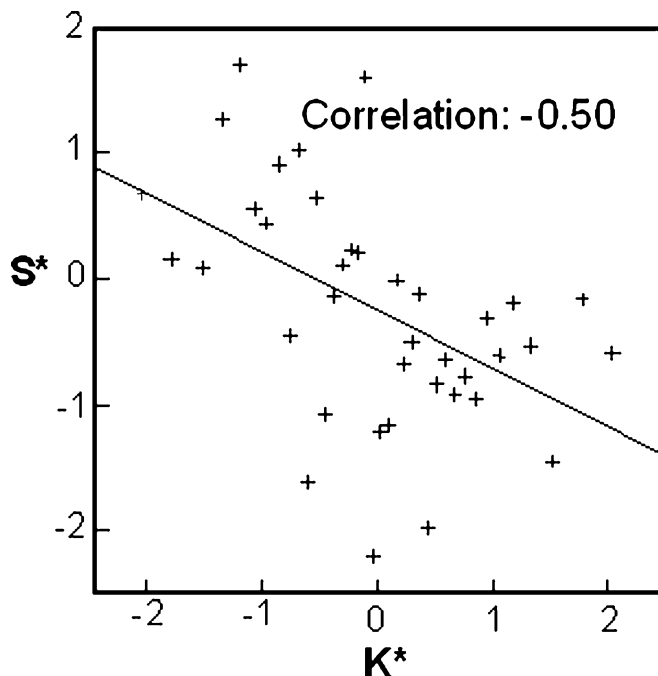


Fig. 7 Scatter plot showing the correlation between Gauss-transformed values of the hydraulic conductivity K^* and the slope of the water table S^*

curring in the late Quaternary as reported by geologists such as de Chevron-Villette (1962), resulting in more permeable sediments in the middle part of the plain.

When checking the estimations of K^* and the error maps, it becomes clear that kriging produces a very smooth map, the result of averaging the measurement values at the pumping test sites, which does not reflect any spatial heterogeneity in locations where there are no measurements. Furthermore, the estimated distribution has the highest error compared to the other methods. Therefore, it is less accurate and the results should be discarded.

The results from cokriging with electrical resistivity and from cokriging with the slope of the water table show more variability even in locations where there are no primary measurements, owing to the presence of the secondary

variable. These estimates are also more reliable since their associated errors are much smaller compared to kriging.

It remains to be decided which of the cokriging estimates is the best. Comparison of the errors resulting from cokriging with K^* and cokriging with S^* leads to the following conclusions. The secondary variable measurements are more abundant in the case of electrical resistivity, especially in the outer parts of the plain where the cokriging errors are smaller than those of cokriging with the slope of water table. Consequently, in the outer areas the results of cokriging with electrical resistivity are better. However, although the number of the available water table slopes is smaller, these measurements are well distributed over the central part of the plain and produce estimates with smaller errors than cokriging with resistivity in the centre of the plain.

Hence, in the centre, cokriging with the slope of the water table is better. Furthermore, because the method considered two separate zones for the cokriging of hydraulic conductivity with electrical resistivity, the variance of each of the separate K^* sets will be smaller than for the total set. This is reflected in the variograms of K^* shown in Fig. 5 compared to Fig. 8. Consequently, the resulting errors from cokriging with electrical resistivity are affected and will be smaller than when the whole data set is considered as in the case of cokriging with the slope of the water table. This is also reflected in the better results for the cross-validation. Nevertheless, in the present application, cokriging with the slope of the water table still yields the smallest error in a large central part of the study area.

Moreover, in practice, one should also take into account that measured groundwater hydraulic heads, from which the slopes of the water table are extracted, are easier and less costly to obtain. On the other hand, electrical resistivity values are estimated from apparent resistivities that are cumbersome to obtain and to interpret. Furthermore, it is believed that slopes of the water table can be measured more accurately and are also more suitable as secondary variables because they are physically related to groundwater movement. Therefore, one concludes that the

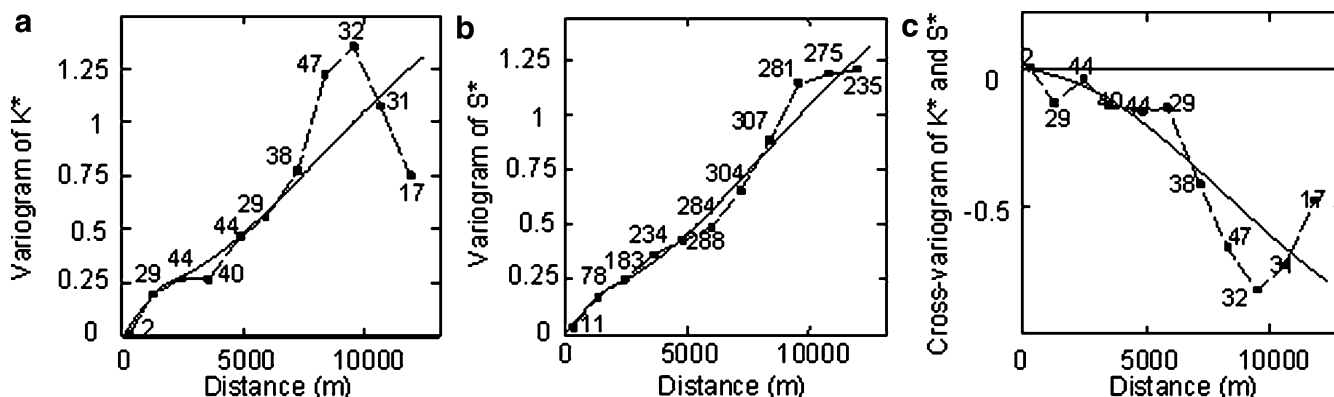


Fig. 8 a Variogram of the Gauss-transformed hydraulic conductivity, K^* , b variogram of Gauss-transformed slope of the water table S^* , and c cross-variogram of K^* and S^* . The calculated experimental variograms are shown in the figures by dashed lines, along with the number of data pairs used in the calculation. The fitted variogram and covariogram models are shown by solid lines

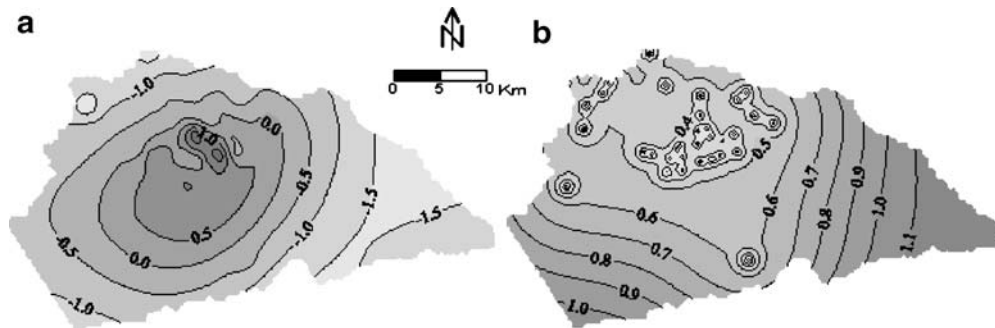


Fig. 9 Results of cokriging with the slope of the water table: **a** estimated Gauss-transformed hydraulic conductivity K^* , and **b** its corresponding standard deviation σ_0^* of the estimation errors

estimations from cokriging with the slope of the water table are preferred for further use, as for groundwater flow and salinity transport models that will be developed afterwards.

The hydraulic conductivity field that results from cokriging with the slope of the water table is obtained by back-transformation and is presented in Fig. 10a. For the estimation error, there is no direct back-transformation, and, therefore, an approximation is used. This approximation consists of calculating $K_{\max}$ and $K_{\min}$ values, where $K_{\max}$ is the back-transformation of the sum of the estimated K^* -value and its corresponding standard deviation σ_0^* of the estimation errors, and $K_{\min}$ the back-transformed value of K^* minus σ_0^* . The possible standard deviation of the errors on the K -estimates is then taken as half the difference between $K_{\max}$ and $K_{\min}$. The map of this standard deviation of the errors is given in Fig. 10b.

One can notice that the resulting map of hydraulic conductivities indicates large conductivity values in the central part of the plain, which is consistent with the field data: (1) the geological setting shows a thick layer of late Quaternary deposits (the main aquifer) in the central part of the plain, while in the outer regions there are only shallow deposits of early Quaternary or Jurassic age; and (2) pumping wells with good yield are situated in the central part, while those with less yield are in outer areas. The error map obtained shows that hydraulic-conductivity values were determined with reasonable accuracy in the centre of the Trifa plain, with relative errors on the order of 30–50%, while in the outer region—for reasons such as those already explained, i.e. mainly lack of field data—the estimates are

highly inaccurate with errors of the same magnitude as the estimated K -values. However, when considering the error magnitude of the estimates, one should also keep in mind that the variation of the measurements (pumping tests data) is very high since the hydraulic conductivity values vary between 0.1 and 200 m/d.

Conclusions

This study investigates three different approaches for estimating the hydraulic conductivity of the Trifa aquifer: (1) kriging of the available hydraulic conductivity values obtained from pumping tests, (2) cokriging of the pumping test data with electrical resistivity data as a secondary variable, and (3) cokriging of the pumping test data with the slope of the water table. Gauss-transformed values of the variables are used in the estimations because they provide smoother and more uniform distributions of the variables and more robust variograms that fit common models. Furthermore, the Gauss-transformed values of the primary and secondary variables show correlations higher than the raw values, which is beneficial in cokriging.

Comparison of the three approaches is carried out based mainly on the estimation errors, and to a lesser degree on the cross-validations of the corresponding variogram models, and on general considerations like the measurements' reliability and aquifer build up. The results show that the spatial variability of the hydraulic conductivity obtained by kriging is rather simplified and merely

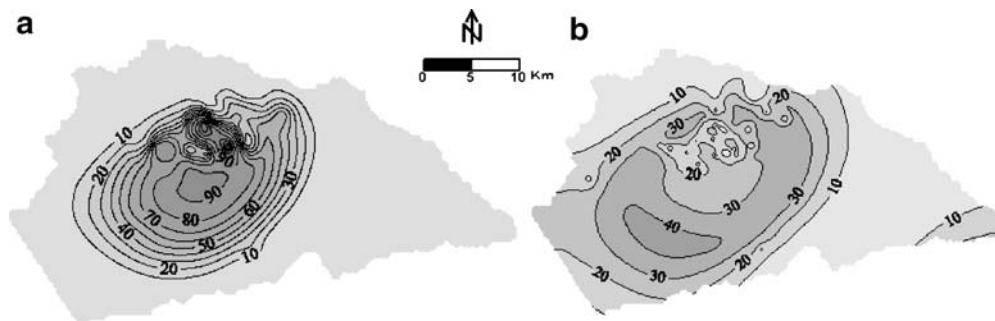


Fig. 10 **a** Back-transformed hydraulic conductivity K (m/d), obtained from the estimated K^* -values obtained by cokriging with S^* , and **b** its corresponding standard deviation of the estimation errors, obtained by approximation

an averaging map of the few values available from pumping tests, mostly situated in the center and north of the plain. These results are discarded because the corresponding errors are the highest compared to the other methods.

In cokriging with electrical resistivity, two zones are considered because the geological deposits are different north and south of the Hassi Smia fault, which is reflected in different correlations between the two variables. The correlation between the hydraulic conductivity and electrical resistivity in the north, where the prevailed sediments are consolidated lacustrine limestone, is negative. However, in the south, where the common sediments that form the aquifer are silts and sands, the correlation is positive.

Contrary to electrical resistivity, the slope of the water table is always negatively correlated with the hydraulic conductivity, which can be seen from Darcy's law. Also, the estimated standard deviation of the cokriging errors obtained using cokriging with the slope of the water table are the smallest in the central part of the aquifer, although the available secondary data are less than in the case of electrical resistivity and the variance of the whole hydraulic conductivity data set is higher compared to the two separated zones data sets.

Taking into consideration that the slopes of the water table are extracted from groundwater heads that were measured in the field, these are considered more reliable than the electrical resistivities, which were estimated from apparent electrical resistivities by curve fitting. Therefore, the estimated hydraulic conductivity by cokriging with the slope of the water table is considered the most accurate for further use in groundwater flow modeling.

This study shows that the slope of the water table can be used as a secondary variable to estimate the hydraulic conductivity of an aquifer. This is convenient when measurements of groundwater heads are available, especially in a heterogeneous aquifer where the variations of the slope of the water table are large enough to capture the hydraulic conductivity variation. Of course, the application of the present approach is limited to two-dimensional groundwater flow problems. Nevertheless, two-dimensional groundwater flow approaches are often used in practice. In such cases, it would be even more appropriate to pose the problem in terms of transmissivity, rather than hydraulic conductivity, as was done in the present study. Moreover, the proposed method might be a viable alternative to inverse-based geostatistical approaches. In this framework, it is possible to obtain an estimate of the hydraulic conductivity field, together with the complete covariance matrix of hydraulic conductivity as well as an estimate of the underlying conductivity variogram shape and parameters. These concepts are elucidated in Hernandez et al. (2006) where the proposed methodology is placed in the general context of the literature dealing with stochastic inverse approaches.

Finally, the comparison of the three methods shows that multivariate estimation techniques are reliable, especially when a more sampled secondary variable is available. As seen from the present results, these tech-

niques reduce considerably the estimation error and allow local variations to be estimated more accurately. Such methods can be very useful, especially in heterogeneous aquifers with lateral variations in lithology.

Appendix

In this section, the theories of kriging and cokriging are briefly reviewed without going into too much detail. The complete derivation of the equations can be found in books such as Journel and Huijbregts (1978), Isaaks and Srivastava (1989), Goovaerts (1997), and Wackernagel (2003).

Kriging consists of estimating a variate from a set of its punctual measurements. First, a variogram-model representing the spatial correlation between the values is generated from the measurement points. This is half the variance between paired data, and for a given variate $Z(x)$ at location x is expressed as follows (Journel and Huijbregts 1978)

$$\gamma_Z^{ij} = \frac{1}{2}E\left\{[Z(x_i) - Z(x_j)]^2\right\} \quad (1)$$

where γ_Z^{ij} is the semi-variogram (generally called variogram), and E stands for expected value, in this case the variance of the difference between Z -pairs separated by a distance $x_i - x_j$. The estimation of the variate Z at a given point x_0 is a weighted linear combination of the available measurements

$$Z_0 = \sum_{i=1}^n \alpha_i Z_i \quad (2)$$

where Z_0 is the estimate of $Z(x_0)$, Z_i are the observed values $Z(x_i)$ of the variable at locations x_i , α_i are the weights, and n the number of measurements. The set of the weighting coefficients α_i corresponding to each measurement Z_i , is determined in such a way that the error variance of the estimation is minimized, respecting the unbiasedness of the estimate. The latter is expressed as

$$\sum_{i=1}^n \alpha_i = 1 \quad (3)$$

and is taken into account by a Lagrange multiplier μ . The minimization and unbiasedness condition results in $n+1$ equations, the solution of which yields the values of α_i and μ . Since the correlation between the values decreases with the distance of separation, the estimation error depends upon the distance to the measurements. The expression of the variance σ_0^2 of the kriging error at location x_0 is calculated as

$$\sigma_0^2 = \sum_{i=1}^n \alpha_i \gamma_Z^{i0} + \mu \quad (4)$$

where γ_Z^{i0} are the variogram values corresponding to the distance from the estimation point x_0 to measuring point x_i .

Cokriging is a multivariate geostatistical method that consists of estimating a variate from a set of its punctual measurements and more densely sampled secondary variable(s). A cross-variogram model representing the spatial correlation between the values of the primary variable and the secondary variables is generated from the measurements. The cross-variogram is half the covariance of the differences between paired sample values of each variable. For a given primary variable Z and a secondary variable Y , the cross-variogram is expressed as follows

$$\gamma_{ZY}^{ij} = \frac{1}{2} E\{[Z(x_i) - Z(x_j)][Y(x_i) - Y(x_j)]\} \quad (5)$$

where γ_{ZY}^{ij} stands for the cross-variogram between variables Z and Y , in this case the covariance between paired data separated by a distance $x_i - x_j$. The estimation Z_0 of the variate Z at a given point x_0 is a weighted linear combination of the available primary and secondary measurements

$$Z_0 = \sum_{i=1}^n \alpha_i Z_i + \sum_{j=1}^m \beta_j Y_j \quad (6)$$

where Z_i and Y_j are the available measurements of Z and Y , n and m are the number of these measurements respectively, and α_i and β_j are the weights for the primary and secondary measurements. Again, the weighting coefficients α_i and β_j are determined such that the error variance of the estimation is minimized, respecting the unbiasedness of the estimate, which implies

$$\sum_{i=1}^n \alpha_i = 1 \quad \text{and} \quad \sum_{j=1}^m \beta_j = 0 \quad (7)$$

This is taken into account by 2 Lagrange multipliers, μ_1 and μ_2 . Minimization of the error variance results in a system of $n + m + 2$ equations that yield the values of α_i , β_j , μ_1 , and μ_2 . The expression of the cokriging error variance σ_0^2 at location x_0 is given by

$$\sigma_0^2 = \sum_{i=1}^n \alpha_i \gamma_Z^{i0} + \sum_{j=1}^m \beta_j \gamma_{ZY}^{j0} + \mu_1 \quad (8)$$

where γ_Z^{i0} and γ_{ZY}^{j0} are respectively the variogram and cross-variogram values corresponding to distances from the estimation point x_0 to a measuring point x_i of the primary variable, or to a sample point x_j of the secondary variable. The error variance in a target location is generally improved (the error reduced) by the presence of a well-correlated secondary variable near that location.

References

- Ahmed S, de Marsily G (1987) Comparison of geostatistical methods for estimating transmissivity using data on transmissivity and specific capacity. *Water Resour Res* 23(9):1717–1737
- Ahmed S, de Marsily G, Talbot A (1988) Combined use of hydraulic and electrical properties of an aquifer in a geostatistical estimation of transmissivity. *Ground Water* 26(1):78–86
- Archie GE (1942) The electrical resistivity log as an aid in determining some reservoir characteristics. *Petroleum Technology*, Technical Report 1422, American Institute of Mining and Metallurgical Engineering, Littleton, CO, p 146
- Archie GE (1950) Introduction to petrophysics of reservoir rocks. *Bull Am Assoc Pet Geol* 34:943–961
- Bleines C, Perseval S, Rambert F, Renard D, Touffait Y (2000) ISATIS software manual. *Géovariances*, Avon, France, p 585
- Dagan G (1985) Stochastic modeling of groundwater flow by unconditional and conditional probabilities 2: the inverse problem. *Water Resour Res* 21(1):65–72
- de Chevron-Villette A (1962) Notes sur la géologie et la morphologie de la région de Triffa (Maroc) [Notes on the geology and morphology of the region of Triffa (Morocco)]. Institut Français de Recherche Scientifique pour le Développement en Coopération, Bondy, France, p 47
- De Lima OAL, Sri Niwas (2000) Estimation of hydraulic parameters of shaly sandstone aquifers from geoelectrical measurements. *J Hydrol* 235:12–26
- Delhomme JP (1976) Application de la théorie des variables régionalisées dans les sciences de l'eau [Application of the theory of regionalized variables in water sciences]. PhD Thesis, Ecole des Mines de Paris, Fontainebleau, France
- Delhomme JP (1978) Kriging in hydrosociences. *Adv Water Resour* 1(5):251–266
- El Idrysy H (2003) Hydrogeological study of the Trifa aquifer, based on a multidisciplinary approach using geostatistics, GIS, and numerical modeling. PhD Thesis, Department of Hydrology and Hydraulic Engineering, Vrije Universiteit Brussels, Belgium, p 204
- EL Idrysy H, De Smedt F (2004) A geostatistical methodology to estimate the base of the Trifa aquifer (Morocco) with scarce drilling and geo-electrical sounding data. *Bull Eng Geol Environ* 63(4):345–351
- EL Mandour A (1998) Contribution à l'étude hydrogéologique de la plaine de Triffa: Salinisation et modélisation [Contribution to the hydrogeological study of the Triffa plain: salinisation and modeling]. PhD Thesis, Mohamed 1st University, Oujda, Morocco
- Goovaert P (1997) *Geostatistics for natural resources evaluation*. Oxford University Press, New York, p 483
- Harvey CF, Gorelick SM (1995) Mapping hydraulic conductivity: sequential conditioning with measurements of solute arrival time, hydraulic head, and local conductivity. *Water Resour Res* 31(7):1615–1626
- Heigold PC, Gilkeson RH, Cartwright K, Reed PC (1979) Aquifer transmissivity from surficial electrical methods. *Ground Water* 7(4):338–345
- Hernandez AF, Neuman SP, Guadagnini A, Carrera J (2006) Inverse stochastic moment analysis of steady state flow in randomly heterogeneous media. *Water Resour Res* 42, W05425, DOI 10.1029/2005WR004449
- Hoeksema RJ, Kitanidis PK (1984) An application of the geostatistical approach to the inverse problem in two-dimensional groundwater modeling. *Water Resour Res* 20(7):1003–1020
- Huntley D (1986) Relation between permeability and electrical resistivity in granular aquifers. *Ground Water* 24(4):466–474
- Isaaks EH, Srivastava RM (1989) *Applied Geostatistics*. Oxford University Press, New York, p 992
- Journel AG, Huijbregts CJ (1978) *Mining geostatistics*. Academic Press, London, p 600
- Keller GV, Frischknecht FC (1970) *Electrical methods in geophysical prospecting*. Pergamon Press, New York, p 517
- Kelly WE (1977) Geoelectric sounding for estimating hydraulic conductivity. *Ground Water* 15(6):420–425
- Kitanidis PK (1997) *Introduction to geostatistics: with applications in hydrogeology*. Cambridge University Press, New York, p 249
- Kitanidis PK, Vomvoris EG (1983) A geostatistical approach to the inverse problem in groundwater modeling (steady-state) and one-dimensional simulations. *Water Resour Res* 20(7):1003–1020
- Kosinsky WK, Kelly WE (1981) Geoelectrical soundings for predicting aquifer properties. *Ground Water* 19(2):163–171

- Kwader T (1985) Estimating aquifer permeability from formation resistivity factors. *Ground Water* 23(6):762–766
- Leonard-Mayer PJ (1981) Discussion of “Geoelectrical soundings for predicting aquifer properties” by Kosinsky WK, Kelly WE (1981). *Ground Water* 19(2):163–171
- Li B, Yeh TCJ (1999) Cokriging estimation of the conductivity field under variably saturated flow conditions. *Water Resour Res* 35(12):3663–3674
- Matheron G (1963) Principles of geostatistics. *Econ Geol* 58(8):1246–1266
- Matheron G (1965) Les variables régionalisées et leur estimation. [The regionalized variables and their estimation] Masson, Paris, pp 192–237
- Matheron G (1971) The theory of regionalized variable and its applications. No. 5, Cahiers du Centre de Morphologie, Mathématique de Fontainebleau, Ecole de Mines de Paris, Fontainebleau, France
- Purvance DT, Andricevic R (2000) On electrical hydraulic conductivity correlation in aquifers. *Water Resour Res* 36(10):2905–2913
- Sri Niwas, de Lima OAL (2003) Aquifer parameter estimation from surface resistivity data. *Ground Water* 41(1):94–99
- Sri Niwas, Singhal DC (1981) Estimation of aquifer transmissivity from Dar Zarrouk parameters in porous media. *J Hydrol* 50:393–399
- Sri Niwas, Singhal DC (1985) Aquifer transmissivity of porous media from resistivity data. *J Hydrol* 82:143–153
- Sun N-Z, Yeh WW-G (1992) A stochastic inverse solution for transient groundwater flow: parameter identification and reliability analysis. *Water Resour Res* 28(12):3269–3280
- Troisi S, Fallico C, Starface S, Migliari E (2000) Application of kriging with external drift to estimate hydraulic conductivity from electrical-resistivity data in unconsolidated deposits near Montalto Uffugo, Italy. *Hydrogeol J* 8:356–367
- Wackernagel H (2003) Multivariate geostatistics: an introduction with applications, 3rd edn. Springer, New York p 387
- Wagner BJ (1992) Simultaneous parameter estimation and contaminant source characterization for coupled groundwater flow and contaminant transport modeling. *J Hydrol* 135:275–303
- Woodbury AD, Smith L (1988) Simultaneous inversion of hydro-geologic and thermal data 2: incorporation of thermal data. *Water Resour Res* 24(3):356–372
- Yeh TCJ, Zhang I (1996) A geostatistical inverse method for variably saturated flow in the vadose zone. *Water Resour Res* 32(9):2757–2766
- Zohdy AAR, Eaton GP, Mabey DR (1974) Application of Surface Geophysics to Ground-water Investigations. In: US Geological Survey Techniques of Water Resources Investigations, Chap. D1, Book. 2, US Geol Surv, Reston, VA, p 116