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GEOPHYSICS

The Choice of Initial Approximation (Reference Velocity Distribution) in Iteration Kinematic Seismotomography

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Kinematic seismotomography is based on the inverse kinematic problem (IKP). It uses kinematic characteristics of wave process to solve its problems: velocity of propagation of oscillations in the medium and arrival times of the corresponding waves. Traditional tomographic algorithms based on inverse integral transformations are not applicable in seismic practice (in particular, owing to incompleteness of projection data). However, iteration methods of algebraic reconstruction allow us to realize the method of computational (computer) tomography (CT) in kinematic seismics. This method is distinguished for its significantly better informative properties compared to the other methods of computational diagnostics. In the organization of iteration procedures, the key aspect is the choice of the initial approximation. In this work, we suggest a new approach to the determination of the reference velocity distribution (initial approximation) for iterative methods in kinematic seismotomography based on the optimization of the parameters of this distribution and the information about the elements of the projection matrix (results of measurements). Unlike the methods of the construction of the reference velocity section based on a priori information, which does not always give the best separation of the sought velocity into the reference (known) and anomalous (to be determined) components from the computational point of view, the method developed by us uses the resources of CT and optimizes such separation. Thus, our method makes it possible to increase significantly the efficiency of realization of the IKP solution in tomographic formulation not only in the seismic conditions but also in the other fields of IKP generation in a linear formulation.

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The solution of inverse kinematic problem of seismics (IKPS) in a linear formulation requires knowledge of a priori reference velocity distribution, i.e., the main component of velocity to be used as the reference one for linearization. Successful solution of the problem depends on the correct choice of this component [1]. The reference velocity distribution is the initial approximation in the iteration computational technology of improving the IKPS solution in tomographic formulation, which consists of the solution of a system of nested linear inverse problems. The best performance of linearization conditions provides convergence of the iteration process [2]. Let us write the main integral relation of kinematic seismics (Fermat functional):

$$\tau(x^0, x^1) = \int\limits_{\Gamma(x^0, x^1)} n ds,$$

where $\tau(x^0, x^1)$ is runtime of a refracted wave along geodetic line $\Gamma(x^0, x^1)$; x^0 and x^1 are locations of the source and receiver of the signal, respectively; ds is length element of this geodetic line; and n is the refraction index (slowness). In the operator form, this relation is written as

$$\tau = A(n).$$

Let us present the refraction index as the infinite sum

$$n = n_0 + n_1 + n_2 + n_3 + \dots, \quad (1)$$

and

$$n_0 \gg |n_1| \gg |n_2| \gg \dots.$$

In addition, the sum of subsequent additives of this series is small compared to the previous additive.

We assume that n_0 is known. Then

$$\tau \approx A(n_0 + n_1),$$

$$\tau \approx A(n_0) + A'(n_0)n_1.$$

The latter relation is the basis for constructing the Newton iteration process for the solution of IKPS, whose realization is given in [2]. However, it is known that convergence of the Newton method depends significantly on the choice of the initial approximation n, which is unknown beforehand in the solution of the inverse problem. It is chosen roughly on the basis of a priori information about the sought velocity distribution. Naturally, this fact negatively influences the solution.

We shall consider media with a quasi-linear increase in velocity with depth

$$V(\mathbf{x}) = A + Bz + V_1(\mathbf{x}), \quad \mathbf{x} = (x, y, z),$$

A and B are positive constants, and the absolute values of V_1 are small compared to the linear component. Such a choice is not accidental because the acoustics of the ocean and the atmosphere satisfies this condition. This choice also takes into account the general tendency of seismic signal velocity increase with depth in the Earth's interior and agrees well with the velocity profile in deep sedimentary basins. In addition, the linear dependence of velocity with depth (in 1D case) allows for explicit relations for the beam (geodetic line) and runtime of the refracted wave along the beam. If constants A and B are known, the initial approximation is determined. However, only their rough estimates can be known in practice, which should be corrected with optimal satisfaction of the linearization requirements.

Let us construct the following functional:

$$F(A, B) = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tau_{ij} - \tau_{0ij}^{(A,B)})^2, \quad (2)$$

where m is the number of tomographic projections, n is the number of elements in each projection, τ_{ij} are data of the direct kinematic problem (runtimes) measured using the tomographic approach, and $\tau_{0ij}^{(A,B)}$ are the corresponding times calculated from the explicit relation (depending on A and B) for the medium with refraction index n. Minimizing this functional in the domain of sought parameters A and B, we find their optimal values.

Thus, we assume that refraction index $n(\mathbf{x})$ is presented in the half-space $z > 0$ by Eq. (1), where $n_0(z) = \frac{1}{(A^* + B^*z)}$ is a function with unknown coefficients A^* and B^* and $n_1(\mathbf{x})$ is a function of the anomalous additive. We note that the following is satisfied:

$$\|n_0(z)\|_{L_1} \gg \|n_1(\mathbf{x})\|_{L_1}.$$

Substituting the relations for τ_{ij} and $\tau_{0ij}^{(A,B)}$ into the relation for functional (2) and taking into account the small value of the anomalous additive, we get the limits from above and below for functional

$$F^-(A, B) \le F(A, B) \le F^+(A, B),$$

where

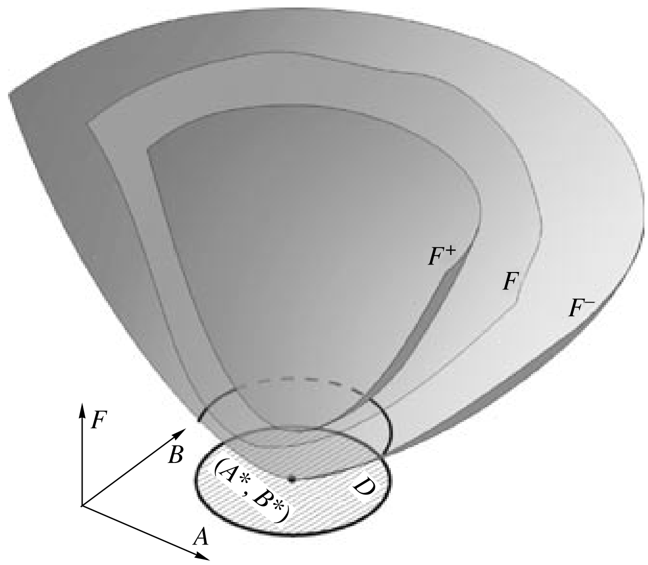

Fig. 1. Illustration of surfaces, F^+, F^-, and $F(A, B)$ in domain D. Domain D is limited by an ellipse, which is a projection on plane $F = 0$ of the intersection between surfaces F^- and the plane parallel to plane $F = 0$ that passes through the point of the functional F^+ minimum.

$$F^-(A, B) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\int_{\Gamma_{0ij}} n_0^{(A^*, B^*)} - n_0^{(A,B)} ds\right)^2,$$

$$F^+(A, B) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\int_{\Gamma_{0ij}} \left|n_0^{(A^*, B^*)} - n_0^{(A,B)}\right| ds + \int_{\Gamma_{ij}} n_1 ds\right)^2.$$

It is seen from these relations that the functions limiting the functional considered here are convex-down functions (Fig. 1) with minima at points $A = A^*$, $B = B^*$. Thus, we can assume the existence of a minimum of the functional. The domain of admissible values of parameters, within which the minimum of the functional is gained, is determined by the inequalities

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\int_{\Gamma_{0ij}} \left|n_0^{(A^*, B^*)} - n_0^{(A,B)}\right| ds\right)^2$$

$$\le \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\int_{\Gamma_{ij}} n_1 ds\right)^2, \quad A > 0, \quad B > 0.$$

Let us consider an illustrative numerical experiment. We assume that $A^* = 0.5$ km/s, $B^* = 0.002$, the function of the anomalous additive is finite, and its carrier is an ellipsoid determined by the inequality

$$\frac{x^2}{R_x^2} + \frac{y^2}{R_y^2} + \frac{(z - Z)^2}{R_z^2} \le 1,$$

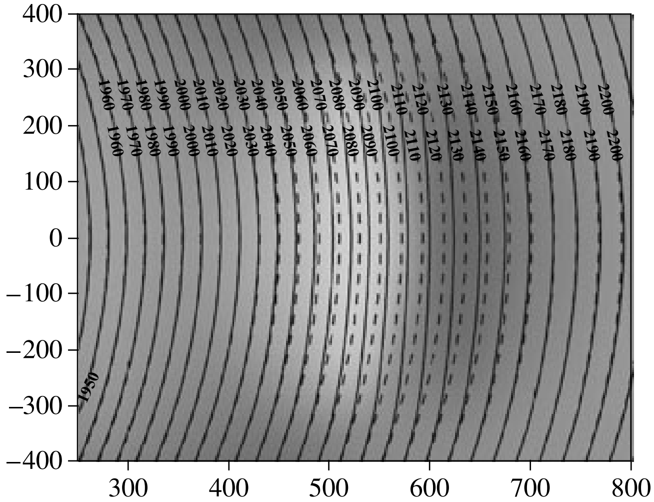

Fig. 2. Fragments of surface time fields for the model with an anomaly (solid contour lines) and without an anomaly (dashed lines).

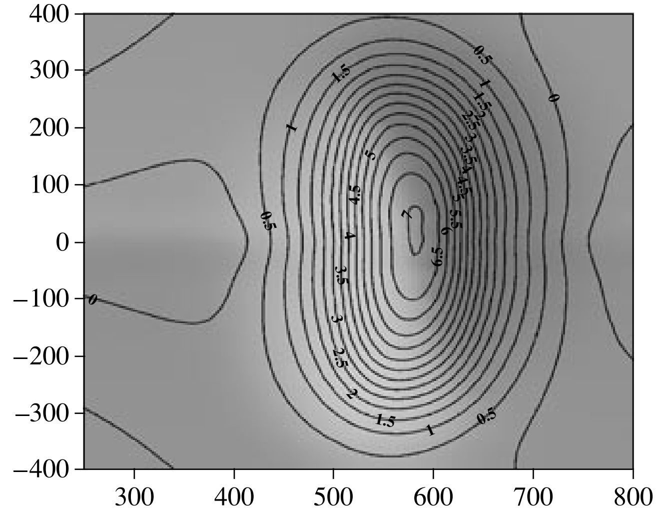

Fig. 3. Fragment of the difference between surface time fields for the model with (and without) an anomaly.

with half-axes $R_x = 200$ m, $R_y = 300$ m, $R_z = 100$ m and the center located at point (0, 0, $Z = 700$ m). Within the carrier, the function takes the following form:

$$n_1(\mathbf{x}) = \frac{5}{1+\cos(\pi S)}, \quad S = \frac{x^2}{R_x^2} + \frac{y^2}{R_y^2} + \frac{(z-Z)^2}{R_z^2}.$$

A surface system of the seismic signal record was chosen as the observation system. The receivers are located at the surface $z = 0$ with a regular step of 50 m in domain $\{-1450 \le x \le 1450; -1450 \le y \le 1450\}$. The source of the seismic oscillations is located at (–1450, 0, 10).

The direct problem for this model and observation system was calculated using the algorithm suggested in [3]. Thus, we obtained experimental values of times τ_{ij}. The time values $\tau_{0ij}^{(A,\,B)}$ for refraction index $n_0^{(A,\,B)}(z)$ are calculated from explicit relation

$$\tau(\mathbf{x}^s, \mathbf{x}^g) = \frac{1}{B}\left[\operatorname{arccoth}\frac{C}{R} + \operatorname{arccoth}\frac{r_g - C}{R}\right],$$

where $\mathbf{x}^s = (x_s, y_s, z_s)$ is the source point, $\mathbf{x}^g = (x_g, y_g, z_g)$ is the receiver point, $r_g = \sqrt{(x_s - x_g)^2 + (y_s - y_g)^2}$, and coefficients C and R are determined from relations

$$C = \frac{r_g}{2} + \frac{(z_g - z_s)(2A + Bz_g + Bz_s)}{2Br_g},$$

$$R = \sqrt{\left(\frac{A}{B} + z_s\right) + C^2}.$$

The minimum of functional (2) found using the Newton method is written as

$$p_{k+1} = p_k - [\nabla^2 F(p_k)]^{-1} \nabla F(p_k), \quad k = 0, 1, \ldots,$$

where $p_k = (A_k, B_k)$ is the value of parameters after iteration k and $p_0 = (A_0, B_0)$ is the initial approximation. At the initial approximation $A = 0.1$, $B = 0.0001$, the iteration process of finding the minimum converged after 44 iterations with parameters $A = 0.49539$ and $B = 0.00198929$.

The time fields at the surface $z = 0$ for refraction indices $n(\mathbf{x})$ and $n_0(z)$ with the found parameters are shown in Fig. 2. The difference between these fields (Fig. 3) will be needed for further determination of refraction index $n(\mathbf{x})$ using the methods of seismotomography according to the following iteration process:

zero iteration: construction of function $n_0(z)$ or calculation of its values at needed points;

first iteration: construction of function $n_1(\mathbf{x})$ (possibly at available complete projection data using explicit inversing relation [4, 5] and calculation of the values of sum $n_0(z) + n_1(\mathbf{x})$;

second iteration: calculation of n_2 on the basis of the knowledge of projection matrix $\|\tau_{ij}\|_{m \times n}$ and method of computational topography, namely the algebraic reconstruction [6] and more precise calculation of refraction index calculated during the first iteration. The medium with the refraction index obtained during the previous iteration (in this case, the first iteration) is assumed as the reference medium.

Further iterations are continued similarly.

If this technique is lacking, the following point is worth noting: starting from the second iteration, one should solve the direct kinematic problem for the medium with a discretely specified velocity.

Remark. The stability of the determination of the sought parameters for complex media can depend significantly on the step of the observation system and number of sources and receivers, as well as on their deepening and dimension of the projection matrix.

The obtained results comprise a numerical model that develops the tomographic approach in seismics, acoustics, and geometric optics (inverse problem to the problem of geometric optics in space [7]). The scope of application of the results of this study is determined by seismotomographic problems of teleseismics, seismic prospecting, and engineering geophysics, as well as seismic, acoustic, and optical monitoring.

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