ISSN 1028-334X, Doklady Earth Sciences, 2007, Vol. 415, No. 5, pp. 799–803. © Pleiades Publishing, Ltd., 2007.
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GEOPHYSICS

Estimate of Maximal Vertical Velocities of Convection in Natural Waters with Lake Baikal as an Example

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Presented by Academician G.S. Golitsyn, July 20, 2006

Received August 6, 2006

DOI: 10.1134/S1028334X07050315

In order to estimate the maximal vertical velocity of convection in natural waters, we propose a formula from [1] modified by multiplier 2 under the radical sign with the assumption that kinetic energy per mass unit does not exceed potential energy. These two formulas agree well with estimates in [2] for vertical velocities of convection based on another method for the case of mixing of the cold Labrador Current and warm Gulf Stream. In the case of Lake Baikal, we consider a real range of water density variation depending on the degree of cabbaling of natural waters due to temperature and suspended matter content with the detailed analysis of anomalous variation near the temperature of the maximal water density.

Density convection in natural waters is one of the most important processes determining the exchange of surface and deep waters. It has geophysical analogs in the atmosphere and solid shell of the Earth [1]. The key issue in the investigation of these processes is the estimate of maximal vertical convective velocity that can be as high as a few kilometers per day in the oceans and seas [2]. This issue is also very interesting for Lake Baikal [3]. The approaches to the research of convection with geophysical applications were developed by Golitsyn [1], and their application to deep natural waters including Lake Baikal is the objective of this work.

BRIEF THEORY

One of the most frequently used formulas for estimating the maximal (vertical) velocity of convective motion W suggested in [1] can be presented as

$$W_{\max} \leq \sqrt{\alpha \Delta T g d}, \tag{1}$$

where α is the coefficient of thermal expansion of water, ΔT is the temperature difference in layer d, and g is acceleration due to gravity. Taking into account that in the Boussinesq approximation

$$\alpha \Delta T \equiv \frac{\Delta\rho}{\rho}, \tag{2}$$

where $\Delta\rho$ is the water density difference in the interacting volumes and ρ is the density of surrounding waters, we can rewrite Eq. (1) as

$$W_{\max} \leq \sqrt{gd\frac{\Delta\rho}{\rho}} \tag{3}$$

or rewriting with the frequently applied reduced acceleration due to gravity $g' = g\frac{\Delta\rho}{\rho}$, we get

$$W_{\max} \leq \sqrt{g'd}. \tag{4}$$

The condition that kinetic energy should not exceed potential energy leads to the appearance of multiplier 2 under the radical sign in the right parts of Eqs. (1), (3), and (4), which would be written as

$$W'_{\max} \leq \sqrt{2\alpha\Delta T g d}, \tag{1'}$$

$$W'_{\max} \leq \sqrt{2\frac{\Delta\rho}{\rho} g d}, \tag{3'}$$

$$W'_{\max} \leq \sqrt{2g'd}. \tag{4'}$$

The dimensionless ratio $\frac{\Delta\rho}{\rho}$ is an obvious parameter of density similarity. When it is necessary to take into the account relative gravity force, we introduce the Froude number, which would be written in our case for Eq. (3') as

$$\mathrm{Fr} = \frac{W^2}{g}d = \frac{2\Delta\rho}{\rho}. \tag{5}$$

For the majority of mixing processes realized in the basins (seas), the Froude number is $\mathrm{Fr} \ll 1$ [2]. Taking this into account, we shall get from Eq. (5)

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$$\frac{\Delta\rho}{\rho} \ll 1. \qquad (6)$$

The most probable process responsible for the increase in $\frac{\Delta\rho}{\rho}$ in natural waters is the temperature-mediated cabbaling $\Delta\rho_{cabb}$, which we shall analyze in detail.

The volumes of natural waters participating in mixing can be located one over the other if convection occurs in a horizontally uniform basin or at one level if the basin is vertically homogeneous [4]. Denser mixed water appears even during mixing of waters with equal density but with different temperatures T_1 and T_2 (temperature difference $\Delta T = T_2 - T_1$) and salinities S_1 and S_2 (salinity difference $\Delta S = S_2 - S_1$). It is calculated as [5]

$$\Delta\rho_{cabb} = -\frac{1}{8}[\rho_{TT}(\Delta T)^2 + 2\rho_{TS}\Delta T\Delta S + \rho_{SS}(\Delta S)^2], \qquad (7)$$

where subscripts at ρ denote partial double and mixed derivatives by T and S. Density differences of natural waters $\Delta\rho_{NW}$ can also be formed due to other components, for example, natural suspended matter of mineral, organic, and other sources. A detailed thermohaline analysis of mixing processes in Baikal water demonstrated that we observe a decrease in density rather than cabbaling in the isothermal case, when $\Delta T \equiv 0$ and $\Delta S \neq 0$. This is also true for seawaters. Hence, cabbaling is more effectively realized in fresh waters compared to seawater [6].

Let us analyze in detail the case with cabbaling $\Delta\rho_{cabb}$, which is reduced to the account for the dependence of density ρ only on temperature ($\Delta T \neq 0$; S = const) and pressure P or depth Z increasing downward. Then (7) is written in the following form [2]:

$$\Delta\rho_{cabb} = -\frac{\rho_{TT}(\Delta T)^2}{8}. \qquad (8)$$

For natural waters, which satisfy the equation of state in Chen–Millero form [7], the methods for calculating $\Delta\rho_{cabb}$, exact values of ρ_{TT}, and other thermodynamic parameters were developed in [8, 9].

RESULTS AND DISCUSSION

Cabbaling $\Delta\rho_{cabb}$. The initial period of convection is when ΔT only slightly exceeds the corresponding adiabatic gradient of water temperature Γ. Vertical convective motions in two neighboring water volumes appear not at any temperature differences but only when the temperature differences ΔT exceed the corresponding adiabatic gradient Γ, i.e., $\Delta T > \Delta T_{ad}$. In the case $\Delta T \leq \Delta T_{ad}$, the condition of no convection appears [10]. Let us present the limiting value of cabbaling $\Delta\rho^{\Gamma}_{cabb}$, after which convection starts with account for the adiabatic gradient Γ in the following form:

$$\Delta\rho^{\Gamma}_{cabb} = -\frac{\rho_{TT}(\Delta T_{ad})^2}{8}. \qquad (9)$$

It is possible to find ΔT_{ad} from the formula for transition from Γ as a function of pressure P (in bars) to adiabatic gradient $\frac{\Delta T_{ad}}{\Delta Z}$ as a function of depth Z (in meters) [10]

$$\frac{\Delta T_{ad}}{\Delta Z} = \rho g\Gamma. \qquad (10)$$

Velocities W_{max}, W'_{max}, and $\frac{\Delta\rho^{\Gamma}_{cabb}}{\rho}$ were calculated using Eqs. (9) and (10) for the case $\Delta T \simeq \Delta T_{ad}$ in the temperature range of natural waters from 0 to 30°C (Fig. 1). The physical condition $\Delta T > \Delta T_{ad}$ means the beginning of convective vertical motions in water.

The behavior of W_{max}, W'_{max}, and $\frac{\Delta\rho^{\Gamma}_{cabb}}{\rho}$ near the temperature of maximum density T_{MD}, which is approximately equal to 4°C or more exactly 3.9646°C in the Baikal surface water, is the most interesting problem.

Parameters $\frac{\Delta\rho^{\Gamma}_{cabb}}{\rho}$, W_{max}, and W'_{max} depend strongly on temperature. At 30°C, they have the following order of magnitude: $5.1 \cdot 10^{-10}$ and $\sim 1 \cdot 10^{-2}$ cm/s. At temperatures 9 and 0°C, their values decrease approximately by a factor of 10 and decrease by one more order of magnitude when the temperatures tend from 9 to 5°C and from 0 to 3°C. At 4°C, they reach an approximate minimum equal to $2.7 \cdot 10^{-15}$ and $\sim 2 \cdot 10^{-15}$ cm/s, and an exact absolute minimum equal to $4.2 \cdot 10^{-22}$ for $\frac{\Delta\rho^{\Gamma}_{cabb}}{\rho}$ and $9 \cdot 10^{-9}$ cm/s for the vertical convective velocity W in Baikal water at the temperature of the maximum density T_{MD} equal to 3.9646°C.

Velocities W equal to $\sim 1 \cdot 10^{-5}$ cm/s or less approach the conditions for diapycnal exchange in the pycnocline [11]. It actually takes place in reality because the order of magnitude of vertical convective velocities W^{Γ}_{max} is 10^{-3} cm/s when the temperatures differ from T_{MD} and significantly lower when the temperature approaches the temperature of maximum density for the surface waters (Fig. 1).

Developed convection when ΔT exceeds significantly the corresponding adiabatic gradient of water temperature Γ. The maximum values of $\Delta\rho_{cabb}$ are realized in the case of the mixing of natural waters with maximum temperature contrast, for example, warm Gulf Stream waters (temperature 24°C) and cold waters of the Labrador Current (temperature 10°C) so that the gradient reaches $\sim 2.34 \cdot 10^{-4}$ g/cm^3 [4, 12] (Fig. 2). Using formulas (3) and (3'), we obtained W_{max} = 4.85

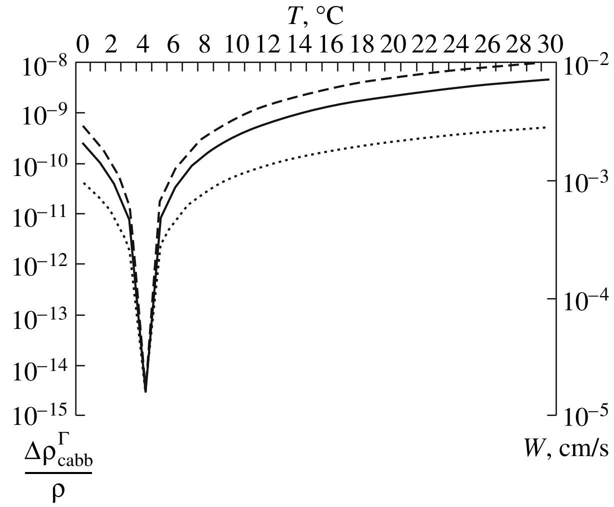

Fig. 1. Density similarity parameter $\frac{\Delta\rho^{\Gamma}_{cabb}}{\rho}$ vs. temperature range 0–30°C for Baikal waters (salinity ~0.096‰). The temperature gradient begins to exceed the corresponding adiabatic gradient Γ (dotted line) and initial vertical convective velocity W after departure from the condition of zero convection. Calculations are based on Eq. (3) (solid line) and Eq. (3') (dashed line).

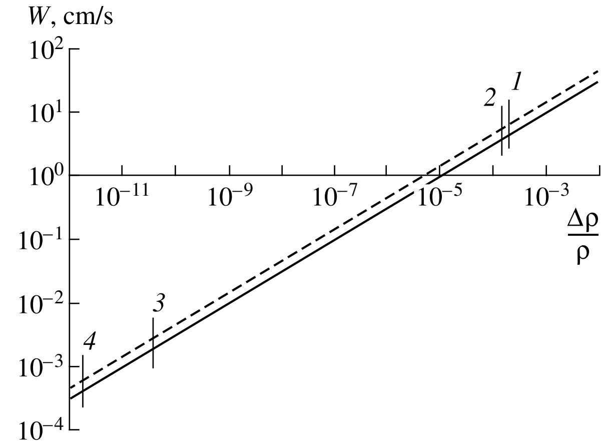

Fig. 2. Maximal vertical convective velocity W vs. density similarity parameter $\frac{\Delta\rho}{\rho}$ ranging from $1 \cdot 10^{-12}$ to $1 \cdot 10^{-2}$ for the natural surface. Calculations are based on Eq. (3) (solid line) and Eq. (3') (dashed line). Vertical tics correspond to the following values of similarity parameter $\frac{\Delta\rho}{\rho}$: (*1*) $2.34 \cdot 10^{-4}$ for cabbaling of waters in the Labrador Current and Gulf Stream [4]; (*2*) $1.6 \cdot 10^{-4}$ or more for incipient suspension (density) flows and cabbaling of Baikal waters with temperatures T; (*3*) $5.06 \cdot 10^{-10}$ at 20°C; (*4*) $\sim 2.1 \cdot 10^{-12}$ ($T \geq 5$°C and $3 \leq T \geq 0$°C), i.e., for the temperatures 1°C higher or lower than the temperature of the maximum density $T_{MD} \approx 4$°C.

and W'_{max} = 6.86 cm/s. Using another formula with account for the real coefficient of turbulent exchange K_Z, Fedorov [2] obtained in this case W^F_{max} = 6 cm/s. Finally we get

$$W'_{max} > W^F_{max} > W_{max}. \qquad (11)$$

This means that Eq. (3') is idealized and does not take into account in the physical sense, for example, water viscosity. Therefore, $W'_{max} > W^F_{max}$. In the right side of Eq. (3) under the radical sign, multiplier 2 is not taken into account for kinetic energy, so that we get $W^F_{max} > W_{max}$. Thus, velocities W'_{max} and W_{max} give the range of estimate of maximal vertical convective velocities. Relation (11) confirms the correctness of the introduction of multiplier 2 under the radical sign in Eqs. (1), (3), and (4). Correspondingly, Eqs. (1'), (2'), and (4') are correct.

High temperature contrasts are also observed in the beginning of summer in Lake Baikal in the places where large rivers run into the lake when the temperature of lacustrine waters is close but smaller than the temperature of maximum density (~4°C), while the temperature of river waters can exceed 14°C. For example, in the observations on May 29, 1960, the temperature difference between lacustrine water (3.6°C) and coastal water (12.5°C) near the Kharaus channel in the shallow-water zone of the Selenga River (spring thermal bar) was equal to ~9°C [13], which corresponded to $\frac{\Delta\rho}{\rho} = 1.24 \cdot 10^{-4}$. High temperature contrasts (but with smaller amplitudes) are also observed in these regions in October–November during the autumnal thermal bar.

The entire range of variations in $\frac{\Delta\rho}{\rho}$ *during cabbaling of natural waters.* The maximum $\frac{\Delta\rho}{\rho}$ values for surface waters with maximal temperature contrast are of the order of 10^{-4} (vertical tic *1* in Fig. 2). The lower threshold of $\frac{\Delta\rho}{\rho}$ (more exactly, lower thresholds) is determined when the temperature difference ΔT decreases and reaches ΔT_{ad} corresponding to the adiabatic temperature gradient Γ, resulting in cessation of vertical water motion.

The first of such thresholds $\frac{\Delta\rho^{\Gamma}_{cabb}}{\rho}$ for waters with temperature 30°C appears at a level of $5 \cdot 10^{-11}$ (vertical tic *3* in Fig. 2). The second threshold appears for waters with temperatures of 5°C (branch from 30 to 5°C) and 3°C (branch from 0 to 3°C) at a level of $\sim 2 \cdot 10^{-12}$ (vertical tic *4* in Fig. 2). The corresponding velocity ranges of $W'_{max} - W_{max}$ are $9.9 \cdot 10^{-3}$–$7.1 \cdot 10^{-3}$ and $6.5 \cdot 10^{-4}$–$4.5 \cdot 10^{-4}$ cm/s, respectively, for the first and second thresholds. According to the calculations, within the domain ±1°C around T_{MD}, when temperature becomes close to T_{MD}, $\frac{\Delta\rho^{\Gamma}_{cabb}}{\rho}$ tends to a value of $\sim 10^{-22}$, while W_{max} tends to 10^{-9} cm/s.

Natural suspended matter as one of the most probable causes of variations in $\frac{\Delta\rho}{\rho}$ *along with temperature.* Variations in the suspended matter of organic and inorganic (mineral) origin in natural waters $\Delta\rho_{sm}$, which occur together with cabbaling due to temperature difference during mixing, make up a unique process that can be written as

$$\frac{\Delta\rho}{\rho} = \frac{\Delta\rho_{cabb}}{\rho} + \frac{\Delta\rho_{sm}}{\rho}. \tag{12}$$

At high concentrations of suspended matter (usually mineral substance), suspension (density, turbid, etc.) flows begin to develop. During their initial stage, $\frac{\Delta\rho}{\rho} > 1.6 \cdot 10^{-4}$ [14] and the corresponding vertical velocities $W_{max} \geq 1.2$ cm/s (vertical tic *2* and higher zone in Fig. 2). According to the observations and calculations for Lake Baikal, $\frac{\Delta\rho}{\rho}$ (written in form [12]) can reach $9 \cdot 10^{-3}$ or more [14] in flood river flows of coastal waters [14]. The total concentration of suspended matter in Lake Baikal changes significantly in the coastal regions and in the open lake: 100 mg/l or more in the Selenga shallow-water zone near the channels of the Selenga River; *n* to 10*n* mg/l at a distance of a few kilometers from the coast; and *n* to 0.*n* mg/l in the central parts of the lake [15]. Parameter $\frac{\Delta\rho}{\rho}$ estimated for suspended matter are as follows: $\sim 1 \cdot 10^{-4}$, $\sim(1–10) \cdot 10^{-6}$, and $\sim(1–10) \cdot 10^{-7}$. W_{max} varies from *n* to 0.*n* cm/s, respectively.

According to [2], fronts appear in the places where horizontal gradients of temperature, salinity, suspended matter (in our case), or the proportional attenuation index of light radiation are significantly greater (approximately by an order of magnitude or more) than the corresponding mean spatial values.

Thus, variations in the concentration of suspended matter by an order of magnitude give practically the same contribution to the variations in $\frac{\Delta\rho}{\rho}$ as variations in temperature (especially in the frontal zones) but with the only difference that significant temperature variations are observed to depths of 100–200 m, while variations in the concentration of suspended matter are observed from the surface to the bottom. The estimates of W_{max} in the oceanic-type fronts in Lake Baikal yield values close to 0.*n* cm/s or less [3], suggesting that variations in the possible real values of $\frac{\Delta\rho}{\rho}$ can be as high as $\sim(1–10) \cdot 10^{-12}$, i.e., occupy the entire range shown in Fig. 2.

Thus, we considered the parameter of density similarity $\frac{\Delta\rho}{\rho}$, which determines the Froude number from Eq. (6), for natural waters in a wide range of density variations depending on temperature and suspended matter concentration: from $\frac{\Delta\rho}{\rho} \sim 1$ to $1.6 \cdot 10^{-4}$, which corresponds to suspension turbid flows; $2.34 \cdot 10^{-4}$, which corresponds to cabbaling in the case of maximal temperature contrasts for the Labrador Current and Gulf Stream (values of this order are recorded at riverine estuaries in Lake Baikal); $\sim 1 \cdot 10^{-10}$ at a temperature of 30°C; and $1 \cdot 10^{-22}$ at the temperature of maximum density (3.9646°C for the Baikal surface waters). Actually, we covered the entire spectrum of variations in $\frac{\Delta\rho}{\rho}$ from turbid flows to the state of weightlessness (hydrospace) in the temperature range from 0 to 30°C with detailed analysis near the temperature of maximum density.

In general, the approach applied for estimating the maximal velocity of convection is a further development of the research of convection with geophysical applications and similarities started by G.S. Golitsyn. This approach provides insight into processes of vertical exchange in Lake Baikal and other media on the Earth.

ACKNOWLEDGMENTS

The author is grateful to Academician G.S. Golitsyn for advice, discussions, and editing of the text and to Prof. K.D. Sabinin for fruitful discussions.

This work was supported by the Russian Foundation for Basic Research, project no. 05-05-64375, 05-05-97242, and 06-05-64685.

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