A Method for Graphically Presenting the Deformation Modulus of Jointed Rock Masses

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Technical Note

A Method for Graphically Presenting the Deformation Modulus of Jointed Rock Masses

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1. Introduction

The deformability of jointed rock masses is associated with the elastic properties of the intact rocks as well as the geometric structure and the stiffness of the geological discontinuities within the rock masses. Determination of the deformation modulus of a rock mass is always a problem in practice. The conventional means to determine the deformation modulus is to perform *in-situ* tests, for instance dilatometer tests, plate-loading tests, flat-jack tests and block tests (Franklin and Dusseault, 1989; Yow Jr., 1993). One can question whether a single evaluation of the deformation modulus obtained in *in-situ* tests is representative of the whole rock mass, considering the relatively small volume of the tested sample and the orientation-related properties of the discontinuities in the rock mass. *In-situ* tests are time-consuming and very costly, so that empirical approaches are more often employed today for estimation of the deformation modulus of rock masses, for instance the methods developed on the basis of rock mass classification systems by Bieniawski (1978), Serafim and Pereira (1983) and Barton et al. (1980).

The deformation modulus of a jointed rock mass may be estimated using the geometric data of the rock joints as well as the elastic parameters of the intact rocks and the rock joints if available. Analytical studies have been made on this subject by a number of researchers, for example Fossum (1985), Amadei (1993) and Huang et al. (1995). Amadei (1993) presented a general solution to the deformability of regularly jointed rock masses. His work shows that the deformation modulus is anisotropic in jointed rock masses. The structure and the stiffness of the rock joints predominately determine the magnitude of the deformation modulus of the rock mass. In Amadei's approach, the regularly jointed rock mass is replaced by an equivalent orthotropic continuum.

This note introduces a method to present the deformation modulus of jointed rock masses in a hemispherical projection. It is assumed in this approach that the

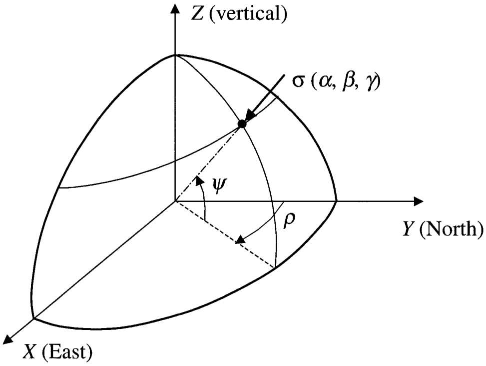


Fig. 1. The global co-ordinate system XYZ

persistence of all rock joints is infinite. The deformation on a single joint is studied first. Then the formulas of the deformation modulus are established for rock blocks containing single joints and for rock masses containing joint sets, respectively. Finally, the graphical presentation of the deformation modulus is introduced via an example.

2. The Displacement on a Single Joint Under Loading

In this approach a global co-ordinate system is employed, which is defined so that the X-axis stands for the East, the Y-axis for the North and the Z-axis for the vertical (see Fig. 1).

The direction of the applied stress σ is represented by the angles α, β and γ with respect to the global co-ordinates X, Y and Z, respectively. The direction of σ can also be represented by its trend (ρ) and plunge (ψ), see Fig. 1. The angles α, β and γ have the following relationships with the trend ρ and the plunge ψ:

$$\begin{aligned} \cos\alpha &= \cos\psi \sin\rho \\ \cos\beta &= \cos\psi \cos\rho \\ \gamma &= \frac{\pi}{2} - \psi. \end{aligned} \tag{1}$$

Let $\mathbf{n}_i(\cos\alpha_i, \cos\beta_i, \cos\gamma_i)$ represent the normal to the plane of the ith joint in the rock mass, see Fig. 2. The angles of the normal, α_i, β_i and γ_i, have the following relationships with the dip angle, λ_i, and the dip direction, ω_i, of the ith joint:

$$\begin{aligned} \cos\alpha_i &= \sin(\lambda_i)\sin(\omega_i) \\ \cos\beta_i &= \sin(\lambda_i)\cos(\omega_i) \\ \gamma_i &= \lambda_i. \end{aligned} \tag{2}$$

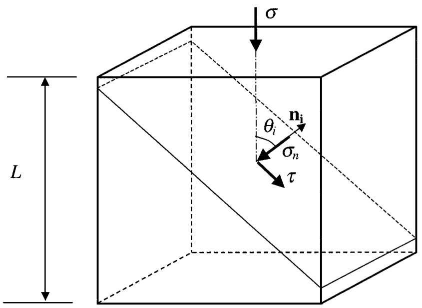


Fig. 2. A rock block containing one joint

The angle between the loading direction and the normal to the plane of the *i*th joint, θ_i, is expressed as:

$$\cos\theta_i = \cos\alpha\cos\alpha_i + \cos\beta\cos\beta_i + \cos\gamma\cos\gamma_i. \tag{3}$$

Assuming that the increment of the applied stress is $\Delta\sigma$, the increments of the normal and shear stresses, $\Delta\sigma_n$ and $\Delta\tau$, on the plane of the *i*th joint have the forms of

$$\begin{aligned} \Delta\sigma_n &= \Delta\sigma\cos^2\theta_i \\ \Delta\tau &= \Delta\sigma\sin\theta_i\cos\theta_i. \end{aligned} \tag{4}$$

The increments of the normal and shear displacements, Δd_{ni} and Δd_{si}, on this joint under the increment of the applied stress, $\Delta\sigma$, are calculated as:

$$\begin{aligned} \Delta d_{ni} &= \frac{\Delta\sigma_n}{k_{ni}(\sigma)} = \frac{\Delta\sigma}{k_{ni}(\sigma)}\cos^2\theta_i \\ \Delta d_{si} &= \frac{\Delta\tau}{k_{si}(\sigma)} = \frac{\Delta\sigma}{k_{si}(\sigma)}\sin\theta_i\cos\theta_i, \end{aligned} \tag{5}$$

where $k_{ni}(\sigma)$ and $k_{si}(\sigma)$ represent the normal and shear stiffness of the *i*th joint, respectively. It should be noticed that the normal and shear stiffness of a rock joint is determined by the geometric profile of the asperities on the joint, the elastic properties of the joint wall material and also the stresses on the joint plane. Based on their experimental results, Bandis et al. (1983) proposed two equations for estimating $k_{ni}(\sigma)$ and $k_{si}(\sigma)$. Their experiments showed that both the normal stiffness and the shear stiffness of the rock joint increases with increasing the normal stress on the joint plane.

Finally, we obtain the displacement increment of the *i*th joint in the loading direction, Δd_i, is calculated as:

$$\Delta d_i = \Delta d_{ni}\cos\theta_i + \Delta d_{si}\sin\theta_i = \Delta\sigma\cos\theta_i\left[\frac{1}{k_{ni}(\sigma)}\cos^2\theta_i + \frac{1}{k_{si}(\sigma)}\sin^2\theta_i\right]. \tag{6}$$

3. Deformation Modulus of Jointed Rock Masses

3.1 Rock Blocks Containing Single Joints

The total deformation of a rock block that contains single joints is the superposition of the deformation of the intact rock and the deformation of the single joints. Under an increment of the applied stress, $\Delta\sigma$, the elastic displacement increment of the intact rock of the block is expressed as:

$$\Delta d^e = \frac{\Delta\sigma}{\mathrm{E}} L, \tag{7}$$

where E is the Young's modulus of the intact rock, and L is the height of the block, see Fig. 2. The total displacement Δd of the block is the sum of Eq. (6) and Eq. (7), i.e.

$$\Delta d = \Delta d^e + \sum_{i=1}^{M} \Delta d_i, \tag{8}$$

where M stands for the number of the single joints contained in the rock block. The corresponding nominal strain of the block is defined as:

$$\Delta\varepsilon = \frac{\Delta d}{L} = \frac{\Delta\sigma}{\mathrm{E}} + \frac{\Delta\sigma}{L} \sum_{i=1}^{M} \cos\theta_i \left[\frac{1}{k_{ni}(\sigma)} \cos^2\theta_i + \frac{1}{k_{si}(\sigma)} \sin^2\theta_i \right]. \tag{9}$$

Thus the deformation modulus of the block in the loading direction, E$'$, has the form of

$$\frac{1}{\mathrm{E}'} = \frac{\Delta\varepsilon}{\Delta\sigma} = \frac{1}{\mathrm{E}} + \frac{1}{L} \sum_{i=1}^{M} \cos\theta_i \left[\frac{1}{k_{ni}(\sigma)} \cos^2\theta_i + \frac{1}{k_{si}(\sigma)} \sin^2\theta_i \right]. \tag{10}$$

3.2 Rock Masses Containing Joint Sets

For a rock block containing a number of joint sets, the total displacement Δd of the block can be expressed as:

$$\Delta d = \Delta d^e + \sum_{i=1}^{N} n\Delta d_i = \Delta d^e + \sum_{i=1}^{N} \frac{L}{S_i} \cos\theta_i \Delta d_i, \tag{11}$$

where N stands for the number of joint sets and $n = (L\cos\theta_i/S_i)$ stands for the number of joint planes of the ith joint set in the length of L, see Fig. 3. The corresponding nominal strain of the block is defined as:

$$\Delta\varepsilon = \frac{\Delta d}{L} = \frac{\Delta\sigma}{\mathrm{E}} + \Delta\sigma \sum_{i=1}^{M} \frac{\cos^2\theta_i}{S_i} \left[\frac{1}{k_{ni}(\sigma)} \cos^2\theta_i + \frac{1}{k_{si}(\sigma)} \sin^2\theta_i \right]. \tag{12}$$

The deformation modulus of the rock mass containing joint sets is then obtained as:

$$\frac{1}{\mathrm{E}'} = \frac{1}{\mathrm{E}} + \sum_{i=1}^{N} \frac{\cos^2\theta_i}{S_i} \left[\frac{1}{k_{ni}(\sigma)} \cos^2\theta_i + \frac{1}{k_{si}(\sigma)} \sin^2\theta_i \right]. \tag{13}$$

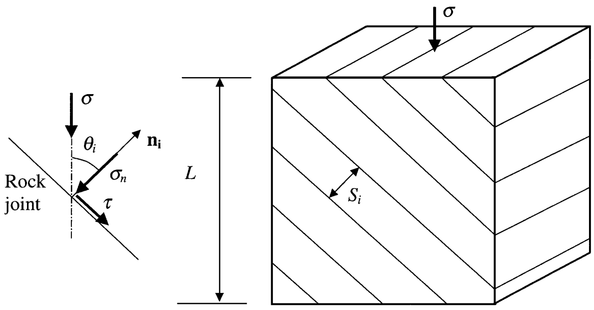
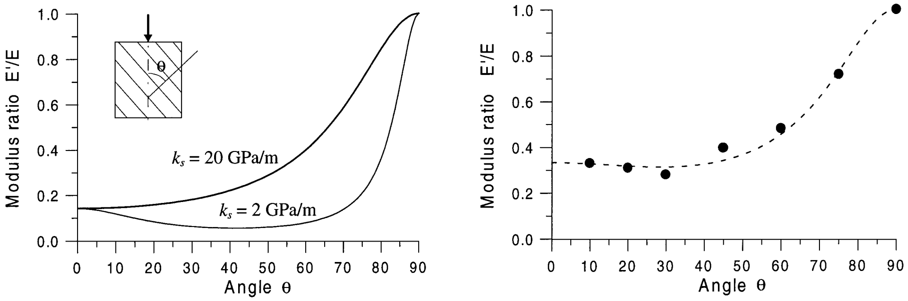


Fig. 3. A rock block containing one joint set



(a) Theoretical solutions of an assumed example ($S = 0.5$ m, $E = 60$ GPa and $k_n = 20$ GPa/m).

(b) The experimental data of the schist specimens (Pinto, 1970).

Fig. 4. The ratio of the deformation modulus of the rock mass to the Young's modulus of the intact rock, E′/E, with respect to the angle θ

4. Demonstrations

In the following demonstrations we assume that the normal and shear stiffness are corresponding to a given stress level, that is, they are constant at that stress level.

4.1 Rock Mass Containing One Joint Set

Assume that:

the spacing of the joint set, S:	0.5 m
Young's modulus of the intact rock, E:	60 GPa
the normal stiffness of the joint planes, k_n:	20 GPa/m.
the shear stiffness of the joint planes, k_s:	20 and 2 GPa/m.

The ratio of the deformation modulus of the rock mass to the Young's modulus of the intact rock, E′/E, is calculated using Eq. (13). The variation of the ratio E′/E with respect to the loading angle θ is illustrated in Fig. 4a for the two different values of the shear stiffness. One of Pinto's experimental results (1970) is shown in Fig. 4b for the sake of comparison. It can be obtained from Eq. (13) that the minimum of the ratio E′/E occurs at an angle of θ in the interval between 0° and 45°. Two extreme cases are that the ratio E′/E is at its minimum at $\theta = 0°$ when the shear stiffness $k_s \geq k_n/2$, while it is at its minimum at $\theta = 45°$ when $k_s = 0$.

4.2 Rock Mass Containing Three Joint Sets

Assume that a rock mass contains the following three joint sets:

Joint set 1: 45°/90° (dip angle/dip direction)
Joint set 2: 60°/220°
Joint set 3: 25°/330°

and let

Young's modulus of the intact rocks:	$E = 60$ GPa,
Spacing of the joint sets:	$S = S_1 = S_2 = S_3 = 1$ m,
Normal stiffness of the joint sets:	$k_n = k_{n1} = k_{n2} = k_{n3} = 20$ GPa/m,
Shear stiffness of the joint sets:	$k_s = k_{s1} = k_{s2} = k_{s3} = 15$ GPa/m.

The direction angles of the joint sets $(\alpha_i, \beta_i, \gamma_i)$ can be obtained using Eq. (2). The direction angles of the loading direction (α, β, γ) can be calculated from its trend ρ and plunge ψ using Eq. (1). Then, the angle θ_i between the ith joint set and the loading direction is calculated using Eq. (3). Finally, the deformation modulus of the rock mass is calculated using Eq. (13). In the calculations, the trend ρ of the loading direction varies from 0 to 360°, and the plunge ψ varies from 0 to 90°.

The results are presented in Fig. 5 and Fig. 6, that are projections of the upper hemisphere. Figure 5 shows the distribution of the deformation modulus of the rock mass when only one of the joint sets is considered. It is seen from the three projection diagrams in Fig. 5 that the minimum modulus occurs in the direction perpendicular to the joint plane. The resultant effect of all the three joint sets on the deformation modulus is shown in Fig. 6. It is seen in this projection diagram that the minimum modulus occurs approximately in the vertical direction. It can be said that the deformation modulus of the rock mass is low in the north-eastern

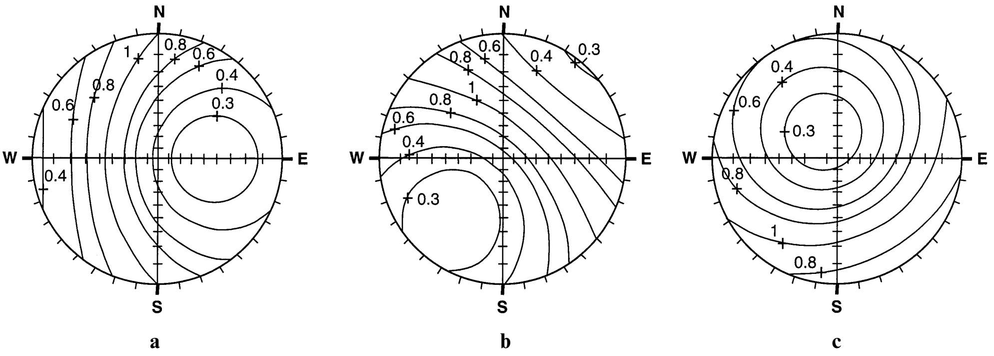


Fig. 5a–c. The effects of the individual joint sets on the modulus ratio E′/E – projections of the upper hemisphere: **a** Containing only joint set 1, **b** containing only joint set 2, **c** containing only joint set 3

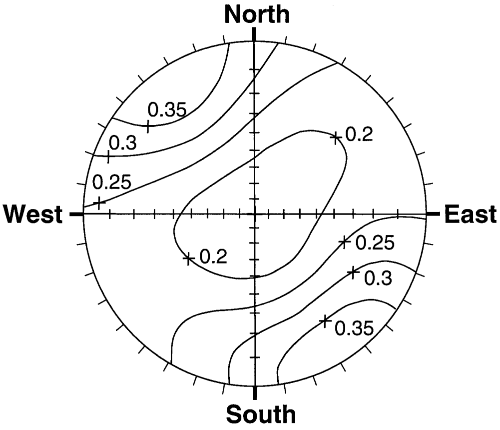


Fig. 6. Distribution of the deformation modulus ratio E′/E of the rock mass containing all the three joint sets in a projection of the upper hemisphere

and the south-western orientations regardless of the magnitude of the plunge, while it becomes relatively large in the north-western and the south-eastern orientations when the plunge becomes small. The range of the deformation modulus of this rock mass is approximately from 15% to 40% of the Young's modulus of the intact rock.

5. Conclusions

The method introduced in this note can be used to graphically present the deformation modulus of jointed rock masses. A global co-ordinate system is used in the approach to define the geometry of rock joints. The deformation modulus of jointed rock masses is determined by the number of joints, the geometry of joints and the stiffness of the joint planes as well as the Young's modulus of the intact rocks. The deformation modulus can be calculated using Eq. (10) for rock blocks containing single joints or Eq. (13) for rock masses containing joint sets. The deformation modulus of a rock mass in the three-dimensional space can be plotted in a projection diagram of the upper hemisphere.

Nomenclature

E – Young's modulus of the intact rock.
E′ – Tangential deformation modulus of the rock block, or rock mass, in the loading direction.
$k_n(\sigma)$ – Normal stiffness of the rock joint at the stress level σ.
$k_{ni}(\sigma)$ – Normal stiffness of the ith rock joint at the stress level σ.
$k_s(\sigma)$ – Shear stiffness of the rock joint at the stress level σ.
$k_{si}(\sigma)$ – Shear stiffness of the ith rock joint at the stress level σ.
L – Height of the rock block.
M – Number of single joints.

$\mathbf{n}_i$ – Normal to the *i*th joint plane.
N – Number of joint sets.
S – Spacing of a joint set.
S_i – Spacing of the *i*th joint set.
X, Y, Z – Axes of the global co-ordinate system.
α – Angle between the loading direction and the X-axis of the co-ordinate system.
β – Angle between the loading direction and the Y-axis of the co-ordinate system.
γ – Angle between the loading direction and the Z-axis of the co-ordinate system.
$\alpha_i, \beta_i, \gamma_i$ – Direction angles of the normal to the *i*th joint plane in the global co-ordinate system XYZ.
λ_i – Dip angle of the *i*th rock joint (set).
θ – Angle between the loading direction and the normal to the joint plane.
θ_i – Angle between the loading direction and the normal to the plane of the *i*th joint.
ρ – Trend of the loading vector.
σ – Applied stress.
σ_n – Normal stress on the joint plane.
τ – Shear stress on the joint plane.
ψ – Plunge of the loading vector.
ω_i – Dip direction of the *i*th rock joint (set).
Δd – Total displacement increment of the rock block in the loading direction under $\Delta\sigma$.
Δd_{ni} – Increment of the normal displacement on the *i*th rock joint (set) under the stress increment $\Delta\sigma$.
Δd_{si} – Increment of the shear displacement on the *i*th rock joint (set) under the stress increment $\Delta\sigma$.
Δd^e – Elastic displacement increment of the intact rock under $\Delta\sigma$.
$\Delta\varepsilon$ – Average strain increment of the rock block, or rock mass, under the stress increment $\Delta\sigma$.
$\Delta\sigma$ – Increment of the applied stress.

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