

Poroelastic finite difference modeling of seismic attenuation and dispersion due to mesoscopic-scale heterogeneity

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[1] Seismic attenuation and dispersion are numerically determined for computer-generated porous materials that contain arbitrary amounts of mesoscopic-scale heterogeneity in the porous continuum properties. The local equations used to determine the poroelastic response within such materials are those of Biot (1962). Upon applying a step change in stress to samples containing mesoscopic-scale heterogeneity, the poroelastic response is determined using finite difference modeling, and the average strain throughout the sample computed, along with the effective complex and frequency-dependent elastic moduli of the sample. The ratio of the imaginary and real parts of these moduli determines the attenuation as a function of frequency associated with the modes of applied stress (pure compression and pure shear). By having a wide range of heterogeneity present, there exists a wide range of relaxation frequencies in the response with the result that the curves of attenuation as a function of frequency are broader than in existing analytical theories based on a single relaxation frequency. Analytical explanations are given for the various high-frequency and low-frequency asymptotic behavior observed in the numerical simulations. It is also shown that the overall level of attenuation of a given sample is proportional to the square of the incompressibility contrasts locally present.

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1. Introduction

[2] A long-standing challenge in seismology is to both measure and interpret seismic attenuation in the Earth. Identifying the attenuation/dispersion mechanisms at work as seismic waves propagate through the porous materials of the Earth's crust remains the subject of ongoing research. Part of the problem in isolating seismic band (1 Hz to 10^4 Hz) mechanisms is that performing laboratory experiments on rocks over these frequencies is itself a difficult endeavor [cf. Batzle *et al.*, 2006]. The present paper is devoted to numerically simulating one loss mechanism that can be important across the seismic band of frequencies when heterogeneity is present within fluid-saturated porous samples.

[3] Most samples of the Earth's crust contain heterogeneity across the mesoscopic scales that range from a few grain diameters to say a tenth of the seismic wavelength. In particular, if the incompressibility of the framework of

grains has heterogeneity across such mesoscales, a wave compressing the material will induce a heterogeneous fluid pressure response that correlates with the incompressibility structure present. Such wave-induced fluid pressure gradients equilibrate via pore pressure diffusion which results in viscous loss. Attenuation, as measured by $1/Q$, is at a maximum when the fluid pressure has just enough time in a wave period to diffuse across the heterogeneous patches present. Peak attenuation thus occurs at a frequency that scales as k/L^2 where k is the local permeability and L a characteristic patch size of the mesoscale heterogeneity. As such, depending on the values of k and L within a sample, it is quite possible to have peak attenuation within the seismic band of frequencies.

[4] In the present paper, the flow induced within a heterogeneous sample by a time-varying stress applied to the sample's surface is numerically determined using the Biot theory of poroelasticity [Biot, 1956, 1962] as the local model controlling the physics. The finite difference algorithm we use is described by Masson *et al.* [2006]. By numerically measuring the strain of the sample, it is possible to determine the complex frequency-dependent elastic moduli of the sample. The imaginary parts of these moduli are due to phase differences between the stress and strain created by the mesoscale viscous flow.

[5] Pride and Berryman [2003a, 2003b] and Pride *et al.* [2004] have obtained analytical models for the seismic

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attenuation and dispersion in the special case where the heterogeneity is due to a mixture of two distinct porous phases and when the heterogeneity has a single dominant length scale. In more general media, involving the mixing of many porous materials or the gradual transition from one material type to another or the presence of many length scales of heterogeneity, no analytical theory presently exists. Numerical solution of the Biot equations with arbitrary heterogeneity in the coefficients is one way to study the seismic response of such materials.

[6] Other analytical models of mesoscopic-scale induced flow are those of *White* [1975] and *Johnson* [2001] in the case of patchy saturation, and *White et al.* [1975], *Norris* [1993], and *Gurevich and Lopatnikov* [1995] in the case of layered porous media. The squirt flow models of *Mavko and Nur* [1979] and *Dvorkin et al.* [1995] are similar in that the fluid within relatively compressible cracks in the grains is allowed to equilibrate with the stiffer main pore space between the grains. However, due to the size of grain-scale cracks, it is more normal for squirt models to produce peak attenuation at frequencies closer to 1 MHz which is well above the seismic band of interest here. The attenuation allowed for in the standard *Biot* [1956] theory is that due to flow at the scale of the wavelength, from the compressions to the dilations. In consolidated rock, such macro flow is well known to produce negligible attenuation in the seismic band. Numerical tests of the *White* [1975] model based on finite difference modeling of the Biot equations have been performed by *Carcione et al.* [1995].

2. Elastic Moduli and Attenuation

[7] Our goal is to numerically compute the complex elastic moduli of numerically created porous samples that contain mesoscale heterogeneity. Over the seismic band of frequencies (1 to 10^4 Hz) that is the focus here, it can be shown [e.g., *Pride*, 2005] that the elastic response is effectively “undrained” which means that the fluid flow into and out of each sample of the porous material can be neglected. However, significant fluid flow may occur within each sample due to the presence of mesoscale heterogeneity.

[8] The porous samples are required to be large enough to contain a statistically significant representation of any mesoscale structure in the porous continuum, but much smaller than the wavelengths. They can be considered to be the finite difference voxels used in the macroscopic forward modeling of a seismic experiment. Saying that the wave response is undrained means that the wave properties would be unaffected if each porous sample were bounded by a vanishingly thin impermeable jacket having comparable elastic moduli and density to that of the surrounding material.

[9] For an isotropic sample, we aim to determine the complex undrained bulk modulus $K_u(\omega)$ and shear modulus $G_u(\omega)$ as defined by Hooke’s law

$$\left[\dot{\tau}_{ik} = 2G_u(\omega) \left(\dot{\epsilon}_{ik} - \frac{1}{3} \delta_{ik} \dot{\epsilon}_{ll} \right) + K_u(\omega) \delta_{ik} \dot{\epsilon}_{ll} \right]_{\zeta=0}, \quad (1)$$

where δ_{ik} is the Kroneker delta function. Here, ζ denotes the fractional changes of fluid mass within the sample, and the

condition $\zeta = 0$ therefore denotes undrained conditions. The bulk stress tensor τ_{ik} represents the average total stress tensor throughout the sample under consideration and we use the notation $\dot{\tau}_{ik}(\omega) = -i\omega\tau_{ik}(\omega)$ to denote the stress rate tensor in the frequency domain. The strain rate tensor of the sample is defined as

$$\dot{\epsilon}_{ik}(\omega) = \frac{1}{2} \left(\frac{\partial v_i(\omega)}{\partial x_k} + \frac{\partial v_k(\omega)}{\partial x_i} \right), \quad (2)$$

where v_i is the average velocity of the solid material within the sample (see *Pride and Berryman* [1998] for a rigorous demonstration of this definition).

[10] For sake of both simplicity and computation time, the present paper only considers two-dimensional (2-D) modeling of the local mesoscopic response within each sample. This is equivalent to assuming a 3-D response in which no strain out of the modeling plane is allowed to develop (plane strain). Using cartesian coordinates, we take $x_1 = x$ and $x_3 = z$ to be the modeling plane, and $x_2 = y$ to be the direction in which no displacements or displacement gradients can occur. Adding the $\dot{\tau}_{xx}$ and $\dot{\tau}_{zz}$ components of equation (1) and using the plane strain condition $\dot{\epsilon}_{yy} = 0$ leads to the relation

$$K_u^{3D}(\omega) = K_u^{2D}(\omega) - \frac{G_u^{3D}(\omega)}{3} \quad (3)$$

for the 3-D elastic modulus of interest, where K_u^{2D} is given by the definition

$$K_u^{2D}(\omega) = \frac{1}{2} \left(\frac{\dot{\tau}_{xx} + \dot{\tau}_{zz}}{\dot{\epsilon}_{xx} + \dot{\epsilon}_{zz}} \right)_{\zeta=0}. \quad (4)$$

The 3-D shear modulus will be obtained by applying different amounts of normal stress to the x and z faces. As such, it can be derived by taking $\dot{\epsilon}_{yy} = 0$ and subtracting $\dot{\tau}_{xx}$ from $\dot{\tau}_{zz}$ in equation (1) to obtain

$$G_u^{3D}(\omega) = \frac{1}{2} \left(\frac{\dot{\tau}_{zz} - \dot{\tau}_{xx}}{\dot{\epsilon}_{zz} - \dot{\epsilon}_{xx}} \right)_{\zeta=0}. \quad (5)$$

The only way there can be a net change in the fluid content of anunjacketed sample during a pure shear is if the sample were anisotropic. We nonetheless focus on the “undrained” shear response of equation (5) because not all of the modeling examples to be considered are isotropic. Note that even when a sample is anisotropic, we only will be measuring the one shear stiffness defined by equation (5) and not the entire suite of shear stiffnesses. *Berryman and Wang* [2001] have shown that this shear stiffness, G_u^{3D} , is the one that has potential dependence on the fluid’s bulk modulus within a porous sample. Further, equations (3)–(5) provide a valid definition of K_u^{3D} even for anisotropic samples.

[11] The viscous relaxation (fluid equilibration) that results in changes of the bulk and shear moduli with frequency is also responsible for wave attenuation. The

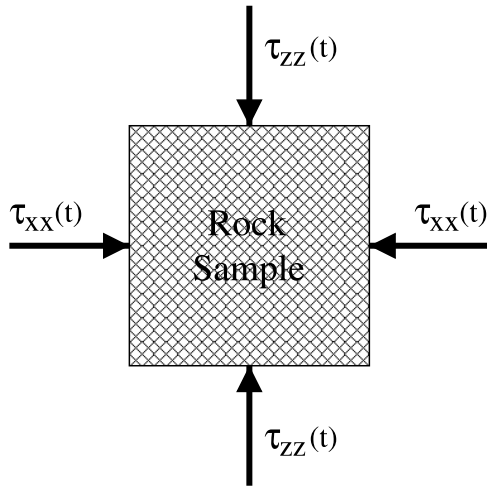


Figure 1. Setup for the numerical experiments. The elastic moduli of the porous sample are obtained by applying a stress on the bounding faces of a sealed sample and measuring the strain response.

inverse quality factor Q^{-1} will be used as our measure of attenuation. The physical definition of Q^{-1} that we employ is

$$\frac{1}{Q} = \frac{\text{total energy lost per stress period}}{4\pi(\text{average energy reversibly stored per period})}. \quad (6)$$

O'Connell and Budiansky [1978] show that this definition requires Q^{-1} to be the ratio of the imaginary and real parts of the elastic modulus involved in the response. We note in passing that this definition, which is the standard one that most seismologists employ, places no upper bound on $1/Q$ since the average energy reversibly stored over one period can fall toward zero for an extremely efficient loss mechanism.

[12] Thus we define Q_{Ku}^{-1} to be the attenuation associated with a pure undrained compression and given by

$$Q_{Ku}^{-1}(\omega) = \frac{\text{Im}\{K_u^{3D}\}}{\text{Re}\{K_u^{3D}\}}. \quad (7)$$

Similarly, Q_G^{-1} is the attenuation of a pure shear and is given by

$$Q_G^{-1}(\omega) = \frac{\text{Im}\{G_u^{3D}\}}{\text{Re}\{G_u^{3D}\}}. \quad (8)$$

For the remainder of the article, the 3-D moduli of a sample will simply be referred to as K_u and G_u .

3. Quasi-Static Modeling

[13] The low-frequency *Biot* [1962] equations, in which viscous boundary layers do not develop in the pores, are solved numerically using a finite difference algorithm that we have recently developed [Masson *et al.*, 2006]. For most consolidated Earth materials, viscous boundary layers may be neglected over the seismic band ($<10^4$ Hz). We empha-

size that the Biot equations are being used to model the local response within a sample. Upon averaging such local poroelastic response, macroscopic viscoelastic moduli are obtained for the sample as a whole that could be used, for example, in the viscoelastic modeling of a seismic experiment. In Appendix A, the local poroelastic equations are given along with all the necessary local input parameters.

[14] The finite differencing algorithm involves explicit time stepping on a staggered grid and requires the local porous medium properties to be discretized onto a finite grid. The spatial sampling interval of the grid is determined by the size of the smallest heterogeneous patch of porous continuum within the sample; we require a minimum of four fourth-order spatial differencing points to be present in the smallest patch in order to avoid numerical artifacts. The time-sampling interval is determined using the stability criteria derived by Masson *et al.* [2006] which, for all ranges of the material properties considered here, amounts to the usual Courant condition.

[15] The mesoscale heterogeneities being allowed for are typically several orders of magnitude smaller than seismic wavelengths. As such, using a full waveform modeling to study the attenuation and dispersion of P and S waves would require several thousand grid points per wavelength, and would be computationally inefficient. Instead, we study the attenuation and dispersion by applying stress quasi-statically to a sample; i.e., the wavelengths over the seismic band in such modeling are always much larger than the sample size.

[16] All the numerical results presented in this study have been obtained using the following three steps:

[17] 1. On a 2-D rectangular mesh, numerically generate a synthetic sample of the material to be studied.

[18] 2. Apply a normal stress in the time domain to each external face of the sample and record both the average stress and average strain throughout the sample. Figure 1 gives a representation of such an experiment.

[19] 3. Take a temporal Fourier transform of the average stress and strain and compute the complex macroscopic elastic moduli $K_u(\omega)$ and $G_u(\omega)$ using equations (3)–(5).

[20] Figure 2 shows the spatial position where the stresses, fluid pressure, particle velocities, and local physical properties are computed within the discretized sample. During step 2, the following sealed boundary conditions are imposed on the external faces of the sample

$$\begin{aligned} \tau_{xx} &= \begin{cases} S_x(t) & \text{on the left and right faces,} \\ 0 & \text{elsewhere;} \end{cases} \\ \tau_{zz} &= \begin{cases} S_z(t) & \text{on the top and bottom faces,} \\ 0 & \text{elsewhere;} \end{cases} \\ \tau_{xz} &= p_f = v_x = v_z = q_x = q_z = 0. \end{aligned} \quad (9)$$

Here, $S_i(t)$ denotes the stress function applied in the i th direction. To properly allow for these boundary conditions, second-order spatial differencing operators are used in the first three layers of grid points around the computational domain, while fourth-order spatial operators are used throughout the remainder of the grid points. This is depicted in Figure 2.

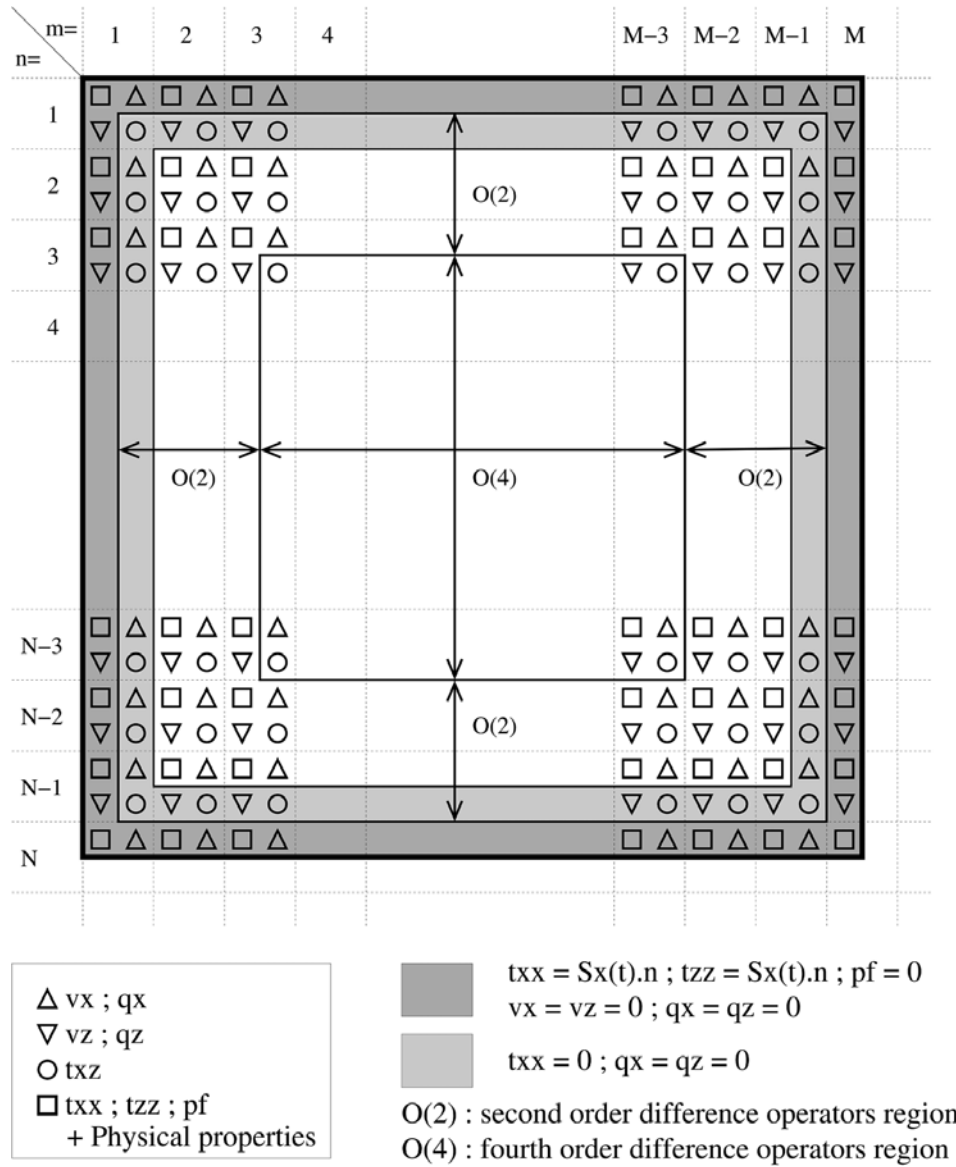


Figure 2. Relative position of the field components on the staggered grids. The stress function is applied in the external dark gray layer of grid points. To simulate undrained experiments, the fluid particle velocity field $\mathbf{q}$ is kept to zero in the light gray area. The boundary conditions are better allowed for by using $O(2)$ spatial finite operators in the external region and $O(4)$ throughout the remaining interior region.

[21] The stress function $S(t)$ has been chosen to offer a wide band of frequency content within a small time duration. For all examples, it is taken to be

$$S(t) = \frac{S_o}{2} \{1 + \tanh[r(t - t_0)]\}. \quad (10)$$

The constant amplitude S_o fixes the stress level, while the time dependence becomes a simple step function when the stretching parameter r goes to infinity. For the experiments, the parameter t_0 is set sufficiently large that $rt_0 \gg 1$ so that $S(t = 0) \simeq 0$. The lowest frequency present in the function $S(t)$ is controlled by the duration of the signal, while the largest frequency is related to the parameter r . To avoid undesirable high-frequency resonance, the stretching para-

meter r is adjusted in order to obtain the largest frequency content without appreciably exciting wavelengths comparable to sample dimensions.

[22] During the numerical experiments, the following average fields are recorded at each time step

$$\langle D_t v_i \rangle(l) = \frac{1}{N-2} \frac{1}{M-2} \sum_{m=2}^{M-1} \sum_{n=2}^{N-1} [D_t v_i]_{l,m,n} \quad (11)$$

$$\langle D_t \tau_{ii} \rangle(l) = \frac{1}{N-2} \frac{1}{M-2} \sum_{m=2}^{M-1} \sum_{n=2}^{N-1} [D_t \tau_{ii}]_{l,m,n}. \quad (12)$$

Table 1. Material Properties of the Samples Shown in Figure 3^a

Property	Phase 1	Phase 2
Grain		
Bulk modulus K_g	36.0 MPa	36.0 GPa
Density ρ_g , kg/m ³	2650	2650
Frame		
Bulk modulus K_d , MPa	621	6.21
Shear modulus μ , MPa	455	455
Porosity ϕ	0.33	0.33
Permeability k m ²	10^{-12}	10^{-15}
Fluid		
Bulk modulus K_f , GPa	2.25	2.25
Density ρ_f , kg/m ³	10^3	10^3
Viscosity η , N s m ⁻²	10^{-3}	10^{-3}

^aPhase 1 is more compressible than phase 2 and has a higher permeability.

Once the modeling is finished, the stress rate and strain rate are computed in the frequency domain using a Fourier transform (FT)

$$\dot{\epsilon}_{ii}(\omega) = \text{FT}\{\langle D_i v_i \rangle\} \quad (13)$$

$$\dot{\tau}_{ii}(\omega) = \text{FT}\{\langle D_i \tau_{ii} \rangle\}. \quad (14)$$

Finally, the undrained bulk and shear moduli, as well as the respective attenuation, are computed using equations (3)–(5). In order to obtain $K_u(\omega)$ and $G_u(\omega)$, two kinds of experiments are performed: (1) pure compression experiments in which $S_x(t) = S_z(t) = S(t)$; and (2) pure shear experiments in which $S_x(t) = S(t)$ and $S_z(t) = -S(t)$. Performing a single mixed experiment with $|S_x| \neq |S_z|$

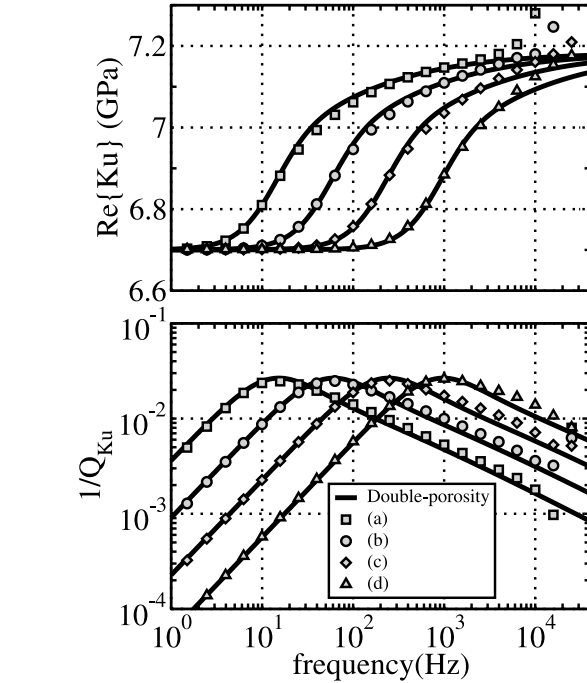


Figure 4. Real part of the bulk modulus and the respective attenuation obtained from the geometries a, b, c, and d in Figure 3. In this example, the black regions in Figure 3 are filled with the softer/more permeable phase 1 and the white regions with the stiffer/less permeable phase 2. The properties of phases 1 and 2 are given in Table 1.

to give both K_u and G_u simultaneously gives noisier results.

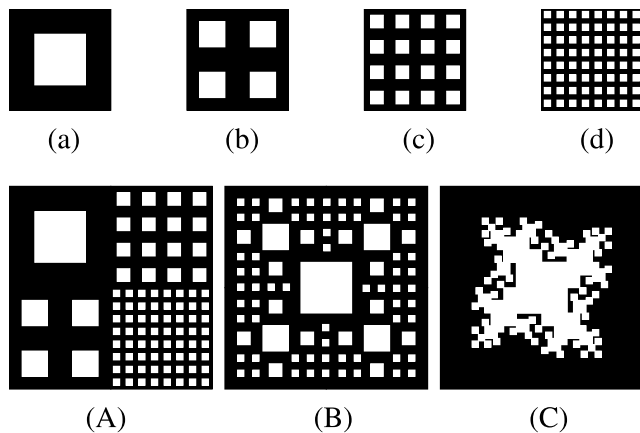


Figure 3. Geometries of the samples used for testing the double-porosity theory. The sizes of the small and the large samples are 64 mm and 128 mm, respectively. The sizes of the white squares for samples a, b, c, and d are 32, 16, 8, and 4 mm, respectively. Each material is made of 1/4 white phase and 3/4 black phase. The three composites A, B, and C are obtained by mixing together a, b, c, and d in different ways.

4. Attenuation and Dispersion due to Pure Compression

4.1. Double Porosity and Simple Geometrical Effects

[23] In order to compare the numerical method to the double-porosity theory [e.g., Pride *et al.*, 2004], two series of four numerical experiments have been realized. In all eight experiments, the samples consist of square-shaped porous inclusions embedded within a homogeneous porous matrix. The physical properties of the two porous phases are given in Table 1 and the spatial positions of the inclusions are shown in Figure 3, samples a, b, c, and d. The number of inclusions in each material has been adjusted to have the same partial volumes of each phase present in all samples; specifically, 1/4 inclusion and 3/4 matrix.

[24] In the first series of experiments, the inclusions are stiffer and less permeable than the surrounding matrix. In this case, the diffusive fluid flow in response to the step change in applied stress advances into the inclusions, with fluid pressure remaining almost spatially uniform throughout the softer and more permeable matrix (the fluid pressure gradually decreases through time in the more permeable phase but remains effectively uniform in space). The undrained bulk modulus K_u and the associated attenuation Q_{Ku}^{-1} numerically measured for the samples a, b, c and d are

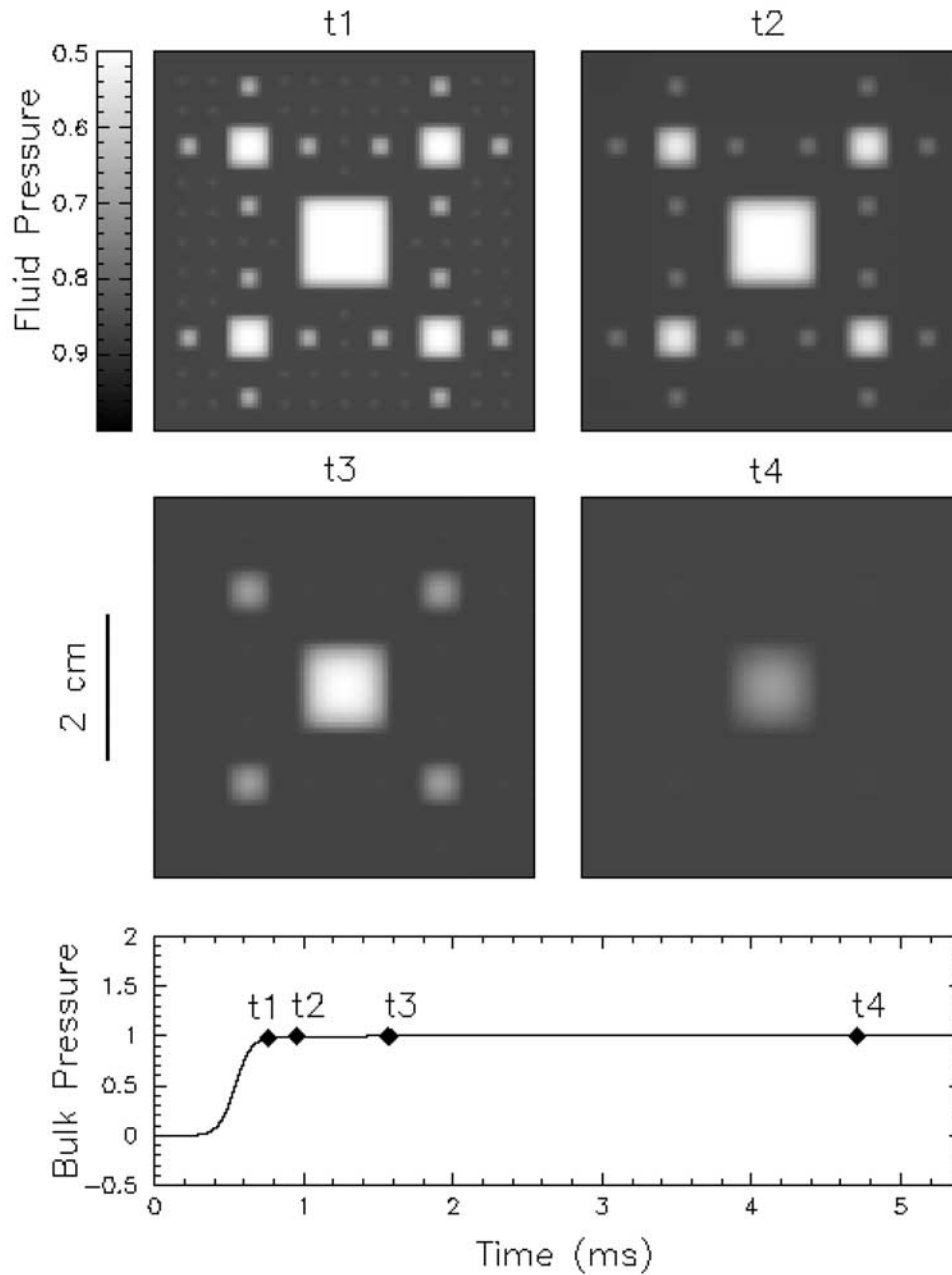


Figure 5. (top) Snapshots showing the fluid pressure at different times (t1–t4) during a purely compressional experiment. (bottom) Applied stress shown as a function of time.

plotted as symbols in Figure 4. The theoretical results, obtained using the analytical double-porosity theory of *Pride et al.* [2004], are plotted as the solid lines. The analytical expressions, along with additional discussion of the double-porosity theory, are given in Appendix B. As seen there, all parameters required by the double-porosity theory are either known or calculated using their theoretical definition. There are no free parameters that are adjusted to achieve a good fit to the numerical data.

[25] The finite difference results show a good agreement with the double-porosity theory. The spatial sampling interval for the finite differencing was taken to be 0.5 mm while

the smallest patch of heterogeneity was 4 mm (see Figure 3, sample d), which corresponds to 8 grid points within the smallest patch. The time-sampling interval was given by the stability criterion of *Masson et al.* [2006]. Fluctuations in the numerical response above 10^4 Hz are due to sample resonance.

[26] At low enough frequencies, the attenuation increases linearly with increasing frequency. This is a consequence of the finite size of the sample (as will be discussed in section 4.2). At peak attenuation, the fluid has just enough time in each period to diffusively penetrate each inclusion. The fluid pressure diffusivity D of a porous material goes as

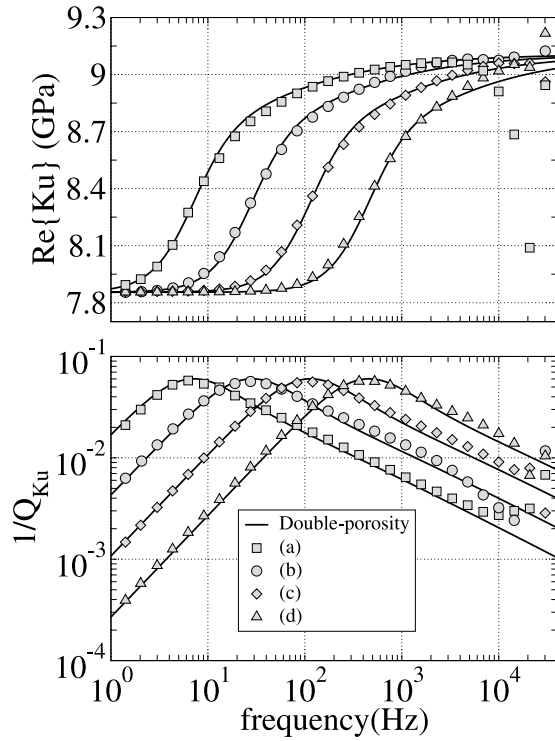


Figure 6. Real part of the bulk modulus and the associated attenuation obtained from the geometries a, b, c, and d in Figure 3. In this example, the black regions in Figure 3 are filled with phase 2 and the white regions with phase 1 from Table 1. Just the opposite from the results of Figure 4.

(see Pride [2005] for a demonstration and Appendix A for definitions of α , B , K , and K_U ; however, this result for D can be found in many places)

$$D = \frac{kBK_U}{\eta\alpha} \left(\frac{K + 4G/3}{K_U + 4G/3} \right) \approx \frac{kK_f}{\eta\phi}, \quad (15)$$

where K_f is the fluid's bulk modulus, ϕ is porosity, and η is the fluid viscosity. Peak attenuation occurs at the relaxation frequency ω_p given by $\omega_p = D/L^2 \propto k/L^2$ where L is a characteristic patch size defined explicitly in Appendix B. Finally, at high frequencies, the attenuation decreases as the square root of the frequency. This is a consequence of the diffusive penetration depth of the fluid pressure into the less permeable inclusion decreasing with increasing frequency as $\omega^{-1/2}$. The permeability is the same in all samples so the relaxation frequency ω_p only varies due to L . Figure 5 gives snapshots of the fluid pressure during an experiment with a sample containing different sizes of inclusions. Looking at the times needed by each size of inclusion to equilibrate gives a visual idea of how the relaxation frequency depends on L .

[27] For the second series of numerical experiments, the two porous phases in a, b, c and d have simply been swapped so that what was inclusion material is now matrix and vice versa (see Figure 3). In this case, the fluid pressure equilibration takes place mainly within the low-permeability matrix. Figure 6 shows the measured values for K_u and $Q_{K_u}^{-1}$

versus frequency. Here again, the finite difference results are consistent with the double-porosity theory.

[28] To begin investigating more complex materials, we construct three composites A, B, and C as depicted in Figure 3. Each of these three materials has exactly the same numbers of each size of square inclusions. The only difference is where these square inclusions are placed inside the sample. Further, the volume fractions of the inclusion and matrix phases in composites A, B, C and a, b, c are all identical. Again, we test the two scenarios in which the inclusions and matrix are either made of material one or two from Table 1.

[29] Figure 7 shows the results obtained when the matrix is softer and more permeable than the inclusions. Interestingly, in this case, the K_u and $Q_{K_u}^{-1}$ curves are very similar for the three composites. The results for A and B are almost equal, while for C, the curves are slightly shifted at lower frequencies. This shifting is due to the fact that the fluid pressure takes more time to equilibrate within the relatively large aggregate in C. The similarity in the results suggests the different inclusions are not strongly interacting so that a theoretical estimate can be made by simply averaging together the double-porosity theory estimates corresponding to examples a, b, c, and d in Figure 3. Figure 7 shows that such a theoretical estimate (solid line) is very consistent with the numerical data.

[30] The results obtained after swapping the roles of the two phases in materials A, B, and C are presented in Figure 8. The numerical results for material A in this case

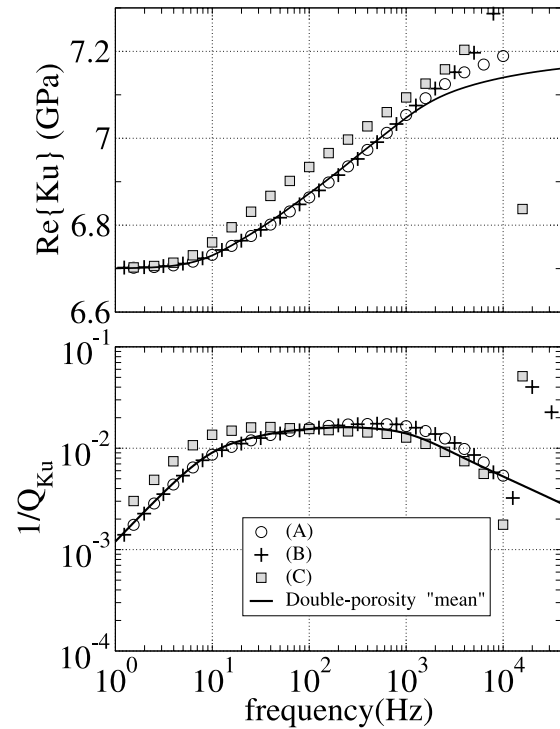


Figure 7. Real part of the bulk modulus and the associated attenuation obtained from the geometries A, B, and C in Figure 3. The black and white areas in Figure 3 where filled with materials 1 and 2, respectively, from Table 1. The solid line is a simple mean of the four theoretical curves associated with samples a, b, c, and d presented in Figure 4.

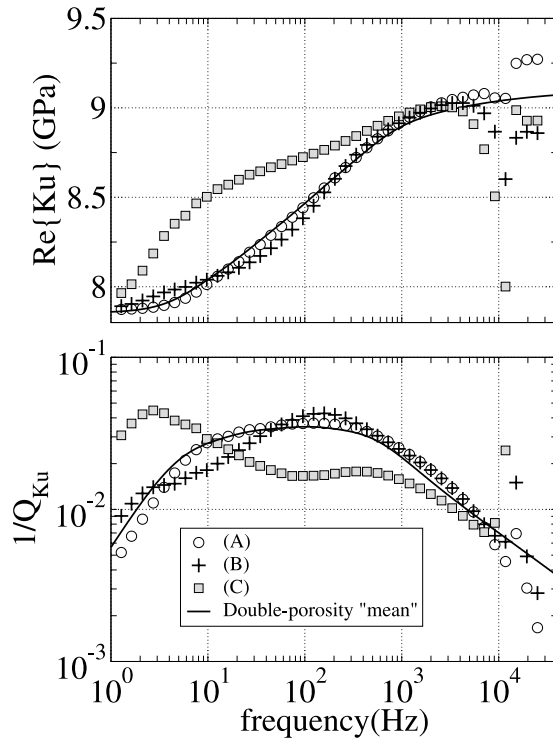


Figure 8. Real part of the bulk modulus and the respective attenuations obtained from the geometries A, B, and C in Figure 3. The black and white areas in Figure 3 were filled with phase 2 and phase 1, respectively, from Table 1. The solid line is a simple mean of the four theoretical curves associated with the samples a, b, c, and d presented in Figure 6.

are again well fit by a simple mean of the Figure 6 results. However, materials B and C show more significant discrepancies between the numerical results and the simple mean curve. This is because the black regions in Figure 3 for samples B and C have a distinctly different geometry than

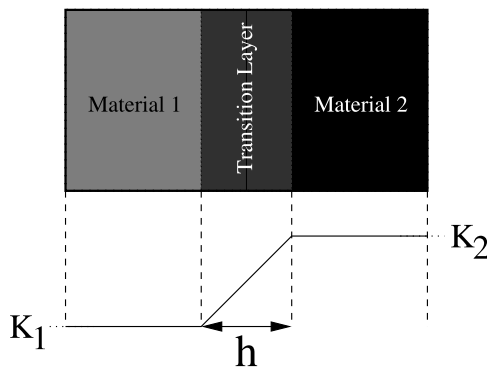


Figure 9. Geometry of the sample used to study the effect of smooth transitions from one material to another. The physical properties of materials 1 and 2 are given in Table 1 as phase 1 and phase 2, respectively, with the exception that the permeability is constant everywhere ($k = 5 \times 10^{-16} \text{ m}^2$). Inside the transition zone, the drained moduli are linearly interpolated between the two materials.

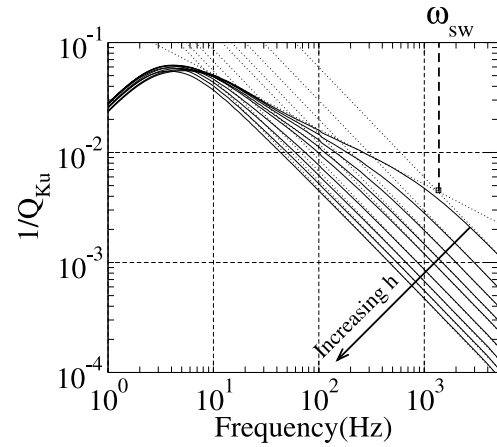


Figure 10. Attenuation curves $Q^{-1}(\omega)$ obtained by varying the thickness of the transition layer h (see Figure 9). The thin dotted lines are obtained by fitting the data in the high-frequency range with a function of the form $Q^{-1}(\omega) = A\omega^{-1}$. For each experiment, the transition frequency ω_{sw} is measured at the intersection between the high-frequency asymptote (thin dotted line) at a given h and the high-frequency asymptote when $h = 0$.

the black region of Figure 3, sample A. In particular, material B has a black area with a narrow range of length scales between the white inclusions which results in the enhanced peak of Figure 8. Similarly, material C has a dominant length scale that is much larger than in any of the other examples which results in the low-frequency peak seen in Figure 8.

[31] The conclusion is that when the diffusion is controlled by the inclusions (i.e., the inclusion material has the smaller fluid pressure diffusivity), then it does not matter where the inclusions are placed within the sample. However, when the diffusion is controlled by the matrix, the fact that the geometry of the matrix changes greatly depending on where the inclusions are placed means that the shape of the attenuation curves will be more sensitive to where the inclusions are placed.

4.2. Gradual Transitions in the Local Properties

[32] We next investigate what is the consequence on the frequency dependence of $K_u(\omega)$ if there is a gradual transition in the local moduli from one patch of material to the next as opposed to the abrupt step changes so far considered.

[33] Consider first the simple situation depicted in Figure 9, in which there is a linear transition in the local drained (frame) moduli from one porous material to another. The transition occurs across a layer that has a thickness h . Figure 10 shows how the frequency dependence of Q_{Ku}^{-1} is affected by changing the transition layer's thickness h . The affect is to make the high-frequency limit of Q_{Ku}^{-1} change from $\omega^{-1/2}$ in the case where no transition layer is present (a step change in local properties) to ω^{-1} when the transition layer is present. So long as the diffusive penetration distance $\delta = \sqrt{D/\omega}$ is larger than the transition thickness h , but smaller than the size L of the main low-permeability patch that is being diffusively penetrated, one has that Q_{Ku}^{-1} will decrease with increasing frequency as $\omega^{-1/2}$. However, at

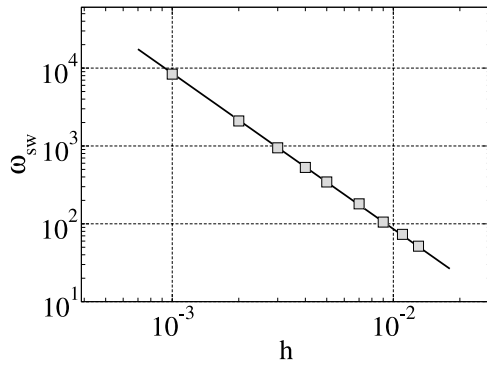


Figure 11. Measured relaxation frequency ω_{sw} as defined in Figure 10 plotted as a function of the thickness of the transition layer (see Figure 9). The solid line is equation (16).

frequencies where $\delta < h$, a transition to a linear decrease with frequency occurs because the fluid pressure gradient is no longer controlled by diffusion into a uniform material, but by the simple gradient in material properties present in the transition layer. A more detailed explanation of this frequency dependence will be provided in section 4.3. The frequency ω_{sw} at which the transition from $\omega^{-1/2}$ to ω^{-1} attenuation falloff occurs can thus be estimated as

$$\omega_{sw} = D/h^2. \quad (16)$$

As shown in Figure 11, this estimate (the solid line) is consistent with the numerical experiments.

[34] In Figure 12, the transition inside the sample takes place with various smooth profiles as shown (not just linear transitions) and the permeability is allowed to vary as well. All conclusions so far obtained about the high-frequency dependence being ω^{-1} in the presence of gradual transitions (instead of $\omega^{-1/2}$ in the presence of step transitions), continue to hold. The explanation for this is now provided.

4.3. Explanation for the Nature of $Q_{Ku}^{-1}(\omega)$

[35] We now provide a simple “back-of-the-envelope” analysis of $Q_{Ku}^{-1}(\omega)$ in order to better explain the observed frequency dependence in the limits of low and high frequencies and to define how the maximum value of Q_{Ku}^{-1} depends on the material properties. The goal is to obtain a simple understanding of Q_{Ku}^{-1} that is nonetheless useful and robust. This is done at the expense of mathematical rigor.

[36] To do so, we exploit the physical definition of Q provided earlier as equation (6). For ease of explanation, we assume the composite is predominantly a mixture of two porous phases (a double-porosity model) and designate using a 1 the porous phase having the smaller fluid pressure diffusivity D . Phase 1 thus controls the timing of the diffusive penetration of the fluid pressure and therefore contains the strongest fluid pressure gradients and viscous losses. Despite this double-porosity context, the effect of smooth transitions between the otherwise homogeneous patches will be considered.

[37] The rate at which energy is locally being lost per unit volume of phase 1 is $(k_1/\eta) |\nabla p_f|^2$. Thus, using equation (6) and assuming that the attenuation is small ($Q^{-1} < 1$) so that

the average stored energy is equal to half the peak stored energy, one has

$$Q^{-1} = \frac{k_1}{\omega\eta} \frac{\int_{\Omega_1} |\nabla p_f|^2 dV}{\int_{\Omega} K_U \epsilon^2 dV} \approx \frac{k_1}{\omega\eta} \frac{\int_{\Omega_1} |\nabla p_f|^2 dV}{V \text{Re}\{K_u\} \epsilon^2}, \quad (17)$$

where $\text{Re}\{K_u\}$ is the macroscopic undrained modulus of the sealed sample, V is the sample volume, ϵ is the applied volumetric strain, and Ω_1 is the domain of phase 1. We now further analyze this expression in the limits of high and low frequencies. In what follows, the fluid pressure gradients in phase 1 are crudely approximated as $\Delta p_f / \Delta x$ where Δp_f is a characteristic pressure contrast between the two phases that exists over a distance Δx within Ω_1 .

4.3.1. High Frequencies

[38] At high frequencies, Δp_f is given by the locally (and globally) undrained response $\Delta p_f = \Delta C \epsilon$ where ΔC is the contrast in the local Biot coupling modulus $C = \alpha M$

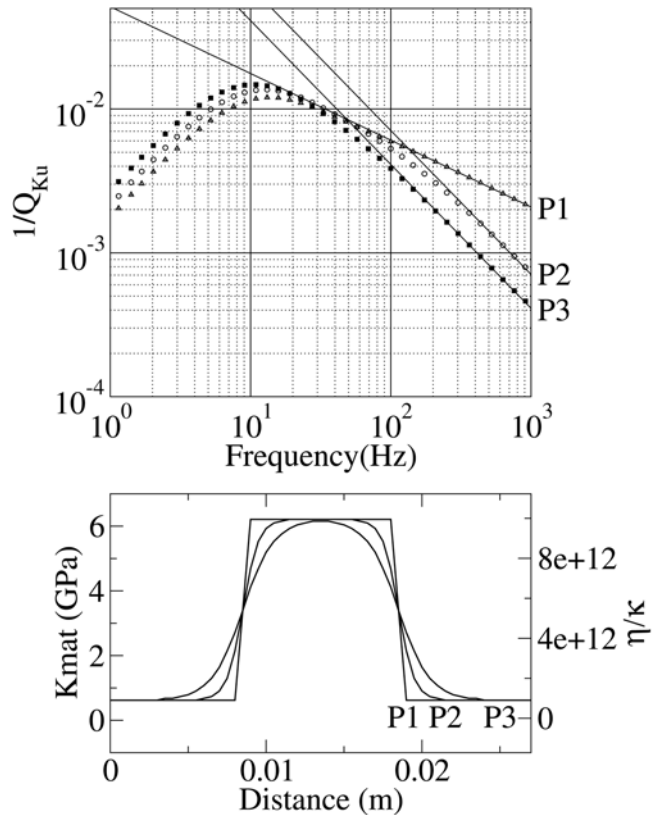


Figure 12. Similar experiment as in Figure 10 but with permeability variations and different profiles for the transition layer. Here the sample consists of a single circular inclusion embedded within a homogeneous matrix. The three curves have been obtained using three different profiles (P1, P2, P3) for the transition layer. The solid lines are obtained by fitting the data in the high frequencies range. The slopes of the lines associated with P2 and P3 are very close to 1, while the one associated with P1 is very close to 1/2. The local drained modulus and permeability profiles along the diameter of the circular inclusion is shown in the bottom plot. The other physical properties correspond to the values shown in Table 1.

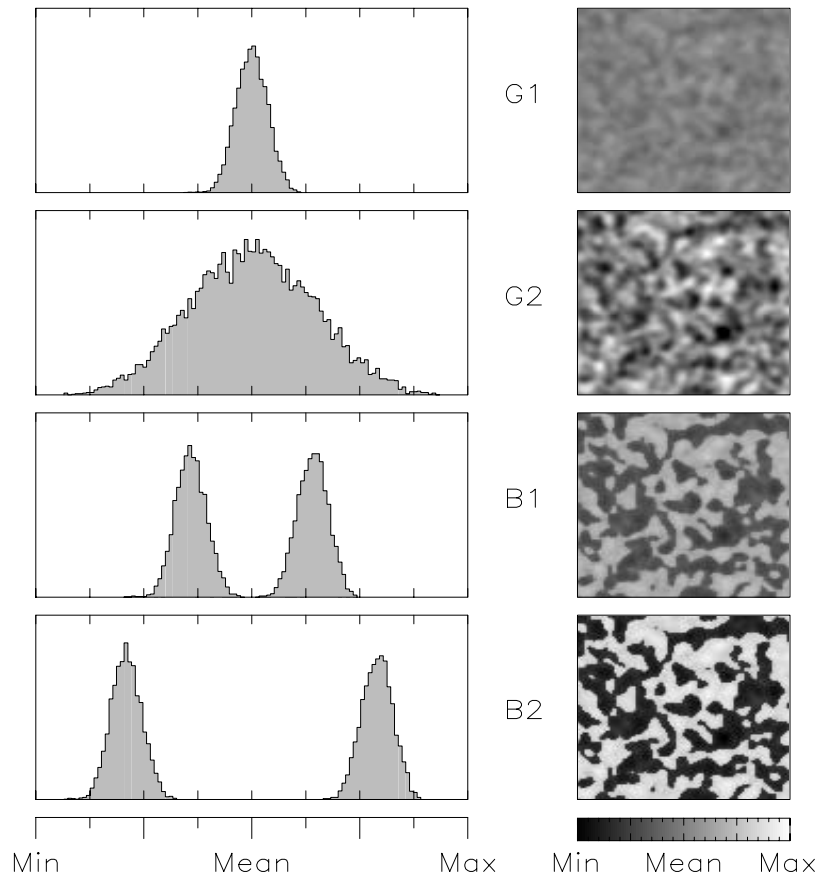


Figure 13. Probability density function (PDF) and spatial distribution of the physical properties after a Gaussian correlation function has been applied. Samples G1 and G2 derive from a Gaussian PDF while B1 and B2 from a bimodal PDF. The sample size is 10 centimeters and the imposed correlation length is about 1 centimeter. The physical properties used for the modeling are obtained using a linear mapping (regular binning) between the minimum and maximum values given in Table 2.

between two adjacent patches (see Appendix A for definitions of α and M). Further, by definition of “high frequencies,” Δx is necessarily smaller than the characteristic size L (defined in Appendix A) of the phase 1 patches. The volume of phase 1 across which the fluid pressure gradients are nonnegligible is roughly $S\Delta x$ where S is the total surface area separating the porous patches. We can thus write

$$Q_{\infty}^{-1} \approx \frac{k_1}{\omega \eta} \frac{S}{V} \frac{(\Delta C)^2}{K_u(\infty)} \frac{1}{\Delta x}, \quad (18)$$

where for a double-porosity material, the high-frequency undrained bulk modulus $K_u(\infty)$ is given by equation (B13) of Appendix B.

[39] In the special case where there is an abrupt step change from one patch to the next, one has that $\Delta x = \delta = \sqrt{D_1/\omega}$ is the diffusive penetration depth into phase 1. In this case, equation (18) gives

$$Q_{\infty}^{-1} \approx \left[\sqrt{\frac{\phi_1 k_1}{K_f \eta}} \frac{S}{V} \frac{(\Delta C)^2}{K_u(\infty)} \right] \omega^{-1/2}, \quad (19)$$

as observed in the numerical experiments. In the case where there is a gradual (but otherwise arbitrary) transition over a distance h from one patch to the next and when $\delta < h$, we have that $\Delta x = h$ so that

$$Q_{\infty}^{-1} \approx \left[\frac{k_1 S}{\eta h V} \frac{(\Delta C)^2}{K_u(\infty)} \right] \omega^{-1}, \quad (20)$$

also as observed in the numerical experiments.

4.3.2. Peak Attenuation

[40] Peak attenuation occurs when the diffusive penetration distance δ just equals the size L of the phase 1 patches; i.e., it occurs at the frequency $\omega_p = D_1/L^2$. At peak attenuation, effectively all of phase 1 has a fluid pressure gradient across it so that

$$Q_{\text{peak}}^{-1} \approx \frac{L^2 k_1}{D_1 \eta} \frac{V_1}{V} \left(\frac{\Delta C}{L} \right)^2 \frac{1}{K_u(\omega_p)} \approx \frac{\phi_1 V_1}{V} \frac{(\Delta C)^2}{K_f K_u(\omega_p)}, \quad (21)$$

where $K_u(\omega_p)$ is approximated in Appendix B. This useful expression says that the magnitude of peak attenuation is proportional to the square of the contrast in the coupling modulus. If the incompressibility contrast is doubled, the attenuation curves will be shifted upward by a factor of four.

Table 2. Material Properties of the Samples Shown in Figure 13

Property	Minimum	Mean	Maximum
Grain			
Bulk modulus K_s , GPa	-	36.0	-
Density ρ_s , kg/m ³	-	2650	-
Frame			
Bulk modulus K_d	500 MPa	5 GPa	9.5 GPa
Shear modulus μ	333 MPa	3.33 GPa	6.33 GPa
Porosity ϕ	0.26	0.33	0.39
Permeability k , m ²	10^{-16}	10^{-15}	1.9×10^{-15}
Fluid			
Bulk modulus K_f , GPa	-	2.25	-
Density ρ_f , kg/m ³	-	10^3	-
Viscosity η , N s m ⁻²	-	10^{-3}	-

4.3.3. Low Frequencies

[41] At low frequencies, the distances over which the fluid pressure gradients in phase 1 exist are independent of frequency and are given by $\Delta x = L$. In such a long-time (low-frequency) limit, the diffusional penetration of fluid pressure across the low-permeability phase 1 material has plenty of time to occur within each stress cycle (i.e., $\omega \ll D_1/L^2$). The remaining pressure gradients are decreasing linearly with decreasing frequency because the average fluid pressure difference between the two phases Δp_f is decreasing. *Pride and Berryman* [2003b] show that at low frequencies, a double-porosity material will have an average difference of fluid pressure between the two phases given by

$$\Delta p_f = \omega \frac{\eta}{k_1} \left(1 - \frac{B_o}{B_1} \right) \frac{\alpha_1}{K_1} L^2 K_u(0) \epsilon, \quad (22)$$

where B_o is the static Skempton's coefficient and $K_u(0)$ the static undrained bulk modulus of the double-porosity composite and expressions for each are given in Appendix B. At zero frequency, no fluid pressure gradients in the composite remain. Thus, at low frequencies, the attenuation goes as

$$Q_o^{-1} \approx \left[\frac{V_1}{V} \frac{\eta L^2}{k_1} \left(1 - \frac{B_o}{B_1} \right)^2 \left(\frac{\alpha_1}{K_1} \right)^2 K_u(0) \right] \omega \quad (23)$$

as seen in the numerical experiments.

4.4. Random Correlated Materials

[42] We now consider the attenuation and dispersion in materials having local properties randomly sampled from a probability distribution function (PDF) and for which some specified correlation function of the properties has been imposed. The algorithm used to create such random correlated materials is presented in Appendix C.

[43] Consider the four materials shown in Figure 13 (right) and having either a unimodal Gaussian (G1 and G2) or bimodal Gaussian (B1 and B2) distribution function as shown in the left column. In all four realizations (B1, B2, G1, and G2), a Gaussian correlation function was imposed having a correlation length of 1 cm. The material properties associated with these distributions are given in Table 2.

[44] The dispersion and attenuation associated with the undrained bulk modulus are plotted in Figure 14. The greatest attenuation and dispersion are associated with the distribution (B2) which has the largest standard of deviation. As shown in section 4.3, the peak value of attenuation is determined by the square of the local contrasts in compressibility present. This is equivalent to saying that the peak attenuation is proportional to the square of the standard of deviation of the PDF used to create the random heterogeneity. Figure 15 shows that the numerical experiments are consistent with such a quadratic relation between peak attenuation and a PDFs standard of deviation. Distributions G2 (unimodal) and B1 (bimodal) have identically the same standard of deviation which results in very similar attenuation and dispersion curves in Figure 14. The main difference is that the bimodal distribution has a $\omega^{-1/2}$ fall off in the high-frequency attenuation, while the unimodal distribution has a ω^{-1} fall off. The reason has been given above in sections 4.2 and 4.3: the bimodal distribution has effectively a step contrast in the local properties and therefore a $\omega^{-1/2}$ high-frequency attenuation dependence, while the unimodal distribution presents smooth transitions of the local properties and a ω^{-1} dependence.

5. Attenuation and Dispersion due to Pure Shear

[45] Mesoscale flow can be created even when a pure shear stress is applied to the sample. The condition required for a pure shear to create local fluid pressure gradients is that the mesoscale geometry have some local anisotropy associated with its shape. A sand lens embedded in shaly sediments or a fracture embedded in a sandstone are natural examples of such geometric anisotropy. Isotropic geome-

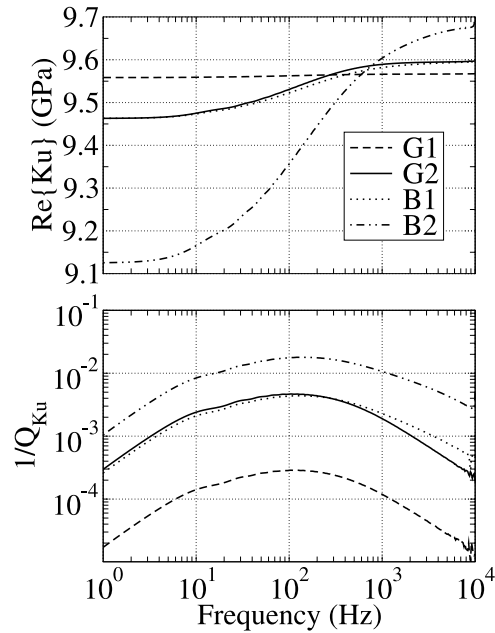


Figure 14. Bulk moduli K_u and the respective attenuations Q^{-1} obtained from the samples shown in Figure 13. The samples that have more important local contrasts in their elastic properties (larger standards of deviations) show higher levels of attenuation and dispersion. Salient features of these results are discussed in the text.

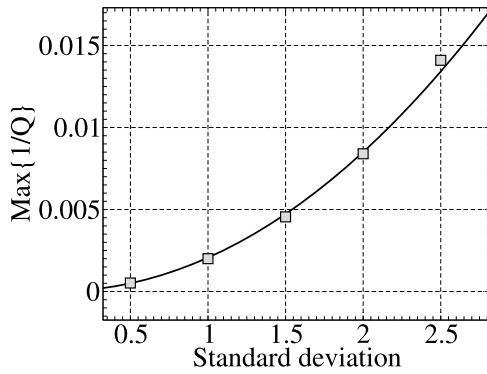


Figure 15. Maximum of Q^{-1} plotted as a function of the contrast in the elastic properties for a bimodal material. The samples used are similar to B1 or B2 in Figure 13 but with varying standards of deviations. For the mapping, we used the properties given in Table 2. The solid line has been obtained by fitting the data as a quadratic function of the standard of deviation.

tries, such as spherical inclusions, do not permit an applied pure shear to create changes in the fluid pressure of the constituents. Since anisotropic mesoscale geometries are more geologically common than are isotropic geometries, shear-induced mesoflow should be considered the rule rather than the exception in porous rock.

[46] An example of the fluid pressure response due to a pure shear (having its maximum (positive) principal stress in the vertical direction and its minimum (negative) stress in the horizontal direction) is given in Figure 16. An ellipse of softer porous material is embedded within a stiffer matrix. The physical properties of the inclusion and matrix are given in Table 3. As the orientation of the ellipse is changed relative to the fixed principal stress directions, there is observed to be a change in the nature of the fluid pressure response.

[47] When the inclusion is perpendicular (or parallel) to the principal stress direction (E1), the dilation (or compression) of the ellipse will create a fluid pressure difference

between the inside of the ellipse and the surrounding matrix. A fluid pressure equilibration then ensues that both attenuates energy and causes the shear modulus of the composite to relax (see Figure 17). In this orientation (E1), the contrast in fluid pressure that is created is due to the contrast in the drained bulk modulus between the inclusion and matrix.

[48] As the ellipse is rotated (E2), there begin to be created lobes of enhanced compression and dilation in the surrounding matrix so long as there is a contrast in the local shear modulus between the inclusion and matrix. This is in addition to the pressure change between the ellipse and matrix caused by the compressibility contrast. When the ellipse is at 45° relative to the principal stress directions (E3), the ellipse no longer has an average fluid pressure difference between itself and the surrounding medium; however, the pressure changes in the shear lobes is at a maximum.

[49] As seen in Figure 17, the greatest amount of shear-induced attenuation and dispersion is created when there develops a fluid pressure contrast between the inside and outside of the ellipse. Again, for this to occur, there must exist an incompressibility contrast between the ellipse and matrix and the ellipse cannot be oriented at 45° relative to the principal stress direction.

[50] Another way to quantify these observations is given in Figure 18. A single ellipsoid having contrasts in both incompressibility and shear modulus with the surrounding matrix is rotated relative to the principal stress and the peak value of Q_G^{-1} is numerically measured. Two curves are given in Figure 18 corresponding to a sealed inclusion (no fluid exchanges between the ellipse and the matrix) and to an open inclusion. For the sealed inclusion, only the equilibration in the matrix between the shear lobes occurs. For the open inclusion, both mechanisms occur. It is again seen that the attenuation, and therefore dispersion, is dominated by the exchanges between inclusion and matrix.

[51] In the mechanism due to fluid exchanges between the inclusion and matrix, the peak attenuation is proportional to the square in the contrast of the incompressibility. While the peak attenuation due to equilibration between the shear

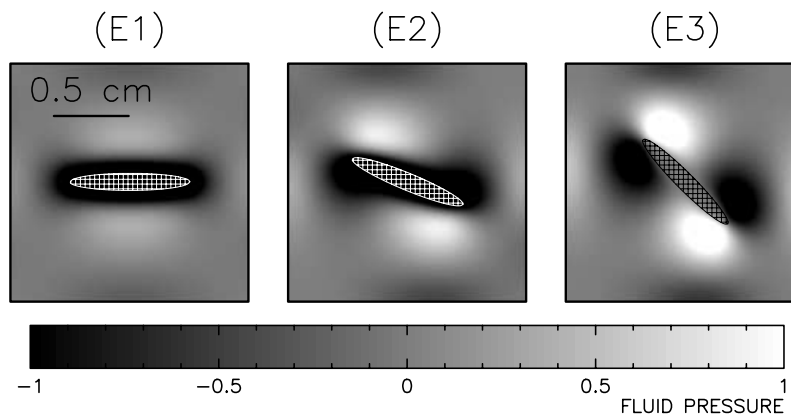


Figure 16. Snapshots showing the fluid pressure recorded during three pure shear experiments (E1–E3). Each sample contains a single heterogeneity simulating a crack. In E1, the fluid pressure equilibration occurs between the inclusion and the matrix. In E3, the fluid pressure equilibrates between the lobes of dilatation and compression in the matrix. Finally, in E2 both kinds of equilibration occur simultaneously.

Table 3. Material Properties of the Samples Shown in Figure 16

Property	Weak Inclusion	Matrix
Grain		
Bulk modulus K_g , GPa	36.0	36.0
Density ρ_g , kg/m ³	2650	2650
Frame		
Bulk modulus K_d	62.1 MPa	6.21 GPa
Shear modulus μ	45.5 MPa	455 GPa
Porosity ϕ	0.33	0.33
Permeability k , m ²	0 or 10 ⁻¹⁵	10 ⁻¹⁵
Fluid		
Bulk modulus K_f , GPa	2.25	2.25
Density ρ_f , kg/m ³	10 ³	10 ³
Viscosity η , N s m ⁻²	10 ⁻³	10 ⁻³

lobes is proportional to the square in the contrast of the shear modulus.

6. Conclusions

[52] In this work, we have seen how mesoscopic-scale geometry in porous samples has an influence on the seismic attenuation and dispersion of the sample. This was done by numerically determining the local poroelastic response inside such samples created by step changes in the stress acting on the sample, and averaging the deformation throughout the sample to obtain the average strain. Taking a Fourier transform of the average stress and strain and dividing gives the complex frequency-dependent elastic moduli of the sample.

[53] The main results are now summarized. For double-porosity materials characterized by a porous inclusion of given size embedded within a homogeneous matrix, the double-porosity theory of *Pride et al.* [2004], with no free parameters, fits the numerical results very well. However,

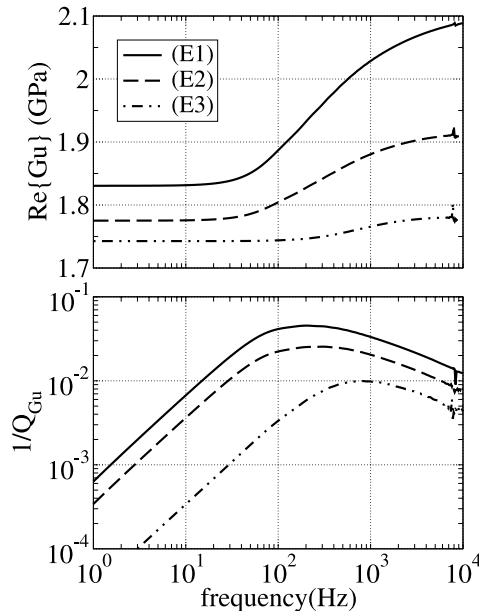


Figure 17. Shear modulus G and the associated attenuation Q_G^{-1} obtained from the three samples E1, E2, and E3 shown in Figure 16.

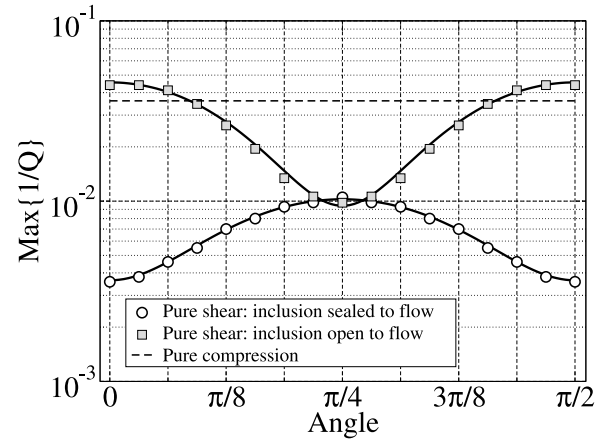


Figure 18. Peak attenuation as an open-to-flow ellipse (gray squares) and sealed-to-flow ellipse (white circles) are rotated relative to the principal stress directions of the pure shear. The horizontal dashed line is the measured attenuation due to a pure compression for the same geometries (ellipse orientations).

for more general media characterized by having continuously variable mesoscopic structures, the assumptions of the double-porosity model breakdown along with the agreement between the analytical model and the numerical results.

[54] In particular, it was shown that when there is a gradual transition between the porous inclusion and the surrounding matrix (as opposed to the step contrast assumed in the double-porosity model), the high-frequency asymptotic behavior of the attenuation is ω^{-1} instead of $\omega^{-1/2}$ in the case of a step contrast. Further, it was shown that having a range of sizes present produces broader curves of attenuation as a function of frequency due to the diffusional relaxation associated with each length scale present.

[55] The peak attenuation in samples containing meso-scale heterogeneity was shown to be proportional to the square of the contrast of the moduli in the case of double-porosity models, and to the square of the standard of deviation in the compressibility distribution for the case of random media.

[56] Last, the attenuation in pure shear was shown to be dominated by fluid exchanges between anisotropically shaped inclusions and the surrounding matrix. Mesoflow between the shear-induced lobes of compression and dilation in the matrix surrounding the inclusion was shown to be much less important.

Appendix A: Local Poroelastic Equations

[57] *Biot's* [1962] equations are used to model the local response within a heterogeneous porous sample that is being stressed in a time-varying manner. As demonstrated by *Masson et al.* [2006], at low enough applied frequencies where $\omega \ll \eta/(\rho_f Fk)$ so that viscous boundary layers do not develop in the pores, *Biot's* [1962] equations in the time domain may be written

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \nabla \cdot \boldsymbol{\tau} - \rho_f \frac{\partial \mathbf{q}}{\partial t} \quad (\text{A1})$$

$$\rho_f(1 + \Phi)F \frac{\partial \mathbf{q}}{\partial t} + \frac{\eta}{k_0} \mathbf{q} = -\nabla p - \rho_f \frac{\partial \mathbf{v}}{\partial t}. \quad (\text{A2})$$

$$\frac{\partial \boldsymbol{\tau}}{\partial t} = (\lambda_U \nabla \cdot \mathbf{v} + \alpha M \nabla \cdot \mathbf{q}) \mathbf{I} + \mu [\nabla \mathbf{v} + (\nabla \mathbf{v})^T] \quad (\text{A3})$$

$$-\frac{\partial p}{\partial t} = M(\alpha \nabla \cdot \mathbf{v} + \nabla \cdot \mathbf{q}). \quad (\text{A4})$$

In sedimentary rocks, these equations can be considered as valid across the seismic band (1 to 10^4 Hz). The various coefficients are all real. Here, ρ is the local bulk density of the material, ρ_f is the fluid density which is taken to be spatially uniform throughout each sample, and F is the electrical formation factor that is modeled here using the *Archie* [1942] law $\phi^{-1.75}$ where ϕ is local porosity, and Φ is a dimensionless pore topology parameter defined and discussed by *Masson et al.* [2006] that is bounded as $\Phi > 1/4$ and will simply be set to 1 in the present article. Over the seismic band of frequencies, the inertial term in the generalized Darcy law of equation (A2) has a magnitude $|\rho_f(1 + \Phi)F \partial \mathbf{q} / \partial t|$ that is always negligible in amplitude relative to the viscous resistance $|\eta/k_0 \mathbf{q}|$; however, the inertial term is entirely responsible for the finite difference scheme to be stable [cf. *Masson et al.*, 2006] and thus cannot be discarded.

[58] The local poroelastic constants used here are the undrained Lamé modulus λ_U , the shear modulus μ (the same for both drained and undrained conditions), the so-called *Biot and Willis* [1957] (Biot-Willis) constant α , and the fluid storage coefficient M . For any porous material, these constants are related to the undrained bulk modulus K_U , the drained bulk modulus K , and *Skempton's* [1954] undrained fluid pressure to confining pressure ratio B as

$$\lambda_U = K_U - 2\mu/3 = K + \alpha^2 M - 2\mu/3, \quad (\text{A5})$$

$$\alpha = (1 - K/K_U)/B, \quad (\text{A6})$$

$$M = BK_U/\alpha. \quad (\text{A7})$$

In the special case considered by *Gassmann* [1951], in which the solid frame is composed of a single isotropic mineral characterized by a bulk modulus K_s , we have as well the so-called “fluid substitution” relations given by

$$B = \frac{1/K - 1/K_s}{1/K - 1/K_s + \phi(1/K_f - 1/K_s)} \quad (\text{A8})$$

$$K_U = \frac{K}{1 - B(1 - K/K_s)}, \quad (\text{A9})$$

where K_f is the fluid bulk modulus and ϕ is the porosity. From these, one further obtains $\alpha = 1 - K/K_s$. We use the *Gassmann* expressions to model the local poroelastic constants in all the numerical experiments.

Appendix B: Double-Porosity Theory

[59] In the special case where the sample is a composite of two distinct porous materials saturated by a single fluid and when the heterogeneity has a single dominant length scale, the double porosity theory of *Pride and Berryman* [2003a, 2003b] is applicable and predicts that the undrained bulk modulus $K_u(\omega)$ of the porous composite is complex (due to the mesoscale fluid equilibration) and given by

$$\frac{1}{K_d(\omega)} = a_{11} - \frac{a_{13}^2}{a_{33} - \gamma/i\omega}, \quad (\text{B1})$$

$$B(\omega) = \frac{-a_{12}(a_{33} - \gamma/i\omega) + a_{13}(a_{23} + \gamma/i\omega)}{(a_{22} - \gamma/i\omega)(a_{33} - \gamma/i\omega) - (a_{23} + \gamma/i\omega)^2}, \quad (\text{B2})$$

$$\frac{1}{K_u(\omega)} = \frac{1}{K_d(\omega)} + B(\omega) \left(a_{12} - \frac{a_{13}(a_{23} + \gamma/i\omega)}{a_{33} - \gamma/i\omega} \right). \quad (\text{B3})$$

Here, $K_d(\omega)$ is the complex drained bulk modulus of the composite (drained in this context means that the average fluid pressure in a sample does not change, which in no way prevents mesoflow from occurring), and $B(\omega)$ is the complex *Skempton's* coefficient. The a_{ij} are real elastic compliances that depend on the elastic moduli of the two porous constituents, while γ is a complex function of frequency given by

$$\gamma(\omega) = \gamma_o \sqrt{1 - i \frac{\omega}{\omega_o}} \quad (\text{B4})$$

that controls the degree of mesoflow between the two phases. Expressions for the real parameters γ_o and ω_o , as well as for the high-frequency elastic compliances a_{ij} have been derived by *Pride and Berryman* [2003a, 2003b] and are also given by *Pride et al.* [2004].

[60] The a_{ij} are given by

$$a_{11} = 1/K_d(0), \quad (\text{B5})$$

$$a_{22} = \frac{v_1 \alpha_1}{K_1} \left(\frac{1}{B_1} - \frac{\alpha_1(1 - Q_1)}{1 - K_1/K_2} \right), \quad (\text{B6})$$

$$a_{33} = \frac{v_2 \alpha_2}{K_2} \left(\frac{1}{B_2} - \frac{\alpha_2(1 - Q_2)}{1 - K_2/K_1} \right), \quad (\text{B7})$$

$$a_{12} = -v_1 Q_1 \alpha_1 / K_1, \quad (\text{B8})$$

$$a_{13} = -v_2 Q_2 \alpha_2 / K_2, \quad (\text{B9})$$

$$a_{23} = -\frac{\alpha_1 \alpha_2 K_1 / K_2}{(1 - K_1/K_2)^2} \left(\frac{1}{K_d(0)} - \frac{v_1}{K_1} - \frac{v_2}{K_2} \right), \quad (\text{B10})$$

where

$$v_1 Q_1 = \frac{1 - K_2/K_d(0)}{1 - K_2/K_1} \quad v_2 Q_2 = \frac{1 - K_1/K_d(0)}{1 - K_1/K_2}. \quad (\text{B11})$$

Here, v_i is the volume fraction of phase i in each sample ($v_1 + v_2 = 1$), K_i is the drained frame modulus of phase i , B_i is the Skempton's coefficient of phase i , α_i is the Biot-Willis constant of phase i .

[61] The one parameter in these a_{ij} that has not yet been modeled is the overall static drained modulus $K_d(0) = 1/a_{11}$ of the two-phase composite. It is through $K_d(0)$ that all dependence on the mesoscopic geometry of the two phases occurs. Although many mixture models for $K_d(0)$ exist, none are exact for arbitrary geometry of the inclusions. In the present paper, we numerically calculate $K_d(0)$ using our finite difference scheme in the long-time limit and use this measured drained bulk modulus in the above double-porosity expressions for all “theoretical” predictions.

[62] The low-frequency and high-frequency limits of $K_u(\omega)$ are determined from equation (B4) to be

$$K_u(0) = a_{11} - \frac{(a_{12} + a_{13})^2}{a_{22} + 2a_{23} + a_{33}} \quad (\text{B12})$$

$$K_u(\infty) = a_{11} - \frac{a_{13}^2}{a_{33}} - \frac{(a_{12}a_{33} - a_{13}a_{23})^2}{a_{33}(a_{22}a_{33} - a_{23}^2)}. \quad (\text{B13})$$

At peak attenuation defined when $\omega = \omega_p$ one can approximate that $K_u(\omega_p) \approx [K_u(0) + K_u(\infty)]/2$.

[63] If phase 2 is defined to be more permeable than phase 1, the low-frequency limit of the internal transport coefficient γ_o is given by

$$\gamma_o = -\frac{k_1 K_1^d}{\eta L_1^2} \left(\frac{a_{12} + B_o(a_{22} + a_{33})}{R_1 - B_o/B_1} \right) [1 + O(k_1/k_2)], \quad (\text{B14})$$

where the parameters B_o , R_1 , and L_1 are now defined. The dimensionless number $B_o = B(0)$ is the static Skempton's coefficient for the composite and is exactly

$$B_o = -\frac{(a_{12} + a_{13})}{a_{22} + 2a_{23} + a_{33}}. \quad (\text{B15})$$

The dimensionless number R_1 is the ratio of the average static confining pressure in the host phase 1 of a sealed sample divided by the confining pressure applied to the sample and is exactly

$$R_1 = Q_1 + \frac{\alpha_1(1 - Q_1)B_o}{1 - K_1^d/K_2^d} - \frac{v_2 \alpha_2(1 - Q_2)B_o}{v_1 1 - K_2^d/K_1^d}, \quad (\text{B16})$$

where the Q_i are given by equation (B11). The length L_1 is the distance over which the fluid pressure gradient still exists in phase 1 in the final approach to fluid pressure equilibrium and is formally defined as

$$L_1^2 = \frac{1}{V_1} \int_{\Omega_1} \Phi_1 dV, \quad (\text{B17})$$

where Ω_1 is the region of an averaging volume occupied by phase 1 and having a volume measure V_1 . The potential Φ_1 has units of length squared and is a solution of an elliptic boundary value problem that under conditions where the permeability ratio k_1/k_2 can be considered small, reduces to

$$\nabla^2 \Phi_1 = -1 \text{ in } \Omega_1, \quad (\text{B18})$$

$$\mathbf{n} \cdot \nabla \Phi_1 = 0 \text{ on } \partial E_1, \quad (\text{B19})$$

$$\Phi_1 = 0 \text{ on } \partial \Omega_{12}, \quad (\text{B20})$$

where $\partial \Omega_{12}$ is the surface separating the two phases within a sample of composite and ∂E_1 is the external surface of the sample that is coincident with phase 1.

[64] In all the examples of the present paper, the boundary value problem for Φ_1 is solved numerically by finite differences. To do so, we add a diffusion term $-\partial \Phi_1 / \partial t$ to the left-hand side of equation (B18) and replace 1 by a step function on the right-hand side, then solve the resulting diffusion equation using explicit time stepping. The long-time steady state response to the imposed step function source term is the solution of equation (B18). We then determine numerically the length L_1 using equation (B17).

[65] Last, the transition frequency ω_o corresponds to the onset of a high-frequency regime in which the fluid pressure-diffusion penetration distance becomes small relative to the scale of the mesoscopic heterogeneity, and is given by

$$\omega_o = \frac{\eta B_1 K_1}{k_1 \alpha_1} \left(\gamma_o \frac{V}{S} \right)^2 \left(1 + \sqrt{\frac{k_1 B_2 K_2 \alpha_1}{k_2 B_1 K_1 \alpha_2}} \right)^2, \quad (\text{B21})$$

where S is the surface area of the interface between the two phases in each volume V of composite.

Appendix C: Synthetic Realization of an Isotropic, Correlated, Gaussian Random Medium

[66] The algorithm used to generate our random correlated materials (as shown, for example, in Figure 13) is now presented:

[67] 1. At each point x on a 2-D square grid, we generate a pseudorandom realization $W(x)$ of the white noise of a given material property having unit standard deviation.

[68] 2. Calculate the 2-D Fourier transform $\hat{W}(\mathbf{k})$ of the white noise.

[69] 3. Multiply the Fourier Transform with the spectral filter $\hat{F}(\mathbf{k})$ that represents the correlation function,

$$\hat{U}(\mathbf{k}) = \hat{F}(\mathbf{k}) \hat{W}(\mathbf{k}). \quad (\text{C1})$$

For an isotropic Gaussian medium, the correlation function is

$$\hat{F}(\mathbf{k}) = \hat{F}_G(\mathbf{k}) = \exp\left(-\frac{a_G^2 \mathbf{k}^2}{8}\right), \quad (\text{C2})$$

where a_G is the Gaussian correlation length.

[70] 4. Calculate the 2-D inverse Fourier transform $U(x)$ of the product $\hat{U}(\mathbf{k})$.

[71] 5. Normalize the appropriate variance and add the appropriate mean to obtain the desired random realization $c_g(x)$ of the material properties possessing a Gaussian correlation function.

[72] 6. The random medium with a bimodal distribution can be obtained from a Gaussian random medium having unit standard of deviation [e.g., Yamazaki and Shinozuka, 1988] using the mapping

$$H_b[c_b(x)] = H_g[c_g(x)]. \quad (C3)$$

Here, $c_g(x)$ and $c_b(x)$ are the field variables in the case of a unimodal Gaussian and bimodal Gaussian realization. The functional maps H_g and H_b are the cumulative PDF of the unimodal Gaussian and bimodal Gaussian distribution and are thus given by

$$2H_g[c_g(x)] = 1 + \operatorname{erf}\left(\frac{c_g(x)}{\sqrt{2}}\right) \quad (C4)$$

$$2H_b[c_b(x)] = 1 + \operatorname{erf}\left(\frac{c_b(x) - \mu_1}{\sigma_1 2\sqrt{2}} + \frac{c_b(x) - \mu_2}{\sigma_2 2\sqrt{2}}\right), \quad (C5)$$

where erf denotes the error function. By substituting equations (C4) and (C5) into (C3), and finding the root $c_b(x)$ for each grid point x , we obtain the desired bimodal medium with the imposed Gaussian correlation function.

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