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GEOPHYSICS

Fractal Properties of Multiple Fracture Patterns

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The authors of [1–3] were admittedly the first to establish self-similarity of multiple fracture patterns for a wide range of materials (metals, polymers, and rocks) under conditions of tension, creep, fatigue, and superplasticity. The self-similarity was confirmed when a unique curve for the distribution of defects was plotted based on the normalization, which corresponds to the relation

$$\frac{N}{N_{\max}} \sim \left(\frac{l}{l_{\max}}\right)^{\alpha}. \qquad (1)$$

The power law describing the distribution of faults in the Earth's crust found by many researchers (in particular, by the authors of [4]) confirms self-similarity of the pattern of discontinuities and Eq. (1) at the global level of development of the process of multiple fractures. The tangent of the inclination angle of these cumulative distributions was accepted in [5] as the fractal dimension (D) of the pattern of fractures observed in the Earth's crust. It was assumed that the fractal dimension can be estimated from the relation

$$D = \frac{3b}{c}, \qquad (2)$$

where b is the exponent in the Guttenberg–Richter power equation ($N \sim E^{-b}$) for the relation between the number of seismic events (N) and their energy (E) (or magnitude $M = \log E$) and c is a constant equal to ~3/2. This means that the fractal dimension of the fracture pattern is proportional to the seismic parameter b estimated from the tangent of the inclination angle of the distribution of the accumulated number of seismic events by magnitude.

The objective of this study included estimation of fractal dimension and parameter b of the real pattern of fractures of metal specimens and establishment of the correlation between them. In addition, taking into account the previously revealed similarity between the kinetics of fractures and different scale levels [6], it was interesting to compare the magnitude dependence of the localization zones of fractures preceding the critical event in the specimen or the Earth's crust.

Let us introduce the following notation for easier description of the results: b_C, b_{AE}, and b_S are exponents in the power laws describing cumulative distributions of microcracks (or faults), acoustic signals, and seismic events, respectively; D_C, D_{AE}, and D_S are fractal dimensions estimated from the data of the analysis of real damage, acoustic, or seismic responses to this damage, respectively. Equation (2) with this notation will be written as

$$b_C = \frac{3b_S}{c}.$$

This information was confirmed in [7, 8], in which acoustic parameter b_{AE} was estimated on the rock specimens by the acoustic emission method. However, in the first case [7], the fractal dimension was accepted as the angular coefficient of cumulative curves of microcrack distribution by length b_C; in the second case [8], it was taken as the fractal (correlation) dimension of the epicenters of acoustic events D_{AE}. In both cases, positive correlation of the estimated parameters was found (although the authors of [8] observed it only at the initial stage of the deformation of specimens) and they corresponded in the accepted notation to relations

$$b_C = 3\frac{b_{AE}}{c}, \quad D_{AE} = 3\frac{b_{AE}}{c}.$$

In this work, we estimated b_C, the parameter of real pattern of microcracks based on the method of replicas obtained at the polished surface of specimens at different stages of loading. We also estimated fractal dimension D_C (corresponding to these patterns) measured by the method of covering the images with squares of different areas and further determination of D_C from relation [9]: $N = \lim_R \rightarrow BR^{-D}$, where N is the number of squares with side R containing black pixels (microcracks were colored black) and B is a constant. Varia-

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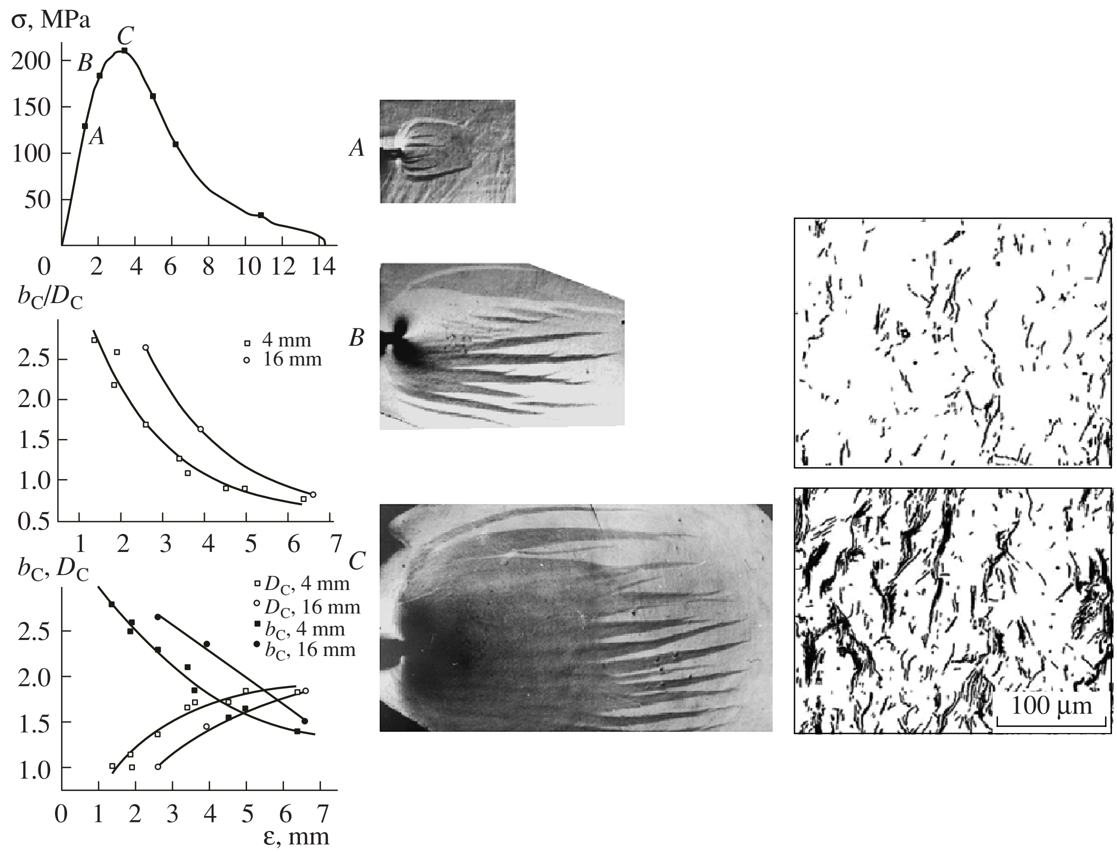

Fig. 1. Deformation dependences of stress in the net-section of specimens with notch, parameters b_C and D_C and their ratio $\frac{b_C}{D_C}$, zones of fracture localization in the notch tip, and patterns of microcracks in zones.

tion in the area of the covered square is no less than two orders of magnitude. The cumulative curve of the distribution of the microcrack length used for determining parameter b_C corresponded to the power relation ($N = Bl^{-bc}$) over the interval of variation in the microcrack length by no less than 1.5 orders of magnitude.

The investigations were carried out with flat specimens of model material (low-carbon steel) with a notch under conditions of static and cyclic loading. The thickness of the specimens ranged from 4 to 16 mm. The replicas allowed us to fix the zone of the localization of fracture and the pattern of microcracks in the zone at each stage of the process. After this, the replicas were analyzed using an optical microscope equipped with a digital video camera and the obtained patterns of multiple fractures were processed by the image analysis program. The maximal number (1500–10 000) of microcracks measured on each specimen depended on the degree of damage of the material at the given stage of loading. The results of measurements were used for plotting cumulative curves of the distribution of microcracks by length in the following coordinates: total number of microcracks (N) with length greater and equal to the current length versus current length of microcracks (L). The details of the experiment are described in [10].

Analysis of the curves revealed variation in the distribution functions in the process of loading. Indeed, at the initial stages of loading, cumulative curves of microcrack distribution were described by an exponential function. When the loads approached the maximal ones, the functions were described by a power law. Moreover, parameter b_C decreased while approaching the final fracture of the specimen.

Figure 1 shows a typical deformation curve for specimens with highlighted points, the corresponding zones of fracture location, and patterns of microcracks in zones. The curves of variations in parameters b_C and D_C with deformation are also shown. It is seen that the zone of localization of fractures is already formed at the linear region of the deformation curve. This initial stage of loading was marked by numerous slip bands that are developed both at the boundaries of the structural elements and within them. Microcracks are absent. Increase of strain leads to increase of the density of slip bands and the appearance of a defect of the next rank—pores formed in the slip bands or places of stress concentrators (scratches at the polished surface of the spec-

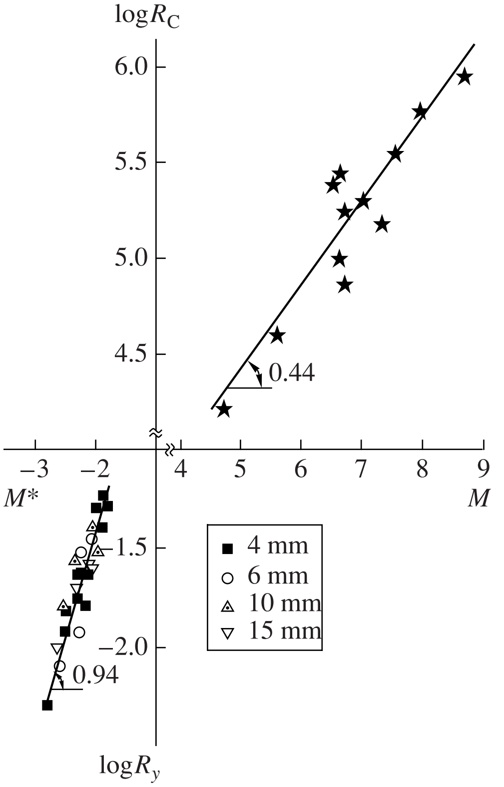

Fig. 2. Correlation radius (R_C) of seismic activity in different regions of the world vs. earthquake magnitude (M) [15] and correlation radius of damage (R_y) in low-carbon steel specimens vs. fracture magnitude (M^*).

imen). When the yield stress is exceeded and the related size of the localization zone increases, a defect of the third rank appears as microcracks formed by the merging of pores along the slip bands.

We can probably assume that the correlation radius of the process of accumulation of fractures is the radius (or length) of the localization zone of fractures with a size one to three orders of magnitude greater than the length of microcracks formed in the zone, because the structure of the material corresponds to the initial one and discontinuities do not accumulate beyond the zone.

Figure 1 also suggests a decrease in the $\frac{b_C}{D_C}$ ratio with the increase in strain accompanied by the variation in parameters b_C and D_C in the opposite phase, unlike in-phase correlation of these parameters corresponding to Eq. (2). Such variation in parameters b_S and D_S was observed in [11, 12]. In [8], such variation in b_{AE} and D_{AE} was observed at the final stage of the fracture of specimens.

In this case, the decrease in parameter b_C is caused by merging of microcracks, while the increase in the fractal dimension D_C is caused by increase in the total area of fractures related to increase in the length and opening of microcracks. Investigation of fractures of specimens with different thicknesses confirmed the noted variation of these parameters with deformation and demonstrated that the dependence of the fractal dimension on parameter b_C corresponds to a linear dependence $D_C = 2.86–0.67b_C$ (correlation coefficient 0.95) for all tested specimens. A similar linear dependence was also found in the analysis of the interrelation between the correlation fractal dimension D_S and parameter b_S estimated from the data of seismic activity: $D_S = 2.3 - 0.73b_S$ (correlation coefficient 0.76) [11] and $D_S = 2.72–1.39b_S$ (correlation coefficient 0.85) [12].

The measurement of the length R_y of the localization zone (or correlation zone of the damage accumulation process) and the corresponding fracture energy E at different stages of loading estimated from the area under the deformation curve allowed us to plot the $R_y - M^*$ dependence, where M^* is the fracture magnitude [13] calculated from the relation used in seismology: $1.5M = \log E - 4.8$ (J). It follows from Fig. 2 (third quadrant of the graph) that the logarithm of the fracture localization zone in the specimens of different thicknesses is proportional to the fracture magnitude ($\log R_y \sim 0.94M^*$) similarly to the logarithm of the correlation radius R_C of seismic activity in different regions of the world proportional to the earthquake magnitude M in the first quadrant ($\log R_C \sim 0.44M$) [14, 15].

Thus, the investigation of the real damage of material with estimation of the fractal dimension confirmed the conclusion about the self-similarity of the damage accumulation process [1] and revealed the scale invariance of the size of the correlation zone of microdamages with respect to the energy characterizing the stage of initiation and development of discontinuities in the specimens of different thicknesses similar to the scale invariance of the correlation zone of seismic activity.

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