
Analysis of radial movement of a confined aquifer due to well pumping and injection

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Abstract Three aspects of radial motion of an aquifer solid matrix, driven by hydraulic forces due to well recharge and discharge, have been investigated mathematically. Firstly, the analytic solutions in the velocity and displacement fields were found using the definition of bulk mass flow, conservation of water and solid mass, the Darcy-Gersevanov law, and the drawdown of hydraulic head for unsteady flow. Secondly, the radial velocity and displacement of aquifer solid matrix were investigated with respect to time and location in response to three pumping-injection schemes (i.e., linear, nonlinear, and sinusoidal well flow rates). Aquifer movement was found to be more responsive to linear and nonlinear pumping rates than to constant pumpage. Aquifer movement in response to periodic well discharge and recharge was governed by an integral sinusoidal function. Finally, the potential risk of earth fissuring at a given location caused by tensile stress, in turn induced by well discharge-recharge activity, has been discussed using the analytic solution. The deformation compression and extension zones derived from analysis of the results indicated the possible occurrence of tensile stress that may cause earth fissures. Sample computations, using assumed parameters, provided a first-hand evaluation of the location of potential fissuring in relation to the well.

Résumé Trois aspects de l'écoulement radial dans un aquifère à matrice solide, du à l'infiltration ou au pompage dans des puits, ont été étudiés mathématiquement. Premièrement, les solutions analytiques des champs de vitesse et de déplacement ont été trouvées suivant la définition de l'écoulement de masse, l'équation de la conservation de la masse, la loi de Darcy-Gersevanov, et le rabattement de la charge hydraulique en régime transitoire. Deuxièmement, la vitesse radiale et l'écou-

ment ont été investis selon le temps et la position, en réponse à trois types de schémas pompage-injection (pompage linéaire, non linéaire, sinusoidale). Les écoulements dans l'aquifère sont apparus plus réactifs aux taux de pompage linéaires et non-linéaires qu'aux pompages constants. Les écoulements en réponse à des pompages et des recharges périodiques sont gouvernés par une fonction intégrale sinusoidale. Finalement, le risque potentiel de fissuration du sol a une localisation donnée et causée par une tension générée par les cycles pompages-recharges, a été discuté suivant la solution analytique. Les zones de déformation, compression et extension, dérivées de l'analyse des résultats sont aussi potentiellement des zones génératrices de fissures. Cette approche utilisant des paramètres choisis avec précautions, permet d'évaluer en première approche la localisation potentielle des fissures autour des puits. Análisis del Movimiento Radial en un Acuífero Confinado debido a Bombeo e Inyección en un Pozo.

Resumen Se han investigado matemáticamente tres aspectos de movimiento radial en un acuífero de matriz sólida el cual está bajo la influencia de fuerzas hidráulicas que se derivan de recarga y descarga. Primero se encontraron las soluciones analíticas en la velocidad y los campos de desplazamiento usando la definición de flujo de masa volumétrico, conservación de agua y masa sólida, la ley Darcy-Gersevanov, y el descenso de la presión hidráulica para flujo no permanente. Después se investigaron como respondían la velocidad radial y el desplazamiento de la matriz sólida del acuífero, en función del tiempo y localización, con tres esquemas de bombeo-inyección (i.e. ritmos de flujo de pozo linear, no linear, y sinusoidal). Se encontró que el movimiento del acuífero respondía mejor a ritmos de bombeo linear y no linear que a bombeo constante. El movimiento del acuífero en respuesta a la recarga y descarga periódica estuvo determinado por la función sinusoidal integral. Finalmente, se ha discutido, usando una solución analítica, el riesgo potencial de fracturamiento del terreno en una localidad dada ocasionado por esfuerzos de tensión que a su vez son inducidos por actividad de recarga-descarga. Las zonas de deformación compresivas y extensionales derivadas del análisis de los resultados indican la manifestación posible de esfuerzos de tensión que pueden causar grietas en el terreno. Los cálculos en muestras

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usando parámetros supuestos aportan una evaluación de primera mano de la localización del potencial de fracturamiento en relación al pozo.

Keywords Earth fissuring · Unsteady flow · Aquifer radial movement · Analytic solutions · Groundwater flow

Introduction

The technology of aquifer storage and recovery (ASR) is widely applied in the United States at over fifty-nine operating sites located in more than 16 states (Pyne et al. 1995). The purposes of ASR applications are many including water resources management and exploitation as well as improvement of water quality. Unfortunately, the applications of the ASR technology may cause earth fissures because artificial recharge and discharge of groundwater induces hydraulic forces that may drive the aquifer matrix to move horizontally toward or away from a pumping well (see Fig. 1). The magnitude of the maximum cumulative horizontal displacement due to ground fluid withdrawal can be the same order as that of the maximum cumulative vertical displacement or subsidence (Yerks and Castle 1969). The mechanism of earth



Fig. 1 The earth fissure with the arrow pointing to the pumping well (Sheng et al. 2003)

fissuring at depth generated by hydraulic forces was previously studied and discussed by numerous investigators (Helm 1994a,b; Burbey 1999; Sheng et al. 2003). Based on the first principles, aquifer deformation can be analyzed in two basic ways that should lead to the same final result. One approach is to emphasize the flow relation in the velocity and displacement fields and the other is to emphasize the constitutive relation of the aquifer skeleton in stress and strain fields. The application of the Darcy-Gersevanov law (Helm 1979, 1987) and the application of poroelasticity (Biot 1941; Terzaghi 1943; Mikasa 1965; Verruijt 1969) are examples of the two methods, respectively. The present paper emphasizes the Darcy-Gersevanov law. Investigation using the flow relation to analyze aquifer horizontal movement in response to well flow was first conducted by Helm (1994b). The Darcy-Gersevanov law (Gersevanov 1937) and the Theis solution for unsteady flow (Theis 1935) were incorporated into Helm's solution to account for the role played by hydraulic forces. The Theis solution, however, requires a constant discharge rate and may be inapplicable to aquifer movement caused by the ASR applications that feature variable pumping-injection cycles. The goals of the present study are to find the analytic solution that can describe aquifer radial movement when the well-flow rate varies with time and to investigate how the aquifer matrix responds to hydraulic driving forces induced by artificial discharge and recharge. Aquifer vertical movement, therefore, is assumed to be insignificant in the present study. The analytic solution may be able to be applied in the displacement and velocity fields for a first-hand evaluation of potential fissuring risk due to aquifer horizontal movement. To analyze radial motion of the aquifer matrix, the definition of bulk flow by mass, mass balance of bulk flux, and the Darcy-Gersevanov law are introduced in the next sections.

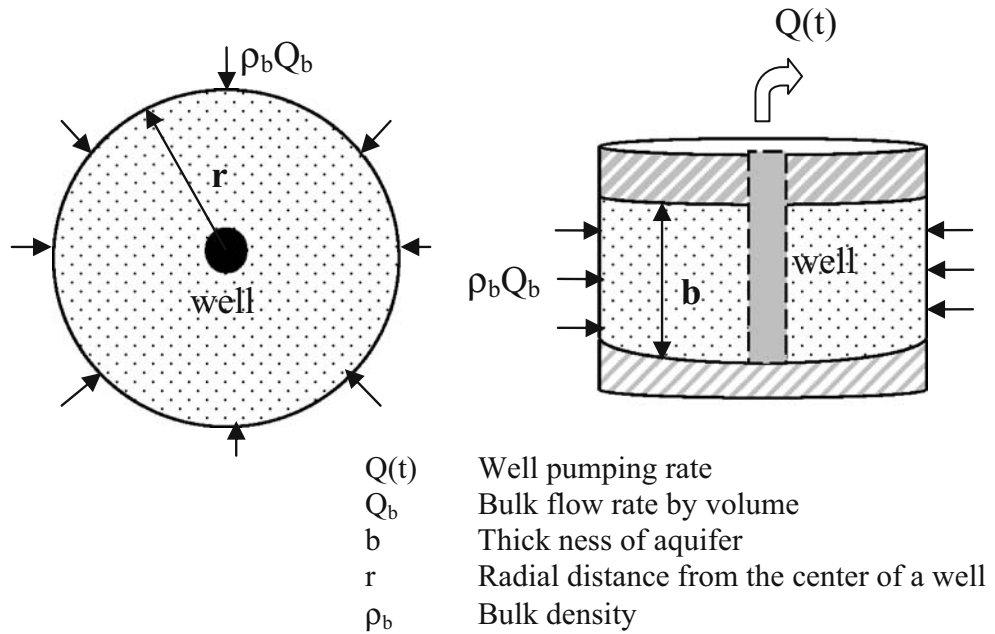
Radial bulk mass flux into a saturated and confined aquifer

If a confined aquifer with thickness b is bounded above and below by impermeable confining beds, the bulk flow by mass into a cylinder at radius r from the center of a fully penetrating pumping well (see Fig. 2) can be defined by the following relation:

$$\rho_b Q_b(r, t) = \rho_w Q_w(r, t) + \rho_s Q_s(r, t), \quad (r_w \leq r < \infty) \quad (1)$$

where Q_i is the radial flow by volume, r_w is the well radius, t is time, and ρ_i ($i=w, s$ or b) is the density of water, solid, or bulk flow respectively. The bulk density ρ_b is defined by $\rho_b = n\rho_w + (1-n)\rho_s$. Subscripts b, w , and s stand for bulk, water, and solid. If the radial flow by volume for Q_b, Q_w , and Q_s are respectively defined by $Q_b = 2\pi r b v_b$, $Q_w = 2\pi r b n v_w$, and $Q_s = 2\pi r b (1-n) v_s$ where n, v_b, v_w , and v_s

Fig. 2 The radial flow in a well in a cylindrical confined aquifer. **a** plan view, **b** side view



are porosity and the radial velocities of bulk, water and solid flow, Eq. (1) alternatively takes the form:

$$v_b(r, t) = v_s(r, t) + \frac{q(r, t)}{n + (1 - n)G} \quad (2)$$

where radial specific discharge $q = n(v_w - v_s)$, G is the specific gravity of solid, n is porosity, and v_b represents the average bulk velocity weighed by mass of water and solid. Introducing the bulk linear momentum (Bear 1972) can derive the same expression in Eq. (2).

Mass balance of radial bulk flux in a saturated and confined aquifer

If mass balance of radial flow is considered within the zone $r_w \leq r < \infty$ (Fig. 2), the accumulated bulk flow by mass (i.e., $\partial m_{\text{accum}} / \partial t$) within the confined aquifer equals:

$$\frac{\partial m_{\text{accum}}}{\partial t} = \rho_b Q_b(r, t) - \rho_b Q_b(r_w, t) + \frac{\partial m_{\text{gen}}}{\partial t} \quad (3a)$$

where on the right-hand side of Eq. (3a), the first term is the bulk flux (a water-solid mixture) out of the aquifer from the well screen at the well radius r_w (Fig. 2), the second term is the bulk flux into the confined aquifer at the radial distance r from the center of the discharge-recharge well, and the last term is changes in mass due to sinks and/or sources within the aquifer. The subscript gen in m_{gen} denotes generated mass within a confined aquifer. If the following four assumptions are applied: (1) there are no springs and drains within the confined aquifer (i.e., $\partial m_{\text{gen}} / \partial t = 0$); (2) soil grains stop at the well screen—i.e., $v_s(r_w, t) = 0$ and $\rho_b Q_b(r_w, t) = \rho_w Q(t)$ where $Q(t)$ is the pumping rate; (3) both the water and solid constituents within the confined aquifer are incompressible; (4) the net

accumulated mass within the aquifer system is due to water immediately derived from aquifer storage and cumulative soil grains crossing the circumferential surface at r and blocked by the well screen—i.e., $\partial m_{\text{accum}} / \partial t = 2\pi b r (1 - n) \rho'_s v_s$ —thus mass balance in Eq. (3a) becomes:

$$2\pi r b (1 - n) \rho'_s v_s = \rho_b 2\pi r b v_b(r, t) - \rho_w Q(t) \quad (3b)$$

where $\rho'_s = \rho_s - \rho_w$. Accordingly, the average bulk velocity can be found from Eq. (3b) in the following expression:

$$v_b = \frac{\rho_w}{\rho_b} \left[\frac{Q(t)}{2\pi r b} + (1 - n)(G - 1)v_s \right] (r_w \leq r < \infty) \quad (4)$$

The well flow rate $Q(t)$ in Eq. (4) can be associated with different schemes of artificial discharge and recharge.

Darcy-Gersevanov law for radial water flow

For saturated and porous sedimentary material, drag force on Newtonian water flow comprises two parts (Li and Helm 1995). One part is related to relative acceleration $\mathbf{a}_w - \mathbf{a}_s$ (the difference between accelerations of pore water and solid grain), and the other part to relative velocity $\mathbf{v}_w - \mathbf{v}_s$ (the difference between velocities of pore water and solid grain). If the former can be dropped due to the minor role played by the negligible relative acceleration when compared to that played by the relative velocity, the simplified relation between the drag and driving forces can be written in terms of specific discharge $\mathbf{q} [=n(\mathbf{v}_s - \mathbf{v}_w)]$, hydraulic head h and water acceleration $\mathbf{a}_w$ (Li and Helm 1998):

$$\mathbf{q} = -\mathbf{K}(\nabla h + \mathbf{a}_w/g) \quad (5)$$

where $\mathbf{K}$, g and ∇ are the conductivity matrix, the gravitational acceleration, and the differential operator

for gradient of the hydraulic head. On the right-hand side of Eq. (5), the driving force comprises two parts. The first is related to seepage force, and the second to inertial force of water flow that allows the term $\mathbf{v}_w^2$ (or $\mathbf{q}^2$ and $\mathbf{v}_s^2$) to be introduced so that Eq. (5) becomes a nonlinear relation between the specific discharge and seepage force (Li and Helm 1998). Equation (5) requires the relative acceleration to be negligible but neither $\mathbf{a}_w$ nor $\mathbf{a}_s$ to be negligibly small. If the second term on the right side in Eq. (5) can be dropped when compared to the first, Eq. (5) reduces to the Darcy-Gersevanov Law. This implies that Darcy-Gersevanov law requires that all phase accelerations or inertial forces are negligible. In particular, for the radial water flow the Darcy-Gersevanov's law becomes:

$$q = -K \frac{\partial h}{\partial r} \quad (6)$$

where K is the radial hydraulic conductivity. It should be pointed out that assuming the relative acceleration to be negligibly small but *not zero* is essential to horizontal transient movement of aquifer skeletal matrix in responses to unsteady flow due to pumping-injection activity. If the relative acceleration is assumed to be zero, for example, the relative velocity becomes a constant that further requires the hydraulic head to be constant through the Darcy-Gersevanov law. Consequently, a transient flow equation reduces to an equilibrium flow equation because of the constant hydraulic head.

Analytic solutions of aquifer transient motion

To investigate how aquifer horizontal motion responds to hydraulic driving forces induced by artificial discharge and recharge, the analytic solutions in the velocity and displacement fields need to be found first.

Radial velocity of the confined aquifer due to discharge and recharge

To describe aquifer radial movement driven by hydraulic forces, the aquifer radial velocity can be found in terms of the well flow rate $Q(t)$ and specific discharge q by inserting Eq. (4) into Eq. (2):

$$v_s(r, t) = \frac{Q(t)}{2\pi br} - q \quad (r_w \leq r < \infty) \quad (7)$$

Introducing the Carslaw-Jaeger analytic solution of drawdown $h_0 - h$ for unsteady flow with $Q(t)$ varying with time (Batu 1998, p 149):

$$h_0 - h = \frac{1}{4\pi bK} \int_0^t Q(t-\tau) \frac{e^{-r^2 S_s / 4K(t-\tau)}}{t-\tau} d\tau \quad (8)$$

and inserting both Eqs. (8) and (6) into Eq. (7) takes the form of the aquifer radial velocity v_s by:

$$v_s(r, t) = \frac{Q(t)}{C_0} \left(1 - e^{-r^2 S_s / 4Kt} \right) \quad (9)$$

where $C_0 = 2\pi br$, h_0 and h are the uniform hydraulic heads at times $t=0$ and $t=t$, $h=h_0$ when r approaches ∞ , τ is an integral dummy variable, and S_s is specific storage. For a special case of constant flow rate $Q(t)=Q_0$, hydraulic drawdown in Eq. (8) reduces to the Theis solution $h-h_0=Q_0 w(u)/4\pi T$ where $T(=bK)$ is the transmissivity, dimensionless variable $u=r^2 S_s / 4Kt$, and $w(u)$ is called the Theis well function in hydrogeology (Theis 1935) or the exponential integral function in applied mathematics (Todd and Mays 2005). The aquifer radial velocity in Eq. (9) can be simplified to the solution found by Helm (1994b) if $Q(t)=Q_0$. The next step is to find the solution in the field of aquifer displacement in response to various ASR applications.

Radial displacement of the confined aquifer due to discharge and recharge

As mentioned in the previous section, applications of the ASR technology require the recharge-discharge flow rate in a well to be a function of time. Keeping this necessity in mind and integrating aquifer velocity Eq. (9) over time gives the cumulative horizontal displacement:

$$u_s(r, t) = \int_0^t v_s(r, t) dt = \frac{C_1}{C_0} \int_u^\infty \frac{Q(\xi)}{\xi^2} \left(1 - e^{-\xi} \right) d\xi \quad (10)$$

where $C_1 = r^2 S_s / 4K$, ξ is an integral dummy variable, and the initial condition $u_s(r, 0)=0$ is assumed and applied to Eq. (10).

In the present paper, the phrase “aquifer” is interchangeably used with aquifer “matrix” when aquifer movement, velocity, and displacement are addressed. Moreover, for simplicity of notation the subscript s will be dropped in the remainder of this paper. It is worth pointing out that as $Q(t)$ in both Eqs. (9) and (10) is a function of time, this opens a door to allow the investigation of transient movement of the confined aquifer in response to pumping and injection. In the following section, three schemes of artificial discharge-recharge activity are to be assumed and discussed.

Analytic solutions of aquifer radial motion in response to different well flow rates $Q(t)$

To analyze aquifer transient motion using solutions Eqs. (9) and (10), the well flow rate $Q(t)$ is assumed to vary with time linearly, nonlinearly and sinusoidally, with

the corresponding solutions that can be found respectively in the following three subsections.

The analytic solutions in response to $Q(t)$ changing linearly with time

After the pump is turned on, if $Q(t) = B_0 t/t_0$ is assumed to simulate a long-term changing flow rate, the radial velocity in Eq. (9) and the cumulative displacement in Eq. (10) are found to be:

$$v_L(u) = \frac{B_0 C_1 / u}{C_0} (1 - e^{-u}) \quad (11)$$

$$u_L(u) = \frac{C_1}{C_0} \int_u^\infty \frac{C_1 B_0}{\xi^3} (1 - e^{-\xi}) d\xi \\ = \frac{C_1^2 B_0}{2C_0} \left[\frac{1 - (1 - u)e^{-u} + u^2 w(u)}{u^2} \right] \quad (12)$$

in which subscript L denotes the linear $Q(t)$, time t is dimensionless as it is normalized by unit time t_0 (e.g., 1 day), and B_0 in m^3/day is a coefficient and can be either a positive or negative value for linearly increasing or decreasing pumping rate. Solution Eq. (11) illustrates that aquifer velocity may include both the linear change and exponential attenuation with time, and Eq. (12) exhibits that cumulative displacement may associate with t^2 . Further analysis and discussion of aquifer movement in response to the linear well flow is to be conducted in a later section using the dimensionless formats of Eqs. (11) and (12).

The analytic solutions in response to $Q(t)$ changing nonlinearly with time

If $Q(t) = B_1 (t/t_0)^{0.5}$ is assumed to simulate a long-term discharge or recharge parabolically varying with time, velocity v_P in Eq. (9) and cumulative displacement u_P in Eq. (10) accordingly take the following two forms:

$$v_P(u) = \frac{B_1 (C_1 / u)^{1/2}}{C_0} (1 - e^{-u}) \quad (13)$$

$$u_P(u) = \frac{C_1}{C_0} \int_u^\infty \frac{C_1^{0.5} B_1}{\xi^{2.5}} (1 - e^{-\xi}) d\xi \\ = \frac{C_1^{1.5} B_1}{1.5 C_0} \left(\frac{1 - e^{-u} + u^{1.5} \int_u^\infty \frac{e^{-\xi}}{\xi^{1.5}} d\xi}{u^{1.5}} \right) \quad (14)$$

in which subscript P denotes the parabolic $Q(t)$, and coefficient B_1 in m^3/day can be either positive or negative. Similar to the previous section, the unit time t_0 in the expression $(t/t_0)^{0.5}$ is to be dropped in the rest of the paper. In the following section, expressions from Eqs. (11) to (14) are to be drawn with MathCad (Larsen 2000), a conventional tool, and discussed in detail with emphasis on dimensionless relations of aquifer velocity and displacement versus pumping time and radial distance from the well.

The analytic solutions in response to $Q(t)$ changing sinusoidally with time

For periodic recharge-discharge activity, $Q(t)$ can be assumed to change sinusoidally with time after pumping starts, namely $Q(t) = B_2 \sin(\omega t)$ where $\omega (= 2\pi/T_p)$ is the angular frequency and T_p is the pumping-injection period in days. Inserting $Q(t) = B_2 \sin(\omega t)$ into Eqs. (9) and (10) gives the forms of horizontal velocity and cumulative displacement by:

$$v_{\sin}(u) = \frac{B_2 \sin(\frac{C_2}{u})}{C_0} (1 - e^{-u}) \quad (15)$$

$$u_{\sin}(u) = \frac{C_1}{C_0} \int_u^\infty \frac{B_2 \sin(\frac{C_2}{\xi})}{\xi^2} (1 - e^{-\xi}) d\xi \\ = \frac{C_1 B_2}{C_0} \left[S_i\left(\frac{C_2}{u}\right) (e^{-u} - 1) - \int_u^\infty \frac{S_i\left(\frac{C_2}{\xi}\right) (1 - \xi e^{-\xi})}{\xi^2} d\xi \right] \quad (16)$$

where $C_2 = C_1 \omega$, subscript sin denotes the variable in response to the sinusoidal well flow, and B_2 in m^3/day is a coefficient of $Q(t)$, and $S_i(u) = \int_0^u \sin(\xi)/\xi d\xi$ is the sinusoidal integral function.

From Eq. (15), radial velocity $v_{\sin}$ responds to sinusoidal well flow and simultaneously attenuates exponentially with pumping-injection time. From Eq. (16), the cumulative displacement versus time may be dominated by the first term related to the sinusoidal integral function $S_i(u)$. This needs to be further investigated in the next section using the dimensionless formats of Eqs. (15) and (16).

Analysis and discussion of radial motion in response to different well flow rates $Q(t)$

In this section, the radial transient movement of the confined aquifer in response to the three schemes of $Q(t)$ is to be analyzed and discussed using the analytic solutions found in the previous sections. For convenience of study, variables such as displacement u , velocity v , time t and radial distance r in the equations from Eqs. (11–16) are converted into dimensionless variables u^* , v^* , t^* and r^*

where superscript * represents a dimensionless variable, $t^* = tK/r^2 S_s$ and $r^* = r(S_s/Kt)^{1/2} = 1/(t^*)^{1/2}$.

Radial movement versus time and location in response to the linear $Q(t)$

Dimensionless radial velocity for the linear pumping rate
When the well flow rate $Q(t) = B_0 t$, aquifer radial velocity Eq. (11) can be written as a function of either t^* or r^* by the two dimensionless forms:

$$v_L^*(t^*) = \frac{2\pi b v_L}{B_0 r} \frac{K}{S_s} = t^* \left(1 - e^{-\frac{1}{4t^*}}\right) \quad (17a)$$

$$v_L^*(r^*) = \frac{2\pi b v_L}{B_0} \sqrt{\frac{K}{t S_s}} = \frac{\left(1 - e^{-\frac{r^{*2}}{4}}\right)}{r^*} \quad (17b)$$

Figure 3a demonstrates that dimensionless velocity v_L^* increases rapidly in early time, slows down significantly when $t^* > 0.5$ due to exponential attenuation, and quickly approaches an ultimate speed of 0.25. Figure 3b illustrates that aquifer radial velocity v_L^* changes with radial distance r^* , and reaches its highest speed $v_{L \max}^* = 0.32$ at $r^* = 2.25$. This implies that after the well pumping starts, aquifer moving speed rapidly increases at $r^* \leq 2.25$ (strain increases as $\partial v^*/\partial r^* = \partial \epsilon / \partial t > 0$), and then slows down at $r^* \geq 2.25$ (strain decreases as $\partial \epsilon / \partial t < 0$) after it reaches its peak speed at the location $r_{\max}^* = 2.25$ (strain equals constant as $\partial \epsilon / \partial t = 0$). It is interesting to note that if aquifer sediments were assumed to behave like poroviscous material, earth fissuring may occur at $r^* \geq 2.25$ because viscous stress related to strain rate $\partial \epsilon / \partial t$ changes its direction at $r_{\max}^* = 2.25$ so that viscous tensile stress may take place within the zone $r^* \geq 2.25$.

Though the maximum $v_{L \max}^*$ is a constant at the dimensionless location $r_{\max}^* = 2.25$, the maximum dimensional velocity $v_{L \max}$ is a function of aquifer property and pumping time. Recalling Eq. (17b) with the peak point $v_{L \max}^* = 0.32$ and $r_{\max}^* = 2.25$ gives $v_{L \max}$ in the form:

$$v_{L \max} = \frac{0.16 B_0 t^{0.5}}{\pi b} \sqrt{\frac{S_s}{K}} \quad (18)$$

For sample computations, if the well pumping coefficient $B_0 = 0.84 \text{ m}^3/\text{day}$ (or 0.15 gal/min), the aquifer thickness $b = 50 \text{ m}$, and the ratio of specific storage to radial conductivity $S_s/K = 0.001 \text{ day/m}^2$, the result from Eq. (18) is listed in Table 1. The maximum dimensional radial speed $v_{L \max} = 0.26 \text{ mm/day}$ for instance, occurs at the location $r_{\max} = 675 \text{ m}$ from the well after 3 months of pumping (Table 1). The definition of $r_{\max}^* = r_{\max}(S_s/Kt)^{1/2}$ is recalled and applied.

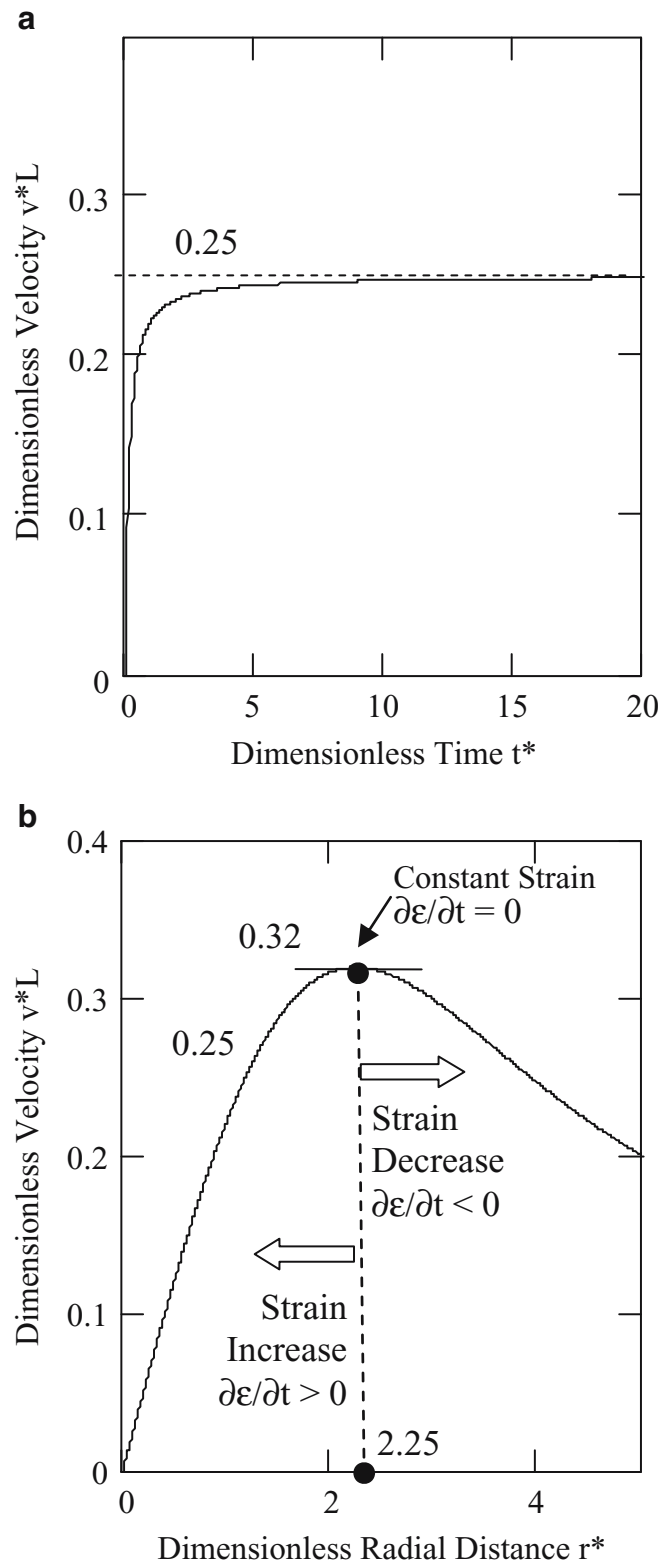


Fig. 3 a Dimensionless velocity v_L^* versus time t^* for the linear $Q(t)$. b Dimensionless velocity v_L^* versus radial distance r^* for the linear $Q(t)$

Table 1 $v_{L \max}$ versus t at different $r_{\max}$ in response to $Q(t)=B_0t$

Maximum radial distance $r_{\max}$ (m)	Maximum radial velocity $v_{L \max}$ (mm/day)	Pumping period t (months)
675	0.26	3
959	0.36	6
1369	0.51	12

Dimensionless radial displacement for the linear pumping rate

For a linear $Q(t)$, recalling Eq. (12) gives two expressions of dimensionless displacement (versus dimensionless time t^* and versus dimensionless distance r^*):

$$u_L^*(t^*) = \frac{4\pi b u_L}{B_0} \left(\frac{K}{S_s} \right)^2 \left(\frac{2}{r} \right)^3 = \int_{\frac{1}{4t^*}}^{\infty} \frac{(1 - e^{-\xi})}{\xi^3} d\xi \quad (19a)$$

$$u_L^*(r^*) = \frac{4\pi b u_L}{B_0 t^{1.5}} \sqrt{\frac{K}{S_s}} = \left(\frac{r^*}{2} \right)^3 \int_{\frac{r^{*2}}{4}}^{\infty} \frac{(1 - e^{-\xi})}{\xi^3} d\xi \quad (19b)$$

The relations of u_L^* versus t^* and r^* in Eqs. (19a) and (19b) are plotted in Fig. 4a,b. Figure 4a shows that the cumulative displacement increases quickly with time after a short initial pumping period. Figure 4b demonstrates that the curve of aquifer displacement against radial distance arrives at the peak point $u_{L \max}^* = 0.4$ at $r_{\max}^* = 1.6$. The positive slope (or strain $\partial u / \partial r = \varepsilon > 0$) of the curve u_L^* versus r^* in Fig. 4b suggests that at the location $r^* \leq 1.6$, the aquifer is radially compacted by hydraulic force. In contrast, the negative slope (or $\varepsilon < 0$) of the curve represents that at the location $r^* \geq 1.6$, the aquifer is extended. At the location $r^* = 1.6$ ($u_{L \max}^* = 0.4$), the aquifer becomes neither compacted nor extended (or $\varepsilon = 0$). It is important to point out that as skeletal strain changes its direction at $r^* = 1.6$, effective skeletal stress related to strain through a constitutive law such as Hooke's law may also change its direction. This may result in tensile stress and cause earth cracks or fissures that develop upward and eventually manifest on the ground surface. From Eq. (19b), the maximum dimensional $u_{L \max}$ can be:

$$u_{L \max}(t) = \frac{0.1 B_0 t^{1.5}}{\pi b} \sqrt{\frac{S_s}{K}} \quad (20)$$

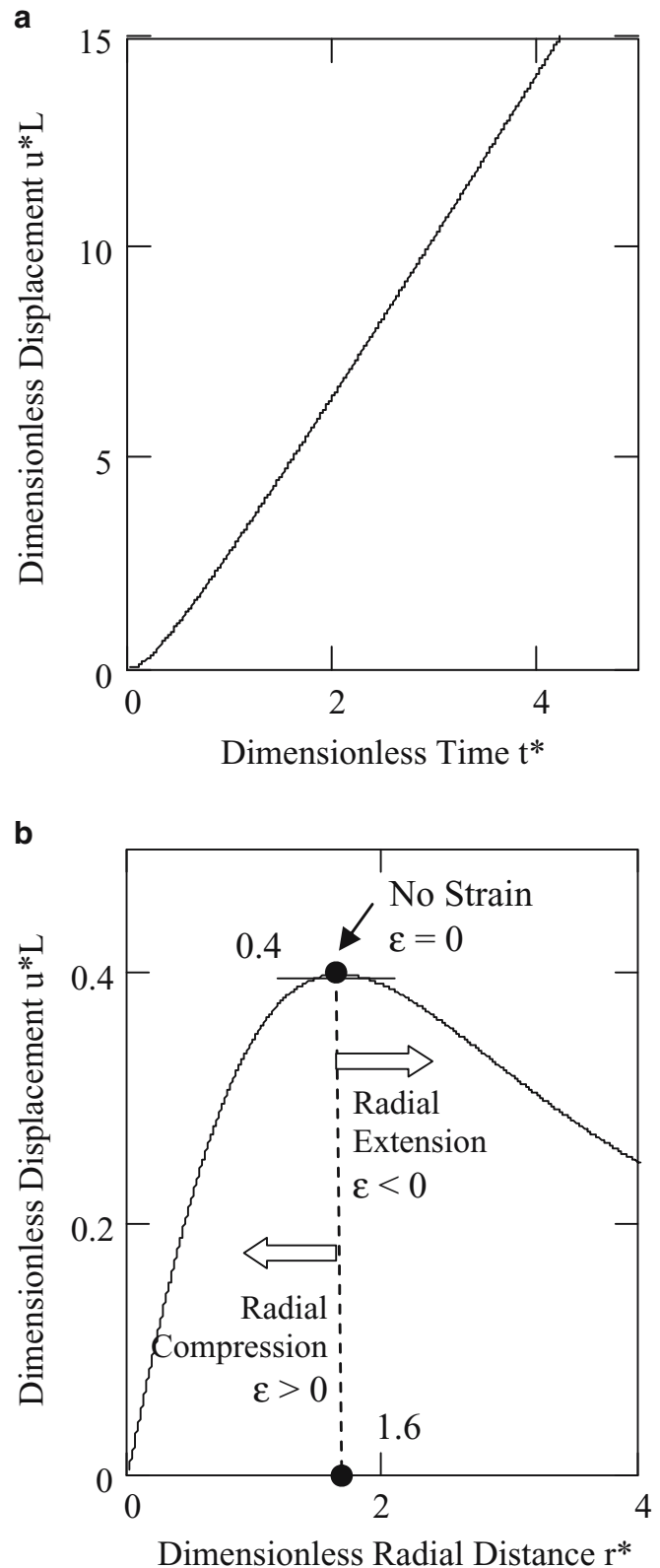


Fig. 4 **a** Dimensionless displacement u_L^* versus time t^* for the linear $Q(t)$. **b** Dimensionless displacement u_L^* versus radial distance r^* for the linear $Q(t)$

Table 2 $u_{L \max}$ versus t at different $r_{\max}$ in response to $Q(t)=B_0t$

Maximum radial distance $r_{\max}$ (m)	Maximum radial displacement $u_{L \max}$ (mm)	Pumping period t (months)
480	14	3
682	41	6
974	118	12

If the same set of parameters is chosen for aquifer property and well discharge, the results calculated from Eq. (20) are given in Table 2. The data show that the maximum dimensional displacement $u_{L \max}=14$ mm takes place at the location $r_{\max}=480$ m after the pump has been turned on for 3 months. The locations $r_{\max}=480$, 682 and 974 m changing with pumping time 3, 6 and 12 months suggests that the possible locations of earth fissuring may radially migrate away from the well during discharge.

Radial movement versus time and location in response to the nonlinear $Q(t)$

Dimensionless radial velocity for the nonlinear pumping rate

For nonlinear well flow rate $Q(t)=B_1t^{1/2}$, dimensionless velocity v_p^* versus time t^* and radial distance r^* can be derived from Eq. (9) or Eq. (13), and expressed individually by:

$$v_p^*(t^*) = \frac{2\pi b v_p}{B_1} \sqrt{\frac{K}{S_s}} = \sqrt{t^*} \left(1 - e^{-\frac{1}{4t^*}}\right) \quad (21a)$$

$$v_p^*(r^*) = \frac{2\pi b v_p}{B_1} \sqrt{\frac{K}{S_s}} = \left(\frac{1 - e^{-\frac{r^{*2}}{4}}}{r^*}\right) \quad (21b)$$

Equations (21a) and (21b) are drawn in Fig. 5a,b to present the dimensionless relations of v_p^* versus t^* and r^* . As Fig. 5b indicates the same relation that is presented in Fig. 3b for the linear $Q(t)$, this section will only discuss Fig. 5a. It is interesting to find that the curve of v_p^* against t^* in (Fig. 5a) is different from that of v_L^* against t^* (Fig. 3a), in which v_L^* versus t^* presents a mono-increasing curve and approaches the ultimate value 0.25. In contrast, the radial velocity v_p^* against t^* reaches its maximum value at $v_{p \max}^*=0.32$ when $t_{\max}^*=0.2$ (Fig. 5a). At the peak point $v_{p \max}^*$, the aquifer acceleration $\partial v^*/\partial t^*=a=0$ indicates that aquifer movement with a constant speed when dimensionless time $t_{\max}^*=0.2$. The domain of dimensionless discharge or recharge time, therefore, is divided into the accelerating period and the decelerating period. When $t^*<0.2$ aquifer movement accelerates as $\partial v^*/\partial t^*=a>0$, and when $t^*>0.2$ aquifer motion decelerates as $\partial v^*/\partial t^*=a<0$. This may imply that at a given location, inertial force associated with acceleration changes its direction at $t^*=0.2$ as aquifer acceleration changes from a

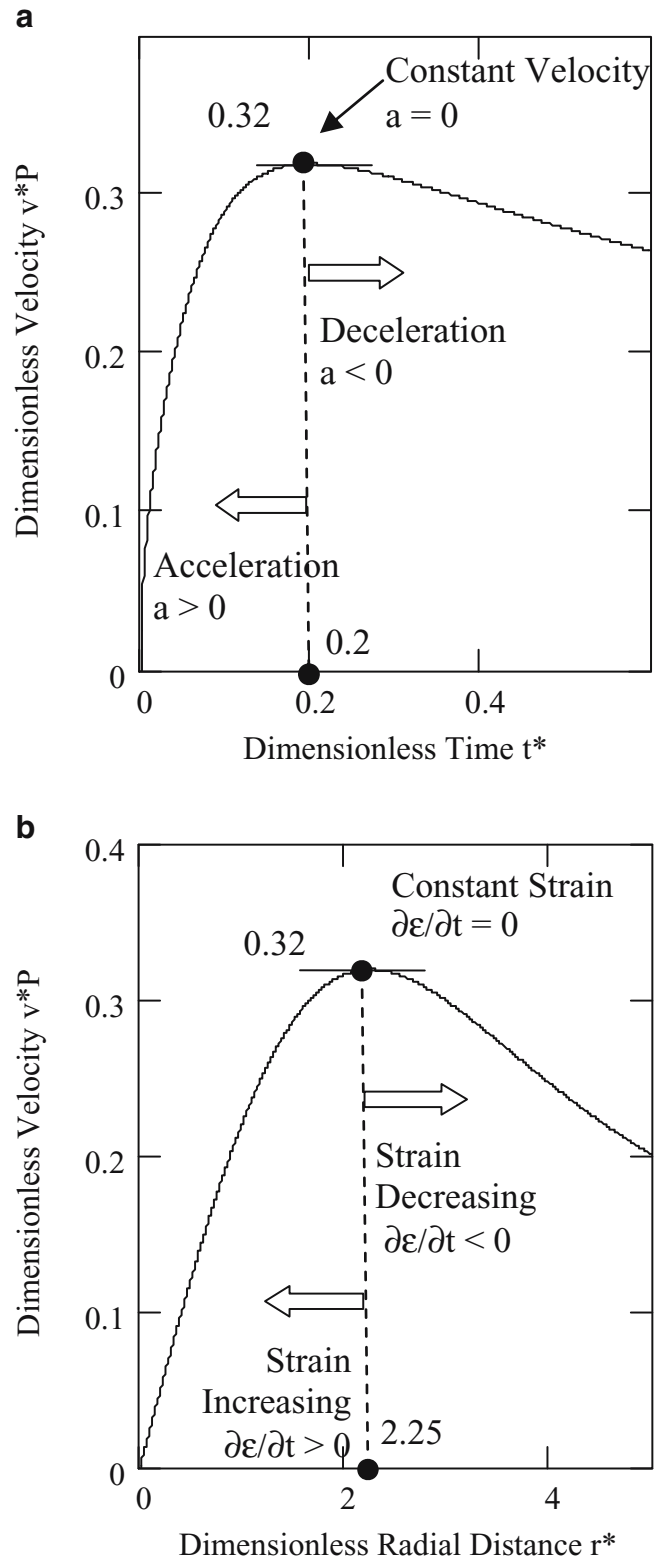


Fig. 5 a Dimensionless velocity v_p^* versus time t^* for the nonlinear $Q(t)$. b Dimensionless velocity v_p^* versus radial distance r^* for the nonlinear $Q(t)$

positive value to a negative value when transitioning from $t^*<0.2$ to $t^*>0.2$. Recalling Eq. (21a) finds $v_{p \max}^*=B_1 v_{p \max}^*/(S_s/K)^{0.5}/2\pi b$ which results in the maximum dimensional

horizontal speed $v_{P \max} = 0.027$ mm/day if the parameters $B_1 = 0.84$ m³/day (or 0.15 gal/min), $b = 50$ m and $S_s/K = 0.001$ day/m² are assumed.

Dimensionless radial displacement for the nonlinear pumping rate

Similarly, substituting $Q(t) = B_1 t^{1/2}$ into aquifer displacement Eq. (10) or recalling Eq. (14) leads to the following two dimensionless forms:

$$u_P^*(t^*) = \frac{4\pi b u_P}{B_1} \left(\frac{K}{S_s}\right)^{1.5} \left(\frac{2}{r}\right)^2 = \int_{\frac{1}{4t^*}}^{\infty} \frac{(1 - e^{-\xi})}{\xi^{2.5}} d\xi \quad (22a)$$

$$u_P^*(r^*) = \frac{4\pi b u_P}{B_1} \frac{\sqrt{K}}{t\sqrt{S_s}} = \left(\frac{r^*}{2}\right)^2 \int_{\frac{r^{*2}}{4}}^{\infty} \frac{(1 - e^{-\xi})}{\xi^{2.5}} d\xi \quad (22b)$$

Equations (22a) and (22b) are plotted in Fig. 6a,b to present how horizontal movement u_P^* responds to $Q(t) = B_1 t^{0.5}$ at different pumping times and locations. Comparing Fig. 6a with Fig. 3a finds that the aquifer displacement responds to $Q(t) = B_{0t}$ considerably faster than to $Q(t) = B_1 t^{0.5}$ for a given pumping time period (i.e., $\partial u_L^* / \partial t \gg \partial u_P^* / \partial t$). This means that aquifer cumulative displacement or deformation is significantly affected by the power of time in $Q(t)$. Figure 6b demonstrates that the radial displacement also changes against radial distance with two zones: the compressed zone and the extended zone. A similar discussion in the previous section, therefore, can be applied to the relation between u_P^* and r^* (Fig. 6b). Applying the same set of aquifer and pumping parameters used previously to the following expression derived from Eq. (22b):

$$u_{P \max}(t) = \frac{B_1 t u_{P \max}^*}{4\pi b} \sqrt{\frac{S_s}{K}} \quad (23)$$

results in the data in Table 3 for $u_{P \max}^* = 0.56$ at $r_{\max}^* = 1.5$. From Table 3, the dimensional displacement $u_{P \max}$ is 2.13 mm at the distance $r_{\max} = 60$ m from the well after 3-months of pumping. As mentioned in the previous section, the locations $r_{\max}$ at 450, 640 and 912 m from the well suggests that the possible locations of earth fissuring within the extensional zone may migrate radially outward from the well with discharge and/or recharge.

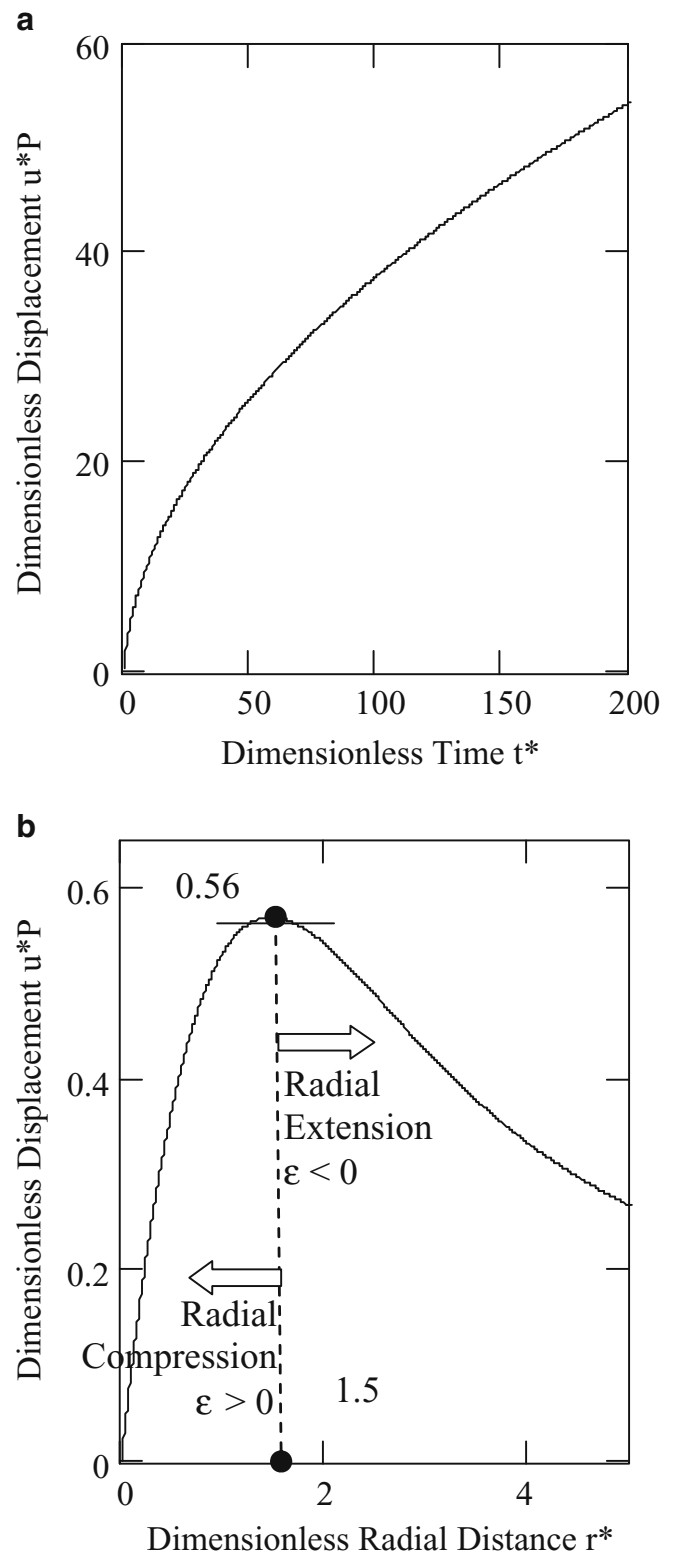


Fig. 6 a Dimensionless displacement u_P^* versus time t^* for the nonlinear $Q(t)$. b Dimensionless displacement u_P^* versus radial distance r^* for the nonlinear $Q(t)$

Table 3 $u_{p \max}$ versus t at different $r_{\max}$ in response to $Q(t)=B_1 t^{0.5}$

Maximum radial distance $r_{\max}$ (m)	Maximum radial displacement $u_{p \max}$ (mm)	Pumping period t (days)
450	2.13	90
640	4.25	180
912	8.61	365

Radial movement versus time and location due to the sinusoidal $Q(t)$

Dimensionless radial velocity for $Q(t)=B_2 \sin(\omega t)$

Inserting $Q(t)=B_2 \sin(\omega t)$ into aquifer velocity Eq. (9) or using Eq. (15) gives the two dimensionless expressions:

$$v_{\sin}^*(t^*) = \frac{2\pi b r v_{\sin}}{B_2} = \sin(4C_2 t^*) \left(1 - e^{-\frac{1}{4t^*}}\right) \quad (24a)$$

$$v_{\sin}^*(r^*) = \frac{2\pi b \sqrt{Kt} v_{\sin}}{B_2 \sqrt{S_s}} = \frac{\sin(4C_2 / r^{*2}) \left(1 - e^{-\frac{r^{*2}}{4}}\right)}{r^*} \quad (24b)$$

Dimensionless velocity $v_{\sin}^*$ against dimensionless time t^* and distance r^* are accordingly drawn in Fig. 7a,b with the pumping-injection period $T_p=365$ days, aquifer property $S_s/K=0.001$ day/m², aquifer thickness $b=50$ m and coefficient $B_2=128$ m³/day (23.5 gal/min). Figure 7a shows that at a given location, aquifer matrix radially moves against time with sinusoidal oscillation and exponential attenuation, and eventually stops. Table 4 indicates that the oscillating frequency increases with increasing radial distance. The previous discussion of the accelerating and decelerating zones can be also applied to each curve (Fig. 7a). Figure 7b illustrates that the velocity $v_{\sin}^*$ sinusoidally increases with r^* . After it arrives at the maximum speed $v_{\sin \max}^*$, aquifer motion slows down exponentially without oscillation. The sample computation in Table 4 from Fig. 7a indicates that the aquifer's highest moving speed $v_{\sin \max}=0.38$ mm/day can be observed at $r_{\max}=300$ m after a pumping period of 45 days. A discussion similar to previous discussions of the strain rate increasing and decreasing zones ($\partial v / \partial r^* = \partial \epsilon / \partial t > 0$ or < 0) and possible earth fissuring caused by viscous tensile stress (Fig. 5b) can be appropriately applied to this section (Fig. 7b; Table 5).

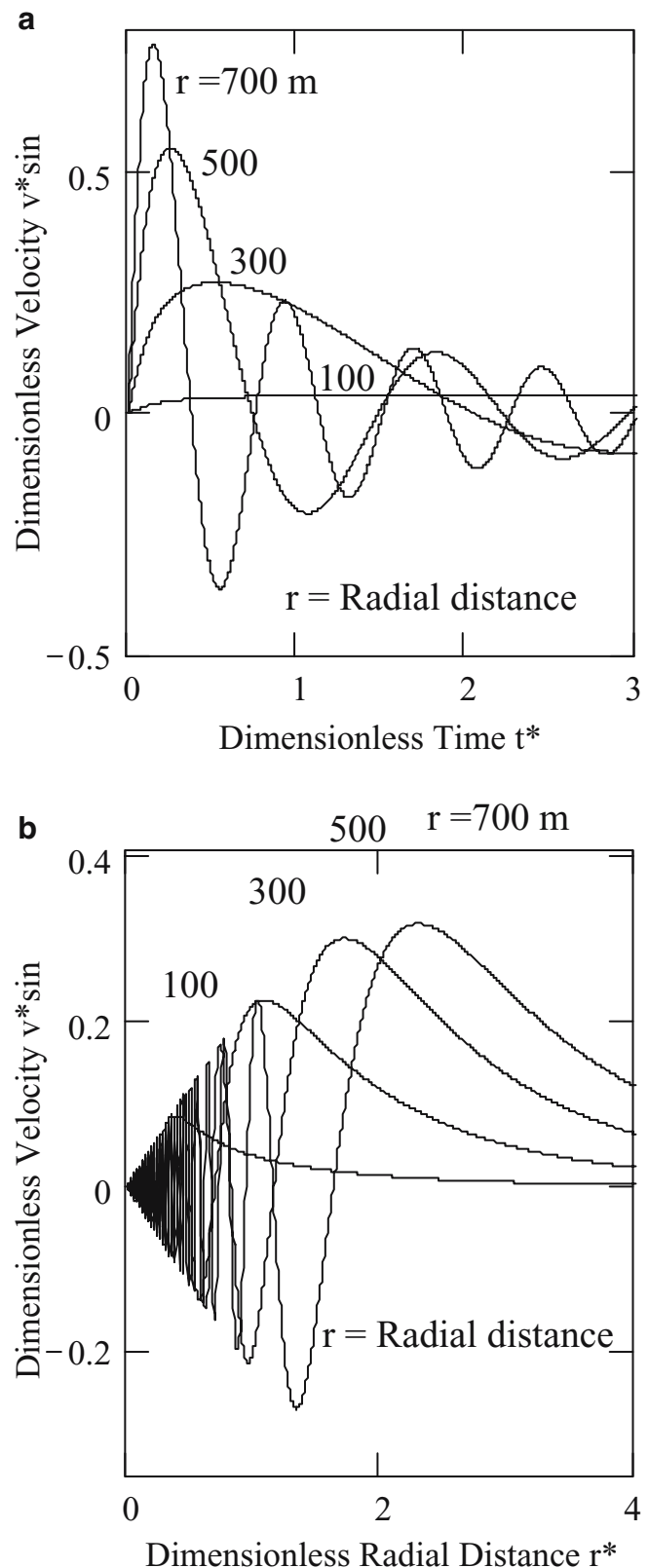


Fig. 7 a Dimensionless velocity $v_{\sin}^*$ versus time t^* for the sinusoidal $Q(t)$. b Dimensionless velocity $v_{\sin}^*$ versus radial distance r^* for the sinusoidal $Q(t)$

Table 4 $v_{\sin \max}^*$ versus $t_{\max}^*$ at different r for $Q(t)=B_2\sin(\omega t)$

$v_{\sin \max}^*$	$t_{\max}^*$	$r_{\max}$ (m)	$t_{\max}$ (day)	$v_{\sin \max}$ (mm/day)
0.28	0.5	300	45	0.38
0.55	0.25	500	63	0.44
0.77	0.15	700	74	0.45

Dimensionless radial displacement for $Q(t)=B_2\sin(\omega t)$

Applying $Q(t)=B_2\sin(\omega t)$ to the aquifer displacement Eq. (10) or employing Eq. (16) takes the dimensionless forms of $u_{\sin}^*$ versus time t^* and versus radial distance r^* by:

$$u_{\sin}^*(t^*) = \frac{8\pi b K u_{\sin}}{B_2 r S_s} = \int_{\frac{1}{4t^*}}^{\infty} \frac{\sin(C_2/\xi)(1 - e^{-\xi})}{\xi^2} d\xi \quad (25a)$$

$$u_{\sin}^*(r^*) = \frac{4\pi b \sqrt{K u_{\sin}}}{B_2 \sqrt{S_s t}} = \frac{r^*}{2} \int_{\frac{r^{*2}}{4}}^{\infty} \frac{\sin(C_2/\xi)(1 - e^{-\xi})}{\xi^2} d\xi \quad (25b)$$

The Eqs. (25a) and (25b) are plotted in Fig. 8a,b using the following parameters: pumping–injection period $T_p=365$ days, aquifer property $S_s/K=0.001$ day/m², aquifer thickness $b=50$ m, the coefficient of the discharge–recharge rate $B_2=128$ m³/day (23.5 gal/min).

For the pumping–injection rate $Q(t)=B_2\sin(\omega t)$, Fig. 8a illustrates that the relation $u_{\sin}^*$ versus t^* is likely governed by the sinusoidal integral function $S_i(x)$. This suggests that aquifer cumulative displacement rapidly reaches its maximum deformation in the first cycle. The aquifer periodic motion then attenuates to different ultimate deformation at different locations. Table 6, for example, shows the maximum cumulative displacement $u_{\sin \max}$ occurs in the first cycle at $r_{\max}=100$ m from the well after 3 days of pumping. The relation between $u_{\sin}^*$ and r^* (Fig. 8b) indicates that the aquifer arrives at its maximum cumulative displacement and then attenuates exponentially. For instance, at $t=3$ days, the aquifer cumulative displacement reaches its largest radial deformation of 7 mm at the radial distance 100 m away from the well, then decreases with increase of radial distance. Similar to the discussion in the

Table 5 $v_{\sin \max}^*$ versus $r_{\max}^*$ at different $r_{\max}$ for $Q(t)=B_2\sin(\omega t)$

$v_{\sin \max}^*$	$R_{\max}^*$	$r_{\max}$ (m)	$t_{\max}$ (day)	$v_{\sin \max}$ (mm/day)
0.23	1.07	300	79	0.33
0.28	1.73	500	84	0.40
0.32	2.27	700	95	0.42

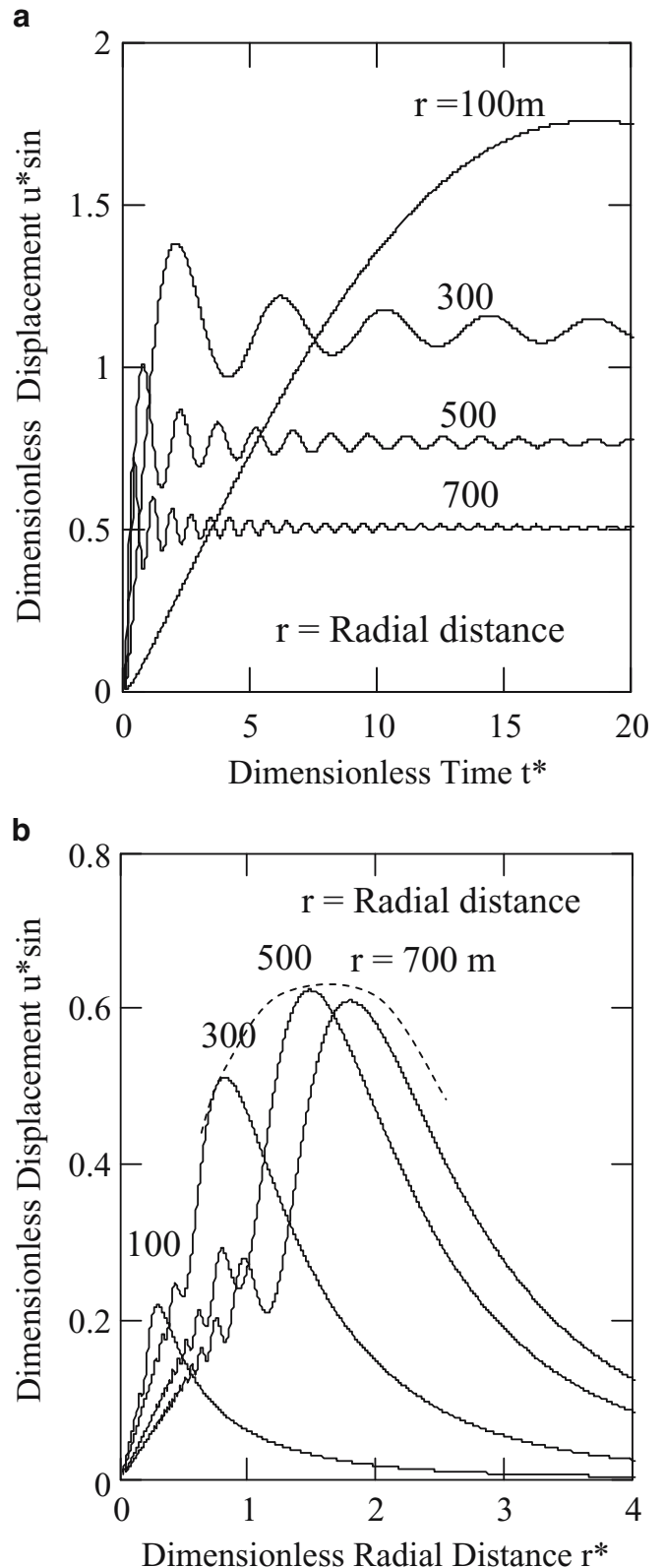


Fig. 8 **a** Dimensionless displacement $u_{\sin}^*$ versus time t^* for the sinusoidal $Q(t)$. **b** Dimensionless displacement $u_{\sin}^*$ versus radial distance r^* for the sinusoidal $Q(t)$

Table 6 $u_{\sin \max}^*$ versus $t_{\max}^*$ at different $r_{\max}$ in response to $Q(t)=B_2\sin(\omega t)$

$u_{\sin \max}^*$	$t_{\max}^*$	$r_{\max}$ (m)	$t_{\max}$ (day)	$u_{\sin \max}$ (mm)
0.71	0.3	100	3	7
1.01	0.8	300	72	31
1.38	1.3	500	325	70
1.85	1.8	700	882	132

previous sections (Figs. 4b and 6b), the deformation compression zone is identified on the left side of the peak point of each individual curve in Fig. 8b. Additionally, minor periodic motion of the aquifer can be observed when $t^* < 1.0$ (Fig. 8a). The deformation extension zone, where fissures may be caused by tensile stress, is found on the right side of the maximum point. As pointed out previously, the boundary of the compressed and extended zones is located at $r_{\max}$, which moves with time as indicated in Table 7. It is interesting to note that among the peak values of all the curves, the maximum values $u_{\sin \max}^* = 0.63$ occurs at $r_{\max} = 570$ m and $r_{\max}^* = 1.45$ (see the dashed line in Fig. 8b).

Summaries and conclusions

The analytic solutions in the velocity and displacement fields for radial aquifer movement within a confined aquifer with a fully penetrating well have been found. The analytic solutions are incorporated with the definition of bulk flow by mass, mass balance of bulk flux, the Darcy-Gersevanov law and the Carslaw-Jaeger analytic solution of hydraulic drawdown for unsteady flow and non-constant pumping-injection rate.

Three schemes of well flow rates have been applied to the analytic solutions in the present work. They are assumed to be the linear, nonlinear and sinusoidal functions of time to simulate the long-term linear, nonlinear and periodic discharge-recharge activity. The three sets of analytic solutions in the velocity and displacement fields can be applied to aquifer radial movement impelled by hydraulic driving forces due to artificial discharge and recharge.

The first hand approximation of earth-fissuring risk due to aquifer movement has been conducted for three pumping-injection schemes. The analytical findings and results from sample computations with assumed aquifer property and pumping parameters are summarized in the following.

For dimensionless aquifer displacement and velocity versus dimensionless pumping time, aquifer displacement for the linear $Q(t)$ is considerably larger than that for the

Table 7 $u_{\sin \max}^*$ versus $r_{\max}^*$ at different $r_{\max}$ in response to $Q(t)=B_2\sin(\omega t)$

$u_{\sin \max}^*$	$r_{\max}^*$	$r_{\max}$ (m)	$t_{\max}$ (day)	$u_{\sin \max}$ (mm)
0.22	0.3	100	111	145
0.51	0.8	300	141	39
0.62	1.5	500	111	42
0.61	1.8	700	151	48

nonlinear $Q(t)$ (i.e., $u_L^* \gg u_P^*$) during a given pumping period (Figs. 4a and 6a). This suggests that if the well flow is considered as a power function $Q(t)=B(t/t_0)^\alpha$, the magnitude of aquifer cumulative displacement is significantly responsive to the power α ($\alpha=1$ and 0.5 in the present paper) after pumping starts. Aquifer motion for a sinusoidal $Q(t)$ is governed by the sinusoidal integral function $S_i(x)$; namely, aquifer cumulative displacement at a given location rapidly increases and reaches its highest magnitude within the first cycle of discharge and recharge, then sinusoidally attenuates, and eventually stops at the ultimate displacement. Inertial tensile stress may occur when aquifer motion changes from acceleration to deceleration.

For dimensionless aquifer velocity and displacement versus dimensionless radial distance with all three types of $Q(t)$, the aquifer cumulative displacement is separated by the point of maximum cumulative displacement into two areas: the deformation compressed and extended zones. The effective elastic stress on aquifer skeleton changes direction at the peak point. This implies that tensile stress may be induced along the boundary of two zones and/or within the deformation extension zone in which earth fissures may occur. For the sinusoidal well flow rate, aquifer radial velocity sinusoidally changes at a given location, and exponentially attenuates to zero. For dimensionless aquifer velocity versus dimensionless radial distance with all three types of $Q(t)$, aquifer radial velocity reaches its highest speed at $r_{\max}^*$, which separates the velocity field into two areas: the strain increasing zone and the strain decreasing zone. Changes in aquifer strain speed up when $\partial \varepsilon / \partial t > 0$, in the strain increasing zone and slow down when $\partial \varepsilon / \partial t < 0$ in the strain decreasing zone. This means that the viscous tensile stress related to the strain rate may occur at $r_{\max}^*$ and may play a role in earth fissuring if the aquifer skeleton were assumed to behave like porous viscous material.

The following limitations should be noted: (1) the solution presented in this paper is limited to the area of continuum mechanics, (2) the application of the analytic solution is only a first-hand evaluation of fissuring potential, (3) tensile failure, crack generation, fissure growth and migration are not considered. In other words, the analytic solution presented in this paper cannot describe the discontinuity of sedimentary material.

Nomenclature

A	($= \partial v^* / \partial t$) acceleration for dimensionless radial velocity
a_i	($i=w, s$): Acceleration of pore water and solid, L/T^2
B	Thickness of the confined aquifer, L
B_i ($i=0, 1, 2$)	Coefficient for linear, nonlinear and sinusoidal well flow rates, L^3/T
C_0	($= 2\pi brn$) a parameter, L^2
C_1	($= S_g r^2 / 4 K$) a parameter, T

C_2	($=\omega C_1$), a dimensionless parameter
g	Gravitational acceleration, L^2/T
h	Hydraulic head, L
h_0	Initial hydraulic head, L
K	Radial hydraulic conductivity, L/T
$\mathbf{K}$	Hydraulic conductivity, L/T
m_i	(i=accum, gen) accumulated and generated mass, M
n	Porosity
q	Radial specific discharge, L/T
$\mathbf{q}$	[$=n(\mathbf{v}_w - \mathbf{v}_s)$] specific discharge, L/T
Q_0	Constant well pumping rate, L^3/T
Q_j (j=b, w, s)	Flow rate by volume for bulk, water and solid flow, L^3/T
$Q(t)$	Well pumping-injection rate by volume, L^3/T
r	Radial distance from the pumping well, L
$r_{\max}$	Radial distance for maximum values of u_j and v_j , L
r_w^*	Radius of the pumping well, L
r^*	[$=r(S_s/Kt)^{1/2}$] dimensionless radial distance for r
$r_{\max}^*$	Dimensionless radial distance for maximum values of u_j and v_j
S_s	Specific storage
t	Time, T
$t_{\max}$	Time for maximum values of u_j and v_j , T
t^*	($=tK/S_s r^2$) dimensionless time
$t_{\max}^*$	Dimensionless time for maximum values of u_j^* and v_j^*
T	($=bK$) transmissivity, L^3/T
T_p	Pumping-injection period, 1/T
u	($=S_s r^2/tK$) a dimensionless integral variable
u_j	Radial displacement, L
(j=L, P, sin)	
$u_{j \max}$	Maximum value of u_j , L
(j=L, P, sin)	
u_s^*	Displacement of solid, L
u_j^*	Dimensionless radial displacement
(j=L, P, sin)	for u_j
$u_{j \max}^*$	Maximum value of u_j^*
(j=L, P, sin)	
v_i	Radial velocities of bulk, water and solid flow, L/T
(i=b, w, s)	
$\mathbf{v}_i$ (i=w, s)	Velocities of water and solid flow, L/T
v_j	Radial velocities in response to linear, parabolic and sinusoidal $Q(t)$, L/T
(j=L, P, sin)	
$v_{j \max}$	Maximum value of v_j , L/T
(j=L, P, sin)	
v_j^*	Dimensionless radial velocities for v_j
(j=L, P, sin)	
$v_{j \max}^*$	Maximum value of v^*
(j=L, P, sin)	
ε	($=\partial u^*/\partial r$) radial strain defined for dimensionless radial displacement u^*
ξ	Dummy variable for integration
ρ_i (i=b, w, s)	Densities of bulk, water and solid flow, M/L^3

ω	($=2\pi/T_p$) angle frequency, 1/T
∇	Differential operator for gradient
$\partial/\partial r$	Partial derivative with respect to r , 1/L
$\partial/\partial t$	Partial derivative with respect to time t , 1/T

Subscripts

0	Initial values of variables at $t=0$
accum	Accumulated mass
b	Bulk flow
gen	Generated mass
L	Linear
max	Maximum values of velocity and displacement
p	Pumping
P	Parabolic
s	Solid
sin	Sinusoidal
w	Water

Superscript

*	Dimensionless variables
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