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# A Procedure for Determining the Reaction Curve of Shotcrete Lining Considering Transient Conditions

By

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**Summary**

The mechanical behaviour of a shotcrete lining is analysed in this paper using the convergence-confinement approach. A calculation procedure is presented which is able to provide the reaction curve of a lining with increasing stiffness, by taking into account the variability due to time of the shotcrete stiffness and strength, as well as the face advancement rate. The proposed procedure is also able to provide the change of the safety factor of the lining in time. This results to be a very useful tool for understanding the behaviour of this widely used support and for assigning its thickness to guarantee stability (also in transient conditions), with a known safety factor, during tunnel excavation.

*Keywords:* Shotcrete lining, convergence-confinement approach, support-reaction curve.

## 1. Introduction

Shotcrete lining is one of the most widely used tunnel support. It is placed close to the excavation face, when the static contribution offered by the face still exists. Its characteristics, the increase of its mechanical parameters in time in the hardening period, make the understanding of its behaviour, and therefore its correct design, difficult. As the shotcrete hardens, the gradual loading of the lining takes place, while the excavation face gradually advances. These transient conditions not only represent critical situations for the stability of the support structure during the construction of the tunnel, but also influence the final equilibrium of the lining (in the long term, far from the excavation face) and its safety factor.

The numerical calculation methods that are currently available are unfortunately not able to simulate the behaviour of the shotcrete lining in a satisfactory way. It results to be problematic, if not impossible, to update the mechanical characteristics of the shotcrete with rational criteria during the loading phase. The convergence-confinement method requires a mean stiffness of the lining to determine the support reaction line. The lining mean stiffness cannot be evaluated a priori (the final equilibrium point between the support and the tunnel being initially unknown). The definition of the mean stiffness does not allow the evaluation of the true stress state in the shotcrete layer and therefore the evaluation of the safety factor, as it does not consider that loads are applied to a structure with variable rather than constant stiffness. A calculation procedure is presented in this paper that is able to define the reaction curve of a shotcrete lining, with a progressively increasing stiffness.

## 2. The Reaction Curve of the Shotcrete Lining

The response of the rock mass around a tunnel is well represented by the "ground reaction curve" (Fig. 1a) (Hoek and Brown, 1980; Lombardi, 1975; Lembo-Fazio and Ribacchi, 1986), which applies to an ideal case of a deep, circular tunnel in an isotropic, homogeneous, elasto-plastic medium. The behaviour of the shotcrete lining can be represented through a reaction curve, with tangent stiffness $k$, which increases in time with the increase in the shotcrete elastic modulus:

$$k = \frac{[R^2 - (R - t_{shot})^2]}{(1 + \nu_{shot}) \cdot [(1 - 2 \cdot \nu_{shot}) \cdot R^2 + (R - t_{shot})^2]} \cdot \frac{1}{R} \cdot E_{shot} \tag{1}$$

where:

$E_{shot}$ and $\nu_{shot}$ are the elastic modulus and the Poisson ratio of the shotcrete
$R$ is the tunnel radius
$t_{shot}$ is the lining thickness.

### *2.1 The Change in the Mechanical Characteristics of Shotcrete in Time*

The shotcrete elastic modulus and uniaxial compressive strength can be expressed, during hardening, as a first approximation, through the following two negative exponential equations (Pottler, 1990; Weber, 1979):

$$E_{shot,t} = E_{shot,0} \cdot (1 - e^{-\alpha \cdot t}) \tag{2}$$

$$\sigma_{c,t} = \sigma_{c,0} \cdot (1 - e^{-\beta \cdot t}) \tag{3}$$

where:

$E_{shot,t}$ is the shotcrete elastic modulus at the time $t$;
$E_{shot,0}$ is the value of the asymptotic shotcrete elastic modulus, for $t = \infty$;
$\sigma_{c,t}$ is the shotcrete uniaxial compressive strength at the time $t$;
$\sigma_{c,0}$ is the value of the asymptotic shotcrete uniaxial compressive strength, for $t = \infty$;
$\alpha$ and $\beta$ are time constants ($t^{-1}$) (many authors propose a linear relationship between $E$ and $\sigma_c$, i.e. $\alpha = \beta$).

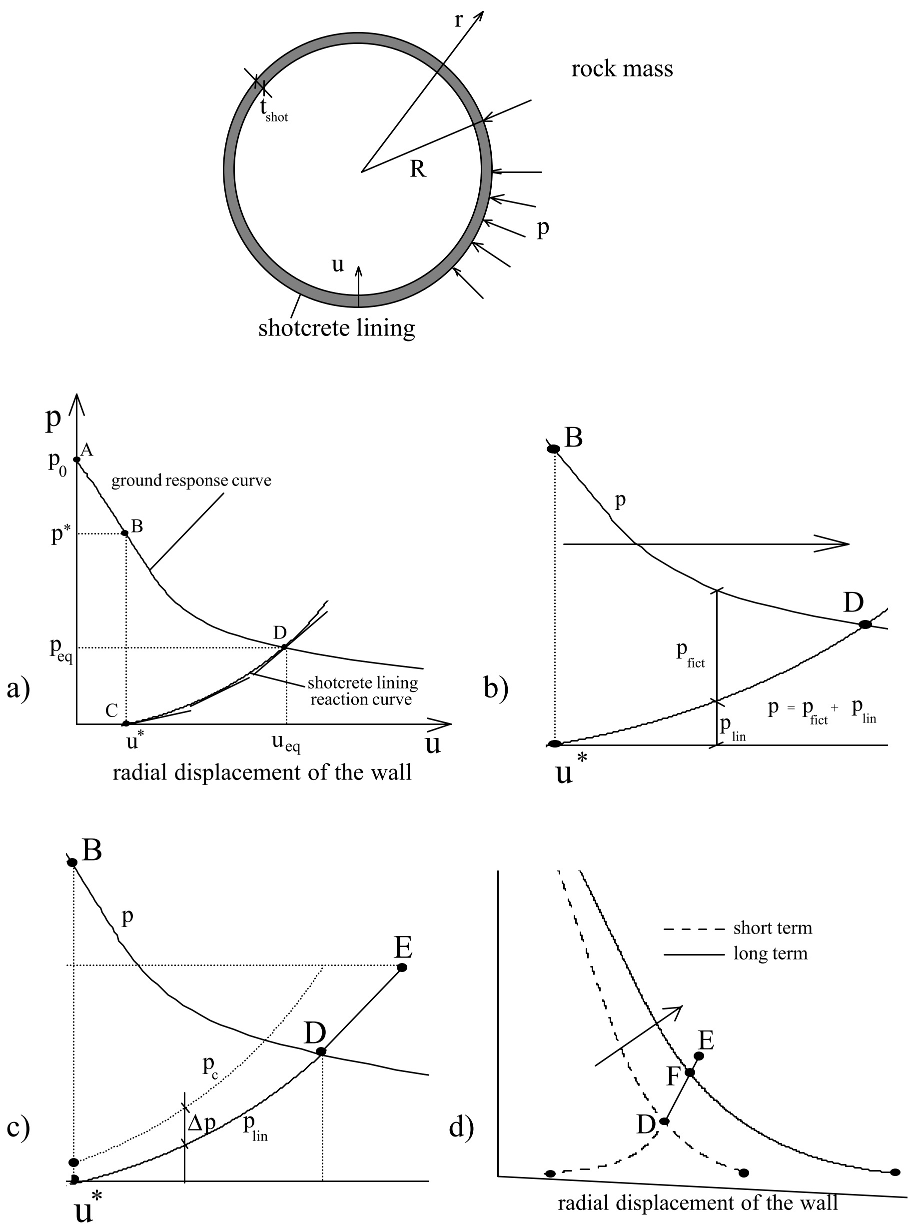


**Fig. 1.** **a**: Ground reaction curve and reaction curve of the shotcrete lining; **b**: details of **a**; **c**: curve of the limit pressures that can be applied to the lining extrados, obtained from the reaction curve of the shotcrete lining; **d**: analysis of the ground-lining interaction in the short and long terms. $p$ is the pressure on the tunnel boundary; $u$ is the radial displacement of the tunnel boundary; $p_0$ is the in situ hydrostatic stress; $p^*$ is the fictitious internal pressure near the excavation face; $u^*$ is the radial wall displacement before the placing of the shotcrete (near the face); $p_{eq}$ and $u_{eq}$ are the pressure and radial displacement at the final equilibrium; $p_{fict}$ is the fictitious internal pressure on the tunnel boundary, that takes the static contribution of the excavation face into account; $p_{lin}$ is the effective pressure produced by the shotcrete lining on the tunnel boundary; A before the tunnel excavation; B and C shotcrete lining installation; D final equilibrium; E represents the shotcrete lining failure; F is the long term equilibrium

**Table 1.** Typical values of uniaxial compressive strength $\sigma_{c,t}$ (MPa) for three types of shotcrete (Hoek and Brown, 1980)

| Type of shotcrete | Hardening time | | | |
|---|---|---|---|---|
| | 1–3 h | 3–8 h | 1 day | 28 days |
| Shotcrete without accelerants | 0 | 0.2 | 5.2 | 41.4 |
| Shotcrete with accelerants (3%) | 0.69 | 5.2 | 10.3 | 34.5 |
| Shotcrete with regulated hardening | 8.27 | 10.3 | 13.8 | 34.5 |

In order to describe the change in time of the shotcrete mechanical parameters, it is possible to refer to the time constant $T_{shot}$: $T_{shot,E} = 1/\alpha$; $T_{shot,\sigma} = 1/\beta$. Studies on the subject (Kuwajima, 1991) have led to a correlation between $E_{shot,t}$ and $\sigma_{c,t}$ in time (this latter parameter is easier to evaluate because of the reduced dimensions of the samples that are necessary and because of the simplicity of the tests). Some typical values of $\sigma_{c,t}$ in time are given in Table 1 for different types of shotcrete.

### *2.2 The Fictitious Internal Pressure*

The reduction of the pressure $p$ on the ground reaction curve before installation of the lining ($p > p^*$, A-B in Fig. 1a) is due to the stress release that occurs in the core of the rock ahead of the excavation face. The shotcrete lining is installed in B close to the excavation face: the rock ahead of the face produces a stabilising effect on the tunnel which can be considered, in a two-dimensional study, through a fictitious internal pressure $p_{fict}$ that acts on the tunnel boundary, and which progressively decreases with the advancement of the face (Fig. 1b). Apart from the fictitious internal pressure, the pressure due to the shotcrete lining $p_{lin}$ also acts on the boundary of the already excavated tunnel. This pressure increases as the face advances and the lining is loaded when the tunnel tends to close (that is, with an increase in $u$).

Initially, in B, the pressure $p$ of the ground reaction curve is entirely supplied by the fictitious pressure produced by the face, while in D, at the final equilibrium point (far from the excavation face), pressure $p$ is caused by the action of the lining. The reduction of the fictitious internal pressure with the distance $x$ from the face, can be estimated, as a first approximation, from Eq. (4) (Panet and Guenot, 1982):

$$p_{fict} = a \cdot p_0 \cdot \frac{b}{x + b}, \tag{4}$$

where: $a = 0{,}72$; $b = 0{,}845 \cdot R$.

Equation 4 was obtained considering a deep circular tunnel in an elastic medium and parameters $a$ and $b$ can therefore vary according to the assumptions introduced (Panet, 1995).

If a cross section of the shotcrete lining is considered, the distance $x$ of this sec-

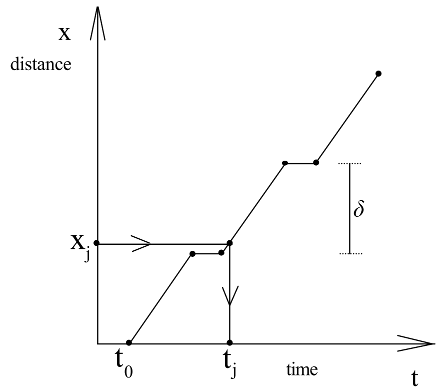


**Fig. 2.** Graph of the excavation face advancements $x$, following the installation of the shotcrete lining. $t_0$ is the dead time due to the installation of the lining and waiting for it to harden; $\delta$ is the advancement step

tion from the face depends (for example, in TBM tunnelling) on the mean advancement velocity $v_m$ of the excavation face and on dead time $t_0$ that is foreseen for the lining installation and hardening (Fig. 2). In drill and blast tunnelling, $v_m$ reaches very high values and $t_0$ increases due to the time needed to prepare blasting. It is possible to consider a time characteristic $T_a = (2 \cdot R)/v_a$, for the advancement rate, where $v_a$ is the average advancement rate: $v_a = v_m/(1 + t_0 \cdot v_m/\delta)$ (Fig. 2). $T_a$ is the time that is necessary to advance for a length equal to the tunnel diameter.

### *2.3 The State of Stress Induced in Shotcrete*

The maximum principal stresses $\sigma_{\max}$ inside the shotcrete lining increase with loading due to the development of the radial displacement $u$ of the tunnel profile. Each infinitesimal increase $\delta u$ of the radial wall displacement produces an infinitesimal increase $d\sigma_{\max}$ of the maximum principal stress (the minimum principal stress is nil at the intrados). As the elastic modulus of the shotcrete varies during loading, the maximum principal stress $\sigma_{\max}$ is given by (Oreste, 1995):

$$\sigma_{\max} = \frac{2 \cdot R}{(1 + v_{shot}) \cdot [(R - t_{shot})^2 + (1 - 2 \cdot v_{shot}) \cdot R^2]} \cdot \int_{u^*}^{u_{eq}} E_{shot}(u) \cdot du. \tag{5}$$

The safety factor of the lining ($F_{s,lin}$) is therefore given by the ratio between $\sigma_c$ (Eq. (3)) and $\sigma_{\max}$ (Eq. (5)):

$$F_{s,lin} = \frac{\sigma_c}{\sigma_{\max}}. \tag{6}$$

At each instant, and therefore for each value of the radial displacement $u$, it is possible to define a limit pressure $p_c = p_{lin} + \Delta p$ for the lining that induces a stress equal to the available uniaxial compressive strength $\sigma_c$. $\Delta p$ is the increase in pressure that should be applied to the lining extrados, for a given value of $u$, to cause failure. $\Delta p$ (Fig. 1c) is obtained, starting from the equations that describe the stress and strain state in the lining, according to the classical theory of elasticity (Oreste, 1995; Hoek and Brown, 1980).

The value of $\Delta p$ at the final equilibrium point D is of particular interest. It permits the drawning of the D-E line in Fig. 1d, which represents the reaction of the lining with reference to the shotcrete elastic modulus in the long term. The D-E line allows one to study the behaviour of the lining after reaching equilibrium in D: it is possible to imagine a reduction of the mechanical characteristics of the rock mass in the long term (due to the chemical-physical interaction between the concrete and the air and water around the tunnel). For this reason the ground reaction curve increases. In some cases, it is necessary to plot two ground response curves: one with reference to the short term, and the other one to the long term with lower geomechanical parameters for the rock mass. In this case there will be two equilibrium points (Fig. 1d): point D, the initial equilibrium on the short term ground reaction curve; and point F, the final equilibrium of the D-E line, on the long term ground reaction curve.

## 3. The Proposed Calculation Procedure

The reaction curve of the lining and its safety factor vs. time with the distance from the excavation face can be obtained according to the following procedure and with reference to Fig. 3:

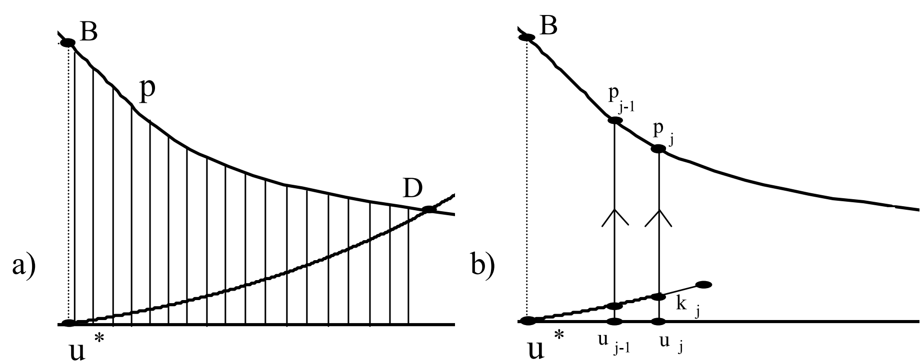


**Fig. 3.** Numerical integration of the reaction curve of the shotcrete lining (**a**) and a calculation step (**b**)

a) Evaluation of the fictitious internal pressure $p^*$ and the associated displacement $u^*$ on installation of the shotcrete lining (point B on the ground reaction curve in Fig. 1a);
b) Choice of the $\Delta u$ interval for the numerical integration of the reaction curve of the lining. This value can also be assumed to vary for each calculation step, according to the calculation technique used for the ground reaction curve.

### 3.1 Calculation Steps

1. Determination of the shotcrete elastic modulus at the previous calculation step $(j-1)$ through Eq. (2), and the lining stiffness through Eq. (1):

$$E_{shot,t_{j-1}} = E_{shot,0} \cdot (1 - e^{-\alpha \cdot t_{j-1}}) \tag{7}$$

$$k_{j-1} = \frac{[R^2 - (R - t_{shot})^2]}{(1 + \nu_{shot}) \cdot [(1 - 2 \cdot \nu_{shot}) \cdot R^2 + (R - t_{shot})^2]} \cdot \frac{1}{R} \cdot E_{shot,t_{j-1}}. \tag{8}$$

2. Determination of the pressure acting on the lining extrados, the values of stiffness $k$ being known at the previous $j-1$ calculation step:

$$p_{lin,j} = \sum_{n=1}^{j} (k_{n-1} \cdot \Delta u_n). \tag{9}$$

3. Evaluation of $p_j$ with $u_j$ being known (ground reaction curve):

$$u_j = u_{j-1} + \Delta u_j \tag{10}$$

$$p_j = g(u_j). \tag{11}$$

4. Calculation of $p_{fict,j}$ as the difference between the pressures $p_j$ and $p_{lin,j}$ (Fig. 1b):

$$p_{fict,j} = p_j - p_{lin,j}. \tag{12}$$

5. Determination of the distance $x_j$ of calculation step $j$, with Eq. (13), obtained from Eq. (4):

$$x_j = b \cdot \left( \frac{a \cdot p_0}{p_{fict,j}} - 1 \right). \tag{13}$$

6. Evaluation of the time $t_j$ associated to the calculation step $j$ (Fig. 2);
7. Calculation of the maximum principal stress in the shotcrete $\sigma_{\max}$ from Eq. (5), rewritten in incremental terms as follows:

$$\sigma_{\max,j} = \frac{2 \cdot R}{(1 + \nu_{shot}) \cdot [(R - t_{shot})^2 + (1 - 2 \cdot \nu_{shot}) \cdot R^2]} \cdot \sum_{n=1}^{j} (E_{shot,t_{n-1}} \cdot \Delta u_n), \tag{14}$$

while the uniaxial compressive strength $\sigma_{c,j}$ is given by Eq. (3), with $t_j$ being known:

$$\sigma_{c,j} = \sigma_{c,0} \cdot (1 - e^{-\beta \cdot t_j}). \tag{15}$$

8. Determination of the safety factor, as a ratio between the uniaxial compressive strength and the maximum principal stress in shotcrete:

$$F_{s,lin,j} = \frac{\sigma_{c,j}}{\sigma_{\max,j}}. \tag{16}$$

9. Evaluation of the limit pressure $p_{c,j}$, having obtained $\sigma_{c,j}$ and $\sigma_{\max,j}$ from step 7 and $p_{lin,j}$ from step 2 (see Appendix 1):

$$\Delta p_j = \frac{1}{2} \cdot \left[1 - \frac{(R - t_{shot})^2}{R^2}\right] \cdot (\sigma_{c,j} - \sigma_{\max,j}), \tag{17}$$

$$p_{c,j} = p_{lin,j} + \Delta p_j. \tag{18}$$

Calculation steps 1–9 are repeated, starting from point C, until point D is reached. To start the procedure ($j = 0$) it is necessary to place $t = t_0$ and $u_0 = u^*$.

The calculation procedure, written in QuickBasic, is given in Appendix 2.

### *3.2 Results to be Obtained*

The proposed calculation method permits one to obtain:

- the effective reaction curve of the shotcrete lining;
- the change of the safety factor in time or with the distance from the excavation face. In this way it is possible to assess the influence of transient state on the final conditions of the shotcrete lining.

Based on the variation of the safety factor versus time, the minimum value can be evaluated, in order to choose the lining thickness that can be considered adequate. As the change of the safety factor of the lining is influenced by the hardening characteristics of the shotcrete and by the loading history, it is necessary to use the evolution in time of the shotcrete elastic modulus, the uniaxial compressive strength and the face advancement rate, as input parameters for the calculation.

The method also allows one to obtain the tunnel convergences versus the distance from the excavation face or versus time, considering the role of the shotcrete lining in controlling the tunnel closure. When the mechanical characteristics of the shotcrete versus time and the face advancement rate are known, it is possible to perform a back-analysis based on in situ measurements. This procedure allows one to derive the original geomechanical parameters of the rock mass, thus adjusting the initial predictions obtained from the preliminary assessment.

## 4. Calculation Examples

Some calculation examples are given in the following sections by using the proposed method.

### *4.1 Example 1. The Influence of the Face Advancement Rate*

The face advancement rate influences the loads transferred to the lining during hardening, when the mechanical parameters of shotcrete rapidly improve. Low advancement rates permit the shotcrete to develop higher mechanical parameters and the lining to appear stiffer (with a steeper reaction curve).

The first example refers to a 2 m radius tunnel, excavated in fair quality rock

**Table 2.** Geomechanical parameters for the rock mass for example 1

| Parameter | Value |
|---|---|
| Elastic modulus E (MPa) | 3160 |
| Poisson's ratio $\nu$ | 0.30 |
| Peak friction angle $\varphi_p$ (°) | 20 |
| Peak cohesion $c_p$ (MPa) | 0.15 |
| Residual friction angle $\varphi_r$ (°) | 16 |
| Residual cohesion $c_r$ (MPa) | 0.12 |
| Dilatancy angle $\psi$ (°) | 16 |

**Table 3.** Shotcrete mechanical parameters for example 1

| Parameter | Value |
|---|---|
| Final elastic modulus $E_{shot,0}$ (MPa) | 12,000 |
| Time constant $\alpha$ ($h^{-1}$) | 0.050 |
| Time constant $T_{shot,E}$ (h) | 20 |
| Poisson's ratio $\nu_{shot}$ | 0.15 |
| Final un. compr. strength $\sigma_{c,0}$ (MPa) | 27 |
| Time constant $\beta$ ($h^{-1}$) | 0.025 |
| Time constant $T_{shot,\sigma}$ (h) | 40 |

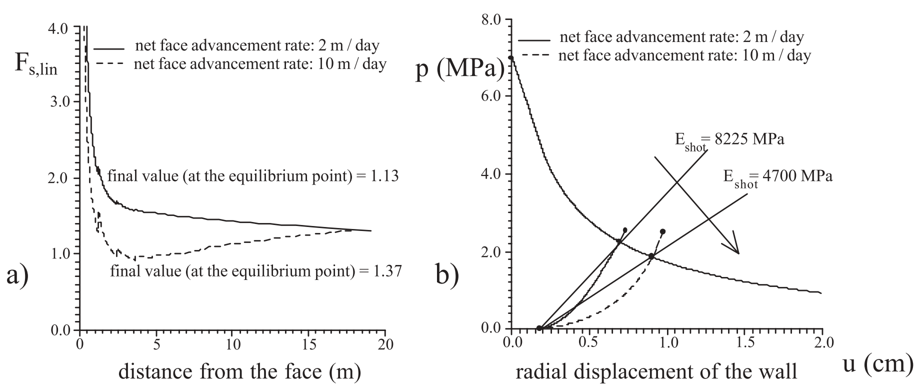


**Fig. 4.** Example 1. **a**: Trend of the safety factor in the lining as a function of the distance from the excavation face, for two face advancement rates; **b**: reaction curve of the shotcrete lining as a function of the face advancement rate and the apparent elastic modulus of the shotcrete

mass. The geomechanical parameters are given in Table 2. The in situ hydrostatic stress $p_0$ is 7 MPa and it is assumed that the fictitious internal pressure $p^*$ is equal to $0.72 \cdot p_0$ near the excavation face, where the lining is installed. The lining has a thickness of 20 cm. The mechanical parameters of shotcrete are given in Table 3.

Two different face advancement rates were assumed (2 m/day and 10 m/day), with $t_0$ and $\delta$ equal to 1 h and 1.2 m respectively. The time characteristic $T_a$ is equal to 35.3 h for $v_m = 2$ m/day and 9.7 h for $v_m = 10$ m/day. The change of the lining safety factor with the distance from the excavation face is shown in Fig. 4a,

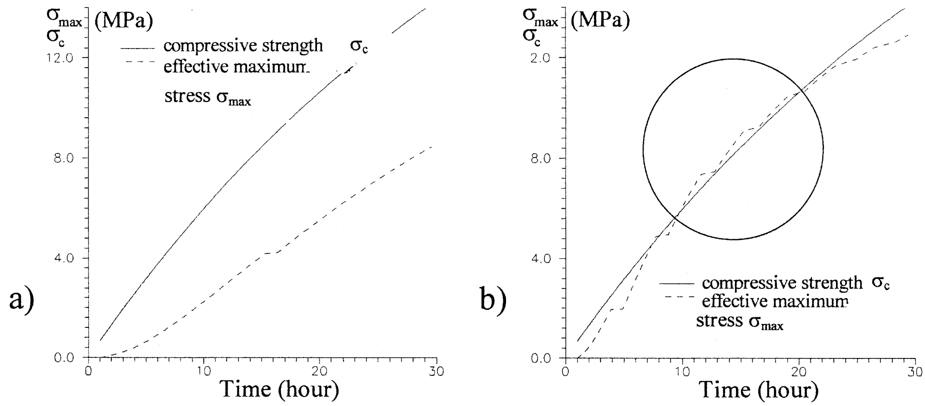


**Fig. 5.** Example 1. Trend of the uniaxial compressive strength of the shotcrete and of the maximum principal stress induced in the lining, for face advancement rates of 2 m/day (**a**) and 10 m/day (**b**).

for the two different advancement rates. It can be seen how the change of the safety factor attains a minimum value close to the face, for high advancement rates, at a distance of about 3 m from the excavation face. Although the minimum value of the safety factor is lower than 1, the calculation has continued in elastic conditions because it is usually more important to identify low values for the safety factor, than studying the behaviour of the shotcrete lining during the yielding phase.

This example shows how the transient state in the shotcrete lining can be very important, with a remarkable decrease of the safety factor, when referring to the equilibrium conditions (final value), far from the excavation face (point D in Fig. 1a). It is also possible to note how the higher advancement rate produces higher safety factors in the lining far from the excavation face, under equilibrium conditions.

The reaction curves of the lining are shown in Fig. 4b for the two cases. Though the geometrical and mechanical parameters of the shotcrete lining were not varied, the reaction curve varies according to the different advancement rates: the apparent stiffness of the lining reduces considerably with an increase of the rate of face advancement. In conclusion, the equilibrium point moves along the ground reaction curve, toward lower pressures, with an increase in the advancement rate, thus explaining the increase of the final safety factor, as shown in Fig. 4a.

The trends of the uniaxial compressive strength of shotcrete and of the maximum principal stress in the lining versus time are shown in Fig. 5. It can be seen how, for the higher advancement rate (10 m/day), the maximum principal stress reaches the uniaxial compressive strength just a few hours after lining installation. The strength versus time increases, thus reading values which are well away from the maximum principal stress, leading to a progressive increase of the safety factor.

It is noted that, when the time constant $T_a$ is lower than $T_{shot,E}$ or $T_{shot,\sigma}$, transient conditions for the shotcrete lining with smaller safety factors are observed. On the contrary, when $T_a$ is similar (or slightly lower) or greater than $T_{shot,E}$ or

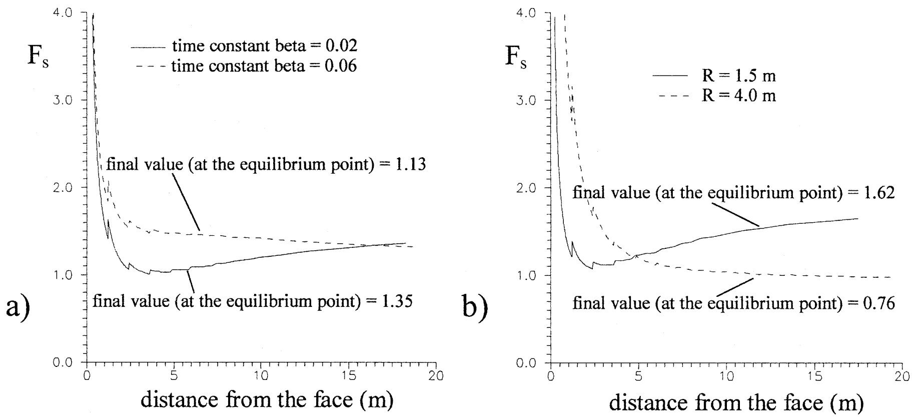


**Fig. 6.** Trend of the safety factor of the lining as a function of the distance from the excavation face, **a** for two different shotcrete hardening time constants (example 2); **b** for two different tunnel radii (example 3)

$T_{shot,\sigma}$, the minimum value for the safety factor of the shotcrete lining is attained, in the long term, far from the excavation face.

### *4.2 Example 2. The Influence of the Shotcrete Mechanical Parameters*

The tunnel described in example 1 was considered in this second example, in order to analyse the influence of the time constants of shotcrete on the lining behaviour. The advancement rate of the face was kept constant (6 m/day, $T_a = 14$ h), and two different values of the time constant $\beta$ (Eq. (3)) were assumed to describe the trend in time of the shotcrete strength. With an increase of the hardening rate ($\beta$ greater), the static conditions of the lining improve (Fig. 6a): even though the final safety factor far from the excavation face is slightly reduced, the minimum value of the safety factor close to the excavation face increases under transient conditions.

From this example it also results, with $T_a \ll T_{shot,\sigma}$, that transient conditions of the shotcrete lining are present with smaller safety factors, while with $T_a \cong T_{shot,\sigma}$, the minimum value of the safety factor occurs far from the face.

### *4.3 Example 3. The Influence of the Tunnel Radius*

In this example, the behaviour of the shotcrete lining is analysed for two tunnels with different radii (1.5 m and 4 m), in the same rock mass as described in example 1 (Table 2). The advancement rate of the face is 6 m/day ($T_a = 10.5$ h for $R = 1.5$ m and $T_a = 28$ h for $R = 4$ m); the shotcrete parameters are the same as reported in Table 3. The change of the safety factor is shown in Fig. 6b versus the distance from the excavation face. It is noted that the tunnel radius affects the stress-strain state of the lining: for tunnels with small radii, the transient effects are more pronounced than for tunnels with larger radii, for the same advancement rate. The static conditions of the lining result to be more critical close to the face, for the

**Table 4.** Operational parameters of the tunnel advancement for example 4

| Parameter | Value |
|---|---|
| $t_0$ after shotcreting (h) | 1.0 |
| Average advancement rate of the face (m/day) | 4.0 |
| Length of the advancement step (m) | 1.2 |

tunnel with 1.5 m radius; however, with the 4 m diameter tunnel these conditions are more critical far from the face. The different behaviour of the lining in the two cases is due to the influence of the tunnel radius on the change of the fictitious internal pressure with the distance from the excavation face (Eq. (4), parameter b).

### *4.4 Example 4. The Influence of the Rock Mass Parameters*

As already mentioned, the reaction curve of the shotcrete lining depends on the ground reaction curve of the tunnel and therefore on the geomechanical parameters of the rock mass. In this example, the behaviour of the lining is analysed for two tunnels of the same size (2.5 m radius), in rock masses with different qualities, identified using the RMR (Rock Mass Rating) classification: RMR = 30 and RMR = 40. The elastic modulus and the strength parameters of the Hoek and Brown criterion (Hoek and Brown, 1980) were obtained by the usual empirical relations reported (Bieniawski, 1989). The in situ hydrostatic stress at the depth of the tunnel was assumed as 5 MPa. The lining thickness was 20 cm. The shotcrete mechanical parameters are the same as given in Table 3. The advancement parameters are reported in Table 4.

The tunnel ground reaction curves for the two rock mass conditions above and the corresponding reaction curves for the shotcrete lining are shown in Fig. 7. It is noted that the shotcrete lining reaction curve is dependent upon the rock mass quality.

Due to the shotcrete stiffness increase versus time and the distance from the excavation face, the rock mass with poor conditions develops larger tunnel wall displacements than are not acceptable with traditional supports. The envelope of the equilibrium points is shown in the same figure with a variation of the rock mass quality (RMR = 20 to 50).

As a result of the calculation the tunnel convergence behind the excavation face is also computed versus the distance from the face.

## 5. Case Studies

### *5.1 Experimental Tunnel in Limestone in the Weardale Valley (GB)*

The Kielder Experimental Tunnel (Ward et al., 1976; Ward et al., 1983; Freeman, 1978) was excavated in 1974 as part of the Kielder Water Project, in the Weardale Valley (England), to study the most suitable supports for the excavation of the

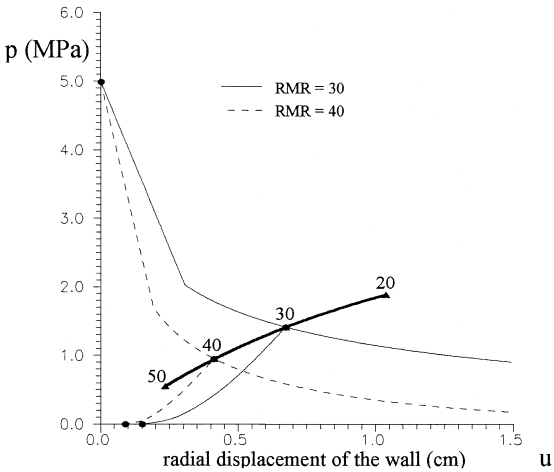


**Fig. 7.** Example 4. Ground reaction curves and shotcrete lining reaction curves for two different rock mass (RMR = 30 and RMR = 40). The envelope of the equilibrium points with the variation of the rock mass quality between RMR = 20 and RMR = 50 is also shown

**Table 5.** Mechanical parameters of the shotcrete used for the Kielder Experimental Tunnel

| Parameter | Value |
|---|---|
| Final elastic modulus $E_{shot,0}$ (MPa) | 28,000 |
| Time constant $\alpha$ ($h^{-1}$) | 0.0187 |
| Time constant $T_{shot,E}$ (h) | 53.5 |
| Poisson's ratio $\nu_{shot}$ | 0.15 |
| Final uniaxial compressive strength $\sigma_{c,0}$ (MPa) | 35 |
| Time constant $\beta$ ($h^{-1}$) | 0.0187 |
| Time constant $T_{shot,\sigma}$ (h) | 53.5 |

main tunnel (32 km in length). The Experimental Tunnel (circular, with a 1.65 m radius, an overburden of 100 m and an in situ hydrostatic stress of 2.56 MPa) was sub-divided into eight sections, each 11 m long, and was excavated in the Four Fathom limestone formation. One of these sections was left unsupported, while the other seven were used to try out different types of supports, including a 14 cm thick shotcrete lining, which is here analysed. The work carried out in the Kielder Experimental Tunnel was well documented and is therefore useful for comparison purposes.

The limestones of the Four Fathom formation are banded and of a dark grey colour, and contain lime and clay materials. These rapidly deteriorate on contact with air and water and form a soil type mass. On the basis of data available in literature, Hoek and Brown (1980) evaluated the Rock Mass Rating (RMR) as 35. The RMR value led to the estimation of the mechanical parameters of the rock mass (Bieniawski, 1984). The shotcrete mechanical parameters used in the calculation are given in Table 5. The face advancement parameters are summarized in Table 6.

**Table 6.** Tunnel advancement parameters in the Kielder Experimental Tunnel

| Parameter | Value |
|---|---|
| $T_0$ after shotcreting (h) | 1.4 |
| Average advancement rate of the face (m/day) | 12.0 |
| Length of the advancement step (m) | 1.5 |
| Time constant $T_a$ (h) | 7.5 |

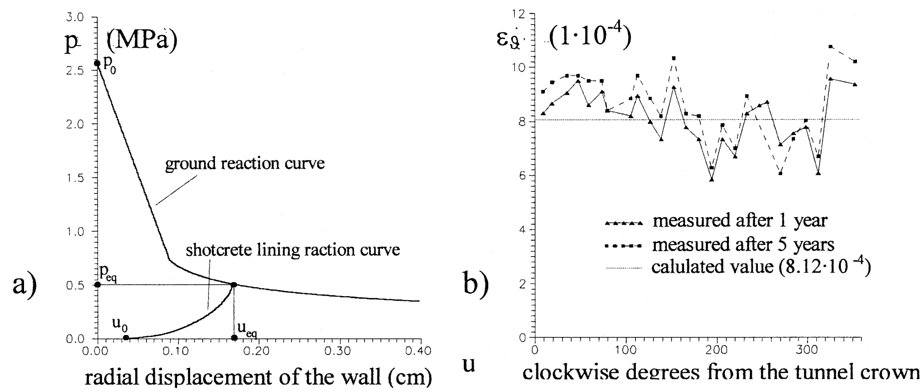


**Fig. 8.** **a**: Ground reaction curve of the Kielder Experimental Tunnel and reaction curve of the shotcrete lining; **b**: circumferential strains in the shotcrete lining, measured one and five years later and calculated using the proposed procedure

The proposed calculation method gives a reaction curve for the shotcrete lining, with the final equilibrium point shown in Fig. 8a. The pressure $p_{eq}$, which represents the load acting on the lining in the final equilibrium conditions, and the displacement $u_{eq}$, which is the final displacement of the tunnel wall, refer to this point. The difference $u_{eq} - u_0$ is the radial displacement of the shotcrete lining. The circumferential strain induced in shotcrete $\varepsilon_\vartheta$ is therefore given by the radial displacement of the lining divided by the radius of the tunnel. The monitored change of the circumferential strains in the shotcrete lining is given in Fig. 8b. The effective circumferential strains is shown to range between 6 and $10 \cdot 10^{-4}$ in the long term, with mean values of about $8 \cdot 10^{-4}$. The value of the circumferential strain in shotcrete, obtained with the proposed method is given in the same figure and is shown to agree in a satisfactory way with the average in situ measured values.

### *5.2 Large Diameter Railway Tunnel in Central Italy*

A large diameter railway tunnel (equivalent diameter 13.7 m) is presently being excavated through the Apennine in Central Italy at a depth of 70 m. The rock mass is a clay shale highly tectonized. The full face excavation method is used with 100 sub-horizontal fibre-glass bars launched ahead of the face to a distance of 22 m. The shotcrete lining is 30 cm thick, with IPN 200 steel ribs installed with a

**Table 7.** Rock mass parameters

| Parameter | Value |
|---|---|
| Elastic modulus $E$ (Mpa) | 450 |
| Poisson's ratio $\nu$ | 0.33 |
| Peak friction angle $\varphi_p$ (°) | 24 |
| Peak cohesion $c_p$ (MPa) | 0.08 |
| Residual friction angle $\varphi_r$ (°) | 19 |
| Residual cohesion $c_r$ (MPa) | 0.06 |
| Dilatancy angle $\psi$ (°) | 0 |

**Table 8.** Mechanical parameters of the "equivalent support"

| Parameter | Value |
|---|---|
| Final elastic modulus $E_{eq,0}$ (MPa) | 9,430 |
| Time constant $\alpha$ ($h^{-1}$) | 0.1 |
| Poisson's ratio $\nu_{shot}$ | 0.15 |
| Time constant $T_{shot,E}$ (h) | 10 |

1.2 m spacing. The face reinforcement is carried out at 10 m interval. The average advancement rate is 3 m/day.

The tunnel was analysed using the procedure above. Also carried out were numerical calculations with an axisymmetric finite difference model using the FLAC code. The ground reinforcement was simulated by using the "improved cohesion criterion" (Grasso et al., 1989; Grasso et al., 1991; Grasso and Mahtab, 1992), with a fictitious increase of the cohesive strength for the rock mass. A continuous updating of the support stiffness with the distance from the face was also introduced.

The rock mass parameters assumed in the calculations are given in Table 7; the mechanical parameters of the 2D "equivalent support" are shown in Table 8. The results obtained for the radial displacements versus the distance from the tunnel face, by using both the proposed approach and the Flac code, are shown in Fig. 9a, where a comparison with the monitored values during excavation is also illustrated. In order to give a further mean for evaulating the effectiveness of the proposed approach, the computed pressure on the lining is compared in Fig. 9b with the values obtained with the Flac code. In all cases, the results appear to be satisfactory.

## 6. The Evaluation of the Lining Thickness

The shotcrete lining thickness can be defined by the calculation procedure proposed above by paying attention to the minimum value of the lining safety factor during tunnel excavation. Once the geometry of the tunnel, the mechanical parameters for shotcrete, and the time-table of the tunnel advancement are known, it is possible to calculate the minimum safety factors versus the RMR in-

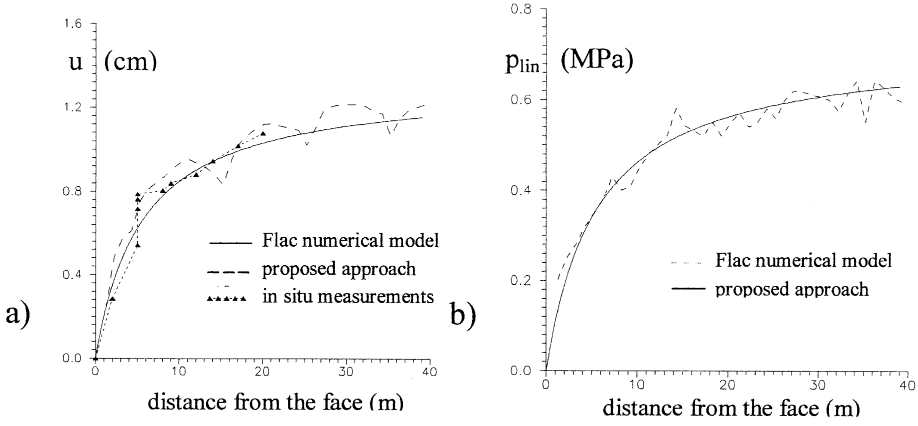


**Fig. 9.** **a**: Radial displacements of the supports measured during the tunnel construction and obtained from the calculation (proposed approach and axisimmetric numerical model); **b**: radial pressures at the support extrados obtained from the used calculation methods

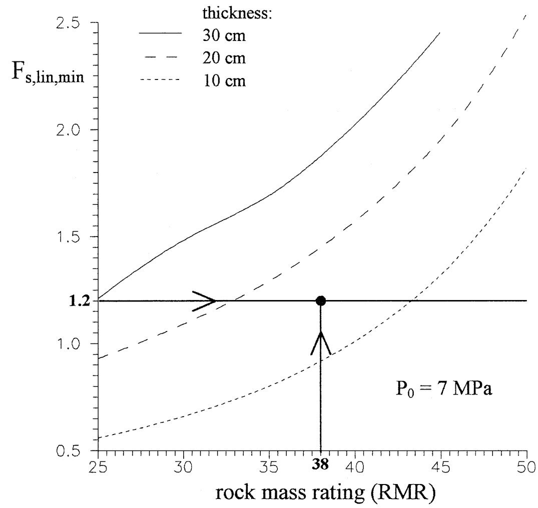


**Fig. 10.** Minimum safety factor in the shotcrete lining with a variation of the RMR for three different thicknesses of the shotcrete lining. The choice of the lining thickness is made by imposing a minimum admissible safety factor (1.2)

dex and the thickness of the lining, for each value of the in situ stress $p_0$ along the tunnel alignment. This approach can also be effectively used during construction as a final evaluation of the design, when the rock mass quality and the tunnel advancement rate of the face are known.

For example, take the case of a 2 m radius tunnel, where the shotcrete parameters are as shown in Table 3, and it is intended to proceed with a time-table as given in Table 4. The results obtained are summarised in the diagrams illustrated in Fig. 10, where the in situ hydrostatic stress is assumed to be $p_0 = 7$ MPa. The lining thickness can be chosen by considering the variation of the RMR index along the tunnel alignment, once a minimum admissible value for the safety factor is defined. For an admissible safety factor of 1.2 and RMR equal to 38, the thickness of the lining is 15 cm.

## 7. Conclusions

The understanding of the behaviour of a shotcrete lining has always been difficult because this support presents a stiffness that varies in time (increasing) due to hardening effects. The widely used convergence-confinement method is only able to supply the reaction curve of the shotcrete lining through an average stiffness. The method, however, is not able to give the state of stress and the safety factors of the lining in transient conditions, that is, in the short period after installation, or in the long term.

A simple and new procedure for the determination of the reaction curve of the shotcrete lining has been presented in this paper which allows one to consider:

– the increase in time of the elastic modulus and uniaxial compressive strength of shotcrete;
– the advancement rate of the excavation face and the time-table during tunnel excavation.

By associating this curve to the ground reaction curve, the final equilibrium point is evaluated and the final pressure acting on the shotcrete lining is computed.

The proposed calculation procedure also allows one to obtain the change, in time and with the distance from the excavation face, of the state of stress in the shotcrete lining and therefore the change of the safety factor. In this way, it is possible to assess any short term critical conditions, that is, just after installation. The minimum safety factor of the lining therefore becomes the fundamental parameter for the choice of the lining thickness.

Some calculation examples have shown the applications of the proposed procedure, by giving relevance to the role of typical parameters on the lining response, both reducing the safety conditions in the short term and modifying the final static equilibrium between the tunnel and the support. Because of the great number of parameters involved, each tunnel results to be a case of specific concern.

One example has illustrated how it is possible to proceed towards a correct choice of the lining thickness. A comparison for a specific case with an axisymmetric numerical model has permitted the calibration of the proposed calculation approach.

## Acknowledgements

The author would like to thank Prof. S. Pelizza and Dr. S. Xu for the help given during the preparation of this paper. The help of the Editor G. Barla is also acknowledged.

## Appendix 1

Equation (17) allows one to obtain the increment ($\Delta p$) of radial stress $p$ at the lining extrados due to the increment of the circumferential stress ($\Delta\sigma_{\vartheta,in}$):

$$\Delta p = \frac{1}{2} \cdot \left[ 1 - \frac{(R - t_{shot})^2}{R^2} \right] \cdot \Delta \sigma_{\vartheta, in}. \tag{A1.1}$$

This equation is derived analysing the structural behaviour of the shotcrete lining on the basis of the general equation of the radial displacements (Eq. (A1.2)):

$$u_r = A \cdot r + \frac{B}{r}, \tag{A1.2}$$

where: $u_r$ is the radial displacement in the lining at the distance $r$ and A and B are integration constants.

From Eq. (A1.2) it is possible to obtain the radial and circumferential strains (Eqs. (A1.3) and (A1.4)) (positive if compression) and from the constitutive law of the elastic material (Eqs. (A1.5) and (A1.6)) the stresses in the lining with the distance $r$ (Eqs. (A1.7) and (A1.8)):

$$\varepsilon_r = \frac{du_r}{dr} = \left[ A - \frac{B}{r^2} \right], \tag{A1.3}$$

$$\varepsilon_\vartheta = \frac{u_r}{r} = \left[ A + \frac{B}{r^2} \right], \tag{A1.4}$$

$$\varepsilon = \frac{1}{E_{shot}} \cdot [(1 - \nu_{shot}^2) \cdot \sigma_r - (\nu_{shot} + \nu_{shot}^2) \cdot \sigma_\vartheta], \tag{A1.5}$$

$$\varepsilon_\vartheta = \frac{1}{E_{shot}} \cdot [(\nu_{shot} - \nu_{shot}^2) \cdot \sigma_\vartheta - (\nu_{shot} + \nu_{shot}^2) \cdot \sigma_r], \tag{A1.6}$$

$$\sigma_\vartheta = C \cdot \varepsilon_\vartheta + D \cdot \varepsilon_r, \tag{A1.7}$$

$$\sigma_r = C \cdot \varepsilon_r + D \cdot \varepsilon_\vartheta, \tag{A1.8}$$

where: $C = \frac{(1 - \nu_{shot})}{(1 - 2 \cdot \nu_{shot}) \cdot (\nu_{shot} + 1)} \cdot E_{shot}$; $D = \frac{\nu_{shot}}{(1 - 2 \cdot \nu_{shot}) \cdot (\nu_{shot} + 1)} \cdot E_{shot}$, and $E_{shot}$ and $\nu_{shot}$ are the shotcrete elastic modulus and Poisson's ratio, respectively.

The following two conditions exist at the lining borders (Fig. A1.1):

1. $r = R$: $u_R = (u - u^*)$:
   the displacement of the lining extrados equal to the displacement of the tunnel wall after the lining installation;
2. $r = (R - t_{shot})$: $\sigma_{rO} = 0$:
   the internal pressure at the intrados is nil.

One can therefore obtain Eqs. (A1.9) and (A1.10) from Eqs. (A1.2) and (A1.8), which give the A and B constants (Eq. (A1.11)).

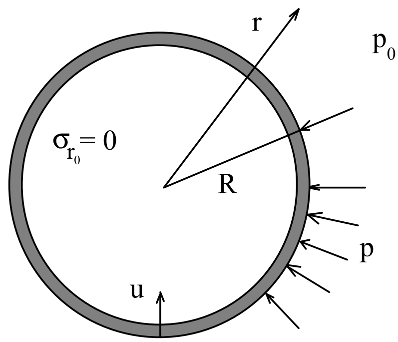


**Fig. A1.1.** Geometrical loading scheme of a shotcrete lining. $r$: radial coordinate; $R$: tunnel radius; $p$: radial pressure acting on the lining extrados; $\sigma_{ro}$: radial stress at the intrados

$$(u - u^*) = A \cdot R + \frac{B}{R} \Rightarrow B = R \cdot [(u - u^*) - A \cdot R], \tag{A1.9}$$

$$0 = C \cdot \left[ A - \frac{R \cdot [(u - u^*) - A \cdot R]}{(R - t_{shot})^2} \right] + D \cdot \left[ A + \frac{R \cdot [(u - u^*) - A \cdot R]}{(R - t_{shot})^2} \right], \tag{A1.10}$$

$$A = \left[ \frac{(1 - 2 \cdot \nu_{con}) \cdot R}{(R - t_{shot})^2 + (1 - 2 \cdot \nu_{con}) \cdot R^2} \right] \cdot (u - u^*);$$

$$B = \left[ \frac{R \cdot (R - t_{shot})^2}{(R - t_{shot})^2 + (1 - 2 \cdot \nu_{con}) \cdot R^2} \right] \cdot (u - u^*). \tag{A1.11}$$

By substituting the constants A and B in Eqs. (A1.3) and (A1.4) and these last two equations in Eq. (A1.8), one obtains the expression of the radial stress $p$ for $r = R$ versus the radial wall displacement after lining installation $(u - u^*)$:

$$p = \frac{E_{shot}}{(1 + \nu_{shot})} \cdot \frac{[R^2 - (R - t_{shot})^2]}{[(1 - 2 \cdot \nu_{shot})R^2 + (R - t_{shot})^2]} \cdot \frac{1}{R} \cdot (u - u^*). \tag{A1.12}$$

Equation (A1.13) is therefore obtained as follows:

- determining expression $(u - u^*)$ from Eq. (A1.12) in function of $p$ and substituting this in Eq. (A1.11);
- evaluating Eqs. (A1.3) and (A1.4) at the lining intrados (for $r = R - t_{shot}$) and substituting these, by simplifying, in Eq. (A1.7).

$$\sigma_{\vartheta, in} = \frac{2 \cdot p}{\left[ 1 - \frac{(R - t_{shot})^2}{R^2} \right]}, \tag{A1.13}$$

from where:

$$p = \frac{1}{2} \cdot \left[ 1 - \frac{(R - t_{shot})^2}{R^2} \right] \cdot \sigma_{\vartheta, in}. \tag{A1.14}$$

## Appendix 2: QuickBasic Program

Input data:

- tunnel radius ($R$) and in situ hydrostatic stress ($p_0$);
- for the rock mass:
  elastic modulus (ED) and Poisson's ratio (nu), strength parameters of the Mohr-Coulomb citerion (peak CP and residual CR cohesion, peak FP and residual friction angle FR) or of the Hoek and Brown criterion (mp, mr, sp, sr and the uniaxial compressive strength of the intact rock sigcrock), dilatancy angle (PSI) or deformative parameter in the plastic field ($f$);
- fictitious internal pressure at the lining installation (pstar) and lining thickness (tcon);
- for the shotcrete:
  elastic modulus in the long term (Econzero), Poisson's ratio (nucon), uniaxial compressive strength in the long term (sigczero), time constants (alfa and beta);
- dead time ($t0$), mean advancement rate of the excavation face (vm), advancement step (delta).

The measurement units of the input parameters are m, MPa, hour; for the advancement rate m/day unit is adopted.

The results are written in a file with the name given at the beginning and the extension ".out".

First the ground reaction curve is given and then the reaction curve of the shotcrete lining.

The ground reaction curve (p-u) is given for points together with the plastic radius of the rock mass for each internal pressure $p$.

The reaction curve of the lining plining-u is given for points; the following results are also given for each point:

- elastic modulus of shotcrete;
- lining stiffness;
- internal fictitious pressure;
- time after the installation of the lining;
- distance from the excavation face;
- maximum stress in the shotcrete;
- uniaxial compressive strength of the shotcrete;
- safety factor of the lining;
- limit pressure pc;
- convergences of the tunnel wall.

This following list of commands refers to the case in which a Mohr-Coulomb strength criterion is adopted for the rock mass.

```
CLS
```

***Attribution of the variables***

```
DIM A, B, C, D, E, G AS DOUBLE
DIM i, PUNT1, PUNT2, z, cont1, cont2, flag1, flag2, j, nn AS INTEGER
PUNT1 = 500: PUNT2 = 500
DIM u(0 TO (PUNT1 + PUNT2)) AS DOUBLE
DIM pre(0 TO (PUNT1 + PUNT2)) AS DOUBLE
DIM RPLAS(0 TO (PUNT1 + PUNT2)) AS DOUBLE
DIM t(0 TO (PUNT1 + PUNT2)) AS SINGLE
DIM Econ(0 TO (PUNT1 + PUNT2)) AS DOUBLE
DIM k(0 TO (PUNT1 + PUNT2)) AS DOUBLE
DIM plin(0 TO (PUNT1 + PUNT2)) AS DOUBLE
DIM pfict(0 TO (PUNT1 + PUNT2)) AS DOUBLE
DIM sigmax(0 TO (PUNT1 + PUNT2)) AS DOUBLE
DIM sigc(0 TO (PUNT1 + PUNT2)) AS DOUBLE
DIM Fslin(0 TO (PUNT1 + PUNT2)) AS SINGLE
DIM deltap(0 TO (PUNT1 + PUNT2)) AS SINGLE
DIM pc(0 TO (PUNT1 + PUNT2)) AS SINGLE
DIM x(0 TO (PUNT1 + PUNT2)) AS SINGLE
```

***Input of data***

```
INPUT "Name of the creating file (without extension)"; file$
INPUT "Tunnel radius (m)"; R
INPUT "Lithostatic pressure Po (MPa)"; p0
INPUT "Young elastic modulus (MPa)"; ED
INPUT "Poisson ratio"; nu
INPUT "Peak friction angle (ø)"; FP
INPUT "Peak cohesion (MPa)"; CP
INPUT "Residual friction angle (ø)"; FR
INPUT "Residual cohesion (MPa)"; CR
INPUT "Dilatancy angle (ø)"; PSI
INPUT "Internal pressure at the lining installation(MPa)"; pstar
INPUT "Thickness of shotcrete lining (m)"; tcon
INPUT "Final elastic modulus of shotcrete (MPa)"; Econzero
INPUT "Elastic modulus time constant alfa (hour -1)"; alfa
INPUT "Poisson's ratio"; nucon
INPUT "Final uniaxial compressive strength of shotcrete (MPa)"; sigczero
INPUT "Uniaxial compressive strength time constant beta (hour -1)"; beta
INPUT "Dead time after shotcrete lining installation (hour)"; t0
INPUT "Mean advancement rate of the face (m/day)"; vm
INPUT "Advancement step (m)"; delta
```

***Attribution of the parameters***

```
FP = FP * 3.1415 / 180
FR = FR * 3.1415 / 180
PSI = PSI * 3.1415 / 180
NR = (1 + SIN(FR)) / (1 - SIN(FR))
kk = (1 + SIN(PSI)) / (1 - SIN(PSI))
```

***Calculation of the critical pressure ($p_{cr}$)***

```
pcrit = p0 * (1 - SIN(FP)) - CP * COS(FP)

u(0) = 0: pre(0) = p0: RPLAS(0) = R
t(0) = t0
plin(0) = 0: sigmax(0) = 0

k1 = (R ^ 2 - (R - tcon) ^ 2)
k2 = (1 + nucon) * ((1 - 2 * nucon) * R ^ 2 + (R - tcon) ^ 2)
sigc(0) = sigczero * (1 - EXP(-beta * t0))
deltap(0) = .5 * (1 - (R - tcon) ^ 2 / R ^ 2) * sigc(0)

j = 1

IF pcrit > 0 THEN
```

```
flag1 = 0
flag2 = 0
cont1 = 0
cont2 = 0
ue = (1 + nu) * R * (p0 - pcrit) / ED

ST1 = (p0 - pcrit) / PUNT1
FOR i = 1 TO PUNT1
```

***Calculation of the ground reaction curve***

```
   pre(i) = p0 - i * ST1
   u(i) = (1 + nu) * R * (p0 - pre(i)) / ED
   RPLAS(i) = R
   IF pre(i) < pstar AND flag2 = 0 THEN
      IF flag1 = 0 THEN
         cont1 = i
         ustar = -(pstar-pre(i)) / (pre(i-1)-pre(i)) * (u(i)-u(i-1)) +
u(i)
         flag1 = 1
      END IF
```

***Calculation of the reaction curve of the shotcrete lining***

```
      Econ(j - 1) = Econzero * (1 - EXP(-alfa * t(j - 1)))
      k(j - 1) = k1 / k2 / R * Econ(j - 1)
      plin(j) = plin(j-1) + k(j-1) * (u(cont1-1+j) - u(cont1-1+j-1))
      IF j = 1 THEN plin(1) = k(0) * (u(cont1) - ustar)
      pfict(j) = pre(cont1 - 1 + j) - plin(j)
      IF pfict(j) > 0 THEN
         x(j) = .845 * R * (.72 * p0 / pfict(j) - 1)
         nn = INT(x(j) / delta)
         t(j) = (nn + 1) * t0 + x(j) / (vm / 24)
      ELSE
         x(j) = 10000
         t(j) = 10000
      END IF
      sigmax1 = 2 * R / k2 * Econ(j-1) * (u(cont1-1+j) - u(cont1-1+j-1))
      sigmax(j) = sigmax(j - 1) + sigmax1
      IF j = 1 THEN sigmax(1) = 2 * R / k2 * Econ(0) * (u(cont1) -
ustar)
      sigc(j) = sigczero * (1 - EXP(-beta * t(j)))
      Fslin(j) = sigc(j) / sigmax(j)
      deltap(j) = .5 * (1 - (R - tcon) ^ 2 / R ^ 2) * (sigc(j) -
sigmax(j))
      pc(j) = plin(j) + deltap(j)

      IF plin(j) > pre(cont1 - 1 + j) THEN
            flag2 = 1
            cont2 = cont1 - 1 + j
         END IF

         j = j + 1
      END IF

   NEXT i

   ST2 = pcrit / PUNT2
   FOR i = (PUNT1 + 1) TO (PUNT1 + PUNT2)
```

***Calculation of the ground reaction curve***

```
      pre(i) = pcrit - (i - PUNT1) * ST2

      RP1 = ((p0 + CR / TAN(FR)) - (p0 + CP / TAN(FP)) * SIN(FP))
      RP = RP1 / (pre(i) + CR / TAN(FR))
      RPLAS(i) = R * RP ^ (1 / (NR - 1))

      A = (1 + nu) / ED
      B = SIN(FP) * (p0 + CP / TAN(FP)) * (RPLAS(i) ^ (k + 1)) / (R ^ k)
```

```
        C = (1-2*nu) * (p0 + CR / TAN(FR)) * (((RPLAS(i) ^ (k+1)) / (R ^ k))
  - R)
        D = (1 + k * NR - nu * (k + 1) * (NR + 1)) / ((NR + k) * (R ^ (NR -
  1)))
        E = pre(i) + CR / TAN(FR)
        G = (((RPLAS(i) ^ (NR + k)) / (R ^ k)) - (R ^ NR))

        u(i) = A * (B + C - D * E * G)

        IF pre(i) < pstar AND flag2 = 0 THEN
           IF flag1 = 0 THEN
              cont1 = i
              ustar = -(pstar-pre(i)) / (pre(i-1)-pre(i)) * (u(i) - u(i-1)) +
  u(i)
              flag1 = 1
           END IF
```

***Calculation of the reaction curve of the shotcrete lining***

```
           Econ(j - 1) = Econzero * (1 - EXP(-alfa * t(j - 1)))
           k(j - 1) = k1 / k2 / R * Econ(j - 1)
           plin(j) = plin(j-1) + k(j-1) * (u(cont1-1+j) - u(cont1-1+j-1))
           IF j = 1 THEN plin(1) = k(0) * (u(cont1) - ustar)
           pfict(j) = pre(cont1 - 1 + j) - plin(j)
           IF pfict(j) > 0 THEN
              x(j) = .845 * R * (.72 * p0 / pfict(j) - 1)
              nn = INT(x(j) / delta)
              t(j) = (nn + 1) * t0 + x(j) / (vm / 24)
           ELSE
              x(j) = 10000
              t(j) = 10000
           END IF
           sigmax1 = 2 * R / k2 * Econ(j-1) * (u(cont1-1+j) - u(cont1-1+j-1))
           sigmax(j) = sigmax(j - 1) + sigmax1
           IF j = 1 THEN sigmax(1) = 2 * R / k2 * Econ(0) * (u(cont1) -
  ustar)
           sigc(j) = sigczero * (1 - EXP(-beta * t(j)))
           Fslin(j) = sigc(j) / sigmax(j)
           deltap(j) = .5 * (1 - (R - tcon) ^ 2 / R ^ 2) * (sigc(j) -
  sigmax(j))
           pc(j) = plin(j) + deltap(j)

           IF plin(j) > pre(cont1 - 1 + j) THEN
              flag2 = 1
              cont2 = cont1 - 1 + j
           END IF

           j = j + 1
        END IF

  NEXT i

ELSE

  flag1 = 0
  flag2 = 0
  cont1 = 0
  cont2 = 0

  ST1 = p0 / PUNT1
  FOR i = 1 TO PUNT1
```

***Calculation of the ground reaction curve***

```
     pre(i) = p0 - i * ST1
     u(i) = (1 + nu) * R * (p0 - pre(i)) / ED
     RPLAS(i) = R
```

```
         IF flag1 = 0 THEN
            cont1 = i
            ustar = -(pstar-pre(i)) / (pre(i-1)-pre(i)) * (u(i) - u(i-1)) +
u(i)
            flag1 = 1
         END IF
```

***Calculation of the reaction curve of the shotcrete lining***

```
         Econ(j - 1) = Econzero * (1 - EXP(-alfa * t(j - 1)))
         k(j - 1) = k1 / k2 / R * Econ(j - 1)
         plin(j) = plin(j-1) + k(j-1) * (u(cont1-1+j) - u(cont1-1+j-1))
         IF j = 1 THEN plin(1) = k(0) * (u(cont1) - ustar)
         pfict(j) = pre(cont1 - 1 + j) - plin(j)
         IF pfict(j) > 0 THEN
            x(j) = .845 * R * (.72 * p0 / pfict(j) - 1)
            nn = INT(x(j) / delta)
            t(j) = (nn + 1) * t0 + x(j) / (vm / 24)
         ELSE
            x(j) = 10000
            t(j) = 10000
         END IF
         sigmax1 = 2 * R / k2 * Econ(j-1) * (u(cont1-1+j) - u(cont1-1+j-1))
         sigmax(j) = sigmax(j - 1) + sigmax1
         IF j = 1 THEN sigmax(1) = 2 * R / k2 * Econ(0) * (u(cont1) -
ustar)
         sigc(j) = sigczero * (1 - EXP(-beta * t(j)))
         Fslin(j) = sigc(j) / sigmax(j)
         deltap(j) = .5 * (1 - (R - tcon) ^ 2 / R ^ 2) * (sigc(j) -
sigmax(j))
         pc(j) = plin(j) + deltap(j)

         IF plin(j) > pre(cont1 - 1 + j) THEN
            flag2 = 1
            cont2 = cont1 - 1 + j
         END IF

         j = j + 1
      END IF

   NEXT i
   pcrit = 0
   ue = (1 + nu) * R * p0 / ED
   PUNT2 = 0
END IF
```

***Reconversion of the angles (only for the Mohr-Coulomb criterion)***

```
FP = FP * 180 / 3.1415
FR = FR * 180 / 3.1415
PSI = PSI * 180 / 3.1415
```

***Opening of the output file***

```
   FILE1$ = file$ + ".out"

   OPEN FILE1$ FOR OUTPUT AS #1
   z = 0

   PRINT #1, "MOHR-COULOMB ELASTIC-PLASTIC SOLUTION (PEAK-RESIDUAL)"
   PRINT #1, ""
   PRINT #1, USING " TUNNEL RADIUS                              ##.##  m "; R
   PRINT #1, USING " LITHOSTATIC PRESSURE                      ###.##  MPa"; p0
   PRINT #1, USING " ELASTIC MODULUS OF THE ROCK MASS       ######.##  MPa"; ED
   PRINT #1, USING " POISSON'S RATIO                             #.##   "; nu
   PRINT #1, USING " PEAK FRICTION ANGLE                        ##.#   degrees";
   FP
   PRINT #1, USING " PEAK COHESION                              ##.##  MPa"; CP
```

```
PRINT #1, USING " RESIDUAL FRICTION ANGLE                    ##.#   degrees";
FR
PRINT #1, USING " RESIDUAL COHESION                          ##.##  MPa"; CR
PRINT #1, USING " DILATANCY ANGLE                            ##.#   degrees";
PSI
PRINT #1, "-----------------------------------------"
PRINT #1, USING "critical pressure = ###.###  MPa"; pcrit
PRINT #1, USING "elastic displacement = ##.####  cm"; ue * 100
PRINT #1, "-----------------------------------------"
```

***Printing of the ground reaction curve***

```
PRINT #1, USING " p (MPa)-u (cm)-Rpl (m) = ###.##  ###.## ###.## "; p0; z;
R
FOR i = 1 TO (PUNT1 + PUNT2)
PRINT#1,USING"p(MPa)-u(cm)-Rpl(m)=###.## ###.##
###.##";pre(i);u(i)*100;RPLAS(i)
NEXT i

PRINT #1, " "
PRINT #1, " "
PRINT #1, USING " INT. PRESSURE AT THE LINING INSTAL.   ###.##  MPa ";
pstar
PRINT #1, USING " THICKNESS OF SHOTCRETE LINING          ##.#   cm "; tcon
PRINT #1, USING " FINAL ELASTIC MODULUS OF SHOTCRETE ######.##  MPa";
Econzero
PRINT #1, USING " ELASTIC MODULUS TIME CONSTANT alfa     ##.#### hour -1";
alfa
PRINT #1, USING " POISSON'S RATIO OF SHOTCRETE            #.##    "; nucon
PRINT #1, USING " FINAL UN. COMPR. STRENGTH OF SHOTCR.  ###.#   MPa";
sigczero
PRINT #1, USING " UN. COMPR. STRENGTH TIME CONST. beta   ##.#### hour -1";
beta
PRINT #1, USING " DEAD TIME AFTER SHOTCRETING            ##.##  hour"; t0
PRINT #1, USING " MEAN ADVANCEMENT RATE OF THE FACE      ##.#   m/day"; vm
PRINT #1, USING " ADVANCEMENT STEP                       ##.#   m"; delta
PRINT #1, "-----------------------------------------"
```

***Calculation of the equilibrium point***

```
i = cont2: j = cont2 - cont1 + 1
deltau = u(i) - u(i - 1)
deltaplin = plin(j) - plin(j - 1)
u(i) = u(i-1) + deltau * (plin(j-1) - pre(i-1)) / (pre(i)-pre(i-1) -
deltaplin)
plin(j) = plin(j - 1) + deltaplin / deltau * (u(i) - u(i - 1))
pfict(j) = 0
x(j) = 10000
t(j) = 10000
sigmax1 = 2 * R / k2 * Econ(j - 1) * deltau
sigmax(j) = sigmax(j - 1) + sigmax1
sigc(j) = sigczero
Fslin(j) = sigc(j) / sigmax(j)
deltap(j) = .5 * (1 - (R - tcon) ^ 2 / R ^ 2) * (sigc(j) - sigmax(j))
pc(j) = plin(j) + deltap(j)

z = 0
zz = 10000
```

***Printing of the reaction curve of the lining***

```
PRINT #1, USING " u (cm) - plining (MPa) - Econ (MPa) - k (MPa/m) - pfict
(MPa) - time (hour) - distance (m) - sigmax (MPa) - sigc (MPa) - Fslining -
pc (MPa) - convergence (cm) =  ###.###  ###.#####  #####.##  #####.##
###.##### ######.##  #####.##  ###.####  ###.####  #####.##  ###.#####
###.### "; ustar * 100; z; Econ(0); k(0); pstar; t0; z; z; sigc(0); zz;
deltap(0); z
```

```
FOR j = 1 TO (cont2 - cont1)
   PRINT #1, USING " u (cm) - plining (MPa) - Econ (MPa) - k (MPa/m) -
pfict (MPa) - time (hour) - distance (m) - sigmax (MPa) - sigc (MPa) -
Fslining - pc (MPa) - convergence (cm) =  ###.###  ###.#####  #####.##
#####.##  ###.#####  ######.##  #####.##  ###.####  ###.####  #####.##
###.#####  ###.### "; u(cont1 - 1 + j) * 100; plin(j); Econ(j); (k(j - 1) +
k(j)) / 2; pfict(j); t(j); x(j); sigmax(j); sigc(j); Fslin(j); pc(j); 2 *
(u(cont1 - 1 + j) - ustar) * 100
NEXT j
j = (cont2 - cont1 + 1)
PRINT #1, USING " u (cm) - plining (MPa) - Econ (MPa) - k (MPa/m) - pfict
(MPa) - time (hour) - distance (m) - sigmax (MPa) - sigc (MPa) - Fslining -
pc (MPa) - convergence (cm) =  ###.###  ###.#####  #####.##  #####.##
###.#####  ######.##  #####.##  ###.####  ###.####  #####.##  ###.#####
###.### "; u(cont2) * 100; plin(j); Econzero; k(j - 1); z; zz; zz;
sigmax(j); sigczero; sigczero / sigmax(j); pc(j); 2 * (u(cont2) - ustar) *
100

Fslin(j) = sigczero / sigmax(j)
```

***Search for the minimum safety factor***

```
Fsmin = 10000
FOR j = 1 TO (cont2 - cont1 + 1)
   IF Fsmin > Fslin(j) THEN Fsmin = Fslin(j)
NEXT j
```

***Printing of the minimum safety factor***

```
CLS
PRINT ""
PRINT USING "Minimum safety factor of the lining during advanc.: ###.##";
Fsmin
```

If one wishes to use the Hoek and Brown strength criterion for the rock mass, the lines of the program written in bold should be substituted with the following lines.

***Attribution of the variables***

```
DIM MM, M, G, N, DDD AS DOUBLE
```

***Input of data***

```
...
INPUT "Peak m strength parameter"; mp
INPUT "Peak s strength parameter"; sp
INPUT "Residual m strength parameter"; mr
INPUT "Residual s strength parameter"; sr
INPUT "uniaxial compressive strength of intact rock (MPa)"; sigcrock
INPUT "deformative parameter f for plastic zone"; f
```

***Attribution of the parameters***

```
MM = ((mp / 4) ^ 2 + mp * p0 / sigcrock + sp) ^ .5
M = .5 * MM - mp / 8
N =(2/(mr*sigcrock))*(mr*sigcrock*p0+sr*sigcrock^2-mr*sigcrock^2*M)^.5
G = ED / (2 * (1 + nu))
```

***Calculation of the critical pressure ($p_{cr}$)***

```
pcrit = p0 - M * sigcrock
```

***Calculation of the ground reaction curve***

```
...
RPLAS(i)=r*EXP(N-(2/(mr*sigcrock))*(mr*sigcrock*pre(i)+sr*sigcrock^2)^.5)
DDD = 1 / (G * (f + 1)) * ((f - 1) / 2 + (RPLAS(i) / r) ^ (f + 1))
u(i) = M * r * sigcrock * DDD
```

***Opening of the output file***

```
...
PRINT #1, "HOEK-BROWN ELASTIC-PLASTIC SOLUTION (PEAK-RESIDUAL)"
...
PRINT #1, USING " PEAK STENGTH PARAMETER m              ##.###  "; mp
PRINT #1, USING " PEAK STENGTH PARAMETER s              ##.#### "; sp
PRINT #1, USING " RESIDUAL STENGTH PARAMETER m          ##.###  "; mr
PRINT #1, USING " RESIDUAL STENGTH PARAMETER s          ##.#### "; sr
PRINT #1, USING " ROCK UNIAXIAL COMPRESSIVE STRENGTH   ###.##  (MPa)";
sigcrock
PRINT #1, USING " DEFORMATIVE PARAMETER f               ##.##   "; f
...
```

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