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Field-scale modeling of subsurface tile-drained soils using an equivalent-medium approach

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Summary Research conducted for the last 35 years has shown that subsurface drainage has a significant impact on hydrology and contaminant transport. This can be observed at the field-scale and also at the watershed scale. Impacts are always associated with modifying otherwise natural flow paths. Most computer model representations of drainage have been drawn at the field-scale. These models require relatively precise data that are usually unavailable when simulating hydrology and water quality in large watersheds. We believe that in this case drainage representation should be simplified and yet closely match observations. As a first step towards incorporating drainage systems into large-scale hydrological models, we propose an equivalent representation of drains buried in a soil profile by using a homogeneous anisotropic porous medium without drains. This representation is based on a "self-consistent" approach and on geometrical considerations. Simplification is such that calculating the equivalent hydraulic conductivity requires only information on the main length and spacing of the tile drains and not on their precise location. This approach also provides a much simpler discretisation of the domain because of the absence of internal boundary conditions on the drainage pipes. Compared to other methods that have simplified drainage representation in existing watershed models, it requires no parameter fitting. Two alternatives to the method are presented: in the first one, the soil profile equipped with the actual drain pipes is represented by an equivalent, horizontally layered system with no pipes; in the second, the layered system has been replaced with an equivalent homogeneous profile. The efficiency of these approaches was tested against a classical representation of tile drains using the SWMS 3D code, which solves the Richards equation for a typical drained plot configuration. The equivalent-medium approach appears to give satisfying results for global water outflow and mean water table elevation.

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Introduction

Subsurface tile drainage systems have been widely used for the past few decades to facilitate crop growth in otherwise unsuitable areas and to provide efficient control of water table conditions at the field-scale.

Drainage function and impacts are rather well known at the plot scale, but its influence extends beyond the field (e.g. Robinson and Rycroft, 1999). In an intensively drained watershed, the overall reduction in surface runoff and the ponding period as well as lowering the water table have been reported (Robinson, 1990; Konyha et al., 1992). The impact of drainage on flows must therefore be considered at two different scales. At the field-scale, the outflow of the system will be directly conditioned by drain spacing, soil properties and the buried network geometry, while drainage influence at the catchment scale depends on the spatial distribution of drained plots and the hydraulic functioning of the collecting network (Robinson and Rycroft, 1999). As a result, large discrepancies exist between drainage system modeling approaches (Skaggs and Chescheir, 1999).

At the field-scale, many drainage models are based on a discretisation of the physical laws describing the evolution of the drained system (e.g. Lorre and Lesaffre, 1994). Depending on their degree of complexity, they can be based either on the Richards equation (Ahmad et al., 1991; Fipps and Skaggs, 1991) or on the Laplace or Boussinesq equations (Bouarfa and Zimmer, 2000), where only the water table elevation is modeled. These models have proven their efficiency in quantifying water fluxes at the field-scale. The most accurate results are generally associated to the most complete descriptions (i.e. the 3D Richards equation), which provide good estimates of velocity and pressure fields in the whole domain.

In these models, the most commonly used tile-drain representation is a seepage-face-type condition at the external boundary of the drain (Saucedo et al., 2002). As pointed out by Fipps et al. (1986) or Tarboton and Wallender (2000), such a realistic representation of tile drains in a two- or three-dimensional numerical model requires a very fine level of spatial discretisation near these singularities, which dimensions are very small compared to those of the global system. The result is either a very large number of discretisation nodes or a wide discrepancy in grid element size, neither of which are generally desired for accuracy and stability reasons. Several methods have been tested to overcome this major difficulty. Fipps et al. (1986) present an overview and a comparison of different techniques for which a drain can be modeled as a single node in a 2D mesh. These methods differ by the condition imposed at this specific node and by the treatment of the surrounding cells. The condition may be a specified flux (Barcelo and Nieber, 1981, 1982) or a specified head at the node. An alternative, in application of Vimokey and Taylor (1962) approach, consists in adjusting the hydraulic conductivities of the elements around the single node representing the drain. The efficiency of these methods has been tested against a classical drain discretisation with satisfying results in some cases (Fipps et al., 1986). In particular, the Vimokey and Taylor approach predicted hydraulic heads (at some distance from the drain) and flow rates with good accuracy. This approach has been imple-

mented in the SWMS codes (2D, Simunek et al., 1999; and 3D, Simunek et al., 1995) and by Buyuktas and Wallender (2002) in their own finite elements code.

At the watershed scale, a few attempts have been made to integrate artificially drained systems into hydrological models. These models were originally designed to represent complex watershed and large systems. They frequently propose a simplified simulation of subsurface flow depending on whether the hydrological model is based on mass balance equations, or on discretised partial differential equations of mass momentum and energy conservation, or on empirically determined equations. Tile drainage can be incorporated by introducing an additional term in mass balance equations of an elementary hydrological unit, as has been implemented, for example, in TOPMODEL (Kim and Delleur, 1997), or, as in MIKE-SHE (DHI, 2000), by an empirical water table height–drainage flow relationship. A comparable approach has been also used by Purkey et al. (2004). In such methods, the geometry of the system is also simplified and represented by a single “drain” per soil cell.

Using a similar approach, Carlier and de Marsily (2003) introduced a virtual drainage layer to incorporate the tile drainage system influence in the distributed hydrological model TOPOG (Vertessy et al., 1993). The hydraulic conductivity of this layer, located at the drains’ depth, was empirically set at a high value to account for the fast water transfer occurring in the drainage network. In all cases, tile drainage systems were incorporated by simplified approaches that require the fitting of empirical parameters.

The main objective of the present study is to find a “physically-based” way of representing a subsurface drained area at the intermediate scale of the drain panel. We propose to represent a drain panel by using a homogeneous anisotropic porous medium without drains. The hydrodynamical properties of this equivalent block should solely depend on soil properties and on the geometry of the subsurface drainage network. This equivalent global parameterization at the plot scale should avoid the need for defining complex internal boundary conditions (drain pipes location) and/or the fitting of empirical parameters.

The proposed approach is based on an effective-medium theory and on geometrical considerations. The original system, composed of a set of parallel drains leading to a collector, is replaced with an equivalent homogeneous medium. Indeed, determining the equivalent properties of heterogeneous and/or fractured porous media has been intensively studied in the past few decades, as it appears to be an efficient way of simplifying modeling of such complex domains. For an extensive review, we refer the reader to Renard and de Marsily (1997). Amongst the numerous applications of these methods, empirical-type approaches have been used in the somewhat similar case of soil “pipes” (Uchida et al., 2001), i.e. large natural macropores responsible for preferential stormflows in various specific areas such as sub-arctic wetland (Woo and diCenzo, 1988), boreal forest (Roberge and Plamondon (1987)) or semiarid land (Newman et al., 1998). However, to our knowledge, there has been so far no attempt to characterize flow in a subsurface drained area through equivalent properties, assuming that the flow in the entire system can be correctly described using the Richards equation.

Application of the equivalent-medium approach to subsurface drained soils

In this study, we modeled an elementary drainage system built as a panel of parallel drains directly connected to a common orthogonal collector. This system can be considered as an elementary unit of a complex drainage system, and hence as an elementary block to be integrated into a large-scale hydrological model.

The approach adopted here lead to the calculation of "equivalent" properties of the system, as defined in Renard and de Marsily (1997). The term "equivalent" is of importance and differs from "effective" properties of this system, since the term "effective" refers to properties involved in ensemble averaged equations (Neuman and Orr, 1993). The term "equivalent-medium" is then used herein that sense, keeping in mind that equivalent properties are always defined for a specific problem having its own geometry and flow conditions.

To apply this type of equivalent-medium theory, assumptions leading to two major hypotheses were made on the hydraulic behavior of the system:

- the pressure head boundary condition at the drains can be effectively replaced without a significant loss of accuracy;
- the flow in the drains can be modeled using Darcy's law.

The first point can be easily justified in the case of a seepage face-type boundary condition at one end of the drain: assuming a "finite" flow in the drain or its porous equivalent, the head gradient will necessarily tend to zero to compensate the infinite hydraulic conductivity. However, the concept of infinite conductivity may be seen simply as a preliminary step in our method: this value is far beyond the bounds of application of Darcy's law, and hence of the Richards equation.

Although simplifying drainage representation provides many advantages, there are some drawbacks. One of them is that there is no clear distinction in this approach between ground and drainage water.

Equivalent parameter calculation

The method is based on the assumption that the influence of the heterogeneity of a medium can be considered as the sum of the individual contributions of each heterogeneity randomly distributed in a surrounding homogeneous medium. The self-consistent theory was first introduced by Maxwell (1891) and further developed in the field of electrostatics (Bruggeman, 1935; Odolevsky, 1951; Landau and Lifshitz, 1960). It was then formulated by Dagan (1979) and Poley (1988) for isotropic and anisotropic porous media, respectively. Fokker (2001) extended this approach to general cases of anisotropy in two and three dimensions in the case when the heterogeneities of the domain can be considered as ellipsoids.

Our approach can be divided in three consecutive steps (S1–S3):

- the first step (S1), following Pozdniakov and Tsang (2004) and assimilating the drain pipes to elliptical inclusions of

infinite conductivity, represents water transfer in the drain panel layer;

- the second step (S2), based on geometrical considerations, is necessary to compensate the absence, in the equivalent-medium approach, of a seepage-face boundary at the drain surface. The equivalent-medium then obtained is a three-layered anisotropic soil with a more conductive (perpendicularly to the collector) intermediate layer zone representing the drain level;
- a third and final step (S3) provides an homogeneous equivalent medium through a vertical averaging of hydraulic conductivities.

First step (S1)

We used a drained system such as the one illustrated in the left part of Fig. 1. The soil constituting this system was considered as a variably saturated homogeneous and isotropic porous medium with specific hydrodynamical properties. In particular, the isotropic saturated hydraulic conductivity tensor of the soil is given by:

$$\overline{K}_s = K_s \bar{I} \quad (1)$$

$\bar{I}$ being the identity tensor.

Our major assumption is that tile drains in this isotropic soil can be modeled as porous entities of infinite conductivity and thus that the resulting flow in the "soil + drains" system can be modeled as a Darcian phenomenon. The soil profile is then divided into three horizontal layers: a layer above the drains, a "drain layer" of thickness d corresponding to drain diameter and a third layer between the drains and the restrictive layer. In the "drain layer", we used the self-consistent approach proposed by Pozdniakov and Tsang (2004) to calculate the effective conductivity of a medium representing both the drains and the soil in between them. In this portion of the domain, the location of the drains can be seen as a realization of a random space function since they are regularly placed in the entire layer.

The system of equations proposed by Pozdniakov and Tsang (2004) for a regularly oriented set of inclusions (Eq. (7) in Pozdniakov and Tsang, 2004) was then used to obtain the longitudinal and orthogonal (with respect to the inclusion orientation) effective hydraulic conductivities. These equations are in our approach written using the appropriate coefficients for the specific case of very long oblate spheroids in 3D (Landau and Lifshitz, 1960; see Eq. (5) in Pozdniakov and Tsang, 2004) of infinite conductivity to represent the drains. We then obtain a system of two nonlinear equations, depending only on:

- The volumetric density of inclusions in the domain p approximated by:

$$p = \frac{\pi d}{4L} \quad (2)$$

where d is the thickness of the soil layer containing the drains (equal to the drain pipe diameter) and L is the half spacing length between drain pipes.

- The aspect ratio of the spheroids ε , being here the ratio between the dimensions of the drains:

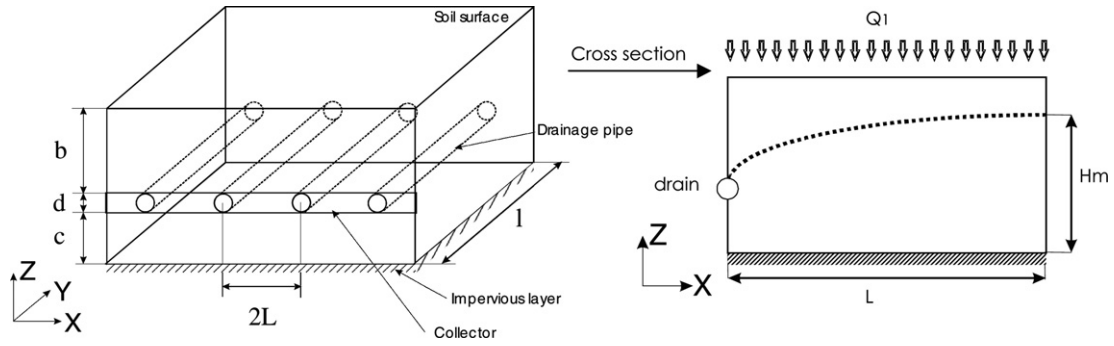


Figure 1 Original system (left) drained by a set of pipes of diameter d , buried at a depth b , at a distance c above the impervious substratum, and section of the elemental sub-domain (right) of the original system. H_m = water table elevation above reference level.

$$\varepsilon = \frac{d}{l} \quad (3)$$

where l is the drain pipe length.

The effective longitudinal hydraulic conductivity $k_{\text{eff}} = k_{\text{ef}}^{\text{yy}}$ corresponds to the conductivity expresses in the Y axis. The effective hydraulic conductivity in the X and Z axes are identical $k_{\text{efo}} = k_{\text{ef}}^{\text{xx}} = k_{\text{ef}}^{\text{zz}}$ and correspond to the orthogonal effective hydraulic conductivity. At this point, these values are rigorously effective properties, since they come from ensemble averaged equations and that no assumption has been made on the direction on the flow. In the drain layer, the hydraulic conductivity tensor (Eq. (1)) becomes:

$$\overline{\overline{K}}_{\text{Layer}} = \begin{bmatrix} k'_{\text{efo}} K_S & 0 & 0 \\ 0 & k'_{\text{eff}} K_S & 0 \\ 0 & 0 & k'_{\text{efo}} K_S \end{bmatrix} \quad (4)$$

with $k'_{\text{eff}} = k_{\text{eff}}/K_S$ and $k'_{\text{efo}} = k_{\text{efo}}/K_S$.

The two other parts of the domain (the upper and lower layers) remain unchanged at this stage.

From a numerical point of view, the value of the relative effective orthogonal conductivity k'_{efo} does not vary much with the geometrical parameters of the system and remains close to 1. On the contrary, the longitudinal coefficient k'_{eff} appears to be very sensitive to the dimensions of the system, especially to the drain spacing $2L$ and to the domain length l in a somewhat quadratic pattern. This is explained

by the fact that the inclusions are assumed to be of infinite hydraulic conductivity. As a result the effective longitudinal conductivity has to reflect the instantaneous transfer in inclusions of increasing length.

The resulting system after this first step (hereafter called the S1 system) is represented in the left part of Fig. 2.

Second step (S2)

From this point, the properties will be calculated from assumptions made upon the flow geometry, and then cannot be considered as effective properties anymore, but rather as equivalent properties of the system for this kind of flow.

In the actual drainage system, the seepage-face boundary is the drain. Flow path is restricted to a plan orthogonal to the drains and is thus of moderate length before reaching the seepage-face boundary. In the equivalent-medium, the seepage-face boundary only exists at the edge of the parcel. Flow path is thus much larger than in the actual drainage system. The transformation made in this second step aims at compensating the absence of internal seepage-face boundary conditions in the equivalent system S1.

To take this into account, we use geometrical considerations analogous to those proposed by Youngs (1986), who presents equivalence relationships between anisotropic and isotropic drained systems. In a similar manner, we consider similarities between the original domain of study and the transformed domain (after Step 1).

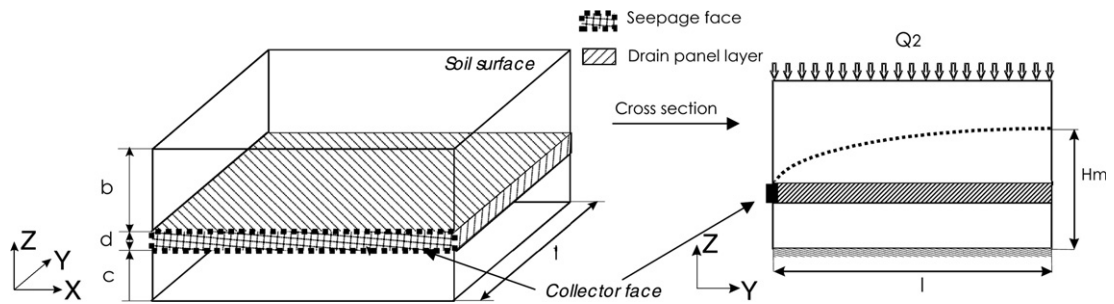


Figure 2 Effective S1 system (left) and section of the elemental sub-domain of the modified (layered) system S1 (right), with L = half length between drains, l = drain pipe length, b = soil depth above drain level, c = soil depth under drain level, d = thickness of the soil layer containing the drains (equal to the drain pipe diameter) and H_m = water table elevation above reference level.

Far enough from the drainage collector, the original domain can be considered, due to symmetry considerations, as the sum of $2n_d$ sub-domains in 2D (with n_d being the number of drains), as shown in the section in the xz plane (plane $y = l$) in the right part of Fig. 1, with $Q_1 = ql$ the constant uniform flux per unit length (to be multiplied by the number of subsections, i.e. $2n_d$ to obtain the total flux).

Similarly, if we make a vertical section in the yz plane (plane $x = (2n_d + 1)L/2$) of the S1 system in Fig. 2, we obtain a representation of this system in 2D, with $Q_2 = 2n_d Lq$ the constant uniform flux per unit length.

From these two sections, and provided that $L \neq l$, it appears that, in the transformed system S1, the distance to a seepage boundary condition, which influences the hydraulic gradient, is significantly different from the original domain.

Using the same approach as that followed by Youngs (1986), we could “rescale” the properties of the equivalent domain S1 to those of the previous one (Fig. 1), by multiplying the horizontal conductivity by a factor given by the transformations of the flow equation.

If this rescaling procedure is applied to the S1 system, for the drain layer we obtain:

$$\overline{\overline{K}}_{\text{drain layer}} = \begin{bmatrix} k'_{\text{efo}} K_S & 0 & 0 \\ 0 & \gamma^2 k'_{\text{efl}} K_S & 0 \\ 0 & 0 & k'_{\text{efo}} K_S \end{bmatrix} \quad (5)$$

and for above and below of the drain layer:

$$\overline{\overline{K}}_S = \begin{bmatrix} K_S & 0 & 0 \\ 0 & \gamma^2 K_S & 0 \\ 0 & 0 & K_S \end{bmatrix} \quad (6)$$

with

$$\gamma = \frac{2n_d Q_1}{Q_2}$$

The result of this transformation is a new system (S2) that is geometrically identical to S1, and that behaves similar to the original drainage system. These two systems would be strictly equivalent (in the sense of Youngs, 1986) if the original system could be fully described in 2D (i.e. in absence of the collector drain) and if S2 would be not layered. The equivalent behavior is assumed and verified in the following sections.

Third step (S3)

At this stage, the equivalent domain S2 is a significant progress in our simplification process since the difficulty brought by internal boundary conditions has been already overcome (there only remains the seepage face at the collector). However, the layered structure calls for further simplifications.

To the best of our knowledge, there is no simple way of replacing a layered drained soil by an equivalent uniform one. Early research on drainage systems (Ernst, 1956; Weseling, 1973; Van Beers, 1976) proposed steady state equations derived for uniform soils involving a geometry factor taken from nomographs (Van der Ploeg et al., 1999). These equations were built to calculate drain spacing in layered formations for a given maximum height of the water table. These formulations do not allow expression of hydraulic

conductivities explicitly as a function of the other variables and parameters of the system; nor do they give a formulation of equivalent conductivity.

More recent studies on ditch drainage in layered soils (Barua and Tiwari, 1996) proposed analytical expressions for hydraulic head in two- and three-layered soils drained by ditches. These formulas could have been applied to our case of tile drainage, but their complex expressions prevented us from finding any explicit expression of equivalent conductivity.

Rogers and Fous (1989) conducted numerical experiments in two-layered soils for different hydraulic conductivity ratios, ranging from 1/2 to 1/500 for K_1/K_2 and K_2/K_1 , K_1 and K_2 being the top and bottom layer conductivities, respectively. Their results show that, although the drain was located in the upper layer, the apparent hydraulic conductivity of the system is much more influenced by K_2 . However, no equations were proposed by the author to quantify this influence.

In a first attempt, we used a vertically arithmetic averaging of saturated horizontal hydraulic conductivities over the entire depth of the domain to obtain a uniform value in the longitudinal (parallel to the direction of the drains) direction. This choice was motivated by the fact that the main flow direction is horizontal and parallel to the layers. For this formula to be exactly valid, the flow should be saturated and one-dimensional in the direction parallel to the drains' axis.

At the end of this step, the values of the equivalent conductivity tensor can be written as (see notations in Fig. 1):

$$\overline{\overline{K}}_{\text{Eq}} = \begin{bmatrix} k'_{\text{efo}} K_S & 0 & 0 \\ 0 & \beta \gamma^2 K_S & 0 \\ 0 & 0 & k'_{\text{efo}} K_S \end{bmatrix} \quad \text{with } \beta = \frac{k'_{\text{efl}} \cdot d + (b + c)}{(b + c + d)} \quad (7)$$

We obtain the final equivalent system S3 containing a single “block” of soil of homogeneous, though anisotropic, saturated hydraulic conductivities.

The accuracy of the approach and the validity of the different assumptions were tested by comparing numerical simulations between the reference and the equivalent systems.

Numerical experiments

Numerical simulations were done using the SWMS 3D model (Simunek et al., 1995), which is a robust and effective model for variably saturated flow in porous media based on a finite element discretisation of the Richards equation. This model is often used as a reference to assess the validity of alternative approaches (e.g. Bruen and Osman, 2004; Lewandowska et al., 2004).

Characteristics of the numerical experiments

The geometrical parameters of the system are defined in Fig. 1 and their numerical values given in Table 1. The boundary conditions consist in a constant recharge flux (R) on the top surface and seepage-faces on the collector and drain surfaces. Any other boundary is considered impervious.

Table 1 Characteristic dimensions of the system

L (m)	l (m)	n_d	c (m)	d (m)	b (m)
5	48	4	0.475	0.05	0.975

L = half length between drains, l = drain pipe length, n_d = number of drains, b = soil depth above drain level, c = soil depth under drain level and d = thickness of the soil layer containing the drains (equal to the drain pipe diameter).

As an illustration, we have selected an arbitrary value of constant flux boundary condition to investigate the hydrological response of the systems in the case where water table does not reach the soil surface, avoiding the problem of the treatment of a ponded boundary condition. The system was set to initially be at hydrostatic equilibrium, and the water table set to be located at the bottom level of the drains ($z = 0.475$ m). At time $t = 0$, a constant recharge of $R = 0.0125K_s$ was uniformly applied on the top boundary.

The dynamic behavior of shallow water table aquifers has been shown to be very sensitive to soil hydraulic parameters (Beaugendre et al., 2004). We have chosen the parametric representation of soil properties of Mualem–van Genuchten (Mualem, 1976; Van Genuchten, 1980). The soil parameters correspond to those of the Yolo Light clay soil from Kao et al. (2001). For this soil, the constant recharge value was evaluated to be $R = 3.25E-5$ m/h.

The equivalent parameters used to define the equivalent systems S1, S2 and S3 were computed by solving the system obtained from Eqs. (5) and (7) in Pozdniakov and Tsang (2004) for k'_{eff} and k'_{efo} , and our Eqs. (6) for γ and (7) for β using data in Table 1.

Unless otherwise stated, the characteristics of the numerical simulations discussed below are identical to those summarized in Table 2.

All calculations were done on the same grid of 28,800 tetrahedral elements. These elements were associated with $40 \times 12 \times 60$ nodes regularly spaced in the X , Y and Z directions.

To compare the results obtained from the different test cases, we chose to focus on variables that have a significant meaning at a larger scale and that influence the hydrological behavior of the larger system where this equivalent block can be integrated. The outflow curves were then computed at the external boundaries of the system: in the reference case, these boundaries include both the drains (located at a set of nodes) and the seepage-face representing the collector, while for equivalent systems S1, S2 and S3, the collector is the only outlet. In the reference simulations describing the original system, drains were characterized using the method proposed by Vimoke and Taylor (1962).

Comparison of successive systems

In Vimoke and Taylor (1962) approach, drains are represented by nodal points (in a regular finite element grid), with a simultaneous modification of the hydraulic conductivity of the elements forming a square surrounding each node in a plane perpendicular to the drains. The value of the modified hydraulic conductivity comes from electrical analogy considerations, and the details of its computation can be found in Simunek et al. (1995).

Calculations were then made until the steady state was reached, using the adaptive time-stepping scheme implemented in SWMS 3D.

Cumulative outflow and mean water table elevation

To evaluate the efficiency of an equivalent-medium representation of this system, we compared the dynamics of cumulative flow and the mean water table elevation. The results are presented in Fig. 3a and b.

As expected, Fig. 3a and b shows that the S1 system is ineffective at representing the reference drainage dynamics

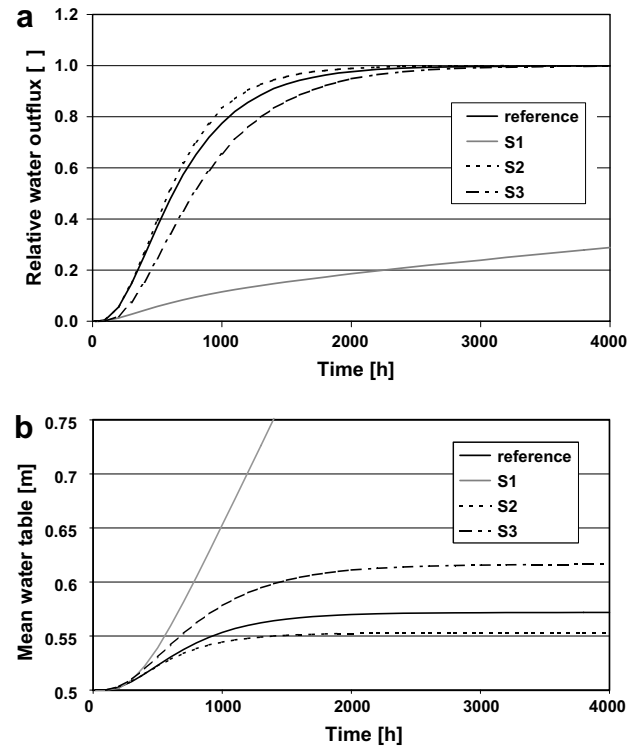


Figure 3 (a) Relative water outflux for each transformed system. (b) Mean water table for each transformed system.

Table 2 Common characteristic parameters for numerical experiments

Soil parameters					Effective parameters			
K_s (m/h)	θ_s	θ_r	n	α (m^{-1})	k'_{eff}	k'_{efo}	γ^2	β
0.0026	0.41	0.1	1.31	1.9	25.4	1.005	92.16	1.813

Soil parameters defined in Van Genuchten (1980), and effective parameters defined in Pozdniakov and Tsang (2004).

as cumulative flow curve is always below the reference and water table is above the reference. Obviously, the capacity of the S1 system to transport water to the collector is much smaller than the capacity of the other three systems (the reference, S2 and S3). This observation confirms that this simplification alone was not sufficient. Both S2 and S3 equivalent representations provide relatively good results in terms of outflow and mean water table level. The layered system S2 appears to predict a slightly faster response, and consequently a lower mean water table than the homogeneous system S3. This can be explained by the fact that flow is not purely horizontal, and that the outlet of the system is located near the most conductive layer of S2.

Water table shapes

Although the response of the system is similar in terms of outflow, the equivalent modeling of the drainage system strongly modifies the flow structure, and then the shape of the water table. Fig. 4a–c shows the shapes of the water table at steady state for the reference, the S2 and S3 cases for the Yolo Light clay soil from Kao et al. (2001).

As expected, the water table shape evolves from a quite complex surface due to the presence of drains (and then of seepage conditions) inside the soil profile in the reference case to a simpler arch-typed shape in both equivalent approaches. The difference between the latter two comes from the fact that the vertical averaging in the homogeneous case favors a more homogeneous distribution of flow (and then of water) over the entire depth of the profile, whereas the highly conductive zone of the layered approach causes a fast drainage of water as soon as it reaches the layer level, and consequently prevents the water table from rising as much.

Unsaturated zone/hydrodynamic parameter influence

In all the preceding developments, the equivalent parameters were those of saturated conditions, which obviously is not the case. Therefore, it seemed important to study the influence of the unsaturated zone in the effectiveness of this approach.

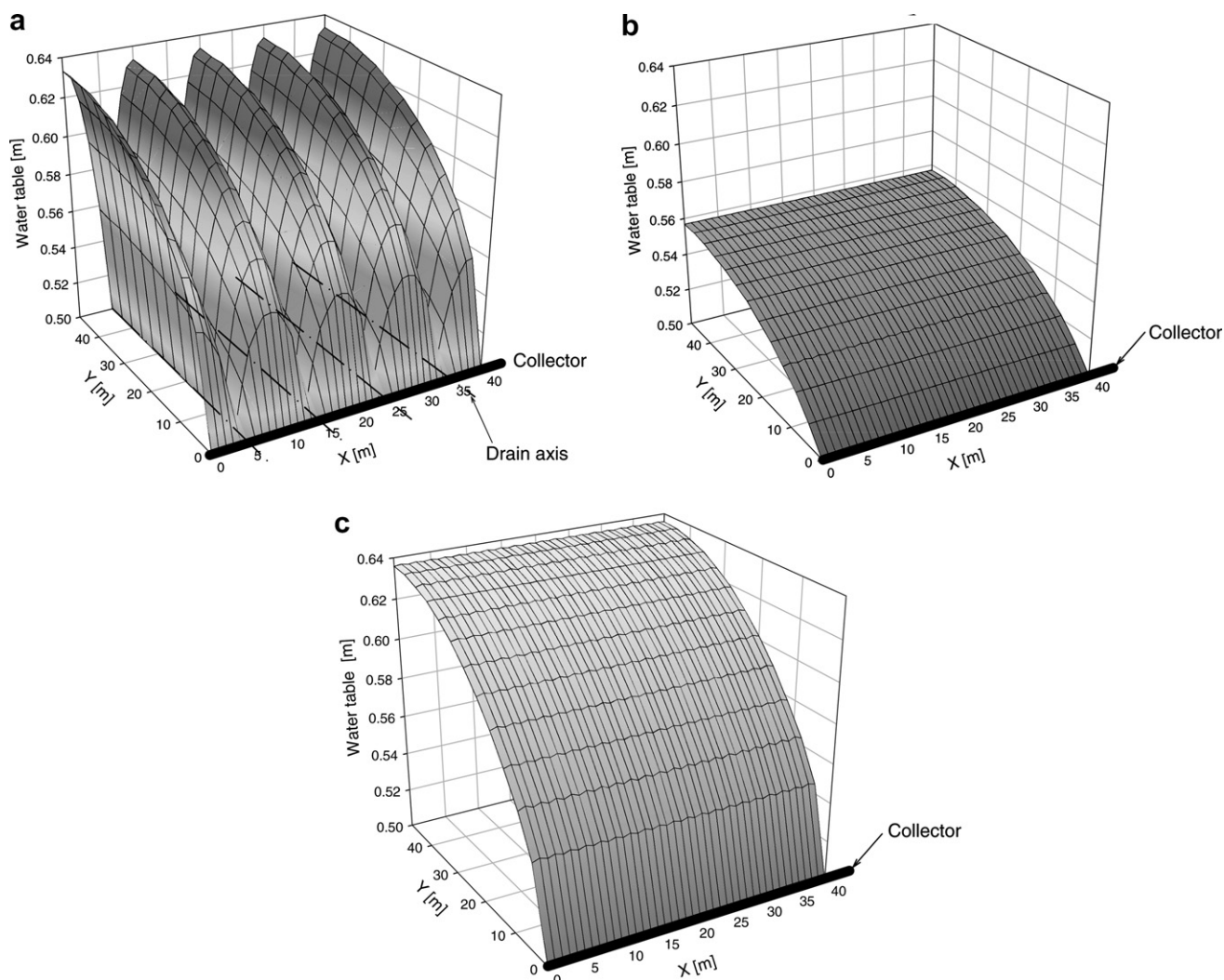


Figure 4 (a) Water table elevation above substratum for Soil no. 1 – reference case. (b) Water table elevation above substratum for Soil no. 1 – S2 case. (c) Water table elevation above substratum for Soil no. 1 – S3 case.

Table 3 Soil hydrodynamic parameters (Van Genuchten, 1980)

Soil	n	α (m ⁻¹)
1	1.31	1.9
2	5	1.9
3	1.31	8

To take the influence of hydrodynamic parameters into account, we selected arbitrary, but realistic, values of n and α (Van Genuchten, 1980) summarized in Table 3.

Varying the hydrodynamic parameters n and α made possible the exploration of a wide range of soil types and thus a wide range of drained soil behaviors. These behaviors are evaluated in the following sections by comparing drainage outflows and mean water table elevation between each set of parameters.

Water fluxes

In Fig. 5a and b are plot the relative outflow of the system for Soils 2 and 3, respectively, as was done with Soil 1 in Fig. 3a.

For all soils, one can observe a fairly good agreement between the outflow predicted by the reference approach and the two representations. As expected, the dynamics of the response is somewhat different in each case.

In particular, the steady state is reached later in time in the case of Soil 3, because of the faster decrease in the relative hydraulic conductivity with the pressure head. Never-

theless, this difference is also present in computations using the equivalent systems.

Modeling results of a layered S2 system predict a slightly faster evacuation of water out of the system, whereas for the homogenous S3 case, results show a slower response to constant recharge. This can be explained by the fact that vertical averaging on the horizontal conductivities, as was performed, is strictly valid only when the flow is purely horizontal, which was not our case. In particular, the position of the outflow of the equivalent system, right at the level of the very conductive layer, emphasizes the “accelerating” effect of this very high conductivity. The presence of outflow over the entire width of the domain may also account for the faster response of the effective layer system as compared to the Vimoke layer system: this highly conductive zone enhances the draining effect of the collector.

The less satisfying results come from simulations performed with Soil 2, which has the specificity of maintaining a high relative hydraulic conductivity in the unsaturated zone up to a pressure head of -0.6 m. Therefore, water may flow faster vertically through this zone directly to the drains in the reference case for this type of soil. This vertical infiltration is not well rendered by the equivalent-medium, because of the disappearance of drains, and appears to become important for this type of soil with large n values.

Mean water table level

To evaluate the influence of our equivalent approaches on water tables on a more quantitative side, we spatially averaged water table elevations above the impervious substratum for the three soils we had selected at steady state. This quantity is an integrated value, at the system scale, and may be of interest for the global moisture content of the soil or for the hydric state of its surface. The results are reported in Table 4 for each modeling system.

Table 4 clearly confirms what was previously observed concerning water table elevation: the layered S2 case appears to slightly underestimate the global elevation of the water table because of the fast transit of water in the draining layer preventing water from accumulating above this layer. On the other hand, the homogeneous S3 approach has the inverse tendency towards higher mean water table values because it facilitates vertical flow above the original drain level, and then a “better” distribution of water through the entire domain. Therefore, it is worth noting that the differences between all values are not very large, approximately 2–4 cm for a domain that is 1.5 m deep. These values are nearly equal for Soils 1 and 3, while the water table is lower for Soil 2, because the unsaturated zone is much more conductive (for the hydraulic head values encountered in this test case) for this soil, which confirms the results obtained for outflows.

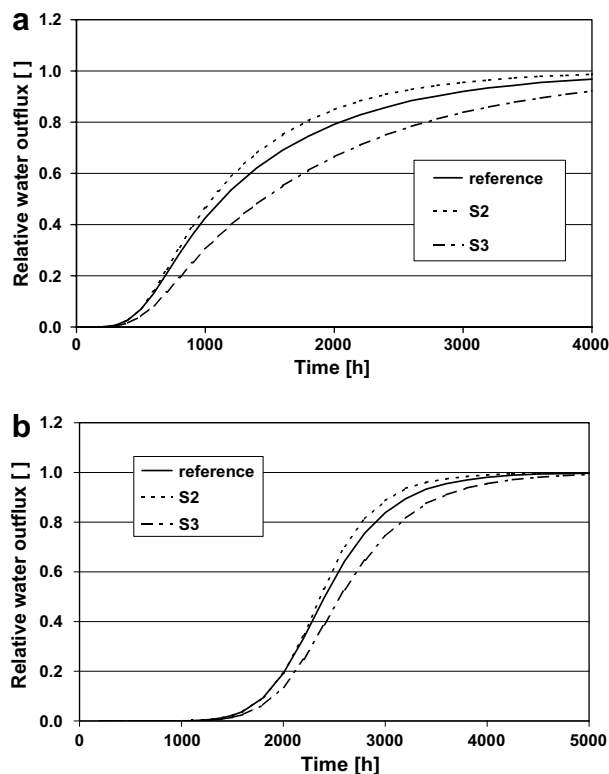


Figure 5 (a) Relative water outflow Soil 2. (b) Relative water outflow Soil 3.

Table 4 Mean water table elevations (m) at steady state

System	Soil		
	1	2	3
Reference	0.572	0.549	0.578
S2	0.553	0.538	0.558
S3	0.616	0.576	0.625

Influence of recharge intensity

To evaluate the influence of recharge intensity on the accuracy of the equivalent approach, we multiplied the original flux R by a factor of 2 and 4 and conducted calculations until steady state was reached using the properties of Soil 1. This modified the value of the dimensionless ratio $\frac{R}{K_s}$ governing the dynamics of drained soils (e.g. Van der Ploeg et al., 1999) from the initial value of 1.25–2.5% and 5%, respectively.

Fig. 6a and b plots relative outflows for two values of inflow for the original and the equivalent systems.

From Fig. 6a and b, and in comparison with Fig. 3a, it can be seen that the value of the inflow on the upper boundary of the domain has no significant influence on the capacity of the equivalent system to reproduce the dynamics of water outflow, despite the changes in the time required to reach equilibrium (when the relative outflow reaches 1).

The discrepancy is, in absolute values, smaller among systems for the highest influx value (Fig. 6b). In this case where R is highest, the saturated zone is extended to the point where the flow domain is largely saturated. The geometrical difference in the water table extension between the systems is smaller, hence the differences in outflows. This validates the assumptions made on this point for the definition of the equivalent homogeneous system S3 in particular.

As one can see in Table 5, the influence of inflow is higher in terms of the mean water table level. The higher the

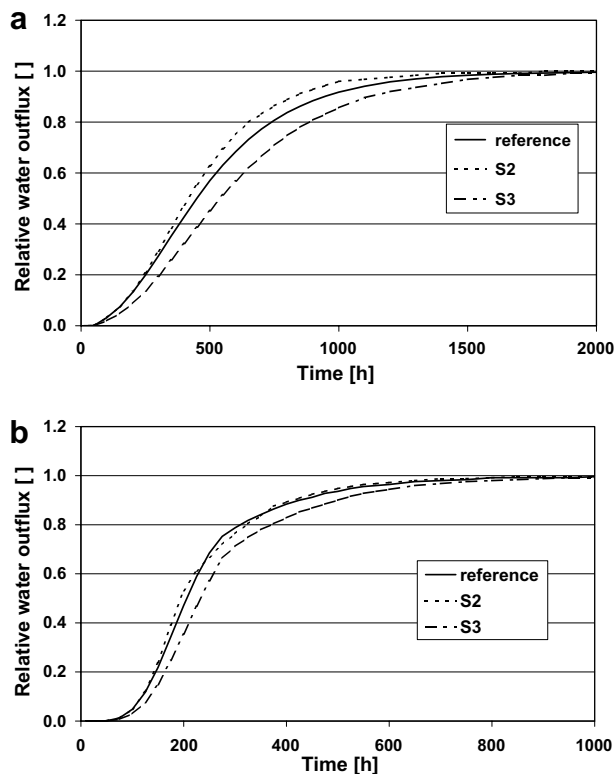


Figure 6 (a) Relative water outflux for $R/K_s = 2.5\%$, with R = constant recharge flux and K_s = saturated soil conductivity. (b) Relative water outflux for $R/K_s = 5\%$, with R = constant recharge flux and K_s = saturated soil conductivity.

Table 5 Mean water table elevations (m) at steady state for different values of R = constant recharge flux and with K_s = saturated soil conductivity

R (m h ⁻¹)	3.25E-05	6.50E-05	1.30E-04
R/K_s	0.0125	0.025	0.05
Water table (m)			
Reference	0.572	0.637	0.763
S2	0.553	0.596	0.683
S3	0.616	0.715	0.884

flux, the more “inflated” the water table, and the larger the discrepancy between values from the different systems.

Conclusions

The equivalent-medium approach we propose here appears to be an attractive first step towards simpler integration of artificially drained soils in watershed models. Although we were able to show that an equivalent-medium can simulate the system using the discretisation of the Richards equation in 3D, it has the advantage of being freed from internal boundary conditions (seepage-faces on the drain surface). This advantage is further consolidated by the fact that the medium can be described by a simplified mesh. Contrary to the electrical analogue method already implemented in the Hydrus 2D or 3D codes, this method also does not need to precisely localize the drainage network, since the only parameters required to calculate equivalent hydraulic conductivities are the drainage density and the drain radius, which generally are known.

The method provides satisfying results in terms of integrated output variables (outflow, spatially averaged water table elevation), i.e. the variables that may be potentially useful in large-scale modeling of a hydrological system. Since it must be considered as an equivalent representation tool, one should not expect accurate modeling of local values such as local velocities or water content.

However, this method requires further development before it can be applied on larger domains such as a network of drain plots. This approach implies further investigations to simplify modeling of the drainage collectors, as well as finding a more accurate way to represent soil with extreme hydrodynamic parameter values. This involves the development of a more precise method of averaging hydraulic conductivities for unsaturated flow, which would better account for the unsaturated part of the flow.

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