

Measuring magnetic field in the ‘diamagnetic’ ionosphere

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SUMMARY

Satellite magnetometers sometimes pass through regions of plasma, such as the terrestrial ionosphere, where the ionization is large enough that some of the original ambient field is excluded from the plasma. This reduction of field inside the plasma region comes from the ‘diamagnetic’ effect of the charged particles in their helical trajectory around the magnetic field lines. The (container of the) magnetometer will exclude the plasma, and a simple-minded approach, treating the ionosphere in the same way as for a conventional diamagnetic fluid, predicts that the field seen by the magnetometer will be somewhat larger than the (reduced) field in the plasma. However, the ‘diamagnetic’ properties of the ionosphere are quite different from those of a conventional diamagnetic. In particular, there is a ‘reflection’ of the ionospheric charged particles at the surface of the magnetometer, and the overall effect is that the magnetometer does actually measure the field present in the plasma before the magnetometer is inserted. Similarly, any leakage fields from the magnetometer have no effect in the magnetosphere.

Key words: diamagnetism, geomagnetic field, instrumentation, ionosphere, satellite.

1 INTRODUCTION

The magnetometers on low altitude magnetometer satellites such as CHAMP can now be calibrated in flight (including the spacecraft field) to give an absolute accuracy of a few nanotesla for the measurement of the ambient field, and in future the SWARM satellites should reduce this to about 1 nT (Olsen *et al.* 2006). When trying to measure the geomagnetic field of internal origin, to avoid most of the ionospheric and magnetospheric ‘noise’, only night-time data in magnetically quiet conditions are used. However, these spacecraft pass through regions of the ionosphere where there can be some ionization, even at night-time. The ionospheric charged particles have a helical motion around the magnetic field lines, and this leads to a ‘diamagnetic’ effect such that the field inside the ionosphere is slightly smaller than that of the original main field. To obtain better measurements of the main field itself, an attempt has been made to estimate and allow for this diamagnetic effect (Lühr *et al.* 2003).

As was pointed out by Lowes (1974a,b), if a magnetometer is inserted into (and hence excludes) a conventional diamagnetic fluid such as water, the field measured by the magnetometer will be slightly larger than that present before the insertion. If this were true also for the ionosphere, there would need to be a correction, of magnitude comparable to that of the ionospheric diamagnetic effect itself. However, ionospheric ‘diamagnetism’ is of a quite different nature to that of conventional fluids, and fortunately it turns out that the inserted magnetometer does, in fact, measure the field that was present before it was inserted.

In the next section, the relevant properties of conventional diamagnetics are summarized, and it is shown why an inserted magnetometer measures a slightly different field. In Section 3, the relevant

properties of the ionosphere are discussed, and it is shown how they differ from those of conventional diamagnetics, and hence why an inserted magnetometer reads the ambient field in the ionosphere before the magnetometer was inserted.

2 CONVENTIONAL DIAMAGNETISM

In a vacuum, the two vector magnetic fields $\mathbf{B}$ and $\mathbf{H}$ are proportional,

$$\mathbf{B} = \mu_0 \mathbf{H}. \quad (1)$$

However, inside a magnetized material there are atomic dipole moments which collectively give an average magnetization (dipole moment per unit volume) of $\mathbf{M}$. In between these atomic dipoles, the field varies rapidly in space, and the fields $\mathbf{B}$ and $\mu_0 \mathbf{H}$ can be thought of as *different* local macroscopic averages, taken over differently shaped regions which *exclude* the atomic dipoles. The two fields are now related by

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}). \quad (2)$$

For para and diamagnetic materials we find

$$\mathbf{M} = \chi \mathbf{H}, \quad (3)$$

with the (constant) susceptibility χ being negative (and small) for diamagnetic materials.

2.1 Field produced by a magnetized region

The field of a dipole having moment $\boldsymbol{\mu}$ is given by

$$\mathbf{H} = (1/4\pi)[- \boldsymbol{\mu} + 3(\boldsymbol{\mu} \cdot \hat{\mathbf{R}})\hat{\mathbf{R}}]/R^3, \quad (4)$$

and it would seem logical to calculate the field given by a volume of magnetized material by putting $\mu = \mathbf{M} d\tau$, and integrating over the volume:

$$\mathbf{H} = (1/4\pi) \int [-\mathbf{M} + 3(\mathbf{M} \cdot \hat{\mathbf{R}})\hat{\mathbf{R}}](1/R^3) d\tau. \quad (5)$$

Unfortunately, this integral is only semi-convergent at points *inside* the material. However it is a standard result (see e.g. Lowes 1974a) that (5) can be transformed to an Ampere-type Law for $\mathbf{B}$,

$$\mathbf{B} = (\mu_0/4\pi) \int_V (\text{curl} \mathbf{M}) \times (\hat{\mathbf{R}}/R^3) d\tau + (\mu_0/4\pi) \int_S (\mathbf{M} \times \mathbf{n}) \times (\hat{\mathbf{R}}/R^3) dS; \quad (6)$$

the surface integral is needed wherever there is a step in the magnetization (here assumed to be from finite to zero). Eq. (6) is convergent at any point, whether inside or outside the material. So, given $\mathbf{M}$, we can calculate the resultant $\mathbf{B}$ anywhere. The local $\mathbf{H}$ is then derived from $\mathbf{B}$ by using (2).

Note that while eq. (6) is *algebraically* identical to that giving the field from a volume current density $\mathbf{j}_m = \text{curl} \mathbf{M}$, and surface current density $\sigma_m = \mathbf{M} \times \mathbf{n}$, in a conventional magnetic material these are *virtual*, or *bound*, current densities, and inside the material the equation is valid only for $\mathbf{B}$. These $\mathbf{j}_m$ and σ_m would *not* occur in the Ampere's Law for $\mathbf{H}$, which contains only *real* current densities. (Similarly, $\mathbf{j}_m$ and σ_m do *not* contribute to $\text{curl} \mathbf{H}$.)

2.2 Magnetometer inserted into a diamagnetic fluid

Consider a large volume of diamagnetic fluid, such as water, in which there is a (locally) uniform field $\mathbf{B}_{\text{water}}$, with

$$\mathbf{B}_{\text{water}} = \mu_0(\mathbf{H}_{\text{water}} + \mathbf{M}_{\text{water}}). \quad (7)$$

For simplicity consider an 'ideal' magnetometer consisting of a small transducer (calibrated, as magnetometers usually are, for use in 'free space') at the centre of a spherical vacuum cavity; the cavity is large enough that the transducer produces negligible magnetic field at its boundary. This magnetometer is then inserted into the water. (It is assumed that the source of the field producing $\mathbf{H}_{\text{water}}$ is sufficiently distant that it is not affected by the insertion of the magnetometer.) After the insertion there will be a uniform field inside the spherical cavity, $\mathbf{B}_{\text{mag}} = \mu_0 \mathbf{H}_{\text{mag}}$, and this is what is measured by the magnetometer; however this $\mathbf{B}_{\text{mag}}$ will be different from the original $\mathbf{B}_{\text{water}}$. We can think of the change of field, from $\mathbf{B}_{\text{water}}$ outside to $\mathbf{B}_{\text{mag}}$ inside the cavity, being given by the virtual surface current corresponding to the resulting step change in $\mathbf{M}$ at the cavity boundary.

Inserting the cavity also produces a dipole field outside the cavity. Conventional magnetostatic theory (see e.g. Lowes 1974a) shows that inside the spherical cavity the uniform field is

$$\mathbf{B}_{\text{mag}} = \mu_0 \mathbf{H}_{\text{mag}} = \mathbf{B}_{\text{water}}/(1 + 2\chi_{\text{water}}/3). \quad (8)$$

The difference between what the magnetometer measures and the original field in the water is

$$(\mathbf{B}_{\text{mag}} - \mathbf{B}_{\text{water}}) = -\mathbf{B}_{\text{water}}(2\chi_{\text{water}}/3)/(1 + 2\chi_{\text{water}}/3). \quad (9)$$

For water χ is about -10^{-5} , so the difference (9) is positive, but small; in the surface geomagnetic field of 30 000–60 000 nT it is less than 0.4 nT.

2.3 Effect of magnetic field produced by the magnetometer

Many real magnetometers produce a magnetic field around them, and if they are immersed in a permeable medium this will produce extra magnetization in this medium; this extra magnetization will itself change the field seen by the magnetometer.

A simple example is the fluxgate magnetometer used by CHAMP (and SWARM). In this the fluxgate element is used as a null detector, adjusting the current in a surrounding compact spherical coil system (approximating surface currents on a sphere) until it cancels the ambient field; it is this feedback current which is used as a measure of the ambient field. If the coil system has radius a , and produces a uniform field $\mathbf{B}_f$ inside it, it will behave externally as a dipole of moment $2\pi a^3 \mathbf{B}_f / \mu_0$. Using (4) and (5), it is easily shown that the magnetization this dipole produces outside a cavity of radius b in turn produces a field $\delta \mathbf{B}_f$ at the centre of the cavity given by

$$\delta \mathbf{B}_f = (2\chi_{\text{water}}/3)(a/b)3\mathbf{B}_f. \quad (10)$$

As a consequence, the magnetometer will slightly change the feedback current, and instead of giving the output $\mathbf{B}_{\text{mag}}$ will give the output $\mathbf{B}_{\text{mag}}^*$ with

$$\mathbf{B}_{\text{mag}}^* - \mathbf{B}_{\text{mag}} = -\delta \mathbf{B}_f = -(2/3)\chi_{\text{water}}(a/b)^3 \mathbf{B}_f. \quad (11)$$

This is the same sign as the correction (9) for the effect of the cavity, and is (almost) the same magnitude as (9) if the cavity has the same size as the coil system. In practice, there is probably not the spherical symmetry assumed here, and the calculation will be more complicated. However, particularly if the cavity radius b is larger than the coil radius a , this correction is also almost certainly trivial for water.

3 THE 'DIAMAGNETIC' IONOSPHERE

The ionosphere can be treated either as a 'fluid' of charged particles, considering the equation of motion in terms of body forces acting on the fluid, or by considering the trajectories of individual charged particles, and then summing the individual contributions; the second approach is used here. These trajectories are complicated, but in the present context we are interested only in the collisionless approximation, and only in that part of a trajectory which looks like a circular orbit in a plane perpendicular to the local $\mathbf{B}$. If the charged particle has mass m , and an orbital speed v_{perp} , it is easy to show that, at a distance large compared with the orbit, this orbiting particle looks like a magnetic dipole having a moment of magnitude $1/2 m v_{\text{perp}}^2 / B$, whatever the magnitude of the electric charge. Also, whatever the sign of the charge, this magnetic dipole is directed opposite to $\mathbf{B}$.

For simplicity, in the rest of this paper only the contribution of one species of ion or electron is considered, and they will be called ions. If there are n ions per unit volume, this corresponds to a 'magnetization'

$$\mathbf{M} = -(1/B^2)(\frac{1}{2} n m v_{\text{perp}}^2) \mathbf{B}. \quad (12)$$

Treating the ions as a gas in thermal equilibrium at temperature T and pressure P , this becomes

$$\mathbf{M} = -(1/B^2) n k T \mathbf{B} = -(1/B^2) P \mathbf{B}. \quad (13)$$

Because of the minus sign, the ionosphere is often referred to as being diamagnetic.

In calculating the magnetic field given by this 'magnetization', if the ionosphere has no sharp boundaries then only the volume

integral of (6) is relevant. If we assume that locally the ambient field $\mathbf{B}$ is uniform, we find that the current density corresponding to the circular motion is

$$\mathbf{j}_{\text{ionos}} = \text{curl } \mathbf{M} = (1/B^2)\text{curl}(-P\mathbf{B}) = -(1/B^2)(\text{grad } P) \times \mathbf{B}, \quad (14)$$

which is indeed the expression obtained by the usual force-density treatment of ionospheric textbooks. But this is now a *real* current density, given by a (net) systematic motion of real ions. In a region where the ion pressure P is constant, there is no current; we can think of the ions as having random phase in their orbits, so that in a given small volume, at any one time as many are travelling in one linear direction as in the opposite direction, so there is no net current. But where there is a gradient of P (effectively of number density) the cancellation is not exact, and there is in consequence a net *real* local current density. In the present magnetic context, the ionosphere behaves like a vacuum, with $\mathbf{B} = \mu_0 \mathbf{H}$, but in which there are the *real* local current densities given by (14). It is now the ‘magnetization’ $\mathbf{M}$ of (12) and (13) which is *fictitious*, because the individual dipole moments are now not of *atomic* scale, but of *macroscopic* scale (with some Larmor radii larger than the size of our magnetometer). So many of the results of conventional magnetostatics are *not* applicable to the ionosphere.

In some respects we can think of the ionosphere as having an effective susceptibility of

$$\chi_{\text{ionos}} = M/H = -\mu_0 P/B^2, \quad (15)$$

but note that this ‘susceptibility’ is not constant, and not necessarily small. Even for the night-time *F*-region at altitude 500 km, local Appleton anomalies can give a ‘susceptibility’ of at least -10^{-4} , which is 10 times that of water.

If the main geomagnetic field is $\mathbf{B}_0$, at a given point inside the ionosphere the actual field will be

$$\mathbf{B}_{\text{ionos}} = \mathbf{B}_0 + \delta\mathbf{B}_{\text{ionos}}, \quad (16)$$

where the diamagnetic disturbance $\delta\mathbf{B}_{\text{ionos}}$ is that given by the currents $\mathbf{j}_{\text{ionos}}$ of (14). Inside an *F*-region Appleton anomaly, $\delta\mathbf{B}_{\text{ionos}}$ can be up to about 5 nT (Lühr *et al.* 2003), a significant amount even at the current magnetometer accuracy.

3.1 Magnetometer inserted into the ionosphere

Now insert our magnetometer into a region where $\mathbf{B}_{\text{ionos}}$ is locally uniform. In this region the $\delta\mathbf{B}_{\text{ionos}}$ of (16) comes from ‘remote’ $\mathbf{j}_{\text{ionos}}$ currents, so is not changed by the insertion. But the magnetometer physically displaces the ionosphere, and if the approach of Section 2.2 were valid, eq. (9) would predict that $\mathbf{B}_{\text{mag}}$ would be larger than $\mathbf{B}_{\text{ionos}}$, by up to a few nanotesla for *F*-region Appleton anomalies. This difference would come from a $\sigma_{\text{ionos}} = \mathbf{M} \times \mathbf{n}$ *real* surface current, just outside the magnetometer cavity, produced by the sharp discontinuity in ion pressure (density) P . However it turns out that such an approach is incomplete, and that this surface current is, in fact, cancelled by an opposite current!

This ‘diamagnetism’ of the ionosphere is essentially the same as an early suggestion (Schrödinger 1912) for the origin of diamagnetism in metals. This was that the free electrons in the metal would behave as described above for the ions in the ionosphere. For a uniform metal, with constant electron pressure, there would be no internal currents, and the diamagnetism would be produced by the corresponding $\mathbf{M} \times \mathbf{n}$ surface currents at the metal boundary. However it was pointed out (von Leeuwen 1921) that these currents

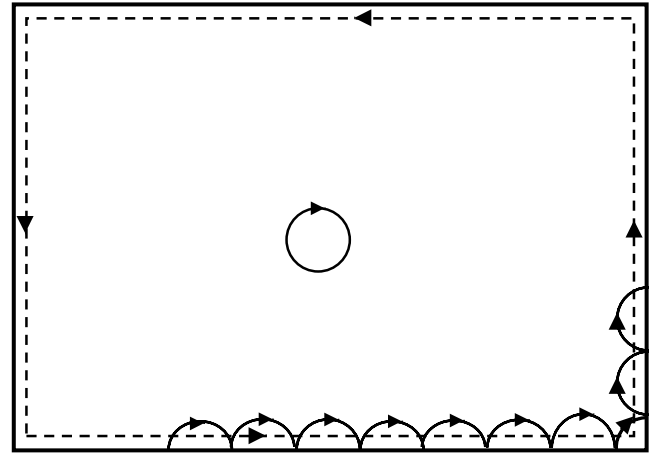


Figure 1. The curve in the centre illustrates the circular (or spiral) motion of a free electric charge, in the presence of a uniform magnetic field directed perpendicularly to the plane of the figure; if the charge is negative (e.g. an electron) the direction of motion, as drawn, corresponds to a magnetic vector *into* the paper. The series of arcs round the edge of the figure illustrate the path of such a charge near the (rectilinear) boundary of the gas, where the charge continually collides with, and is reflected from, the boundary; the magnetic field bends the path round again so as to produce further collisions, with the result that the charge travels round near the boundary, in a sense (as illustrated by the dotted line) opposite to the sense of the circular motion of the charges (such as the one near the middle of the diagram) that do not meet the boundary. This motion of the boundary charges neutralizes the diamagnetism due to the central charges. (Figure and legend copied by permission from fig. 23–8 of Chapman & Bartels 1940.)

would be exactly cancelled by another effect, coming from the internal ‘reflection’ of the electrons at this sharp outer boundary. These reflected electrons still see the ambient magnetic field, so start another, backward, trajectory until they again hit the surface, and are reflected again, see Fig. 1. It turns out that the overall effect of the reflection is that there is a drift of these reflected electrons along the surface, which is just such as to exactly cancel the $\mathbf{M} \times \mathbf{n}$ currents.

But this is exactly what will happen to ionospheric ions at the sharp boundary of our magnetometer. There will therefore be *no* local net surface current. The ‘remote’ $\mathbf{j}_{\text{ionos}}$ currents are unchanged, so the cavity field $\mathbf{B}_{\text{mag}}$ seen by the magnetometer will be the same as the $\mathbf{B}_{\text{ionos}}$ that was present before the magnetometer was inserted.

3.2 Effect of magnetic field produced by the magnetometer

Given the larger ‘susceptibility’ of the ionosphere, at first sight the correction (11) obtained above could now be significant. However it turns out that the magnetometer leakage field also has no effect in the ionosphere. Eq. (11) was obtained by using (5), but it could equally well be obtained by using (6). However, in the latter formulation the volume integral is zero, as this $\mathbf{M}$ is proportional to a curl-free dipole field. So *all* $\delta\mathbf{B}_f$ comes from the *surface* integral over the cavity surface. While in the conventional diamagnetic this (fictitious) surface current will be finite, in the ionosphere the corresponding (real) surface current will be exactly cancelled by the reflection process discussed above.

3.3 Historical note

Interestingly, this cancellation of surface effects is discussed in the first modern textbook of geomagnetism, that of Chapman & Bartels

(1940, section 23.14); Fig 1 is a copy of their fig. 23–8. (The formal mathematical argument is to show that the velocity distribution of the electrons remains symmetrically Maxwellian right to the surface, so there can be no net drift of the electrons to give a macroscopic surface current, but the Chapman & Bartels explanation is more appropriate to a ‘trajectory’ approach.)

It had been suggested that the Sq currents in the ionosphere might be produced by the ‘diamagnetic’ action discussed above. Workers in (what is now called) condensed matter physics had rejected a possible free-electron origin of diamagnetism in metals because of this ‘reflection’ effect; because of this, several geomagnetic authors spent quite a lot of time showing that the effect did *not* apply at the diffusive outer ‘boundaries’ of the ionosphere. However in his own work Chapman showed that while the geometry of the resultant current distribution was of the right sort of shape to give Sq, he concluded (correctly) that it was most unlikely that the effect would be large enough.

4 DISCUSSION

There have been several experiments (e.g. Lühr *et al.* 1986, Pfaff *et al.* 2004) in which a large local increase in ionization has been produced deliberately, and in which the resultant ‘diamagnetism’ was so large that the ambient magnetic field was observed to be almost entirely excluded. However, even if the ionosphere behaved like a conventional diamagnetic, the approach of Section 2.2 is valid only for small χ , and is not applicable to these extreme conditions. The only ‘observations’ I know of relevant to the present paper are those of Lühr *et al.* 2003. They found that (after removing main-field and crustal-field models) applying an approximate correction for the diamagnetic reduction (based on measurements of the local ion density) significantly reduced the CHAMP magnetometer field residuals in the regions of Appleton anomalies; this strongly supports the conclusion of this paper that the results of Section 2.2 are *not* applicable.

Until now there has been an implicit assumption, by those measuring magnetic field in the ionosphere, that their magnetometer did, in fact, indicate the field that was present before it was inserted. It turns out that we are in the happy situation that this assumption is, in fact, justified. This is particularly useful, because it presumably follows that many other possible sources of perturbation can also be neglected. These include, for example: shadowing of the magnetometer by the (much larger) satellite of which it is a part; the problem that while the satellite and/or magnetometer might be large compared with the Larmor radius of the electrons, it can be small compared with the Larmor radius of the ions; and the production of shock waves in the ions, if not electrons, by the satellite motion.

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