

# ***Computational modeling of the 1991 block and ash flows at Colima Volcano, Mexico***

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## **ABSTRACT**

**A new numerical code for simulating flows of granular material, TITAN2D, is used to model the Merapi-type block and ash flows resulting from the 1991 eruption of Colima Volcano, Mexico. The 1991 block and ash flows reached distances of up to 4 km from the vent with a total estimated volume of  $8 \times 10^5 \text{ m}^3$ . The block and ash flows were modeled using a digital elevation model (DEM) of the region and compared to field data using a quantitative center-line comparison method. Input parameter values, which dictate flow dynamics, were varied to demonstrate the effect these parameters have on the program output. Analysis showed that the TITAN2D model performs best in replicating the center lines of the 1991 Colima block and ash flows when using an initial volume value representative of a single flow event rather than a total deposit volume.**

**Keywords:** Mexico, Colima, block and ash flows, granular flow, modeling.

## INTRODUCTION

Block and ash flows result from gravitational collapse or explosion of a lava dome or lava front (Francis, 1993; Freundt and Bursik, 1998; Bursik et al., 2005). They range in size from inconsequential rockfalls to the catastrophic pyroclastic flows responsible for over 29,000 deaths at Mount Pelee, West Indies in 1902 (Fisher and Heiken, 1982; Blong, 1984). There is interest in developing computer models that are able to accurately simulate these events to reduce future loss of life and property. The ability to model relic flows in predicting the paths of future flows not only aids government authorities in their hazard mitigation planning efforts through the creation of hazard maps but also provides a tool for the development of emergency management scenarios and public awareness. Recent advances have been made in creating computational models of these flows for the purpose of hazard mitigation (McEwen and Malin, 1989; Iverson et al., 1998; Takahashi and Tsujimoto, 2000; Denlinger and Iverson, 2001; Patra et al., 2005). One well-accepted validation method used to test a geophysical flow model is to use data from natural flows and attempt to simulate them in a digital environment (McEwen and Malin, 1989; Iverson et al., 1998; Takahashi and Tsujimoto, 2000; Denlinger and Iverson, 2001; Patra et al., 2005). To realistically model geophysical flows, model development has involved the use of digital elevation models (DEMs). DEMs are digital three-dimensional representations of natural terrain.

An early modeling attempt was made by McEwen and Malin (1989), who developed the FLOW model to simulate pyroclastic flows, rockslide-avalanches, lahars, and the blast from the 1980 eruption of Mount St. Helens, Washington. The FLOW model calculates the velocity and simulated flow path of an event over a digital terrain model. The movement of the flow is determined by initial conditions, gravitational acceleration, and resistance to motion. The resistance to motion is described by FLOW as Coulomb, viscous, or turbulent. This model takes a simplified approach to solving the complex physics driving such flows in nature. This model does not account for conservation of mass, and individual masses do not interact with one another. McEwen and Malin (1989) also found that calculated flow velocities were much too high, creating flow paths that responded poorly to topography.

A new class of geophysical flow models began with the work of Eglit and Sveshnikova (1980) and later Savage and Hutter (1989). The Eglit model assumes a one-to-one relationship between the strain in the down-flow and cross-flow directions, whereas the Savage and Hutter model assumes this relationship to be a more complex function. Savage and Hutter (1989) developed a one-dimensional model regarding these phenomena as depth-averaged granular flows, governed by Coulomb-like interactions between the individual grains. Savage and Hutter later developed the one-dimensional model into a two-dimensional model through the introduction of a simple curvilinear coordinate system that controlled flow propagation. Iverson and Denlinger (2001) further developed this model into a depth-aver-

aged, two-dimensional model able to account for solid and fluid phase forces and interactions. This model is capable of simulating dense granular flows across three-dimensional terrain. This model of granular flow physics, in the dry limit (without interstitial fluid), was used in the development of the TITAN2D model.

Takahashi and Tsujimoto (2000) created a numerical model of Merapi-type pyroclastic flows for Unzen Volcano, Japan. This model is based upon the granular flow physics developed by Savage and Hutter (1989). The 1991 eruption of Unzen Volcano in Kyushu, Japan, resulted in Merapi-type block and ash flows similar to those emitted from Colima Volcano that same year. As the emerging lava dome collapsed, large lava blocks were crushed as they moved down steep slopes. The crushed lava blocks created an inertial granular flow in which the dominant particle size was greater than 1 mm in diameter. At this point, stress from the collision of individual particles was the dominant mechanism of flow. Due to the large size of the particles, juvenile gas emitted from the colliding lava blocks had a minimal effect on the flow. Most of the large particles that comprised the inertial granular flow broke apart, dropped out, and deposited a short (0–1.6 km) distance from the starting point on steep slopes. The bulk of the remaining particles were smaller than 1 mm. These fine grained particles continued to travel, supported by the upward flow of gas emitted from the breaking particles. This upward flow of gas resulted in a small resistance to flow, which complimented the continued momentum of the flow. The flow at this point was composed of a lower poorly fluidized layer and an upper fluidized layer that was supported entirely by the upward flow of emitted gas. When the flow reached a gentler slope, the larger particles dropped out of the bottom layer due to increased resistance. The deposition of larger gas-emitting particles left little upward flow of gas to support the fluidized second layer, which caused the entire flow to stop. The remaining small particles and gas formed a hot ash cloud above the main body of the flow. The hot ash cloud continued to travel on its own, fueled by the supply of particles and gas from the main body.

The analysis of the simulation results of Takahashi and Tsujimoto (2000) consisted of qualitative comparisons between the actual flow limits and the modeled results. The researchers performed a simple visual analysis of their results by overlaying the numerical model results on top of the observed flow outline. Takahashi and Tsujimoto (2000, p. 112) remarked that “. . . the general tendencies of width and length are well reproduced,” and that the deposit areas of the calculated and natural flows “almost completely overlap.”

Heinrich et al. (2001) simulated the debris avalanche emplaced on 26 December 1997 in Montserrat, Lesser Antilles. The debris avalanche originated in response to collapse of the southwestern flank and the expanding lava dome of Soufrière Hills volcano. The estimated volume of this flow deposit was  $40 \times 10^6 \text{ m}^3$  to  $50 \times 10^6 \text{ m}^3$ . The flow traveled ~3 km, stopping just short of the coastline. As a means of comparing the calculated numerical flow results with the observed debris avalanche, Heinrich et al. (2001) performed a simple visual analysis to judge

goodness-of-fit. The observed flow versus the calculated flow displays a similarity in shape and runout length, yet there is no quantification of these observations.

The lack of quantitative analyses of simulation outputs from the aforementioned models seems to be commonplace. In general, for simulations of past events, a visual comparison between observed flow outlines (typically mapped in the field after the event) with the simulation results has been deemed sufficient.

Simulations of the 1991 block and ash flows of Colima Volcano, Mexico, were created with the TITAN2D model developed by the Geophysical Mass Flow Group (GMFG) at the State University of New York, Buffalo (Patra et al., 2005). The TITAN2D model is an improvement over other geophysical flow models such as Laharz (Iverson et al., 1998) and Flow3D (McEwen and Malin, 1989; Kover, 1995) in that it is based on an incompressible Coulomb continuum model rather than a much-simplified “sliding-block” model or “valley-filling” empiricism. TITAN2D is based on depth-averaged conservation of mass and momentum equations, allowing for computational tractability.

The goal of the present work is to use topographic and geologic data to test the ability of the TITAN2D program to accurately model the 1991 block and ash flows of Colima Volcano, Mexico. The primary hypothesis being tested is that the TITAN2D model can be utilized to replicate the flow center lines of the 1991 block and ash flows of Colima Volcano. Simulation output will be qualitatively analyzed for goodness-of-fit to the observed flow data by analyzing center-line comparison statistics.

### BLOCK AND ASH FLOWS OF 16–17 APRIL 1991 AT COLIMA VOLCANO, MEXICO

In the early months of 1991, Colima Volcano began an episode of effusive activity, reactivating for the first time since 1982. Primary indicators of reactivation included increased seismic activity, increased fumarolic activity, and the appearance of new fractures on the summit dome (Connor et al., 1991; Núñez Cornú et al., 1994). On 1 March 1991 eyewitnesses observed the formation of a new lava dome on the summit. This new dome was composed of scoriaceous, dark-gray andesite (59.7 wt% SiO<sub>2</sub>; Saucedo et al., 2004). By the next day the new dome was 27 m wide and 6.5 m high. Measurements taken on 27 March revealed that the dome was now 100 m in diameter and 30–40 m high (Connor et al., 1991; Rodríguez-Elizarrarás et al., 1991; Núñez Cornú et al., 1994; Saucedo et al., 2004). On 16 April 1991, bystanders witnessed the first pyroclastic flows resulting from partial collapse of the newly formed dome. At 16:00 h (local time, Greenwich mean time –6 h) a series of relatively large, shallow volcanic earthquakes initiated a gravitational collapse of the growing dome. The dome collapsed in a series of prolonged Merapi-type pyroclastic flows onto the southern slopes of the volcano (Fig. 1). A small number of avalanches occurred throughout 17 April until around 19:00 h (Rodríguez-Elizarrarás et al., 1991). Núñez Cornú et al. (1994) report that between

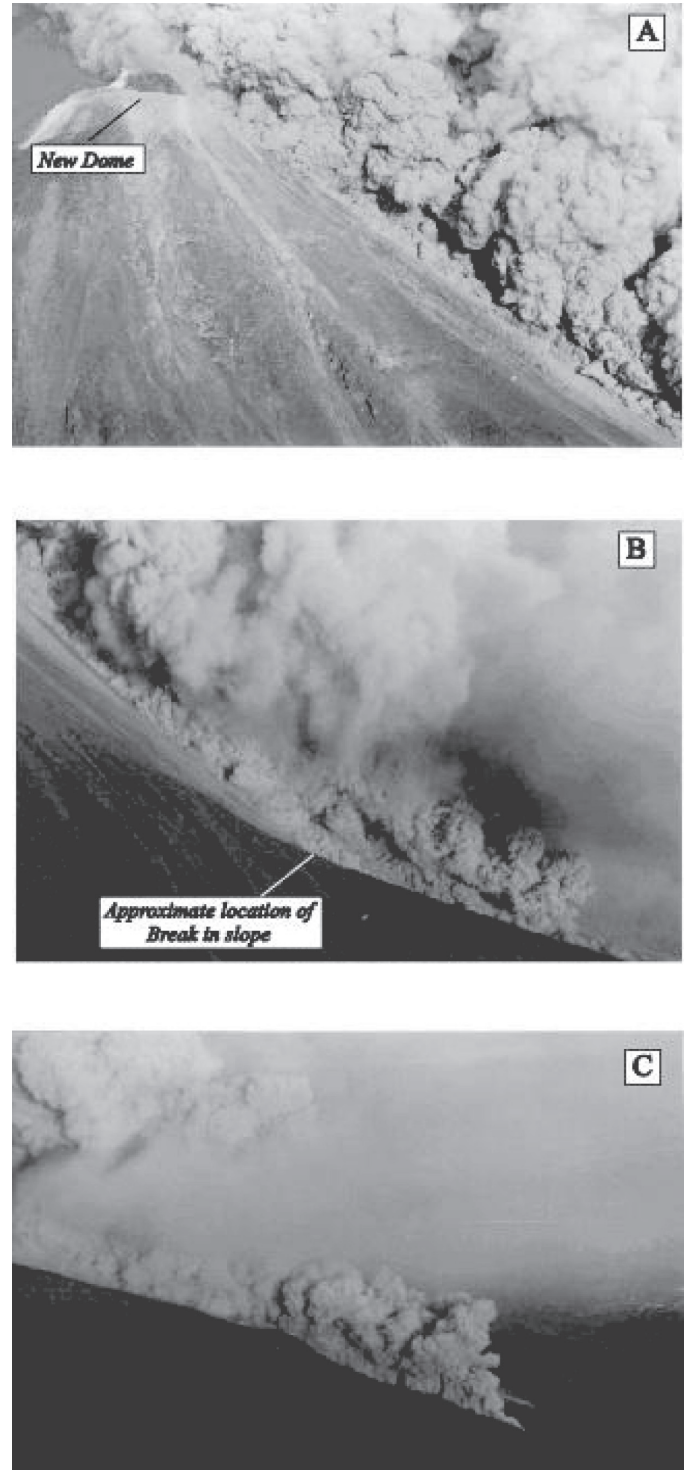


Figure 1. Aerial photos of the 16 April 1991 pyroclastic flow. (A) Partially collapsing 100-m-wide lava dome with block and ash flow moving down the  $>30^\circ$  slope. (B) Flow has moved through the break in slope from  $>30^\circ$  to  $<20^\circ$ . (C) Flow continues to travel along the  $<20^\circ$  slope. Photographs by Ernesto Gomez Hoffman (from Saucedo et al., 2004).

February and April more than 1800 rock falls occurred in addition to the Merapi-type flows of 16–17 April.

After flowing down the steep upper southwestern slope of the volcano, the 1991 block and ash flows entered the upper branches of the Barranca El Cordobán (West, Central, and East) (Fig. 2). At this point the flows were clearly influenced by topography. Rodríguez-Elizarrarás et al. (1991) report the flows followed the three channels to a maximum runout distance of 4 km at an altitude of 2100 m. The same authors estimate the total volume of the deposits to be  $8 \times 10^5 \text{ m}^3$ . Block and ash flow deposits within the three Cordobán gullies have a recorded thickness of up to 10 m (Saucedo et al., 2004). The extent of the 1991 pyroclastic flows can be seen in Figure 3. These data provide the background information needed to establish initial conditions forming the parameter data set used in the TITAN2D program. Observed

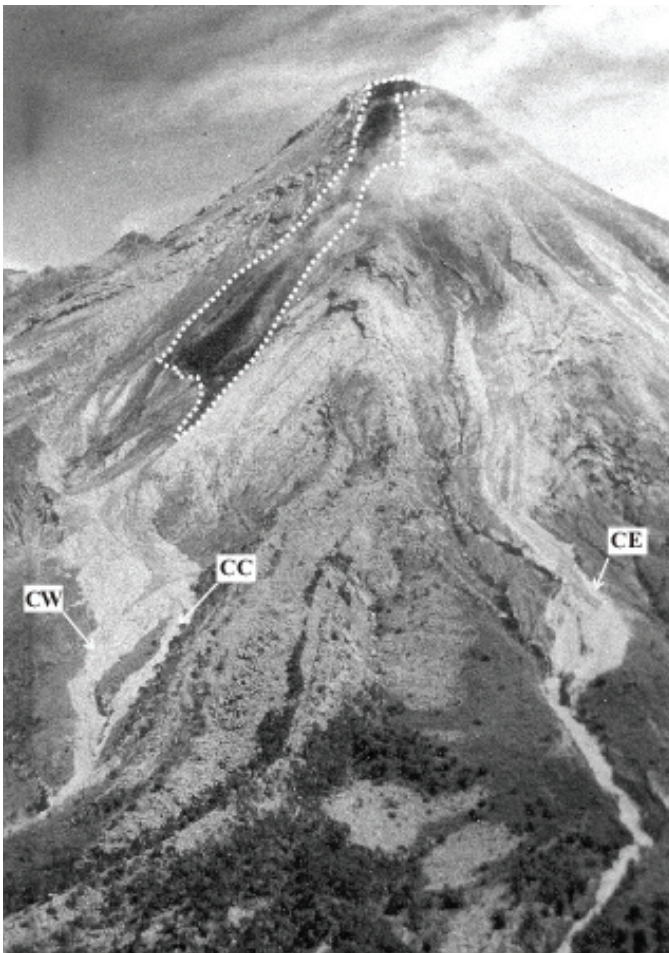


Figure 2. 16 April 1991 pyroclastic flow deposits on Colima Volcano. A later dark-gray lava flow descending from the summit is traced in white. CW—Cordobán West; CC—Cordobán Central; CE—Cordobán East gullies. A map view of these pyroclastic flow deposits can be seen in Figure 3. Photograph by C. Siebe, 1991 (from Saucedo et al., 2004).

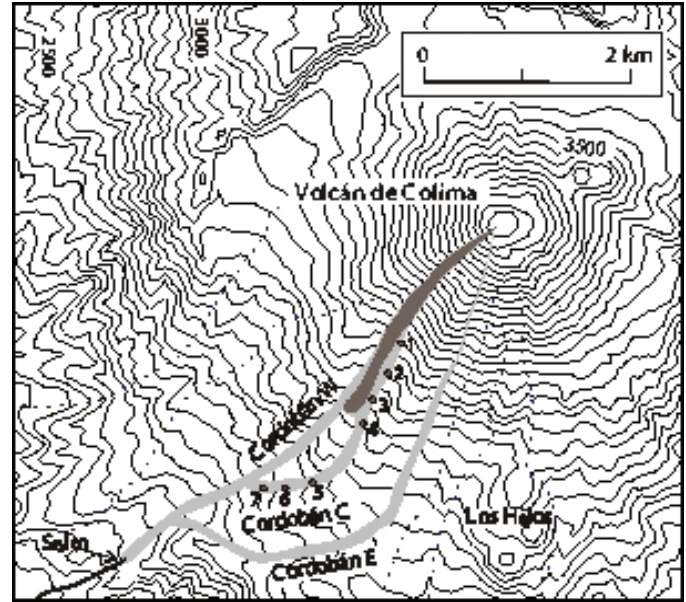


Figure 3. Mapped extent of pyroclastic flow products from the 16 April 1991 Colima volcano eruption (from Saucedo et al., 2004).

1991 Colima block and ash flow data are also used to quantify how well TITAN2D simulations model the event.

Deposit stratigraphy was studied in 1998 by Saucedo et al. (2004). Seven sites along a 4 km longitudinal section of El Cordobán Central gully were analyzed. Three distinct block and ash flow deposits were noted from the 16–17 April 1991 eruption: F1, F2, and F3.

## TITAN2D

### Description of the Model

TITAN2D is a program developed for the purpose of simulating dry granular avalanches over digital representations of natural terrain. The program is designed for simulating geological flows such as debris avalanches and landslides. TITAN2D combines numerical simulations over representations of natural terrain supported through a geographic information system (GIS) interface. TITAN2D represents the latest research in numerical modeling and parallel computing with integrated support through a GIS (Patra et al., 2005).

The TITAN2D program is based upon a model for an incompressible Coulomb continuum, a “shallow-water” granular flow (Denlinger and Iverson, 2001). The conservation equations for mass and momentum are solved with a Coulomb-type friction term for the interactions between the granular material and the basal surface (Savage and Hutter, 1989). It is assumed that conservation of energy can be neglected to first order because the coarse grain size, typical of block and ash flows, results in minimal thermal effects on avalanche propagation.

The resulting hyperbolic system of equations is solved using a parallel, adaptive mesh, Godunov scheme (Berger and Colella, 1989; Davis, 1988). The Message Passing Interface (MPI) (<http://www-unix.mcs.anl.gov/mpi/>) Application Programming Interface (API) allows for computing on multiple processors, increasing computational power, decreasing computing time, and allowing for the use of large data sets. Adaptive gridding allows for the concentration of computing power on regions of special interest. Mesh refinement captures the leading edge of the flow, as well as locations where the topography changes rapidly. Mesh unrefinement is applied where solution values are relatively constant or small, to further improve computational efficiency.

The model used in TITAN2D assumes a continuum volume, parcels of which are pulled downslope by gravity. Friction between particles and the ground resist this movement. Governing equations for this model, the conservation of mass and conservation of momentum, are solved using a finite-volume scheme with a second-order Godunov solver. Fluxes at the cell edges are evaluated using an approximate Riemann solver. The outputs of TITAN2D are flow momentum, depth, runup height, and inundation area.

TITAN2D operates via a Python-scripted graphical user interface (GUI). Through this interface the user inputs the parameters needed to successfully run the program such as pile dimensions, starting coordinates, internal and bed friction angles, and simulation time. The simulation is computed on a DEM of the desired region, and results can be displayed through the TITAN2D viewer utility or other visualization software packages.

TITAN2D performs flow simulations on a DEM of a user-defined region. The DEM file containing X, Y, Z data (typically latitude, longitude, and elevation in meters) must be properly configured to operate with TITAN2D through the use of a header. The DEM data file is then formatted to operate in a GRASS (Geographic Resources Analysis Support System) GIS environment. GRASS GIS is an open source GIS with raster, topological vector, image processing, and graphics production functionality. It operates on various platforms through a graphical user interface and shell in XWindow. GRASS is available free of charge under the GNU General Public License (GPL; <http://grass.ibiblio.org/index.html>).

The GRASS GIS interface used in conjunction with TITAN2D allows the user to adjust the area of the initial DEM. This capability decreases output file size, thus increasing visualization speed while allowing the user to focus attention upon a specific region of interest within a much larger DEM.

Simulation accuracy is highly dependent on the level of DEM resolution and quality. DEMs exist in a wide range of resolutions that relate to the level of accuracy to which the digital surface emulates actual terrain. DEMs with higher resolutions (e.g., 5–30 m) render more accurate representations of actual geophysical flow events, especially in situations where channelized flow is involved.

## Inputs

Before attempting to simulate a flow with TITAN2D, data regarding the physical nature of the flow must be gathered. These data are entered into TITAN2D by the user. The user must enter the values for the following parameters: (1) internal friction angle, (2) basal friction angle, (3) maximum initial pile thickness, (4) center of initial volume (in Universal Transverse Mercator [UTM] coordinates), (5) northern and eastern extent of the pile (pile geometry), and (6) elongation angle of pile from northern axis to eastern axis.

The internal and basal friction angles of granular material are important in controlling flow movement. The internal friction angle is a measure of the strength of the material's bulk and is attributable to two factors, the friction between individual grains and the geometrical interlocking between particles known as Reynolds dilatancy (Daerr, 2001). Basal friction angle is a measure of the friction between the grains and the bed surface. Friction angles were chosen based upon table-top grain flow experiments and trial and error.

In the table-top experiments the internal friction angle was measured by increasing the angle of a platform upon which a pile of sand was placed and measuring the angle at which visible internal movement occurred. The average internal friction angle achieved by performing this experiment was 37.3°. The basal friction angle was measured by performing a series of experiments, the first of which measured the angle at which a plastic block began to move when placed upon a thin layer of sand. The average basal friction angle achieved by performing this experiment was 18.2°. The second experiment measured the angle at which sand released from a pipet onto an inclined plane ceased to move down the plane. The average basal friction angle achieved by performing this experiment was 28.9°. The final experiment measured the slope at which sand inside a rectangular container came to rest after the container was tilted onto its side. The average basal friction angle achieved by performing this experiment was 34.4°. The sand used in the friction angle experiments contained  $2\phi$  (187–250  $\mu\text{m}$ ) particles. The angles measured in these experiments were used as a starting point for the internal and basal friction angle input parameters. Various angles were used to study the effects on the simulated flows.

Initial geometry is based on published 1991 dome volume data (Rodríguez-Elizarrarás et al., 1991; Saucedo et al., 2004). Using the published volumes of the lava dome and of the pyroclastic flow deposits, initial pile thickness ( $Z$ ) and the northern and eastern extents were established using the TITAN2D “calculate volume” utility. If the final volume is known, the X, Y, and Z values can be chosen through trial and error until the simulation volume matches the published volume. The starting location of the initial mass (in UTM coordinates) was chosen based on video recordings, aerial photographs, and eyewitness observations of the 16 April 1991 event. Choosing the precise coordinate in the situation of a dome or lava front collapse is somewhat arbitrary, however one can surmise that the point of actual collapse was

100–200 m southwest of the geographic summit of Colima Volcano. There is no published information on the elongation of the collapsing dome so this angle was set to the default value of 0°.

## DIGITAL ELEVATION MODELS

Simulations performed by TITAN2D were done upon a DEM of the Colima Volcanic Complex. The age and resolution of the DEM used is an important issue when testing the accuracy of any terrain-dependent model. The DEM used in this project was created from the Shuttle Radar Topography Mission (SRTM) of February 2000. In the SRTM a specially modified radar system was mounted on to the Endeavor. The radar system collected elevation data on a near-global scale as it orbited the Earth during an 11-day mission. The mission resulted in the most complete global high-resolution digital topographic data set ever assembled. SRTM data for areas outside of the United States are available in 3 arc-sec (~90 m) resolution. The SRTM DEM was used to test the applicability of this global data set to TITAN2D flow simulations. Although using a finer resolution DEM of Colima Volcano would provide more precise data, none exist. The DEM was not modified in any way to represent the actual topography present during the 1991 events. This leaves a margin of error due to the fact that the simulations were performed upon topography representing the posteruption conditions.

Despite its 90 m resolution the SRTM DEM reasonably defines the channels present in the Colima Volcano Complex. Experimentation showed that there were some problem areas where the channel involved in a TITAN2D simulation run was poorly defined. Overall, this particular DEM was well suited to the needs of this research.

## ANALYSIS

### Comparing Simulated Flow Path with the Observed Path

The main method of analysis is a comparison of simulated and observed flow center lines. The software package used for the analysis is the Sistema de Processamento de Informações Georeferenciadas (SPRING) geographical information system (GIS). This free GIS package was developed by Brazil's National Institute for Space Research (INPE; Câmara et al., 1996). This procedure could of course be performed using any GIS package.

First, the 1991 flow deposit map from Rodríguez-Elizarrarás et al. (1991) was digitized and uploaded into SPRING GIS. The digitized flow deposit map was georectified in order for it to be precisely geographically oriented. A center line for this flow outline was then calculated using SPRING GIS. Next, the TITAN2D output files were converted to a grid file containing the maximum calculated flow depths for each cell in the DEM area. The SPRING slicing utility was then used to establish various bins of flow depth. Slicing intervals were established through trial and error until the best visual representation of the

pile depths was displayed. Using the displayed pile depths for the entire length of the flow, the SPRING GIS vector drawing tool was used to create a flow center line by connecting the points of maximum pile depth. Completion of this step resulted in a center line of the simulated flow. The simulation center line could then be compared to the center line of the observed 1991 flows.

Comparison of the simulated and observed flow center lines involved the following steps. First, using SPRING GIS, contour lines based on the SRTM DEM were established at 100 m intervals over the region covered by the flow. At every 100 m elevation stop a measurement was made from the point where the contour line intersected the measured flow center line to the point where the same contour line intersected the center line of the simulated flow. These same measurements were made at all contour lines that intersected both the measured and the simulated center lines along the entire length of the flow.

The data set of these center-line measurements was then used to perform a statistical analysis of the offsets. The center-line offsets were also graphed to observe general trends in the simulated versus observed flow center-line offsets.

### Altering Input Parameters in TITAN2D

A critical test of any geophysical flow model is to compare and contrast numerous simulation runs to see how the model outputs change as input parameters are adjusted. The input parameters that have the largest effect on simulated flow propagation are: (1) internal and basal friction angles of the flow, (2) depth of the initial mass or pile to be modeled, (3) X, Y dimensions of the initial mass, and (4) starting position of initial mass. This research involved hundreds of TITAN2D simulation runs while analyzing the accuracy of the program and its applicability to modeling block and ash flows. The Results section below highlights the effects of altering input parameters on five TITAN2D simulation runs of the 1991 block and ash flows.

Table 1 displays the input data for the five example TITAN2D simulation runs of the 1991 block and ash flows. Simulation runs were computed using varying internal and basal friction angles, starting locations, and initial starting volumes. The goal was to discover what effect modifying each parameter would have on the simulation as well as to decide which simulation best fit the observed flow outline. The TITAN2D-generated output files for these runs were further analyzed using SPRING GIS.

Each TITAN2D simulation was performed on a Dell desktop with a Pentium 4, 1.80 Ghz processor. Output files for each time step of the simulation were converted into an X, Y, Z (latitude, longitude, pile depth) data file using a program that extracts the maximum pile heights for the entire simulation. This file was then imported into SPRING GIS as a grid file. The grid file contains the maximum pile depth calculated for each cell within the calculation area for the duration of the simulation run. Using this file and the SPRING slicing utility, georeferenced flow maps showing the maximum depth of the flows at all points were produced. The flow maps provide the user with other important flow

TABLE 1. TITAN2D SIMULATIONS USED TO MODEL 1991 BLOCK AND ASH FLOWS OF COLIMA VOLCANO

Run name	$\phi$ internal (°)	$\phi$ basal (°)	Starting position	Volume	Digital elevation model
Jan_26_d	30	20	260 m SW of Summit	$8 \times 10^5 \text{ m}^3$	SRTM
Feb_10a	37	20	260 m SW of Summit	$8 \times 10^5 \text{ m}^3$	SRTM
Feb_14a	37	28	260 m SW of Summit	$8 \times 10^5 \text{ m}^3$	SRTM
Feb_16a	37	20	Summit	$8 \times 10^5 \text{ m}^3$	SRTM
Feb_22a	37	20	260 m SW of Summit	$1.4 \times 10^5 \text{ m}^3$	SRTM

Notes: SRTM— Shuttle Radar Topography Mission

information such as runout length, runup height, and basic flow geometry.

### Statistical Analysis

To quantify the significance of the center-line offset measurements, statistical information was extracted from each set of measurements. The list of statistics below was calculated from each set of measurements. The arithmetic mean is the sum of the center-line offset measurements divided by the number of offset measurements collected. If the mean is high, the channel followed by the model may have been systematically far from the measured channel, either to the east or to the west.

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i, \quad (1)$$

where  $x_i$  is each measure of distance between contour crossing of simulated flow and of flow deposit outline, and  $N$  is the number of measurements of this distance. The root-mean square (RMS) distance is the square root of the sum of the squares of the sampled values:

$$RMS = \sqrt{\frac{1}{N} \sum_{i=1}^N x_i^2}. \quad (2)$$

This is of use when a value goes between negative and positive (west and east), allowing useful averages other than zero to be found, despite the quantity having spent equal amounts of time being positive and negative. This value is important in this research as westerly center-line offsets were assigned a negative value and easterly offsets were positive. If the RMS is high, the channel followed by the simulated flow was far from the measured channel. RMS differs from the mean in that it is a different measure of the central tendency. For example, the mean could be zero because of alterations between east and west, but the root mean square could still be high.

High standard deviations suggest a great deal of variability in the measured offsets between the model runs and the observed events:

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2}. \quad (3)$$

Average absolute deviation is the average of the absolute differences of data points from their mean. Average absolute deviation is a measure of the systematic variability in the center-line measurements.

$$\alpha = \frac{1}{N} \sum_{i=1}^N |x_i - \mu| \quad (4)$$

## RESULTS

### TITAN2D Block and Ash Flow Simulations

The outputs of five example TITAN2D simulation runs of the 1991 block and ash flows of Colima Volcano were used to create the simulated flow maps seen in Figures 4A–E. Out of the hundreds of TITAN2D simulations attempted for the 1991 block and ash flows, these five were selected to illustrate the process one would use to get a reasonable output. The five runs show a progression from a first attempt at modeling the event, toward a convergence with a more reasonable simulation. The maps display the maximum pile depths calculated by TITAN2D for the entire duration of the simulation run. Each run was computed on a 90 m SRTM DEM of the region using varying input parameters (Table 1). The simulated flow maps were generated in SPRING GIS using the TITAN2D output files. The sequence of simulation outputs represents the typical trial-and-error path a TITAN2D user may follow while attempting to create a “best-fit” flow simulation.

#### TITAN2D Simulation “feb\_16a”

The flow map for feb\_16a (Fig. 4A) represents a typical first attempt at simulating an observed flow. In this run the internal friction angle was set to  $37^\circ$  and the basal friction angle to  $20^\circ$ . These values were chosen based on values obtained in table-top experiments performed to measure the internal and basal friction angles.

The initial volume was set to  $8 \times 10^5 \text{ m}^3$  (the total deposit volume from Rodríguez-Elizarrarás et al., 1991), and the starting point of the initial pile was chosen to be a coordinate directly on the summit of Colima Volcano. As observed in the flow map, the simulated run travels a path east of the known channels

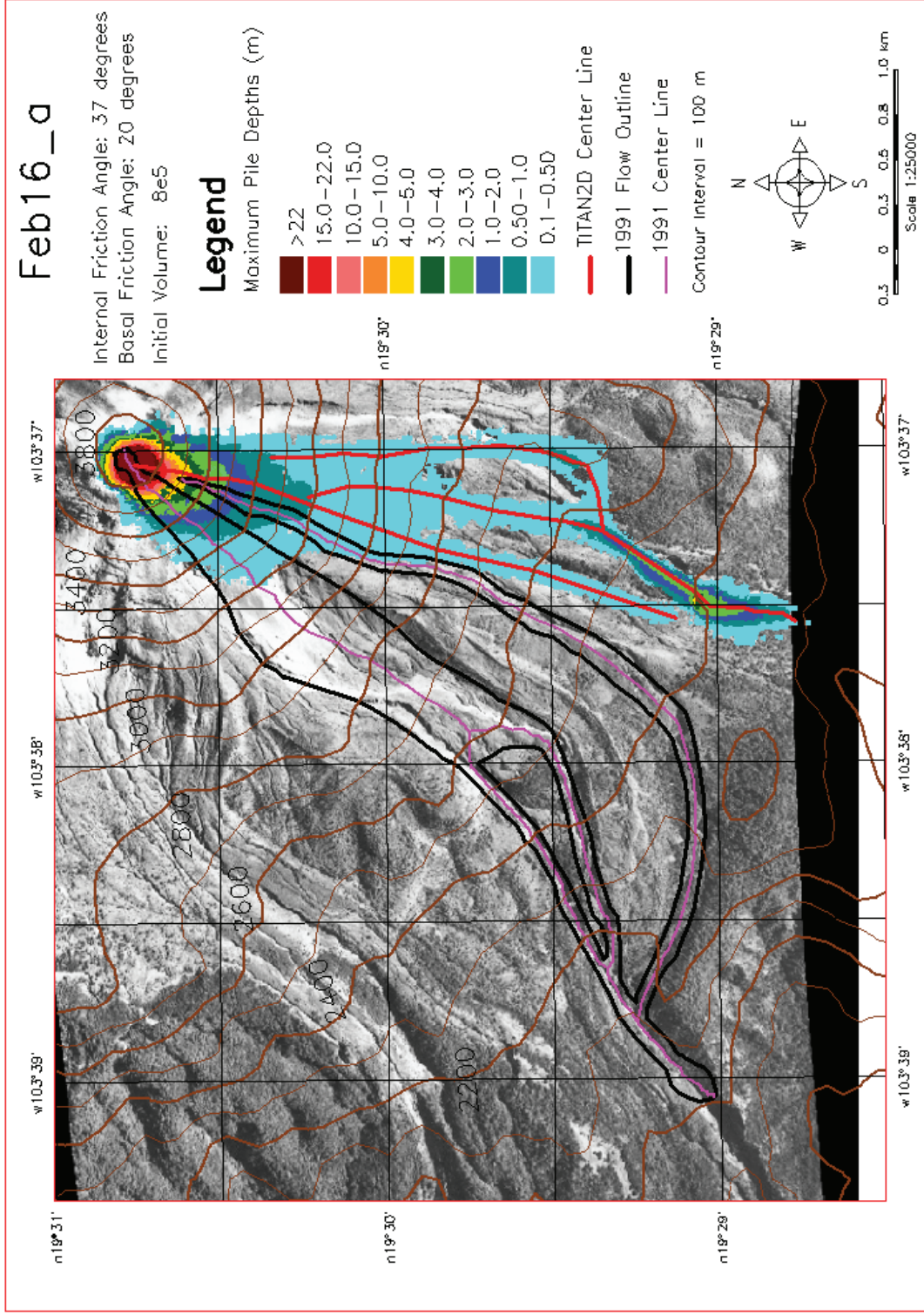


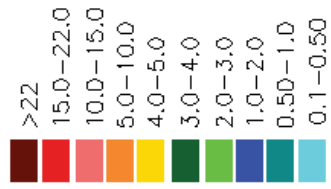
Figure 4. (A) TITAN2D simulation run feb\_16a. (B) TITAN2D simulation run feb\_14a. (C) TITAN2D simulation run jan\_26\_d. (D) TITAN2D simulation run feb\_10a. (E) TITAN2D simulation run feb\_22a.

# Feb\_14a

Internal Friction Angle: 37 degrees  
Basal Friction Angle: 28 degrees  
Initial Volume: 8e5

## Legend

Maximum Pile Depths (m)

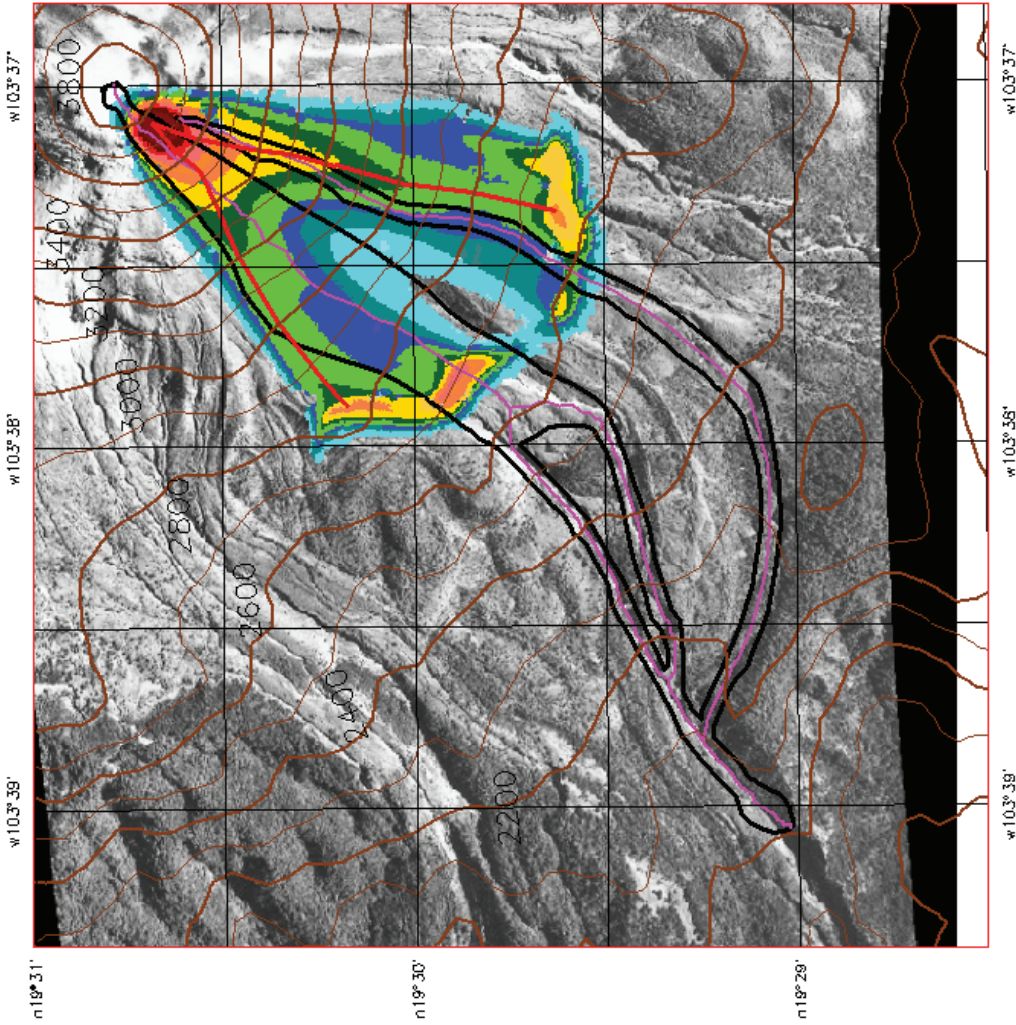
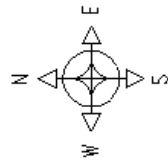


TITAN2D Center Line

1991 Flow Outline

1991 Center Line

Contour Interval = 100 m

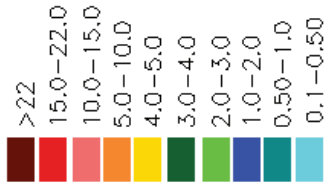


# Jan26\_d

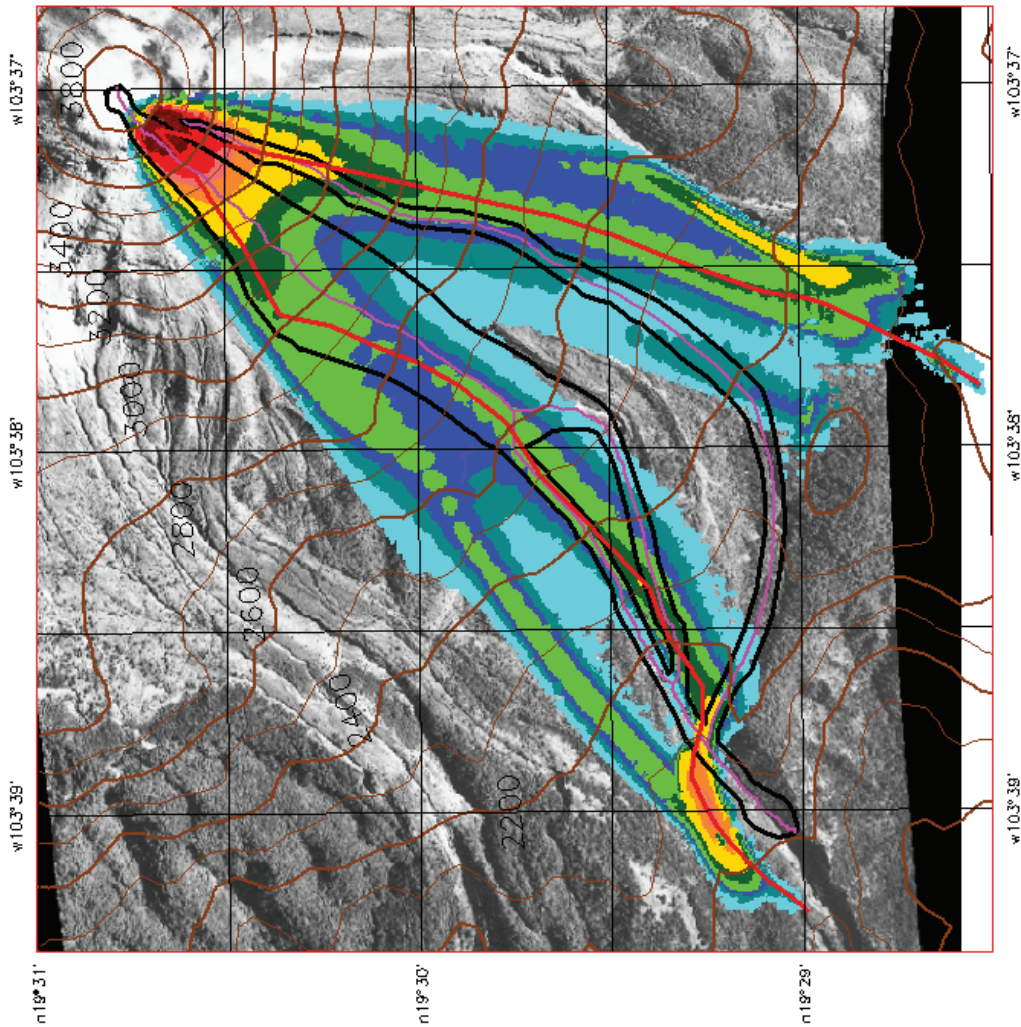
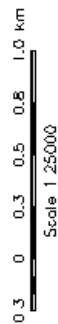
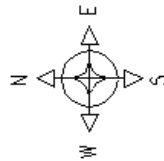
Internal Friction Angle: 30 degrees  
Basal Friction Angle: 20 degrees  
Initial Volume: 8e5

## Legend

Maximum Pile Depths (m)



- TITAN2D Center Line
- 1991 Flow Outline
- 1991 Center Line
- Contour Interval = 100 m



# Feb\_10a

Internal Friction Angle: 37 degrees  
Basal Friction Angle: 20 degrees  
Initial Volume: 8e5

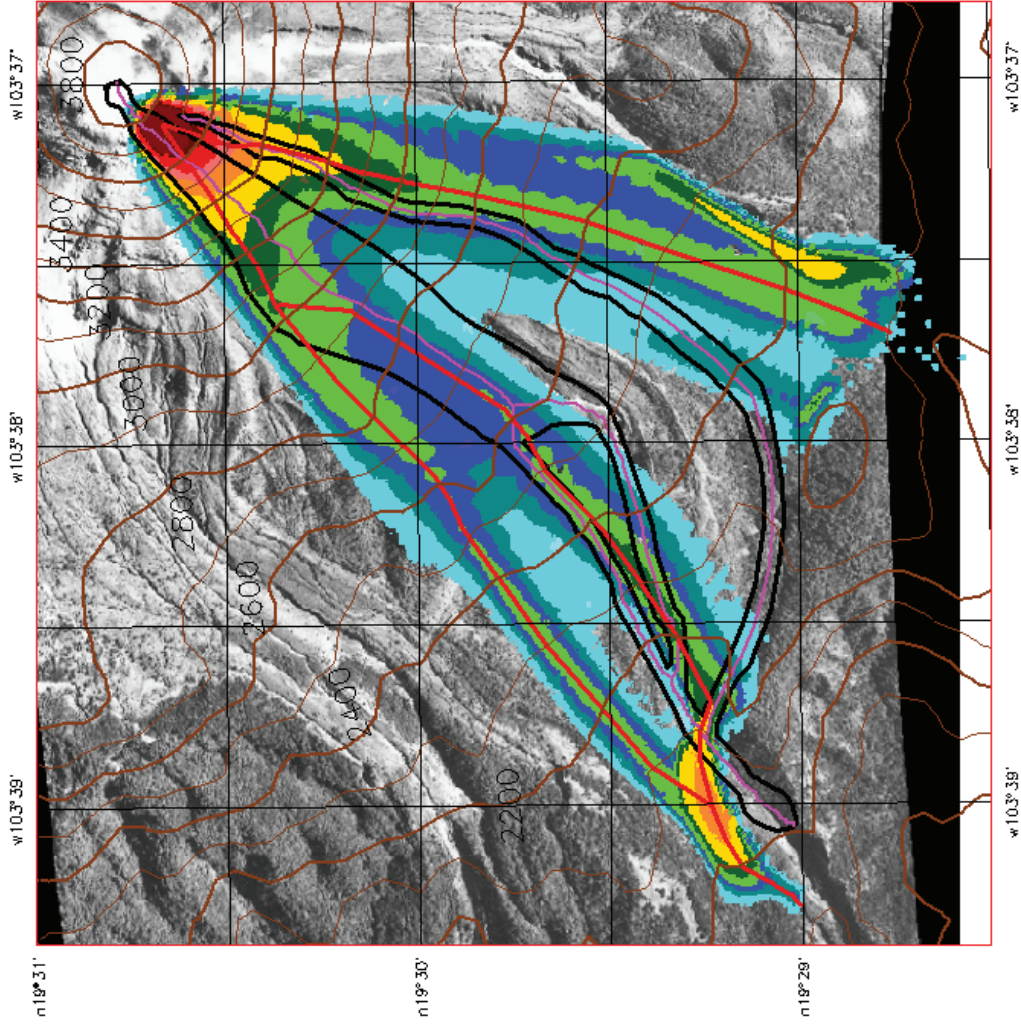
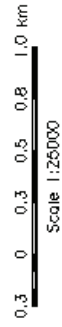
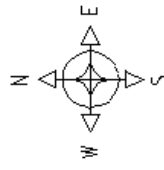
## Legend

Maximum Pile Depths (m)



- TITAN2D Center Line (red line)
- 1991 Flow Outline (black line)
- 1991 Center Line (purple line)

Contour Interval = 100 m

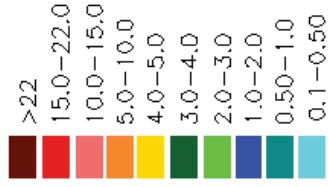


# Feb22\_a

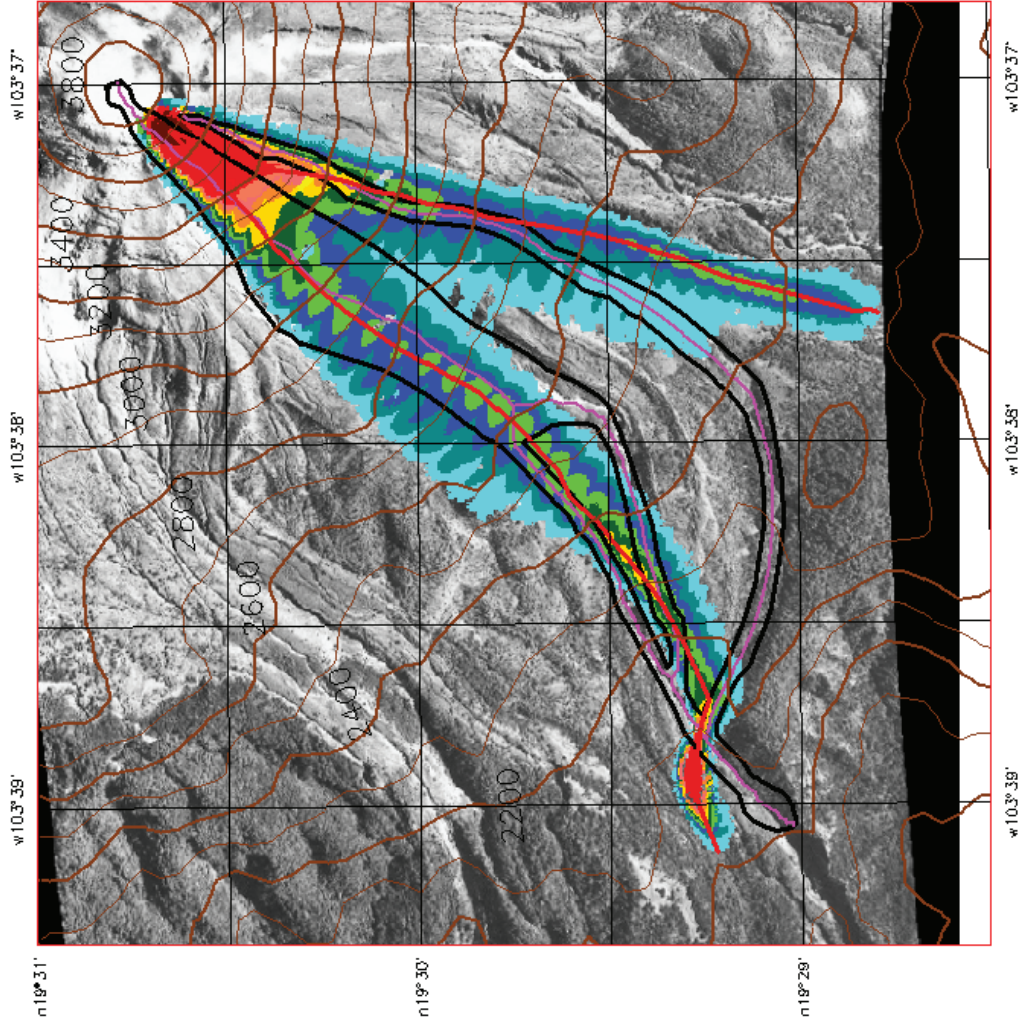
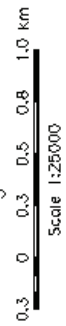
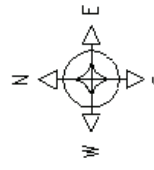
Internal Friction Angle: 37 degrees  
Basal Friction Angle: 20 degrees  
Initial Volume: 1.4e5

## Legend

Maximum Pile Depths (m)



- TITAN2D Center Line
- 1991 Flow Outline
- 1991 Center Line
- Contour Interval = 100 m



(Cordobán West, Central, and East). The initial mass travels down the nonchannelized upper slope along the most direct fall line.

Due to the extreme offset between the simulated flow path and the observed flow path, there was no need to report statistics quantifying the center-line offsets. While the runout length appears to be a reasonable match, this flow is not following the same channels as the observed 1991 block and ash flow. More experimentation is needed to adjust the coordinates of the initial mass.

#### **TITAN2D Simulation “feb\_14a”**

The flow map for feb\_14a (Fig. 4B) was constructed using an internal friction angle of  $37^\circ$  and a basal friction angle of  $28^\circ$ . In this run the initial volume was set to  $8 \times 10^5 \text{ m}^3$  (from Rodríguez-Elizarrarás et al., 1991), and the starting coordinate of the initial pile was chosen to be a location  $\sim 260$  m southwest of the geographic summit of Colima Volcano.

The flow now travels in the desired southwestern direction, heading toward Cordobán West, Central, and East. However, due to the high basal friction angle of  $28^\circ$ , the flow stops short of the observed 4 km path traveled by the 1991 block and ash flow. The high basal friction angle prevents the simulated flow from passing the break in slope present on Colima Volcano (and on the DEM). This break in slope can be noted in Figure 1. A lower basal friction angle would allow the flow to pass this break in slope and continue on through the lower gullies (as observed in Fig. 4A).

Statistics regarding the center-line offset measurements were not recorded for this simulation. The flow does not propagate far enough to justify a comparison between the two center lines.

#### **TITAN2D Simulation “jan\_26\_d”**

The flow map for jan\_26\_d (Fig. 4C) was constructed using an internal friction angle of  $30^\circ$  and a basal friction angle of  $20^\circ$ . In this run the initial volume was set to  $8 \times 10^5 \text{ m}^3$  (from Rodríguez-Elizarrarás et al., 1991), and the starting coordinate of the initial pile was chosen to be a location  $\sim 260$  m southwest of the geographic summit of Colima Volcano.

The lower basal friction angle used in this simulation allows the flow to travel past the break in slope and propagate a distance reasonably close to that of the observed 1991 block and ash flow. To quantify the goodness-of-fit, the center-line comparison statistics were applied. Offset measurements were taken (in meters) at every 100 m contour line that intersected the two center lines. Negative values reflect a westerly offset, while positive values denote an easterly offset. Statistics calculated from the offset measurements are reported in Table 2. Statistics show that the simulated run flows best through Cordobán West with a mean center-line offset of  $-39.62$  m (westerly) and a standard deviation of 95.16 m.

#### **TITAN2D Simulation “feb\_10a”**

The flow map for feb\_10a (Fig. 4D) was constructed using an internal friction angle of  $37^\circ$  and a basal friction angle of  $20^\circ$ .

In this run the initial volume was set to  $8 \times 10^5 \text{ m}^3$  (from Rodríguez-Elizarrarás et al., 1991), and the starting coordinate of the initial pile was chosen to be a location  $\sim 260$  m southwest of the geographic summit of Colima Volcano.

In this simulation, the internal friction angle was increased to  $37^\circ$ . This increase appears to have had little effect on the propagation of the simulated flow when compared visually to run jan\_26\_d (Fig. 4C). To quantify the goodness-of-fit and the actual effect increasing the internal friction angle had on the flow, the center-line comparison statistics were applied. Offset measurements were taken (in meters) at every 100 m contour line that intersected the two center lines (Table 2). Negative values reflect a westerly offset, while positive values denote an easterly offset.

Statistics (Table 2) show that the simulated run flows best through Cordobán West with a mean center-line offset of  $-26.72$  m (westerly) and a standard deviation of 88.89 m. When compared to run jan\_26\_d, the statistics show that increasing the internal friction angle yields a simulated flow center line that better fits the observed center line in Cordobán West.

#### **TITAN2D Simulation “feb\_22a”**

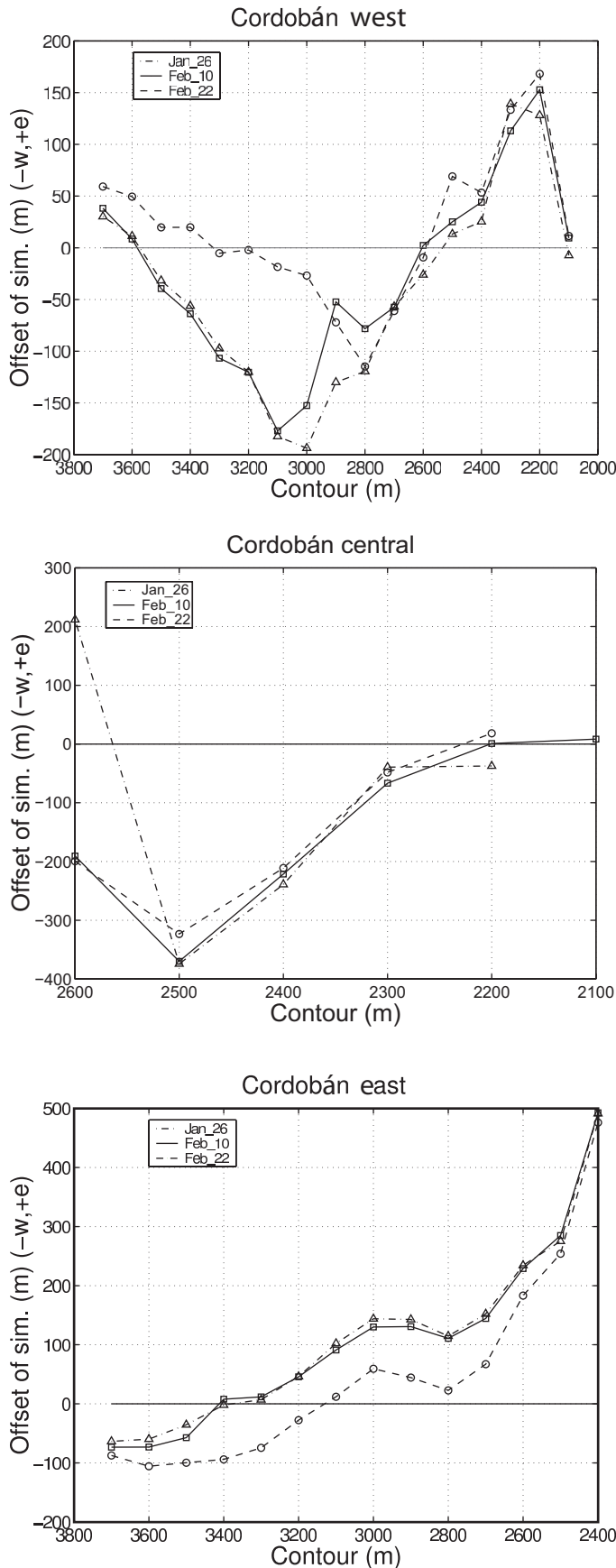
The flow map for feb\_22a (Fig. 4E) was constructed using an internal friction angle of  $37^\circ$  and a basal friction angle of  $20^\circ$ . In this run the initial volume was changed to  $1.4 \times 10^5 \text{ m}^3$  (the volume of the dome as observed before block and ash flow eruption from Rodríguez-Elizarrarás et al., 1991). The volume was decreased to compare the simulation output of an initial volume equal to the dome ( $1.4 \times 10^5 \text{ m}^3$ ) rather than an initial volume equaling the estimated volume of all the block and ash flows for this event ( $8 \times 10^5 \text{ m}^3$ ). The starting coordinates of the initial pile were chosen to be a location  $\sim 260$  m southwest of the summit of Colima Volcano.

The smaller initial volume resulted in a simulation that visually appears to be a better match to the observed flow outline. To quantify the goodness-of-fit and the actual effect decreasing the initial pile volume had on the flow, the center-line comparison statistics were applied. Offset measurements were taken as previously described (Table 2).

The series of graphs printed below (Figs. 5A–C) represent center-line offset measurements taken for simulation runs jan\_26\_d, feb\_10a, and feb\_22a. There are three graphs, one for each of the main gullies through which the simulated and observed 1991 block and ash flows traveled (Cordobán West, Central, and East). Center-line offset measurements for each simulation, through each of the gullies, are compiled for comparison. The graphs show a slight difference in the offsets of simulations jan\_26\_d and feb\_10a. The center line of simulation run feb\_22a appears to best fit the observed flow center line (seen on the graphs at 0 m offset).

## **DISCUSSION AND CONCLUSIONS**

The final product of this research was a comparison of measurements of the 1991 block and ash flows with realizations of the TITAN2D model. The TITAN2D output maps seen in Figures 4A–E show the effect that altering the input parameters has on the



output of the program. This suite of five runs represents a sampling of the hundreds of TITAN2D simulations performed for this research. Runs feb\_16a and feb\_14a do not compare well with the observed 1991 flow but are still important, as they exhibit the effects of changing the initial starting coordinate and friction angles. These runs prove that unrealistic input parameters will yield unfavorable results. Neither center-line offset measurements nor statistics were calculated from these two runs.

The next three runs (jan\_26\_d, feb\_10a, and feb\_22a) are better matches with the observed flow-path center line of the 1991 block and ash flow. For these runs, center-line offset measurements were made to quantify the goodness-of-fit. From these measurements a series of statistics were calculated to better judge how well the simulated flows matched up to the observed flow and to one another.

The flow simulation feb\_22a (Fig. 4E) best matches the limits of the observed flow. Statistics and offset measurements show that the center line from feb\_22a is the best match with the observed center line. The mean difference between model and data varies between a small fraction of a single computational grid cell to over two grid cells in this, as in the other two simulations. However, the RMS varies from just less than a grid cell to over two grid cells, suggesting that part of the lower mean difference results from the model flow “jumping” from east to west of the real flow deposit. Furthermore, the RMS values suggest that a typical error is generally between one and two computational grid cells in areas with relatively poorly resolved channels.

Assuming that the best simulation output had the best input parameters, feb\_22a is the best fit to the observed flow outline. The initial volume used in this simulation ( $1.4 \times 10^5 \text{ m}^3$ ) is equal to the estimated dome volume of Colima Volcano in 1991. It is therefore

Figure 5. (A) Comparison of the center-line offset measurements collected for Cordobán West for runs jan\_26\_d, feb\_10a, and feb\_22a. The smallest initial volume (run feb\_22a) provides the flow center line closest to the observed center line. Negative values reflect a westerly offset, while positive values denote an easterly offset. (B) Comparison of the center-line offset measurements collected for Cordobán Central for runs jan\_26\_d, feb\_10a, and feb\_22a. The smallest initial volume (run feb\_22a) provides the flow center line closest to the observed center line. Negative values reflect a westerly offset, while positive values denote an easterly offset. (C) Comparison of the center-line offset measurements collected for Cordobán East for runs jan\_26\_d, feb\_10a, and feb\_22a. The smallest initial volume (run feb\_22a) provides the flow center line closest to the observed center line. Negative values reflect a westerly offset, while positive values denote an easterly offset. The run with the lowest initial volume (feb\_22a) also had the lowest initial pile height. The smaller pile height results in a lower shear stress, so it thins slower. Because it thins slower, run feb\_22a remains thicker longer than runs jan\_26\_d and feb\_10a. This causes the flow to travel further from the desired center lines than the other two runs initially. At the 3200 m contour line, feb\_22a thins out because there is less mass, causing the model center line to be closer to the observed center line than the other two runs. After the flow thins out, it “sloshes” less than the two larger flows, staying closer to the observed path.

the best estimate of the volume of material that was flowing at any given time during the eruption. This finding is significant for a number of reasons. Rodríguez-Elizarrarás et al. (1991) report the total volume of the block and ash flows deposited on 16–17 April 1991 to be  $8 \times 10^5 \text{ m}^3$ . This final volume was the result of numerous smaller flows depositing over a two-day span. There was never a single flow equal to this volume propagating down Colima Volcano. Using the total deposit volume as an initial volume for the simulation yields a flow that propagates too far. To estimate flow paths, it is best to begin with a volume equal to that of a hypothetical single event. The results suggest that the maximum volume flowing at any one time may have been controlled by the volume of the dome. As this best-fit volume is less than the volume of the most extensive separate block and ash unit (flow deposit F3 with a volume of  $6 \times 10^5 \text{ m}^3$ ), the results may mean that F3 is not the result of emplacement from a single flowing mass.

The results from each of the Colima Volcano simulation runs strongly suggest that the TITAN2D program is useful in replicating historic granular flows. The ability to simulate a historic flow and to quantify statistically the accuracy of the model output provides geoscientists and civil protection authorities with a valuable tool. The TITAN2D program can be used to make reasonable predictions as to what the flow dynamics of any potential natural granular flow may be. The flow maps created by TITAN2D show the maximum pile depths calculated. This information essentially depicts the worst-case scenario or maximum possible hazard for a particular flow. TITAN2D has also shown that it is useful in determining whether or not an existing flow deposit could reasonably have been deposited by a single flow event. This information is helpful when trying to reveal the events leading up to the deposition of a past flow event.

This research has provided an improved and strengthened understanding of the ability of the TITAN2D program to model granular flow as well as of the effect input variables have on simulation output. The TITAN2D tool could prove extremely useful to geoscientists and civil protection authorities in risk assessment and mitigation of hazards due to natural granular flows. Authorities can use the output generated by the TITAN2D model to create hazard maps, develop emergency management scenarios, and increase public awareness, in turn helping to ensure the future safety and well-being of both lives and property.

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