

# External Ballistic of Volcanic Explosions

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## ABSTRACT

In order to determine the kinetic energy of an explosion it is necessary to know the initial velocities of ejected fragments.

Calculations of initial velocities made earlier with few exceptions did not take into account the resistance of the air and, therefore, greatly underestimated the initial velocities, and consequently the energy of the explosions. A solution of the inverse problem of ballistics which takes into account air resistance makes it possible to find precise values of initial velocities of ejection, determined from the distance of ejected fragments and their ballistic coefficients. The method makes it possible to determine the kinetic energy of explosions for eruptions which were not directly observed.

It is shown that the velocity field in volcanic explosions is not spherical, *i.e.* the velocities of ejection of fragments differ markedly in different directions. Accordingly, the maximum distance of ejection of fragments depends not only on the initial angle of ejection, but also on the character of distribution of initial velocities and the ballistic coefficients of the fragments. For fragments with diameters of 40 cm to 2 m the maximum distance of ejection is realized in the case of angles of ejection of 30-36°. For smaller fragments the optimal angles of ejection are still smaller.

The ballistics of explosive eruptions of the volcanoes Asama, Arenal, and Shiveluch are considered.

## INTRODUCTION

Explosive activity is one of the most characteristic manifestations of volcanic processes, and with respect to Kamchatkan volcanism the most impor-

tant type of eruption. However, the physics and mechanism of volcanic explosions and especially of directed blasts have not been adequately studied. By physics we mean the physical and physico-chemical processes leading to explosions and by mechanism the internal and external ballistics of volcanic explosions.

In respect to the physical processes it is important to distinguish between two main distinctly different processes: 1. the explosive exsolution of a gasphase from a magma leading to a «normal» volcanic explosion and 2. the contact process between a magma and external water which represents a powerful coolant leading to a water vapour explosion, *i.e.* a phreatomagmatic explosion. A recent example for the first type is the plinian eruption of the May 18, 1980 eruption of Mount St. Helens (LIPMAN and MULLINEAUX, 1981) and the second type is represented by the 1963/1964 eruption of Surtsey (THORARINSSON *et al.*, 1964) and the 1965 eruption of Taal volcano/Philippines (MOORE *et al.*, 1966). In the present study we will not differentiate between the two different physical processes of origin of volcanic explosions but concentrate on the external ballistics.

The mechanism of volcanic explosions has been the subject of only a few investigations and attention has been paid to it to get some idea of the mechanism to solve specific volcanological problems: the formation of ash (GUSHCHENKO, 1965; VERHOOGEN, 1951), of pyroclastic flows (SMITH, 1963), estimation of the amount of water and volatiles in the magma

(KUZNETSOV, 1967; MARKHININ, 1967; WILSON, 1980), estimation of the pressure, ejection velocity, and kinetic energy of volcanic explosions (FUDALI and MELSON, 1968; GORSHKOV and BOGOYAVLENSKAYA, 1965; GORSHKOV and DUBIK, 1969; HEDERVARY, 1963; LORENZ, 1970, 1971; RICHARDS, 1965; WILSON, 1980), etc. Due to the limited nature of experimental data and the arbitrary choice of initial conditions, in respect to volcanic explosions, the cited investigations contain substantial simplifications, which often lead to serious errors. Some of these errors, passing from publication to publication have even become constant, which also indicates the need for special examination of the various aspects of a volcanic explosion. Problems of the development of volcanic explosions have been discussed (LEGLER and STEINBERG, 1975; LORENZ, 1980; STEINBERG, 1974); therefore in this paper we will limit ourselves to an examination of the external ballistics of volcanic explosions. An investigation of the velocity and density of the volcanic gases during volcanic eruptions has recently been published by STEINBERG and BABENKO (1978).

#### COMPARISON BETWEEN VOLCANIC AND NONVOLCANIC EXPLOSIONS

The end results of volcanic explosions (ratios of representative crater dimensions and their relation to energy, depth of the center of the explosions, volume and character of distribution of ejected material, etc.) are well described by experimental and theoretical relationships established for point explosions (DOKUCHAYEV *et al.*, 1963, RYABININ and RODIONOV, 1966; SADOVSKIY *et al.*, 1967; SAKHAROV *et al.*, 1959; STEINBERG, 1973). However, it should be pointed out that in volcanological studies, the energy of the explosion is usually understood to be the kinetic energy, the magnitude of which rarely was compared to the total energy of the explosion, and which according to our estimates and estimates by others (VERHOOGEN, 1951) constitutes only 3-10%

of the total energy of the explosion, analogous to the proportions established for ordinary explosions. However, there are essential differences between volcanic and ordinary explosions which should be taken into account by making use of the voluminous experimental data obtained from chemical and nuclear explosions.

Field observations and films of volcanic explosions made at the time of volcanic eruptions (KARYMSKIY, 1962-65; KLYUCHEVSKIY, 1966; ALAID, 1972; TOLBACHIK, 1975-76) showed that the basic difference between volcanic explosions and chemical and nuclear ones is the different rate of occurrence of the process, and also the fact that for the latter the energy of the explosion (or mass of the «effective body») and depth of the explosion center are constant quantities, but for volcanic explosions they vary during the course of development of the explosion (LEGLER and STEINBERG, 1975; STEINBERG, 1973, 1974). The rate of development of a nuclear explosion is determined by the rapidity of occurrences of the fission chain reactions, the kinetics of a chemical explosion by the rate propagation of the detonation wave (6-9 km/s for solid explosives, 3-4 km/s for gases).

The development of a volcanic explosion is determined either by the kinetics of the liberation of volatiles dissolved in the magma upon abrupt decrease in pressure (a non-phreatomagmatic explosion) or by the kinetics of the contact process between magma and external water which triggers a water vapour explosion (a phreatomagmatic explosion, LORENZ, 1980).

Despite the sufficiently high rate of reaction of liberation of water vapour from water-silicate melts (KATSIK *et al.*, 1971), the rate of propagation of this process is by one or two orders of magnitude slower than detonational wave velocities. Therefore a volcanic non-phreatomagmatic explosion is markedly extended in time compared to chemical and nuclear ones. Phreatomagmatic explosions have not been studied yet in this respect but phreatomagmatic explosions observed in recent years (*e.g.* Surtsey) show similar extension in time.

Due to the slower rate of development, the pressures created in the shock wave of volcanic explosions are not as great as those in chemical and nuclear explosions, and correspondingly we do not encounter the formation of minerals typical of explosions with high pressures in the enclosing rocks and products of volcanic explosions. Calculations show that in very large volcanic explosions, with a kinetic energy of  $\sim 10^{17}$ J, a pressure of 0.075-0.45 GPa is realized (FUDALI and MELSON, 1972; GORSHKOV and BOGOYAVLENSKAYA, 1965; GORSHKOV and DUBIK, 1969; GUSHCHENKO, 1965; MELSON and SAENZ, 1968), according to WILSON (1980) even lower pressures are realized, while in nuclear explosions, with a power of  $\sim 10^{13}$ J, the pressure reaches 100 GPa (JOHNSON *et al.*, 1959). *I.e.*, even without reduction to equal power, the pressure at the focus of a nuclear explosion is 2-3 orders of magnitude greater than the pressure at the focus of a volcanic explosion.

In character of development, a volcanic explosion is similar to the bursting of a steam boiler or eruption of a geyser (MERZHANOV *et al.*, 1970, 1973), in which the energy concentrated in the gas phase is very insignificant compared to the energy accumulated in superheating of the water. When the liberated steam reaches an exit and ejects the water from the outlet channel, the pressure in the channel (chamber, focus) drops abruptly and there occurs volumetric (explosive) boiling up – a geyser eruption (or boiler explosion). A volcanic, non-phreatomagmatic eruption develops according to an analogous scheme: first the gases liberated already from the magma escape to the surface («they open the system»), after which the pressure in the conduit (chamber) drops and volumetric liberation of the volatiles dissolved in the magma occurs – causing a volcanic explosion. Due to the viscosity of the magma the system silicate melt-volatiles is more inertial than the system water-steam, therefore the lag between the initial escape of gas from a system closed to that extent and the explosion itself may be rather appreciable. For instance, in the satellite eruption of Klyuchevskiy volcano in 1966 – the break-

through of Piyp – the interval was several hours. In most cases, however, the time interval between the initial «gas» explosion which opens the roof and allows the «discharge» of the gas cap given off already by the magma and eruption proper – the volumetric liberation of gas («boiling up») in the upper part of the magma column – is very small.

The similarity of volcanic explosions to geyser eruptions was noted long ago: «the rhythm of frequently repeated detonations evinced by many volcanoes must be considered analogous and mechanically similar to the rhythm of a geyser. To a certain extent it confirms the common nature of the eruptive mechanism, which must be thought to be the result of similar factors» (GRATON, 1949). This was pointed out later too (McBIRNEY, 1973) and it can be added that detailed observations of the variation in the discharge of water in geysers before their eruptions have shown that a geyser eruption is always preceded by a strong surge which ejects water from the upper part of the geyser channel (MERZHANOV *et al.*, 1973). This in turn causes an abrupt reduction in pressure in the lower parts of the channel which leads to the geyser eruption.

In respect to the character of movement of juvenile material in space, volcanic explosions also are similar to geyser eruptions, within which water is expelled, carried out by steam, rather than flowing out, due to the change in hydrostatic pressure in the system. Very powerful volcanic eruptions are accompanied by ejection (not outflow) of pyroclastic flows of highly degassed pumice-like or pumice material carried out by powerful (eruptive) emission of gases.

Thus in the eruption of Bezmyannyi in 1956 some pyroclastic flows had no apparent relationship to the main pyroclastic flow and to the crater (GORSHKOV and BOGOYAVLENSKAYA, 1965). In the eruption of Shiveluch in 1964 pyroclastic flows were observed not only on the south slope below the newly formed crater but also on the northwest slope, separated from the crater by a ridge more than 200 m high (GORSHKOV and DUBIK, 1969). These facts indicate that the pyroclastic

flows did not flow out but were ejected (SPARKS and WILSON, 1976; SPARKS *et al.*, 1978). An analogous mechanism of propagation of pyroclastic flows was observed in the eruption of Komagotake in 1929 and was suggested by WILLIAMS (1942) for the Crater Lake pumice flow and several other eruptions.

#### EXTERNAL BALLISTICS

The dispersal of fragments in a volcanic eruption is due mainly to the expansion of the gas phase. Therefore, the internal ballistics of a volcanic explosion are intermediate between the ballistics of a concentrated explosion, where a substantial part of the kinetic energy is concentrated in the shock wave, and a steam explosion, where the kinetic energy is concentrated in the expanding gas phase. In the first case the initial velocities of the fragments depend but little on the size of the fragments (SHOEMAKER, 1962), but in the second, on the contrary, they are inversely proportional to the size (STEINBERG and BABENKO, 1977). For more rigorous judgments concerning the internal ballistics of volcanic eruptions it is advisable to examine their external ballistics.

The rate of escape of gases and velocity of ejection of fragments are important characteristics of a volcanic explosion. They have independent significance and have to be known in order to determine the kinetic energy of an explosion. Direct determination of these parameters from films is possible (BLACKBURN *et al.*, 1976; CHOUET *et al.*, 1974; STEINBERG and BABENKO, 1976) but powerful volcanic explosions are rare and, moreover, to choose a point from which a film would provide the necessary information is rather complicated, *i.e.* the zone affected has a radius of several kilometers. Therefore the use of films is limited to the recording of weak explosions and gas detonations. For powerful explosions the needed parameters can be determined by solving the inverse problem of external ballistics: by finding the initial velocities of ejection

from the distribution of fragmental material over the ejection area.

In numerous studies by volcanologists (GORSHKOV and BOGOYAVLENSKAYA, 1965; GORSHKOV and DUBIK, 1969; LORENZ, 1970, 1971; MARKHININ, 1967; MELSON and SAENZ, 1968) the inverse problem was solved without taking into account air resistance and the size of the ejected fragments, as a result of which the initial velocities of ejection and correspondingly the kinetic energy of the eruption were greatly underestimated.

Ballistic calculations of the dispersal of fragments which take into account air resistance have been made for nuclear explosions (SHERWOOD, 1967) and also for volcanic explosions accompanied by pyroclastic ejecta (FUDALI and MELSON, 1972; WALKER *et al.*, 1971; WILSON, 1972). The defect of these mathematically quite correct studies is the somewhat idealized approach to the solution, which did not take into account several essential points.

In SHERWOOD's (1967) study the standard wind accepted for ballistic and aerodynamic calculations in the earth's atmosphere of 10-15 m/sec is not taken into account. The result was the conclusion that all fragments ejected with a diameter of more than 2 mm move on ballistic trajectories. Numerous observations of eruptions have shown that fragments with diameters of up to 3-5 cm, and sometimes more, fall in the zone of wind-carrying, and that wind has an appreciable effect on the trajectories of fragments up to 20-30 cm in diameter, so that the trajectories differ essentially from the ballistic ones. In the eruption of Heimaey volcano in 1973 the distance of ejection was correlated with the direction and force of the wind and it was established that only fragments with a diameter of more than 40 cm were not subject to wind action and could be used for ballistic calculations (SELF *et al.*, 1974). For ballistic calculations the standard wind has to be taken into account.

In addition it must be realized that the field of initial velocities is not spherical and that the maximum distance of ejection for any initial dimensions is not realized at  $\theta = 45^\circ$ .

The field of velocities in explosions with ejections can be described by the formula

$$v = v_0 \sin^{3/2} \text{ or } v = v_0 \sin^{1.8}$$

(DOKUCHAYEV *et al.*, 1963; SHOEMAKER; 1962; STEINBERG; 1975) (for conventional symbols, see below), as a result of which the maximum distance of dispersal of fragments in eruptions with a vertical axis of ejection is realized at  $\delta_0 = 63-65^\circ$ , rather than  $45^\circ$ , even in the absence of air resistance. This theoretical conclusion is in good agreement with results of experiments on explosions with ejecta under atmospheric conditions, where the maximum distances were obtained for angles of  $55^\circ \leq \delta_0 \leq 75^\circ$  (SAKHAROV *et al.*, 1959). Films of industrial and nuclear explosions with ejecta have made it possible to describe the character of distribution of initial velocities of fragments as a function of the angle relative to the axis of the explosion. Films of the «Scooter» (KNOX, 1969) and «Cabriolet» (TEWES, 1969) explosions show that only at the

very first moment the earth's surface, raised by the explosion, is unbroken and approximately spherical with respect to its center, i.e. the velocity of dispersal (displacement) of the rocks is the same in different directions. In the further development of the explosion there is surface rupture along its axis and spatial distribution of velocities changes abruptly: in the axial parts the velocities of the fragments increase greatly.

At present we have no data available on the character of the velocity field for powerful volcanic eruptions, but films of weak volcanic explosions show that the the maximum velocities of fragments are observed in the axial part. In addition, the similarity of distribution of the ejected fragmental material in space, and also the closeness of the reduced depths and indices of ejection for ordinary and volcanic explosions (Table 1), indicate that their velocity fields are qualitatively the same.

To this it should be added that even in the case of different initial velocities in all directions, the maximum distance

TABLE 1 — Characteristics of nuclear, chemical, and volcanic explosions.  
H = depth of center of explosion; R = radius of crater.  
1\*: SHERWOOD (1967); 2\*: KNOX (1969).

	E, J	E, kg TNT	H, m	R, m	$\frac{H}{E^{1/3.3}}$	$n = \frac{R}{H}$
Cabriolet 1*	$9.68 \cdot 10^{12}$	$2.3 \cdot 10^6$	52	54	0.700	1.04
Danny Bay 2*	$1.77 \cdot 10^{12}$	$4.2 \cdot 10^5$	33	33	0.732	1.00
Preshoener-II 2*	$3.57 \cdot 10^{11}$	$8.5 \cdot 10^4$	22	29	0.781	1.32
Bezmyannaya	$1 \cdot 10^{17}$	$2.4 \cdot 10^{10}$	700	1000	0.620	1.43
Shiveluch	$2.5 \cdot 10^{17}$	$6.0 \cdot 10^{10}$	420-700	1100	0.284-0473	2.62-1.57
Mendeleyeva	$4 \cdot 10^{17}$	$9.5 \cdot 10^9$	530	700	0.616	1.32
Tau Rusyr	$2 \cdot 10^{19}$	$4.8 \cdot 10^{12}$	2500	3750	0.466	1.50
Matua	$2 \cdot 10^{14}$	$4.8 \cdot 10^7$	90	130	0.496	1.45
Galya Maar	$3 \cdot 10^{16}$	$7.2 \cdot 10^9$	450	620	0.568	1.38
Valentia Maar	$5 \cdot 10^{15}$	$1.2 \cdot 10^9$	320	470	0.684	1.47
Preve Peak	$2 \cdot 10^{15}$	$4.8 \cdot 10^8$	220	270	0.613	1.23
Avacha (souma)	$2 \cdot 10^{18}$	$4.8 \cdot 10^{11}$	1750	2050	0.642	1.17

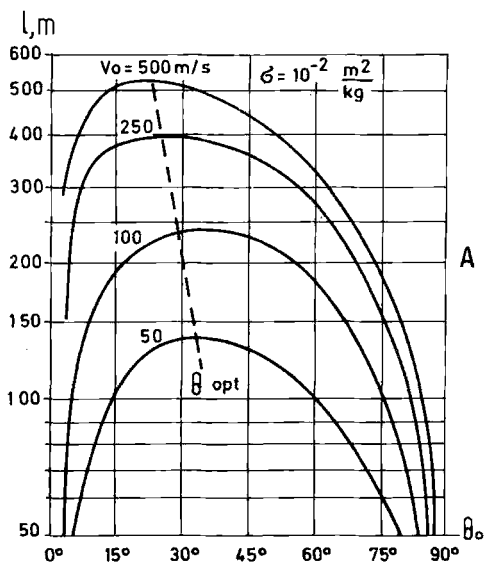
depends not only on the angle of ejection but also on the size, shape, and density of the fragments, which are taken into account by the ballistic coefficient  $\sigma$  ( $\sigma = 3c/2\rho_k D$ , see list of symbols on page 346). When  $\sigma \geq 10^{-3} \text{ m}^2/\text{kg}$ , its effect on the distance of ejection is dominant compared to the ejection angle  $\delta$ , and at velocities of more than 500 m/s the region of «dominant effect» is broadened to  $\sigma \geq 10^{-4} \text{ m}^2/\text{kg}$  (Fig. 1). From this figure it is seen that the optimal angles of ejection for fragments with  $D$  4 m ( $\sigma < 10^{-4} \text{ m}^2/\text{kg}$  for  $\rho_k = 2.5 \cdot 10^3 \text{ kg/m}^3$  are essentially different from  $45^\circ$ ). Thus the resulting distance of ejection depends «doubly» on the angle of ejection: on the distribution of initial velocities by angle (velocity field, internal ballistics) and on the character of distribution of distances by angle of ejection taking into account  $\sigma$  (external ballistics) (Fig. 1).

Data on the distance of ejection of fragments in high-level initial phreatomagmatic explosions of the volcano Asama in 1935, 1937, and 1938 (MINAKAMI, 1942) show that the initial velocities of ejection of fragments depend little or not at all on their size (Table 2). From this, taking into account the important effect of aerodynamic drag, an important inference follows: if the distribution of fragments by size is the same for all angles of ejection  $\delta$ , then the distance of ejection in every direction depends only on  $\sigma$ . Accordingly from such explosions large fragments (small  $\sigma$ ) will be concentrated in the explosive deposits in places farthest from the crater. The physical significance of this conclusion is rather simple: for equal density and initial velocity of fragments, their kinetic energy is proportional to the mass or cube of the diameter, air resistance is proportional to the cross-section of the fragment, i.e. the square of the diameter; distance of ejection (for  $\delta = \text{const}$ ) is proportional to kinetic energy ( $D^3$ ) and inversely proportional to the air resistance ( $D^{-2}$ ), i.e. proportional to the diameter.

The validity of this conclusion is confirmed by volcanologists (MCBIRNEY, 1973) and is well illustrated by photo-

graphs of the deposits of the directed blast from Shiveluch volcano, taken right after the eruption of 1964 (Fig. 5, 6), and also by field observations: fragments 3-5 m were concentrated chiefly in the frontal part of the deposits, in a strip 1-1.5 km wide at a distance of 8-10 km from the crater (STEINBERG, 1975). Similar distribution of coarse fragmental material was observed by one of us (G.S.S.) in the eruption of Alaid volcano in 1972: for fragments 1-2 m in diameter the maximum distance from the crater was 2-2.5 km, and for fragments with a diameter less than a meter it was not more than 1.5 km. The same regularity is noted for the Arizona meteor crater: the largest fragments of the meteorite were found on the plain far from the crater and small fragments (up to 5 kg, i.e.  $\sigma > 3 \cdot 10^{-3} \text{ m}^2/\text{kg}$ ) on the wall of the crater (NININGER, 1956; VDOVYKIN, 1971). In the present level of investigation of volcanic explosions the approximation is admissible that the internal ballistics of many volcanic explosions and the ballistics of a concentrated explosion are similar.

To calculate the distances, trajectories, terminal velocities, angles of incidence,



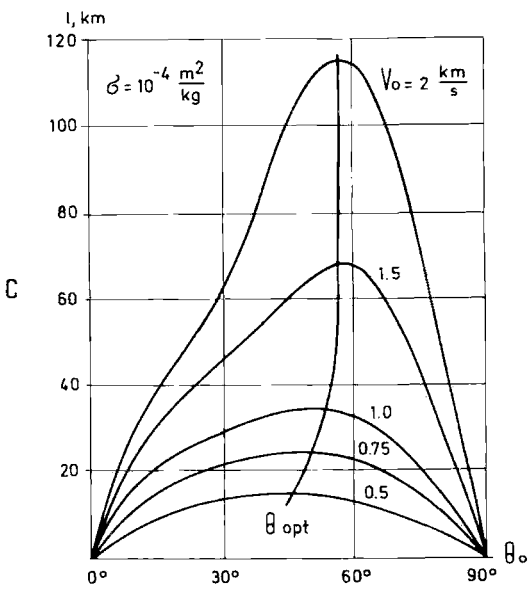
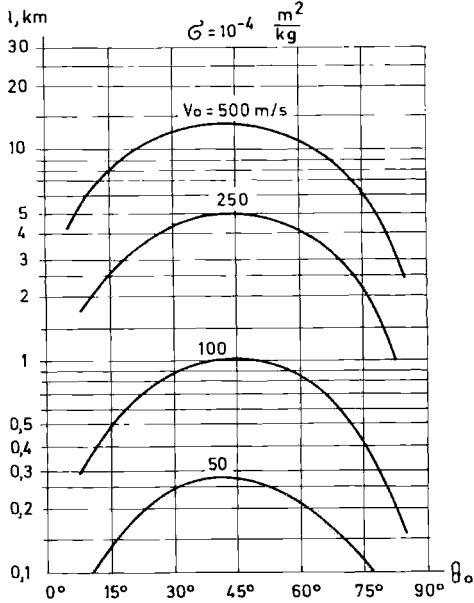
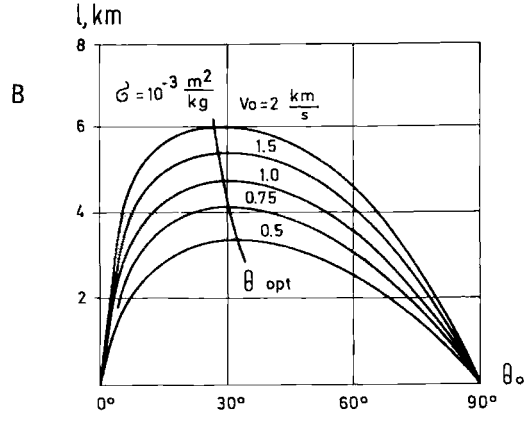
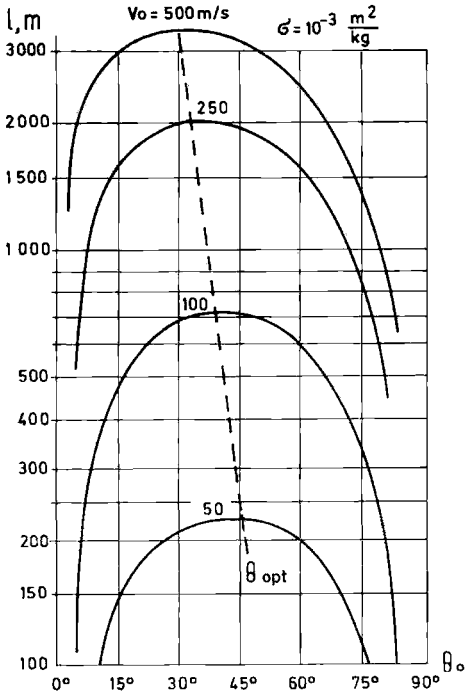


FIG. 1 — Dependence of distance of ejection of fragments on initial velocities  $V_0$  and angle of ejection  $\theta_0$ .

- A:  $\sigma = 10^{-2} \text{ m}^2/\text{kg}$ ,  $D = 0.04 \text{ m}$
- B:  $\sigma = 10^{-3} \text{ m}^2/\text{kg}$ ,  $D = 0.40 \text{ m}$
- C:  $\sigma = 10^{-4} \text{ m}^2/\text{kg}$ ,  $D = 4.00 \text{ m}$

heights, and flight time of fragments, the system of equations of ballistic trajectories was used - a body in the gravitation field of the earth subjected to the resistance of the atmosphere - in a velocity system of coordinates (STEINBERG and STEINBERG, 1975):

$$dV/dt = -g_0 \sigma q - g \sin \delta$$

$$V \frac{d\delta}{dt} = -g \cos \delta + \frac{V^2}{R+h} \cos \delta \quad (2)$$

$$dh/dt = V \sin \delta$$

$$\frac{dl}{dt} = \frac{V \cos \delta}{R+h} R \quad (\text{for symbols used see p. 346})$$

The calculations, e.g. by the Runge - Kutta method, were made on a BESM-6 computer and the solutions obtained are given in Fig. 1-4. Having the results of a

solution of the direct problem of external ballistics, the initial data and boundary conditions needed for solving the inverse problem can be determined.

In the general case of deposits of a volcanic explosion the available data are density  $\rho_b$ , size  $D$ ,  $c$ , and distance of ejection  $l$ . As  $\sigma = f(\rho_b, D, c)$ , assuming for simplicity that the shape of the fragments is isometric and the density constant (i.e.  $c = \text{const}$ ,  $\rho_k = \text{const}$ ), we have  $\sigma = f(D)$  and henceforth we will chiefly use the coefficient  $\sigma$  to determine the size of fragments.

With known  $l$  and  $\sigma$ , not any fragments can be used to solve the inverse problem, but only those corresponding to the condition of the maximum range: fragments with a given  $\sigma$  (diameter) found at the maximum distance from the center of the crater, assuming that such fragments were ejected at the angle of  $\delta_{opt}$  providing maximum distance.

In field mapping of explosive deposits, a profile or map of isopleths is drawn

TABLE 2 - Parameters of explosive eruptions of Asama volcano in 1935-1938, in part from MINAKAMI (1942).

	$\sigma \cdot 10^{-4}$ m <sup>2</sup> /kg	$D$ , m	$l_{\text{meas}}$ , km	$l$ , km	$\theta_{opt}$	$V_0$ , m/s
20. IV .35	4	1.0	3.0	2.7	35°	230
	6	0.65	2.6	2.3	34°	230
	8	0.5	2.6	2.3	34°	250
16. VI .37	4	1.0	1.68	1.53	35°	150
	6	0.65	1.6	1.45	34°	155
	8	0.5	1.53	1.4	34°	160
16. IV .37	4	1.0	3.42	3.1	35°	255
	6	0.65	3.35	3.0	34°	295
	8	0.5	3.30	2.95	34°	340
7. VI .38	4	1.0	4.5	4.1	35°	330
	6	0.65	4.1	3.7	34°	375
	8	0.5	4.0	3.6	34°	450

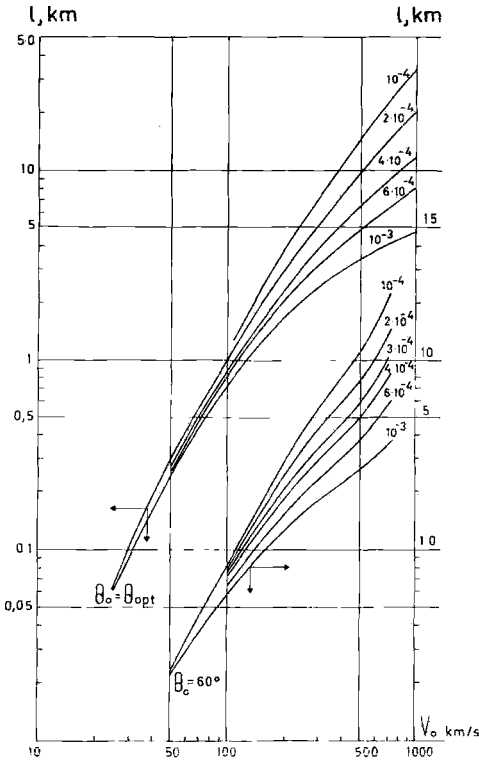


FIG. 2 — Dependence of distance of ejection of fragments on the initial velocity  $V_0$  and  $\sigma$ , for  $\theta_0 = \theta_{opt}$  and  $\theta_0 = 60^\circ$ .

(LORENZ, 1970, 1971; SELF *et al.*, 1974; WALKER *et al.*, 1971) in contours of maximum distance from the crater of fragments of a given size. From the determined values of  $l$  and  $\sigma$ , and using the solution given in Fig. 2, we determine  $V_0$ , and then with knowing  $V_0$  we find the angle of ejection from the solution given in Fig. 1. In this work graphic solutions are given for wide ranges of  $V_0$  and  $\sigma$ , as it would substantially increase the volume of the paper to give greater details. For working calculations, solutions of system (2) are tabulated with an interval of  $(2.4-6.8-10) \cdot 10^{-n}$ ;  $2 \leq n \leq 5$  for  $\sigma$  and 50 m/s for  $V_0$ .

In using solutions of the direct problem, it is necessary to determine the field of

their application and also to take into account the necessary corrections. As has been reported (SELF *et al.*, 1974), the trajectories of fragments with  $\sigma > 10^{-3}$  ( $D < 40$  cm) are subject to wind action and in addition, for  $\sigma > 10^{-3}$  the solutions become unstable for distances of ejection of more than 2-3 km: very substantial variations in initial velocity correspond to small fluctuations in distance, as a result of which even slight wind action leads to essential errors in the determination of  $V_0$ . Therefore fragments with  $\sigma > 10^{-3}$   $m^2/kg$  are unsuitable for solving the inverse problem of external ballistics and they are left out of consideration.

The correction for overshooting the crater ( $\Delta h$ ) (Fig. 7) over the surrounding locality can attain a substantial value, particularly for explosions with low initial velocities and low angles of ejection. However, for high-velocity explosions ( $V_0 \geq 200$  m/s :  $10^{-4} < \sigma < 10^{-3}$ ) and overshooting of more than 500 m, this correction asymptotically approaches the limit of  $l_h/l = 1.1$  or  $l = 0.9l_h$ , i.e. the ballistic distance reduced to a horizontal plane is about 0.9 of the maximum recorded distance of ejection.

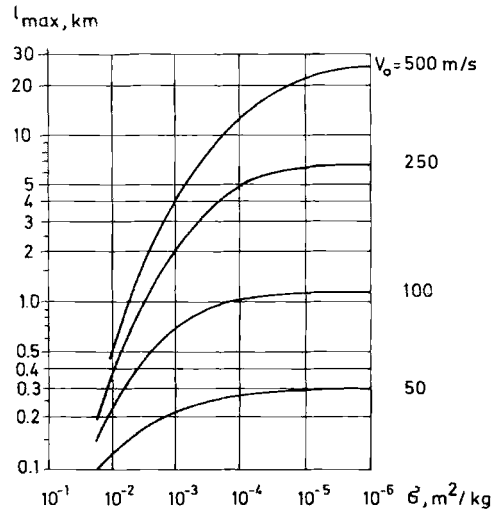


FIG. 3 — Dependence of maximum distance of ejection on  $V_0$  and  $\sigma$  ( $\theta_0 = \theta_{opt}$ ).

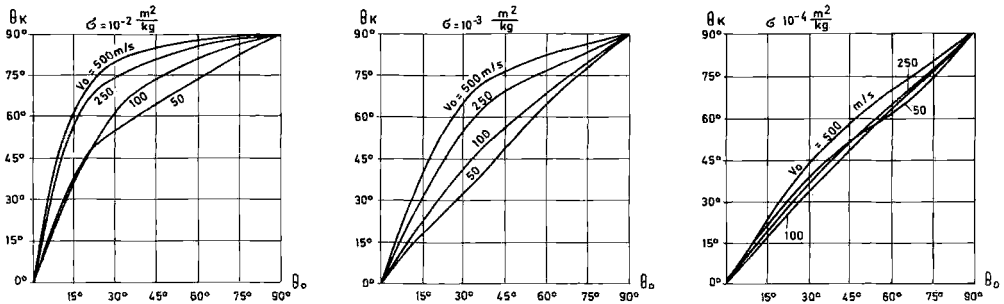


FIG. 4 — Dependence of angle of incidence of a fragment,  $\theta_k$ , on angle of ejection,  $\theta_0$ , and initial velocity  $V_0$ .

- 4a:  $\sigma = 10^{-2} \text{ m}^2/\text{kg}$   
 4b:  $\sigma = 10^{-3} \text{ m}^2/\text{kg}$   
 4c:  $\sigma = 10^{-4} \text{ m}^2/\text{kg}$

The precision of determination of initial velocities of fragments in solving the inverse ballistic problem depends on the precision of determination of the density and distance of ejection of the fragments. The density and also the size of the fragments can be determined with the necessary precision in every case. The distance of ejection of fragments is determined from their location on the site or, more precisely, from the position of the impact craters formed when they fell (Fig. 5, 6). Impact craters are preserved for 2-5 years after an eruption: Study of more than a hundred and fifty impact craters in the eruption of Alaid volcano (1972) showed that the fragments remained in the craters in 96% of the cases. In rare cases, for low angles of incidence, the fragments bounced and were found outside the impact crater. Analogous conclusions were reached in a study of impact craters on Kilauea volcano (HARTMAN, 1967).

The displacement of fragments after they fell is observed on rather steep slopes ( $> 15^\circ - 20^\circ$ ) and in the case of powerful directed blasts with low (flat) angles of inclination of the axis of the explosion. For such explosions a precise determination of the distance of ejection and, correspondingly, calculation of the initial velocities and kinetic energy of an explosion is rendered difficult. However,

in those cases a rigorous estimate of maximum and minimum values is possible. An example of such an estimate is given below for an explosion of Shiveluch volcano.

#### EXAMPLES FOR THE APPLICATION OF THE SOLUTIONS

##### *Asama Volcano*

The explosive eruptions of Asama volcano (1935, 1937, 1938) consisted of vertically oriented or slightly directed explosions. Data on the distances of ejection as a function of fragment size were taken from MINAKAMI (1942) and treated by the procedure discussed above; the results are given in Table 2, from which it is seen that there is little or no differentiation of the fragments according to velocity ( $V_{\max}/V_{\min} \leq 1.35$  when  $D_{\max}^3/D_{\min}^3 = 8$ ), which supports the assumption made as to the nature of the internal ballistics of a volcanic explosion. The velocity values obtained are 1.5–2 times higher than the determination in CHOUET *et al.* (1974), which does not take into account air resistance. The optimal angles of ejection given in MINAKAMI (1942), determined visually and from photographs of erup-

tions ( $37^\circ$ ), are close to those obtained ( $34^\circ$  -  $35^\circ$ ) in solving the inverse problem.

$l = 4.5$  km. Using the solution (Fig. 2) we find that

$$V_0 = 360-410 \text{ m/s}$$

#### *Arenal Volcano*

In an explosive satellite eruption of the volcano in July 1968, a considerable number of fragments 0.8 - 1.0 m in diameter were ejected to a distance of more than 5 km. The distance of ejection was determined from a large number of impact craters averaging 2 m in diameter. Overshooting of the satellite crater over the surrounding locality was 500-550 m. Calculations of the initial velocity of ejection (without taking air resistance into account gave  $V_0 = 220$  m/s (MELSON and SAENZ, 1968). Calculations made without taking into account overshooting of the crater but with air resistance taken into account and under the assumption that the maximum distance was realized when  $\delta_0 = 45^\circ$ , gave  $V_0 = 600$  m/s (FUDALI and MELSON, 1972).

However, for fragments with diameters of 1.0-0.8 m and  $\sigma = (4-5) 10^{-4} \text{ m}^2/\text{kg}$  the optimal angle of ejection is  $\delta_{\text{opt}} = 34-35^\circ$ , and the ballistic distance taking into account the correction for overshooting is

Thus, failure to take air resistance into account led to underestimation of the initial velocity by 1.6 - 1.85 times, and calculations that took it into account but not overshooting of the crater, and which choose the optimum angle of ejection incorrectly, overestimated the minimum necessary initial velocity by 1.45-1.65 times (accordingly, the kinetic energy in these calculations is 2-3 times different from reality). This example shows that failure to take, or only partially to take, into account the factors determining the distance of ejection leads to appreciable errors in solving the inverse ballistic problem.

#### *Shiveluch Volcano*

The deposits of the directed blast in the 1964 eruption consist of fragments of resurgent material from a few centimeters to 5-9 m in diameter (STEINBERG, 1977). Fragments with a diameter of less than 40



FIG. 5 — Volcanic impact craters from Shiveluch eruption 1964. (Photo by G.S. Steinberg).



FIG. 6 — Typical volcanic impact crater from Shiveluch eruption 1964. (Photo by G. S. Steinberg).

cm could not be used in the ballistic calculations ( $\sigma < 10^{-3} \text{ m}^2/\text{kg}$ ), moreover small fragments were formed chiefly due to breakup of larger ones upon impact and subsequent displacement, which was considerable on Shiveluch. Fragments with a diameter of more than 4 m ( $\sigma < 10^{-4} \text{ m}^2/\text{kg}$ ) occur relatively rarely and therefore it is impossible to assume that their distribution is the same in all directions of ejection. For the sake of simplicity the fragments of this directed blast are treated in terms of ballistics (see however KIEFFER, 1980). Consequently, fragments suitable for ballistic calculations, satisfying the assumption as to even distribution in all directions of ejection, lie in the range of  $10^{-4} < \sigma < 10^{-3}$ . Fragments with  $\sigma = (2-6) \cdot 10^{-4} \text{ m}^2/\text{kg}$  (or diameter of 2-0.6 m) were used in the calculations, assuming that they constituted the main mass of primary fragments of comminution in the explosion of 12 November 1964.

#### DISTANCE OF EJECTION AT SHIVELUCH VOLCANO

The boundary of deposits of the directed Shiveluch blast is drawn 8-10 km from the crater; a concentration of large fragments in the frontal part of the deposits was observed (GORSHKOV and DUBIK, 1969; ПЫП and MARKHININ, 1967; STEIN-

BERG, 1975, 1977: Fig. 3). Analysis of radial grooves on the surface of the explosive deposits, easily traced on aerial photographs and formed by movement of the ejected material after it fell to earth, shows that the distance of displacement was 2-3 km. This is confirmed by geologic observations: «dead» zones arising on small eminences are observed at a distance of 6-7 km from the crater (GORSHKOV and DUBIK, 1969). These zones arose due to by-passing of the eminences by the directed-blast deposits as they moved on the ground, and are not due to the fact that these eminences were obstacles for fragments with low angles of incidence, as it was suggested by GORSHKOV and DUBIK (1969). Calculations show that to form the observed «dead» zones by fallout rather than by subsequent movement of the fragmental material, the final angles of incidence  $\theta_k$  would have to have been not more than  $10^\circ$  to the horizontal; this in turn requires initial angles of ejection of  $\theta \leq 5^\circ$  (Fig. 4). At such low initial angles, as is seen from Fig. 1, the observed distances of ejection could not have been attained with the actually possible initial velocities.

Thus the distance of ejection without correction for overshooting of the crater is  $l_h = 6-7 \text{ km}$  for Shiveluch. As overshooting of the crater was more than 1 km, the correction is 0.6-0.7 km, and the maximum ballistic distance reduced to a horizontal plane is  $l = 6.3 \text{ km}$ . The initial velocities and optimal angles of ejection

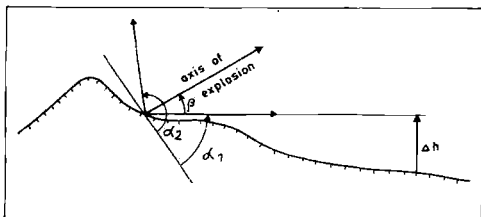


FIG. 7 — Sketch of angles of ejection in the directed blast of Shiveluch volcano.

$\beta$  = angle of axis of eruption with horizontal plane

$\alpha$  = angles of ejection with plane perpendicular to axis of eruption.

needed to provide that distance were found from the solutions (Fig. 1,2) and are given in Table 3.

To estimate the kinetic energy of the explosion taking into account that the initial velocities are not the same in all directions, it is necessary to determine the integral value of initial velocity of ejection,  $V$ , and also the mass of ejected material and inclination of the angle of the eruption to the horizontal. The axis of the eruption made an angle of  $\beta = 30^\circ$  with the horizontal (GORSHKOV and DUBIK, 1969). As is seen from Table 3, for the main mass of fragments the optimal angle of ejection  $\vartheta_{opt}$  is  $30-36^\circ$ , i.e.  $\vartheta_{opt} - \beta \leq 6^\circ$ . Taking into account that the initial velocities are distributed according to (1), the velocities of ejection given in Table 3 based on the optimal direction,  $V_{opt}$ , differ from the maximum initial velocities  $V_0$  by not more than  $\sin^{3/2} 6^\circ = 0.03$ , i.e. less than 3%. With the precision applicable to our calculations, it can be considered that the velocities  $V_{opt}$  given in Table 3 are equal to the maximum velocities  $V_0$  for fragments with corresponding  $\sigma$ .

The full angle of ejection in the explosion of 12 November 1964 was  $90-95^\circ$  (Fig. 7), and if it is counted off from the plane perpendicular to the axis of the eruption the range of variation is  $60-155^\circ$ . Accord-

ingly, the integral value of velocity of ejection is:

$$V = V_0 \int_{60^\circ}^{155^\circ} \sin^{3/2} \alpha d\alpha = 0.805 V_0 \approx 0.805 V_{opt} \quad (3)$$

The volume, density, and mass of directed-blast deposits are estimated as:

$$v = 1.5 \cdot 10^9 \text{m}^3, \quad \rho_k = 2.0 \cdot 10^3 \text{kg/m}^3 \\ M = 3 \cdot 10^{12} \text{kg} \quad (\text{GORSHKOV and DUBIK, 1969});$$

$$v = 0.9 \cdot 10^9 \text{m}^3, \quad \rho_k = 2.45 \cdot 10^3 \text{kg/m}^3 \\ M = 2.3 \cdot 10^{12} \text{kg} \quad (\text{STEINBERG, 1975}).$$

In the first case the data were obtained by measuring the area of distribution, thickness, and density of the directed-blast deposits, and in the second they were found from the size of the crater and density of the rocks in undisturbed occurrence.

For a precise determination of the energy of the explosion, besides the mass of ejected material and integral velocities it is necessary to know the distribution of the ejected mass by fragment size,  $M(\sigma)$ , or - what is the same thing - by velocity,  $M(V)$ . In view of the lack of that distribution we can determine the upper and

TABLE 3 - Parameters of the explosive eruption of Shiveluch volcano, 12.11.1964.

$\sigma \cdot 10^{-4}$ m <sup>2</sup> / kg	$\beta$ , m	$z$ , km	$\vartheta_{opt}$	$V_{opt} \approx V_0$ m/s	$\bar{v}$ , m/s
2	2.0	6.3	$36^\circ$	355	285
4	1.0	6.3	$35^\circ$	510	410
6	0.65	6.3	$34^\circ$	720	580

TABLE 4 — Variants of calculations of energy of the explosion of Shiveluch volcano.

Energy	Ejected mass, kg	
	2.3.10 <sup>12</sup>	3.10 <sup>12</sup>
$E_{\min}$ , J	9.3.10 <sup>16</sup>	1.2.10 <sup>17</sup>
$E_{\max}$ , J	3.9.10 <sup>17</sup>	5.2.10 <sup>17</sup>
$\bar{E}$ , J	2.2.10 <sup>17</sup>	2.9.10 <sup>17</sup>
$E_a$ , J	2.3.10 <sup>17</sup>	3.3.10 <sup>17</sup>

lower limits of kinetic energy of the explosion which fit the established distance of ejection (6.3 km).

It should be mentioned that the velocities  $V_{\text{opt}}$  given in Table 3 were obtained under the assumption that the maximum distance of ejection of all fragments, regardless of size, is 6.3 km. The need for that assumption arose because the fragmental material on Shiveluch was subjected to intensive break-up and displacement after it fell to earth, as a result of which it was impossible to make isopleth maps or determine the  $l(\sigma)$  dependences analogous to those given in Table 2. As a result of that assumption, a distinct differentiation of velocities as a function of  $\sigma$  is obtained (Table 3) which is not typical of volcanic explosions in general or of Shiveluch in particular, where a tendency toward concentration of large fragments in the frontal part is very clearly manifested. The determination of the minimum kinetic energy is equivalent to the presumption that all the fragments have  $\sigma = 2 \cdot 10^{-4} \text{ m}^2/\text{kg}$  ( $D = 2 \text{ m}$ ) and, correspondingly,  $\bar{V} = 285 \text{ m/s}$ . In determining the maximum kinetic energy  $E_{\max}$  we assume  $\sigma = 6 \cdot 10^{-4} \text{ m}^2/\text{kg}$  ( $D = 0.65 \text{ m}$ ) and  $\bar{V} = 580 \text{ m/s}$ . If an even distribution of mass by fragment size is given,  $M \sigma_1 = M \sigma_2 = \dots = M \sigma_n = M \sigma$ , then  $\bar{E} = \frac{M}{2n} \sum_{i=1}^n \bar{V}_i^2$  or for our case,  $\bar{E} = \frac{M}{6} (\bar{V}_1^2 + \bar{V}_2^2 + \bar{V}_3^2)$ . If a

distribution of velocities as a function of  $\sigma$  analogous to that obtained for Asama is adopted for Shiveluch ( $V_{\max}/V_{\min} = 1.35$ ), then the kinetic energy of the explosion is  $E = (2.3-3.3) \cdot 10^{17} \text{ J}$ . The results of the calculation of kinetic energy for different variants are given in Table 4.

In conclusion it should be stressed that the purpose of this study is not to give more precise values of initial velocities and kinetic energy for the cases considered, but to show how ballistic calculations for volcanic explosions can be improved and what factors should be taken into account in doing so.

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#### SYMBOLS

- $V$  = velocity
- $V_0$  = initial velocity
- $V_k$  = terminal velocity
- $h$  = height of ejection
- $g_0$  = 9.81 m/s<sup>2</sup>, acceleration of gravity at sea level
- $g$  = acceleration of gravity
- $\sigma$  =  $S/G = 3c/2\rho_b D$  = ballistic coefficient (in m<sup>2</sup>/kg)
- $c$  = drag coefficient
- $S$  = cross section of fragment (m<sup>2</sup>)
- $G$  = weight of fragment (kg)
- $D$  = diameter of fragment (m)
- $\rho_b$  = density of fragment (kg/m<sup>3</sup>)
- $\rho$  = density of air
- $q$  =  $\rho V^2/2$  = form drag
- $t$  = time
- $\delta_0$  = initial angle of ejection
- $\delta_{\text{opt}}$  = angle of ejection providing maximum distance
- $\delta_k$  = final angle of incidence
- $\alpha$  = angle between direction of ejection and plane perpendicular to axis of explosion (in normal explosions  $\alpha = \delta$ )
- $l$  = ballistic distance of ejection

- $l_h$  = distance of ejection without correction for overshooting the crater  
 $\Delta h$  = overshooting of the crater  
 $R$  = radius of the earth

## REFERENCES

- BLACKBURN, E. A., WILSON, L. & SPARKS, R. S. J., 1976, *Mechanism and Dynamics of Strombolian Activity*. Jour. geol. Soc. Lond., 132, p. 429-440.
- CHOUET, B., HAMITSEVICZ, N. and MCGETCHIN T. R., 1974, *Photoballistics of Volcanic Jet Activity at Stromboly, Italy*. Jour. Geophys. Res., 79, p. 4961-4976.
- DOKUCHAYEV, M. M., RODIONOV, V. N. and ROMASHEV, V. D., 1963, *Explosion with Ejecta*. Izd. AN SSSR, (in Russian).
- FUDALI, R. F., MELSON, W. G., 1972, *Ejecta Velocities, Magma Chamber Pressure and Kinetic Energy Associated with the 1968 Eruption of Arenal Volcano*. Bull. Volcanol., 35, p. 383-401.
- GORSHKOV, G. S. and BOGOYAVLENSKAYA, G. Y., 1965, *Bezmyannyy Volcano and the Particulars of Its Last Eruption*. «Nauka», (in Russian).
- and DUBIK, Y. M., 1969, *Directed Blast on Shiveluch Volcano*. In: *Volcanoes and Eruptions*. «Nauka», (in Russian).
- GRATON, L. C., 1949, *Conjectures Regarding Volcanic Heat*. IL, (in Russian).
- GUSHCHENKO, I. I., 1965, *Ashes of Northern Kamchatka and Conditions of Their Formation*. «Nauka», (in Russian).
- HARTMANN, W. K., 1967, *Secondary Volcanic Impact Craters at Kapoho (Hawaii) and Comparisons with the Lunar Surface*. Icarus, 7-1.
- HEDERVARI, P., 1963, *On the Energy and Magnitude of Volcanic Eruptions*. Bull. Volcanol., 25, p. 372-385.
- JOHNSON, G. W., HIGGINS, G. H. and VIOLET, C. E., 1959, *Underground Nuclear Detonations*. Jour. Geophys. Res., 64, p. 1457-1469.
- KATSIK, A. A., LEBEDEV, Y. B. and KHITAROV, N. I., 1971, *Water in Magmatic Melts*. «Nauka», (in Russian).
- KIEFFER, S. W., 1980, *Fluid Dynamics of the May 18 Blast at Mount St. Helens*. U.S.G.S. Prof. Papers, 1250, p. 379-400.
- KNOX, I. B., 1969, *Nuclear Excavation: Theory and Applications*. Nuclear Appl. and Technology, 7, p. 182-231.
- KUZNETSOV, G. N., 1967, *Determination of the Volatile Content in a Magma in Volcanic Eruptions*. Geologiya i Geofizika, 4 (in Russian).
- LEGLER, V. A. and STEINBERG, G. S., 1975, *On the Origin of the Kamchatka Maars*. In: *Geology of the Far East*. Vladivostok, (in Russian).
- LIPMAN, P. W. and MULLINEAUX, D. R. (eds.), 1981, *The 1980 Eruptions of Mount St. Helens, Washington*. Geol. Soc. Prof. Paper, 1250, 844 pp.
- LORENZ, V., 1970, *Some Aspects of the Eruption Mechanism of the Big Hole Maar, Central Oregon*. Geol. Soc. Amer. Bull., 81, p. 1823-1830.
- , 1971, *An Investigation of Volcanic Depressions. Part IV. Origin of Hole-in-the-Ground, a Maar in Central Oregon*. Nasa progr. Rep. (NGR-38-012), Houston, Tex., pp. 113.
- , 1980, *Explosive Volcanism of Alkali-basaltic to Kimberlitic Melts*. Lithos, 13, p. 217-219.
- MARKHININ, Y. K., 1976, *The Role of volcanism in the Formation of the Earth's Crust*. «Nauka», (in Russian).
- MC BIRNEY, A. R., 1973, *Factors Governing the Intensity of Explosive Andesitic Eruptions*. Bull. Volcanol., 37, p. 443-453.
- MCGETCHIN, T. R., SETTLE, M. and CHOUET, B. A., 1974, *Cinder Cone Growth Modeled after Northeast Crater, Mount Etna, Sicily*. Jour. Geophys. Res., 79, p. 3257-3272.
- MELSON, W. G. and SAENZ, 1968, *The 1968 Eruption of Volcano Arenal: Preliminary Summary of Field and Laboratory Studies*. Smithsonian Center for Short-Lived Phenomena, Report 7/1968, 35.
- MERZHANOV, A. G., RAZINA, A. A., FUNDAMENSKY, V. S., STEINBERG, A. S., STEINBERG G. S., 1973, *Investigation of Geyser Systems by the Hydrosounding Method*. Byull. Vulkan. St., no. 49, (in Russian).
- , STEINBERG, A. S. and STEINBERG, G. S., 1970, *On the Theory of the Geyser Process*. Doklady AN SSSR, 194-2, (in Russian).
- MINAKAMI, T., 1942, *On the Distribution of Volcanic Ejecta (Part. I). The Distribution of Volcanic Bombs Ejected by the Recent Explosions of Asama*. Bull. Earth. Res. Inst. Tokyo, 20, p. 65-92.
- MOORE, J. G., NAKAMURA, K. and ALCARAZ A., 1966, *The 1965 Eruption of Taal Volcano*. Science, 151, p. 955-960.
- NININGER, H. H., 1956, *Arizona's Meteoritic Crater: Past - Present - Future*. American Meteor. Museum, Sedona, Arizona.
- PIYP, B. I. and MARKHININ, Y. K., 1965, *Gigantic Eruption of Shiveluch Volcano*. Byull. Vulkan. St., no. 39, (in Russian).
- RICHARDS, J. H., 1965, *Linear Relationship between Energy and Pressure of Volcanic Explosions*. Nature, 207, p. 1382-1383.

- RYABININ, Y.N. and RODIONOV, V.N., 1966, *Estimation of the Energy of Volcanic Explosions*. In: *Volcanism and the Deep Structure of the Earth*, Proc. of 2nd All-Union Volcanological Conference, 3, «Nauka», (in Russian).
- SADOVSKIY, M.A., ADUSHKIN, V.V., RODIONOV, V.N. and STARTSEV, G.N., 1967, *On a Method of Modeling a Point Explosion with Ejecta*. *Fizika zryva*, no. 1, (in Russian).
- SAKHAROV, V.N., KOLESNIKOV-SVINAREV, V.I. and ZABIDAROV, Y.I., 1959, *Distribution over a Locality of Ground Ejected in Underground Explosions*. *Doklady AN SSSR*, 124, no. 2, (in Russian).
- SELF, S., SPARKS, R.S., BOTH, B. and WALKER, G.P.L., 1974, *The Heimey Strombolian Scoria Deposit, Iceland*. *Geol. Mag.*, 111, p. 539-548.
- SHERWOOD, A.E., 1967, *Effect of Air Drag on Particles Ejected during Explosive Cratering*. *Jour. Geophys. Res.*, 72, p. 1783-1791.
- SHOEMAKER, A.E., 1962, *Interpretation of Lunar Craters*. In: KOPAL, Z. (ed.), *Physics and Astronomy of the Moon*. Acad. Press, New York, London, p. 283-359.
- SMITH, R.L., 1963, *Volcanic Ash Flows*. In: *Problems of Paleovolcanology*. IL, p. 321-316 (Russian translation).
- SPARKS, R.S.J. & WILSON, L., 1976, *A Model for the Formation of Ignimbrite by Gravitational Column Collapse*. *J. geol. Soc. Lond.*, 132, p. 441-451.
- , WILSON, L. & HULME, G., 1978, *Theoretical Modelling of the Generation, Movement and Emplacement of Pyroclastic Flows by Column Collapse*. *J. geophys. Res.*, 83, p. 1727-1739.
- STEINBERG, G.S., 1973, *On the Explosive Formation of Calderas*. *Doklady AN SSSR*, 208-6 (in Russian).
- , 1974, *Volcanic Explosion - Physics of the Process*. In: *Geodynamics of Volcanism and of the Hydrothermal Process*. Petropavlovsk-Kamchatski, (in Russian).
- , 1975, *On the Determination of the Energy and Depth of Origin of Volcanic Explosions*. *Geologiya i Geofizika*, no. 7, (in Russian).
- , 1977, *On the Determination of the Energy and Depth of Volcanic Explosions*. *Bull. Volcanol.*, 40, p. 116-120.
- and BABENKO, J.I., 1976, *Experimental Determination of Velocity and Density of Volcanic Gases at the Time of an Eruption*. *Doklady AN SSSR*, 226-6, (in Russian).
- and ———, 1978, *Experimental Velocity and Density Determination of Volcanic Gases during Eruption*. *Jour. Volc., Geotherm. Res.*, 3, p. 89-98.
- and STEINBERG, A.S., 1975, *On Possible Causes of Volcanic Tremor*. *Jour. Geophys. Res.*, 80, p. 1600-1604.
- TEWES, H.A., 1969, *Results of the Cabriolett Excavation Experiment*. *Nuclear Appl. and Technology*, 7, p. 232-234.
- THORARINSSON S., EINARSSON, T., SIGVALDASON, G. and ELLISON, G., 1964, *The Submarine Eruption of the Vestmann Islands 1963/64*. *Bull. Volcanol.*, 37, p. 435-445.
- VDOVYKIN, G.P., 1971, *The Cañon Diablo Meteorite*. «Nauka», p. 31, (in Russian).
- VERHOOGEN, J., 1951, *Mechanics of Ash Formation*. *Amer. Jour. Sci.*, 249, p. 729-739.
- WALKER, G.P.L., WILSON, L. and BOWELL, E.L.G., 1971, *Explosive Volcanic Eruptions. I. The Rate of Fall of Pyroclasts*. *Geophys. Jour. Roy. Astr. Soc.*, 22, p. 377-383.
- WILLIAMS, H., 1942, *The Geology of Crater Lake National Park, Oregon*. Carnegie Inst. Wash. Publ., 540, pp. 162.
- WILSON, L., 1972, *Explosive Volcanic Eruptions. II. The Atmospheric Trajectories of Pyroclasts*. *Geophys. Jour. Roy. Astr. Soc.*, 30, p. 381-392.
- , 1976, *Explosive Volcanic Eruptions. III. Plinian Eruption Columns*. *Geophys. Jour. Roy. Astr. Soc.*, 45, p. 543-556.
- , 1980, *Relationships between Pressure, Volatile Content and Ejecta Velocity in Three Types of Volcanic Explosion*. *J. Volc. Geothermal Res.*, 8, p. 297-313.

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