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Journal of Hydrology 274 (2003) 145–159
Journal of Hydrology
www.elsevier.com/locate/jhydrol

Evaluation of the use of reach transmissivity to quantify exchange between groundwater and surface water

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Received 5 March 2002; revised 7 November 2002; accepted 29 November 2002

Abstract

A method to quantify the exchange of water between surface water channels and the groundwater aquifer based on the concept of reach transmissivity was evaluated for use under transient conditions and incorporated into a numerical model. This method provides significant benefits relative to the common existing method of calculating leakage, which assumes flow of water through a discrete, low permeability layer at the bed of the surface water channel. The reach transmissivity leakage relationship employs input parameters that are more commonly available from published sources, which reduces the effort required for model calibration; also, the calibrated values of the reach transmissivity relationship input parameters are less dependent on model grid spacing. Furthermore, the reach transmissivity relationship allows calculation of flow to each side of the surface water channel, rather than calculating only the net exchange between the surface water and groundwater systems. The reach transmissivity leakage relationship is based on the assumption of steady state conditions; a method was developed to estimate the error associated with its use to simulate transient conditions. The reach transmissivity relationship was further evaluated through its use in analytical and numerical models to simulate conditions in a field study area.

Keywords: Surface water; Groundwater; Leakage; Seepage; Numerical model

1. Introduction

Quantification of the exchange of water between surface water channels and the groundwater aquifer is frequently of scientific or practical interest. This calculation is of particular value as a means of coupling numerical models of surface water and groundwater. Development of combined surface water and groundwater models is difficult because most computer models of groundwater flow are based on Darcy's law, whereas most surface water models employ the Saint Venant equations. Additionally, surface water systems, such as rivers and canals, tend to respond more quickly to changing boundary conditions than groundwater systems; the time scale of interest is often significantly longer for analysis of groundwater than for analysis of surface water.

A method that used a relationship that assumed that leakage occurred as vertical flow through the bed of the surface water channel has been used to link previously independent numerical models of

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doi:10.1016/S0022-1694(02)00419-5

Nomenclature

b'	thickness of the low permeability layer of the channel boundary (m)
B	top width of the channel (m)
h	groundwater head, saturated thickness of the aquifer (m)
k	hydraulic conductivity of the aquifer (m/s)
k'	hydraulic conductivity of the low permeability layer of the channel bottom (m/s)
L	horizontal distance from surface water channel (m)
q	leakage to the aquifer from the channel (volume of water per unit channel length per unit time) (m^2/s)
s	dimensionless variable (h/Z_0)
s_1	dimensionless variable (Z_1/Z_0)
S	specific yield of the aquifer
t	time (s)
T	aquifer transmissivity (m^2/s)
V_x	Darcy velocity (m/s)
V_0	Darcy velocity at reservoir boundary (m/s)
x	distance from the reservoir boundary to the point at which at which groundwater head is measured (m)
Z	surface water elevation in the surface water reservoir (m)
Z_0	initial surface water elevation in the surface water reservoir (m)
Z_1	surface water elevation in the surface water reservoir after an instantaneous change in elevation (m)
Γ_r	reach transmissivity or volumetric flow rate of water per unit drawdown per unit channel length
ξ	Boltzmann variable ($x/\tau^{1/2}$)
ξ_0	dimensionless variable ($\xi(Z_0)^{-1/2}/2$)
ξ_I	value of inflection point in ξ curve
τ	Boltzmann variable (kt/s)
ψ	variable in Darcy velocity equation
ψ_0	value of ψ at reservoir boundary

groundwater and surface water by calculating leakage between surface water channels and the groundwater aquifer (Swain and Wexler, 1996). This vertical flow leakage relationship often requires extensive model calibration because the field conditions frequently do not conform to the conceptualization on which its derivation is based. An alternate leakage relationship, based on the concept of reach transmissivity, was developed to provide a more convenient means of linking groundwater and surface water models. The derivation of the reach transmissivity relationship required the assumption of steady state conditions, so a method to estimate the error associated with its use to simulate transient conditions was developed. The performance of the reach transmissivity relationship was tested in analytical and numerical models of a field problem.

2. Formulation of leakage relationships

The leakage relationship previously employed to quantify exchange between the groundwater and surface water systems will be designated as the vertical flow relationship; it assumes flow through a discrete, low permeability layer at the bottom of the channel. The mathematical formulation of this relationship is based on Darcy's law and may be expressed as follows:

$$q = \frac{k'}{b'}B(Z - h) \quad (1)$$

where

q is leakage to the aquifer from the channel (volume of water per unit channel length per unit time), k' is hydraulic conductivity of the low permeability layer of the channel bottom, b' is thickness of the low permeability layer of the channel bottom, B is top width of the channel, Z is surface water elevation in the channel, and h is elevation of the groundwater table immediately adjacent to the channel.

A schematic representation of the vertical flow leakage relationship is shown in Fig. 1. The common conceptualization of the vertical flow relationship assumes that the sides of the channel are impermeable and that all leakage occurs through the bottom of the channel in the vertical direction, which results in a discontinuity in water-surface elevation between the channel and aquifer at the lateral boundary of the channel. However, the relationship is valid for any situation in which exchange of groundwater and surface water occurs through a discrete, low permeability layer at the boundary of the surface water channel. The low permeability layer may have any geometry, so long as it forms a constant-thickness boundary between the groundwater and surface water systems through which all exchange must occur. Consequently, leakage does not necessarily flow only in the vertical direction. Nevertheless, the conditions depicted in Fig. 1—which do assume that leakage occurs only vertically–have frequently served as the basis for development and analysis of this leakage relationship.

An alternate leakage relationship can be developed using the concept of reach transmissivity (Morel-Seytoux, 1975; Morel-Seytoux and Daly, 1975; Illangasekare and Morel-Seytoux, 1986). This relationship was selected because of its simplicity and the availability of independent physical measures of its coefficients; additionally, it responds differently to local changes in surface water elevations than the vertical flow relationship. Furthermore, the relationship has been shown to function well under steady-state conditions (Chin, 1991), and it has been hypothesized to be suitable for analysis of transient conditions (Mishra and Seth, 1988). In general form, the reach transmissivity relationship is expressed as

$$q = \Gamma_r(Z - h) \tag{2}$$

where Γ_r is the reach transmissivity or volumetric flow rate of water per unit drawdown per unit channel length, and h is the groundwater head measured at a distance L away from the channel. The parameter Γ_r is dependent on the geometry of the channel cross-section and the characteristics of the channel bed. Illangasekare and Morel-Seytoux, (1982) developed an expression for Γ_r for a condition in which symmetrical conditions exist on each side of the surface water channel; in this relation, Γ_r is a function of the wetted perimeter of the channel and the aquifer transmissivity and saturated thickness. However, it is possible to use reach transmissivity to develop a leakage expression for asymmetrical conditions by making certain simplifying assumptions. When the channel width is large relative to the distance from

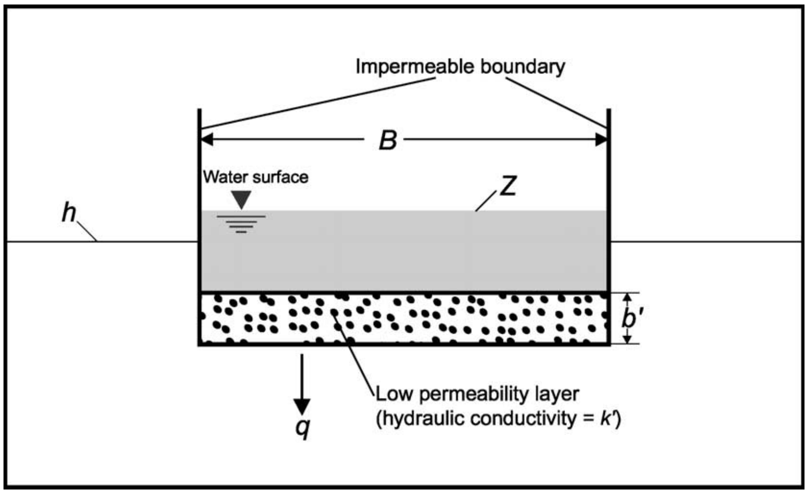

Fig. 1. Vertical flow leakage relationship.

the bottom of the channel to an impervious layer underlying the aquifer, the channel can be assumed to be fully penetrating (Mishra and Seth, 1988). In such a case, the Dupuit–Forcheimer assumption (Bear, 1979) can be used to help derive an estimate of reach transmissivity.

The derivation begins with Darcy's law, written for one-dimensional flow per unit width:

$$q = -kh\frac{dZ}{dx} \tag{3}$$

where k is the hydraulic conductivity of the aquifer, h is the saturated thickness of the aquifer, and dZ/dx is the horizontal head gradient. The finite-difference form of this equation is

$$q = \frac{T(Z-h)}{L-\frac{B}{2}} \tag{4}$$

where the aquifer transmissivity, T, is the product of the hydraulic conductivity and thickness.

In a surface water channel, leakage occurs to two sides (Fig. 2). The total leakage from the channel is the sum of the leakage to the two sides and is expressed as

$$q = \frac{T_R(Z-h_R)}{L_R-\frac{B}{2}} + \frac{T_L(Z-h_L)}{L_L-\frac{B}{2}} \tag{5}$$

where the subscripts R and L indicate the right and left sides of the channel, respectively.

3. Theoretical performance of the reach transmissivity leakage relationship under transient conditions

The reach transmissivity equation was originally derived on the assumption of steady state conditions; its use introduces error under transient conditions where values of the component parameters of Γ_r are not constant with time. However, this error becomes negligible as conditions in the system approach steady state. In many applications, the error introduced by the steady state assumption of the reach transmissivity relationship may be insignificant, particularly when the expression for Γ_r contains parameters that can be recalculated at each time step. However, in cases where conditions deviate greatly from steady state, or when numerical model time steps are unusually small, the reach transmissivity relationship may produce undesirably large error. Quantification of this error is therefore necessary to ensure that the reach transmissivity relationship is not employed in an application for which its use is inappropriate. The suitability of the reach transmissivity relationship for a particular application can be evaluated by comparing results obtained by the reach transmissivity equation to the solution of the Boussinesq equation, which is accurate for both transient and steady state conditions.

For a semi-infinite unconfined aquifer adjacent to a reservoir (Fig. 3), the reach transmissivity equation may be stated as

$$q = T\frac{(Z_1-h)}{x} \tag{6}$$

where T is the transmissivity of the aquifer and Z_1 is the subsequent stage of the reservoir. In a numerical model, h is the groundwater head in the reference cell, and x is the distance from the boundary of the reservoir to the center of the cell. For an analytical model relying on field measurements of h, x is the distance from the reservoir boundary to the point at which at which h is measured. Dividing by the aquifer thickness, the Darcy velocity obtained with the reach transmissivity relationship is then given as

$$V_x = k\frac{(Z_1-h)}{x} \tag{7}$$

where V_x is the Darcy velocity and k is the hydraulic conductivity of the aquifer.

In applications where a numerical model has already been developed, values of h can easily be obtained from the model's results. However, use of Eq. (7) does not require that the groundwater head (h) be obtained by any particular method. When an analysis of potential error of the reach transmissivity relationship is conducted prior to model development, obtaining reliable estimates of the groundwater head at a particular location may be problematic. If they are available, field measurements of groundwater head and channel stage can be used as input to Eq. (7). If neither field measurements nor numerical model results are available, as is frequently the case at the inception of a project, the groundwater head at any

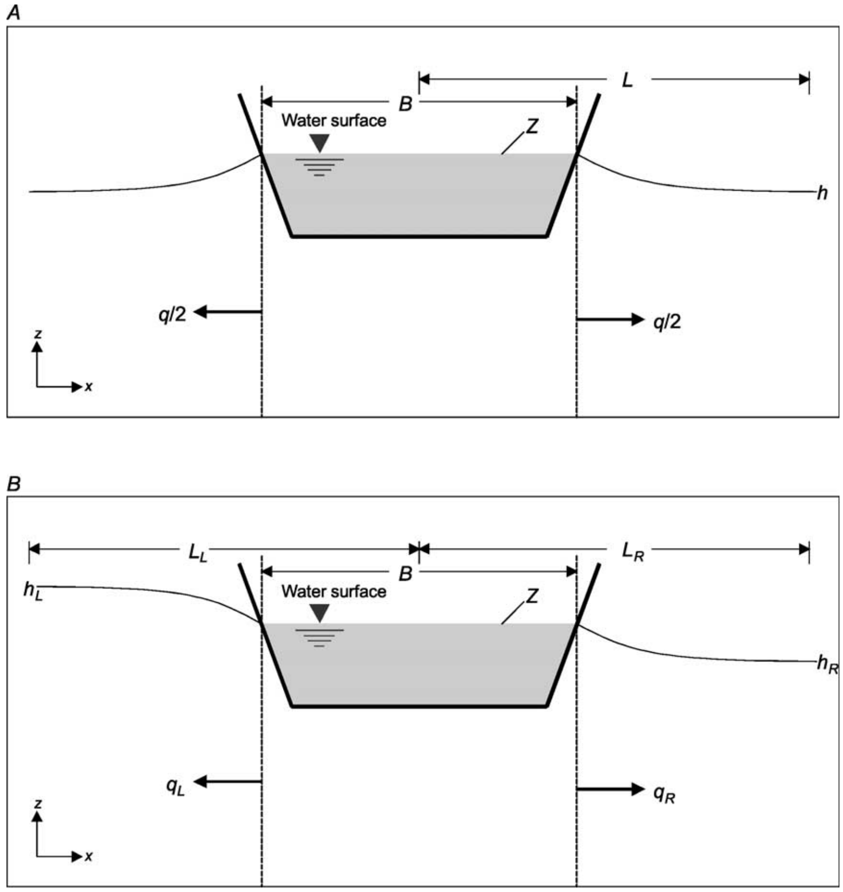

Fig. 2. Reach transmissivity leakage relationship for (A) symmetrical conditions and (B) asymmetrical conditions.

distance from the channel can be determined by solving the Boussinesq equation.

The Boussinesq equation describes the true solution to the problem of transient head in a semi-infinite unconfined aquifer adjacent to a reservoir:

$$\frac{\partial h}{\partial t} = \frac{k}{S}\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) \tag{8}$$

where h is the head in the aquifer, t is time, S is the specific yield of the aquifer, and x is the distance from the reservoir. This equation is for one-dimensional flow and is subject to the following assumptions: vertical gradients, pumping, recharge, and evapotranspiration are negligible; the aquifer is homogeneous and isotropic; the release of water from storage is instantaneous; and the aquifer bottom is horizontal.

The initial and boundary conditions are

$$h(0,t) = Z_1 \tag{9}$$

$$h(\infty,t) = Z_0 \tag{10}$$

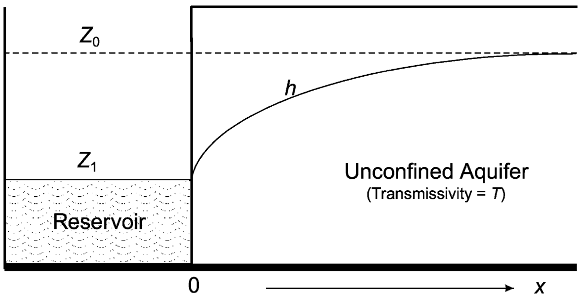

Fig. 3. Semi-infinite unconfined aquifer adjacent to a reservoir.

and

$$h(x,0) = Z_0 \tag{11}$$

where Z_0 and Z_1 represent the initial and subsequent stage in the reservoir (measured from the bottom of the aquifer), respectively.

Guo (1997) developed a solution to the Boussinesq equation that includes introduction of the following two variables:

$$\tau = \frac{kt}{S} \tag{12}$$

and

$$\xi = \frac{x}{\sqrt{\tau}} \tag{13}$$

This solution technique uses the Boltzmann transformation and an application of the mean value theorem for integrals to derive the following equation describing head in the aquifer under transient conditions:

$$h^2 = Z_1^2 + (Z_0^2 - Z_1^2)\mathrm{erf}\left(\frac{\xi}{\sqrt{4h}}\right) \tag{14}$$

where 'erf' denotes the error function. This equation cannot be explicitly solved for h and must be solved numerically. Conversion of the equation to a dimensionless form makes the solution procedure more convenient (Guo, 1997); the equation may be expressed as

$$s^2 = s_1^2 + (1 - s_1^2)\mathrm{erf}\left(\frac{\xi_0}{\sqrt{s}}\right) \tag{15}$$

where

$$s = \frac{h}{Z_0} \tag{16}$$

$$s_1 = \frac{Z_1}{Z_0} \tag{17}$$

and

$$\xi_0 = \frac{\xi}{2\sqrt{Z_0}} \tag{18}$$

However, even the dimensionless form of the equation requires numerical solution by the Newton–Raphson method or a similar technique, which may be inconvenient for some applications. A plot of solutions to Eq. (15) is presented in Fig. 4; use of dimensionless parameters is necessary to develop a two-dimensional plot applicable to all configurations of parameters. In most cases, all the parameters comprising ξ_0 can be determined a priori, and Fig. 4 can be used to provide an estimate of h without requiring an iterative solution to Eq. (15). When regarding Fig. 4, it is important to note that large values of ξ_0 correspond to large deviations from steady state (i.e. highly transient conditions); ξ_0 is equal to zero at steady state.

The asymmetrical nature of Fig. 4 is evident from cursory examination. Curves for the recharging case (i.e. when the reservoir level is higher than the aquifer) exhibit different characteristics than the curves for the discharging case. Errors resulting from the steady state assumption of the reach transmissivity relationship will therefore also differ for the recharging and discharging cases. Furthermore, the curvature of the plots is worthy of examination. The curves for the discharging case are concave downward over their entire range. In contrast, the curves for the recharging case have an inflection point at $\xi_0 = \xi_I$; the curves are concave downward for $0 < \xi_I < \xi_0$ and concave upward for $\xi_0 > \xi_I$. The portions of the recharging curves that are concave downward are very nearly linear. The discharging and recharging curves asymptotically approach $h/Z_0 = 1$ as ξ_0 approaches infinity, which occurs with increasing values of S and x and decreasing values of $t,k,$ and Z_0.

When the aquifer head at a specified distance from the reservoir has been obtained, the Darcy velocity at that distance can be calculated by the following equation (Guo, 1997):

$$V_x = -\frac{\Psi k}{\sqrt{\tau h}\left(2h + \dfrac{x\Psi}{2h\sqrt{\tau h}}\right)} \tag{19}$$

in which

$$\Psi = \frac{(Z_0^2 - Z_1^2)}{\sqrt{\pi}}\exp\left(-\frac{\xi^2}{4h}\right) \tag{20}$$

The usual application of the reach transmissivity relationship is to calculate leakage at the boundary of a reservoir, such as a canal or river. Therefore, the Darcy velocity calculated by the reach transmissivity equation should be compared to the actual Darcy

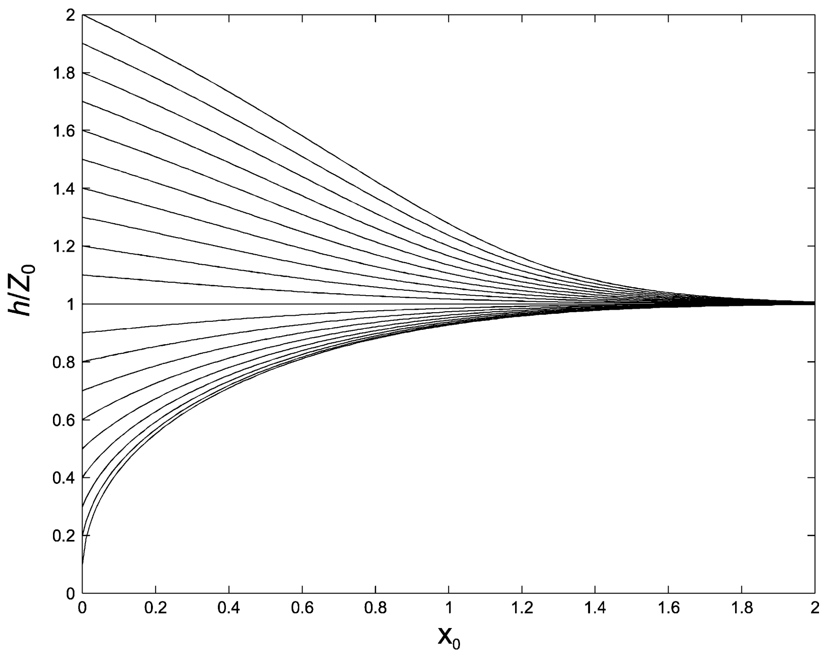

Fig. 4. Dimensionless head distributions corresponding to different boundary conditions.

velocity at the boundary of the reservoir, where h is equal to Z_1, and ξ and x are both zero. This leads to the following equation for calculation of the Darcy velocity at the reservoir boundary:

$$V_0 = -\frac{\Psi_0 k}{2Z_1\sqrt{\tau Z_1}} \tag{21}$$

in which

$$\Psi_0 = \frac{(Z_0^2 - Z_1^2)}{\sqrt{\pi}} \tag{22}$$

Comparison of results obtained by the reach transmissivity relationship to the solution of the Boussinesq equation can be used to determine whether the reach transmissivity relationship is suitable for a particular application. When making such a comparison, it is advisable to use the set of expected or historical conditions that least approximates steady state so that the evaluation of the reach transmissivity relationship will reveal the maximum error that might occur. However, it is important to recall that the comparison is dependent on several assumptions that must be valid to ensure applicability of the Boussinesq equation; if a particular field application significantly violates these assumptions, the analysis may be inaccurate. This analytical procedure can be used to determine whether the expected error resulting from use of the reach transmissivity relationship under transient conditions is within acceptable tolerances, which will depend upon the particular application.

The preceding equations are sufficiently complex in formulation that cursory examination does not reveal all the details of how variations in parameters affect the accuracy of the reach transmissivity equation. The influence of individual parameters on error associated with the reach transmissivity relationship is summarized in Table 1.

Although a smaller distance from the aquifer (x) reduces error, the distance must be large enough to ensure that the Dupuit–Forcheimer assumption is valid. This distance depends on the characteristics of a particular site, but previous research has developed guidelines for determination of the minimum x at which the Dupuit–Forcheimer assumption is valid. In a study of open channels, Bouwer (1965) notes that the Dupuit–Forcheimer assumption was not violated at any distances beyond ten bottom-widths of the channel. Chin (1991) states that

Table 1
Influence of input parameters on error associated with the reach transmissivity relationship

Parameter	Relationship to error	Explanation
Hydraulic conductivity, k	Inverse	High hydraulic conductivity results in less resistance to flow and lower hydraulic gradients, so steady state conditions are restored more quickly.
Time, t	Inverse	Steady state conditions are approached as time increases.
Aquifer thickness, Z_0	Inverse	As aquifer thickness increases, a given change in reservoir stage yields a smaller percentage change in the aquifer head and seepage velocity, resulting in a smaller deviation from steady state.
Distance from reservoir, x	Positive	Groundwater head nearest the reservoir responds most quickly to changes in reservoir stage, resulting in a faster return to steady state conditions.
Specific yield, S	Positive	Larger specific yield causes decreased values of τ and slower return to steady state conditions following a change in reservoir stage.
Change in reservoir stage, $\|Z_0\text{-}Z_1\|$	Positive	A larger change in reservoir stage causes greater hydraulic gradients in the aquifer, which produce greater deviation from steady state.

the Dupuit–Forcheimer assumption is applicable at distances greater than four aquifer depths from the reservoir. As long as x is large enough to ensure that the Dupuit–Forcheimer assumption is valid, the minimum x will yield the least error from the reach transmissivity relationship.

An example of evaluation of the reach transmissivity equation for a particular application illustrates the analytical process. Consider a canal and aquifer with the following characteristics: initial canal stage and saturated aquifer thickness (Z_0) of 18.29 m, hydraulic conductivity (k) of 6,096 m/day, specific yield (S) of 0.15, and canal top width of 30.48 m. Use of the reach transmissivity relationship in a numerical model with a time step (t) of 1 day and a grid size of 152.4 m is under evaluation; the maximum expected stage variation within a one-day period is an increase of 0.1524 m ($Z_1 = 18.44$ m). Suppose the canal is to be located in the center of its model cell and that the reference groundwater head is to be obtained from the immediately adjacent cells. The distance from the channel boundary to the location of the reference groundwater head (x) is therefore 137.16 m: the grid size minus half the top width of the canal. These input parameters yield values of τ and ξ of 40,639.9 m and 0.68017 $m^{0.5}$, respectively; ξ_0 is therefore 0.0795. Solution of Eq. (15) results in an estimated groundwater reference head (h) of 18.42684 m. Using this head as input to Eq. (7) yields a Darcy velocity (V_x) of 0.60220000 m/day, according to the reach transmissivity relationship. In comparison, solution of Eq. (21) results in a Darcy velocity of 0.60216288 m/s at the channel boundary. The error associated with use of the reach transmissivity equation is therefore −0.14 percent, which should be sufficiently accurate for most modeling efforts. The calculations in this example are very sensitive to round-off error, so as high a level of numerical precision as possible should be used.

If the time step is decreased to one hour, the error increases to −4.49 percent. If the grid size is increased to 304.8 m (and the time step remains one day), the error increases to −0.77 percent.

In the preceding example, estimates of error are based on comparisons of Darcy velocity, which is a function of hydraulic conductivity Eq. (7). However, leakage is a function of transmissivity, which is variable under transient conditions because of changes in the saturated thickness of the aquifer. Consequently, a representative saturated thickness must be used for calculations; an analysis of various transmissivity-averaging techniques was conducted by Guo (1997).

4. Evaluation of the reach transmissivity leakage relationship using field data and numerical modeling results

One of the primary applications of the reach transmissivity leakage relationship is in numerical modeling. Accordingly, for the reach transmissivity relationship to be of any practical use in calculating leakage between groundwater and surface water, it is essential that it yields accurate results when used in analytical or numerical models to analyze field problems. Therefore, the performance of the reach transmissivity relationship was evaluated through comparisons to field data and numerical model results.

Field testing of the validity of the reach transmissivity relationship under transient conditions requires measurements of leakage, channel stage, and groundwater stage at relatively short time intervals. For evaluation of the reach transmissivity relationship, a study site with an area of approximately 130 km^2 was selected in southeastern Florida (Fig. 5). The western portion of this area consists of wetlands of the Florida Everglades, and the eastern portion is a non-wetland region of suburban Miami. A levee and canal (designated as the L-31N Canal) form the boundary between wetland and non-wetland areas. The L-31N Canal flows through the longitudinal center of the study area from north to south. Measured values of leakage between the canal and the groundwater aquifer were obtained by taking the difference in recorded discharge from gaging stations at the upstream and downstream ends of a 6-mile reach of the canal. Canal and aquifer characteristics of a portion of this reach were evaluated by Chin (1991), who found that the bottom of the canal was covered with an effectively impermeable sediment layer; thus, nearly all the leakage occurs through the sides.

An analytical model of the study area using the reach transmissivity relationship was developed using Eq. (5). Records of groundwater level were obtained from monitoring wells and were used as the groundwater reference head input. Aquifer transmissivity was estimated from previous research (Fish and Stewart, 1991; Chin, 1991). Leakage was calculated on a daily basis using data from the 1996 calendar year; positive leakage values indicate flow from the channel into the aquifer (Fig. 6). Leakage to each side of the channel was calculated separately; the net leakage is the sum of these two components. Over the six-mile reach of the canal, the absolute value of differences between measured leakage and leakage calculated with the reach transmissivity relationship was 0.314 m^3/s, a difference of 4.4 percent of the annual mean measured flow at the two gaging stations. These differences are on the order of the accuracy of the discharge measurements.

A numerical model of the study area was developed to further evaluate the performance of the reach transmissivity relationship. This model used the MODBRANCH program (Swain and Wexler, 1996), which uses the vertical flow leakage relationship to link MODFLOW (McDonald and Harbaugh, 1988), a three-dimensional finite-difference groundwater model, to BRANCH (Schaffranek et al., 1981), a one-dimensional model of unsteady flow in open channel networks. The MODBRANCH program was modified to use the reach transmissivity relationship to link MODFLOW and BRANCH (Nemeth et al., 2000). The study area was modeled using both the original (vertical flow) and modified (reach transmissivity) versions of MODBRANCH. Leakage was calculated using daily time steps for 1996 input data. The groundwater reference heads used as input to the reach transmissivity version of the model were obtained from the cells immediately adjacent to the cells containing the canal. Inspection of plots of the modeled leakage (Fig. 7) reveal that the vertical flow and reach transmissivity models provide very similar estimates of net leakage. A tabular comparison of each model's errors (the difference between modeled and measured results) in groundwater head, canal stage, and canal discharge reveals similar performance for the vertical flow and reach transmissivity models (Table 2). Groundwater head comparisons were made using 13 monitoring wells, and canal stage and discharge comparisons were made using data from three gaging stations. The reach transmissivity model has the advantage that leakage to each side of the surface water channel can be computed individually. Finally, the reach transmissivity version of MODBRANCH provided an unexpected benefit; the total number of iterations, as well as run time, was reduced by approximately 40 percent, relative to the original version of MODBRANCH. This decrease was observed in every model run that was performed.

5. Discussion of functional differences between leakage relationships

It is apparent that the vertical flow and reach transmissivity relationships are based on differing

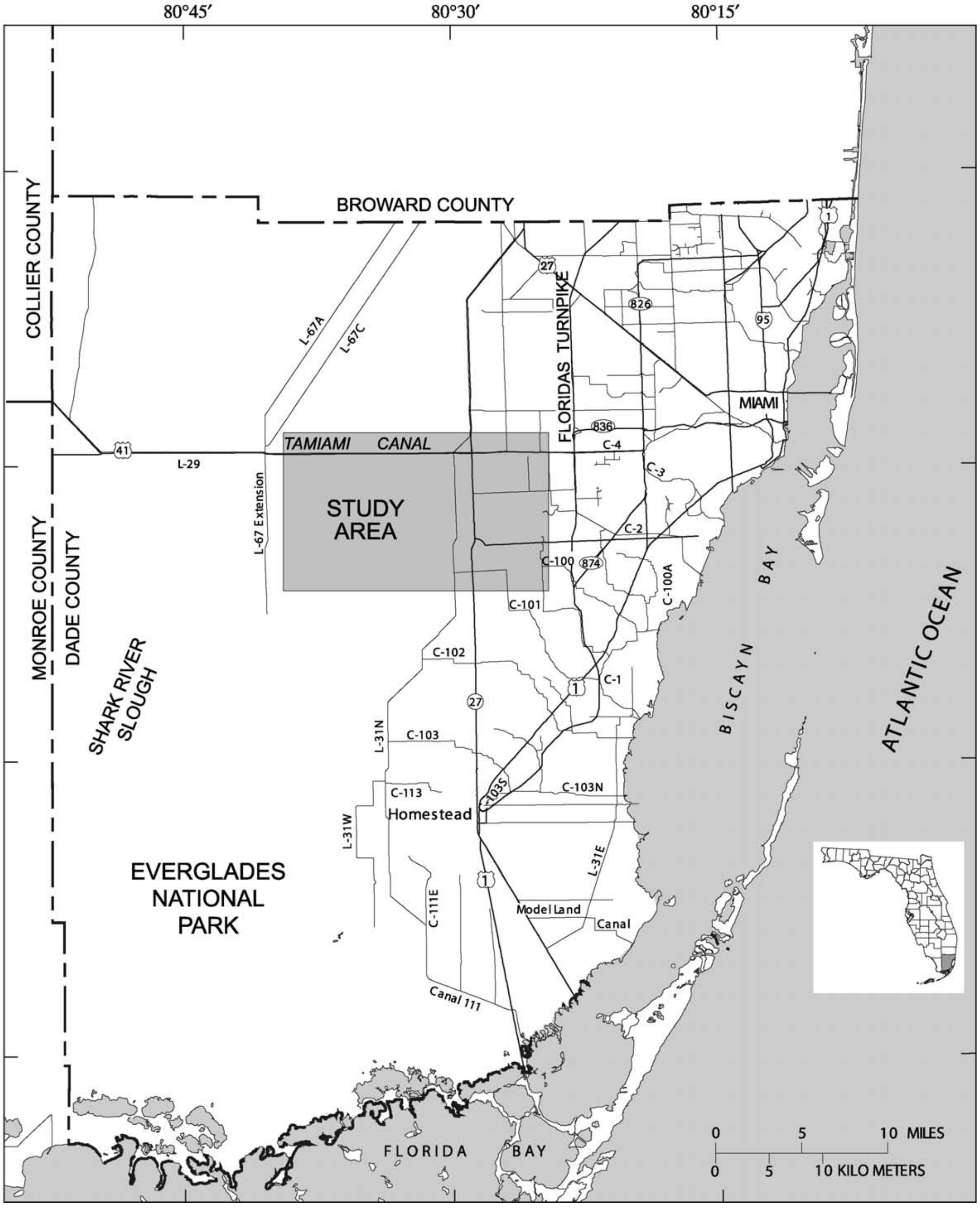

Fig. 5. Location of study site, canals, and wetland areas within Miami–Dade County, Florida, USA.

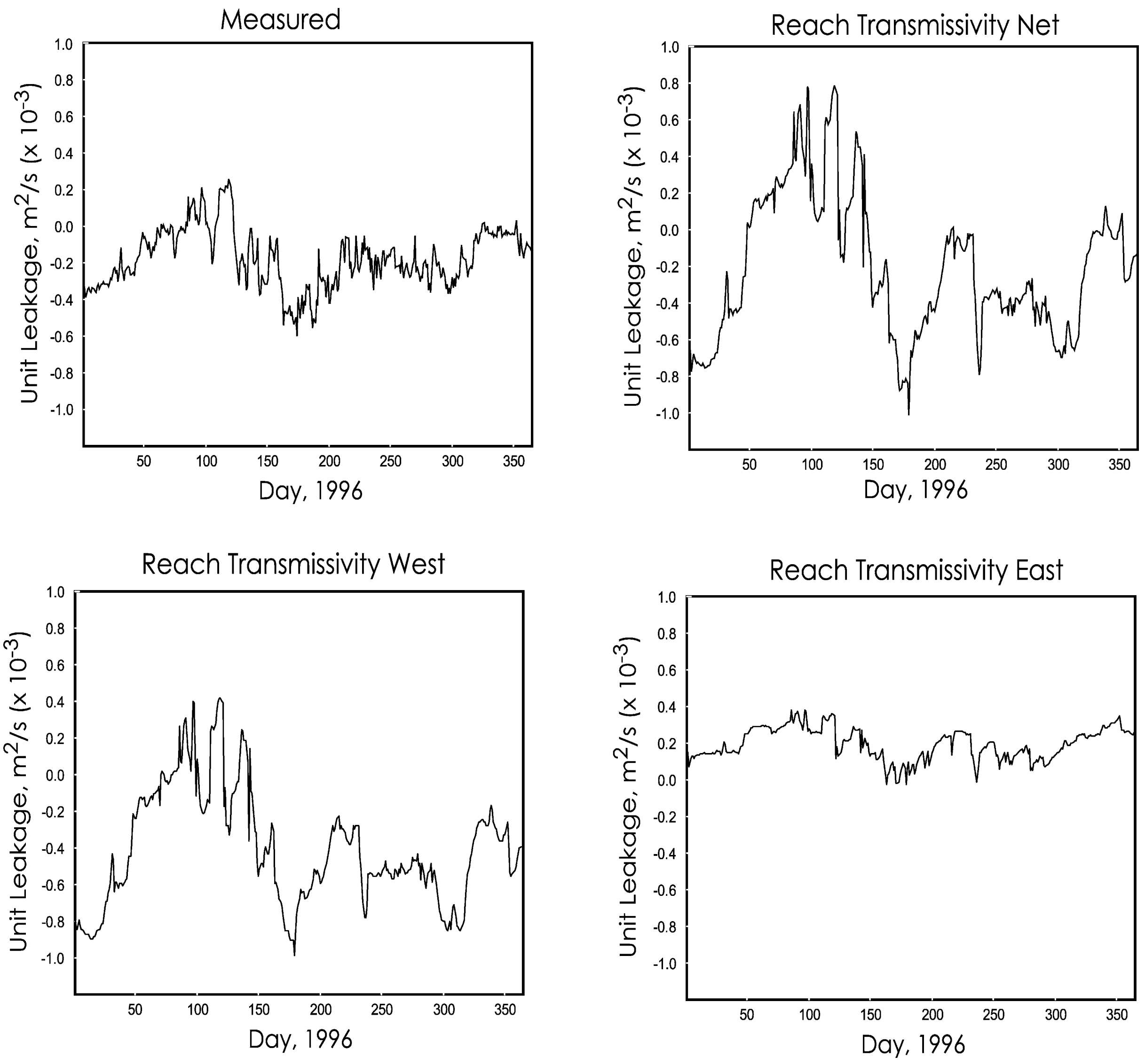

Fig. 6. Directional leakage on the L-31N Canal calculated with the reach transmissivity relationship (input from 1996 measured field data).

conceptualizations of leakage. The vertical flow relationship assumes that all leakage occurs through a discrete, low permeability layer at the channel boundary. In contrast, leakage in the reach transmissivity relationship occurs through the sides of the channel, and resistance to flow is based on the aquifer transmissivity. To determine the head differential between the channel and groundwater in the vertical flow relationship, the water table elevation immediately adjacent to the channel is measured. In the reach transmissivity relationship, the water table elevation is measured a significant distance away from the channel (although the exchange of water between the groundwater and surface water models still occurs in the model cell in which the channel is located). This results in a notable discontinuity in water surface elevation at the channel boundary in the vertical flow conceptualization. Another difference

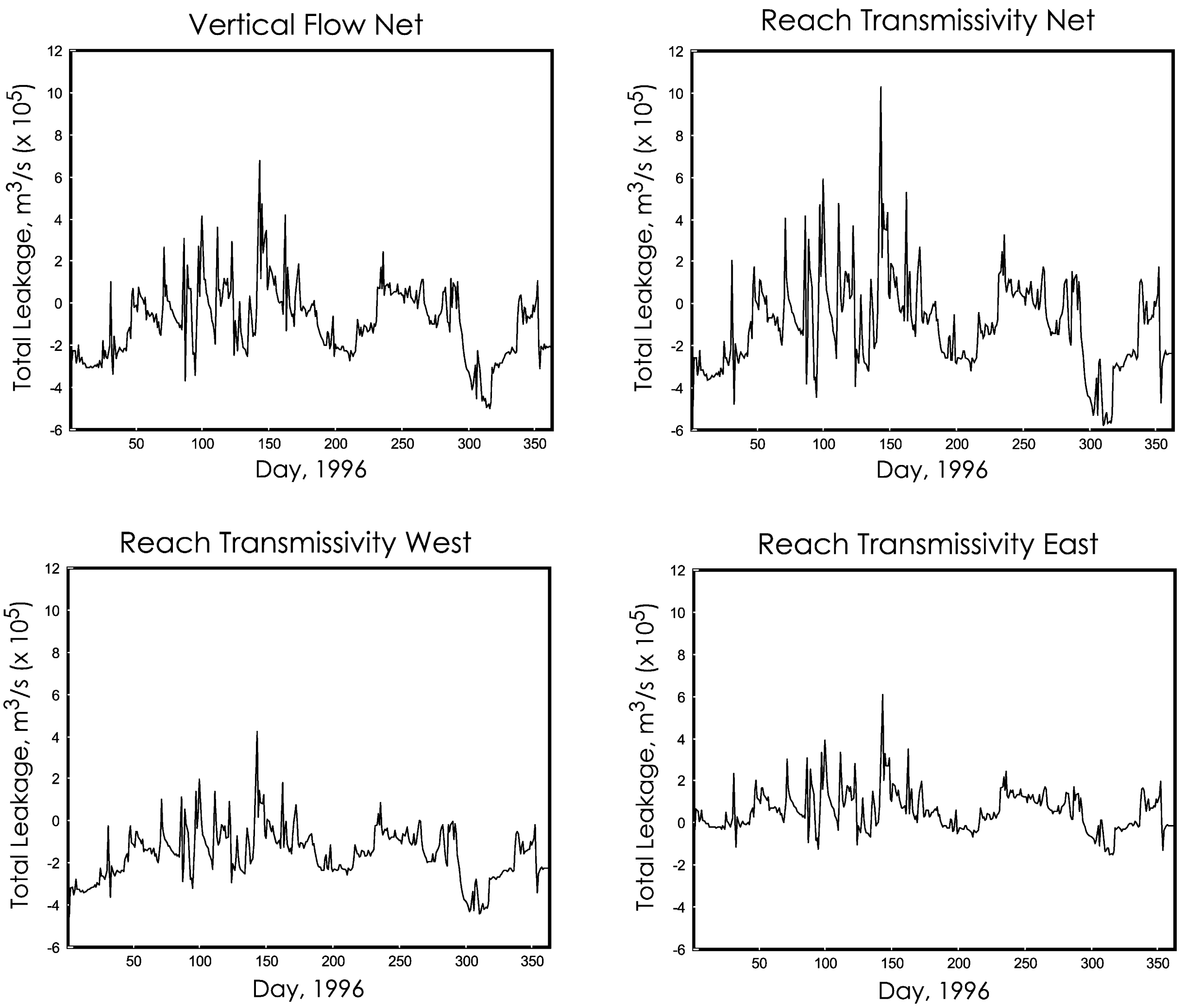

Fig. 7. Net leakage determined by the vertical flow and reach transmissivity models, and directional leakage determined by the reach transmissivity model (input from 1996 modeled data).

between the leakage relationships is that, to make coefficient determination practical, the reach transmissivity relationship requires that the channel be assumed to fully penetrate the aquifer, though violation of this assumption might not result in significant error (Nemeth, 2000). The assumption of full penetration is not necessary for use of the vertical flow relationship.

Table 2
Comparison of vertical flow and reach transmissivity model results

Leakage relationship	Groundwater head		Canal stage		Canal discharge	
	Mean error, ft	Std. dev. error (ft)	Mean error (ft)	Std. dev. error (ft)	Mean error (ft^3/s)	Std. dev. error (ft^3/s)
Vertical flow	0.01	0.13	0.04	0.00	8.6	12.7
Reach transmissivity	0.03	0.13	0.04	0.00	7.9	9.6

In most cases, the stage in surface water channels fluctuates on a much shorter time scale than the water table. As a result, the largest fluctuations of the water table usually occur immediately adjacent to the channels. The vertical flow and reach transmissivity leakage relationships may provide different results when modeling short-term canal fluctuations. When used in a finite difference model, the vertical flow relationship employs a reference aquifer head within the same model cell as the channel; the head in this cell will be the first to respond to changes in the canal stage and will fluctuate more than the head of cells more distant from the canal. The reach transmissivity relationship, however, uses a reference aquifer head in a cell more distant from the channel and is consequently less affected by fluctuations in channel stage. This particular difference between the two leakage relationships has the potential to result in differences in the quantity and temporal distribution of exchange between the groundwater and surface water.

Another difference between the leakage relationships is that the parameters in the reach transmissivity relationship have a different physical basis than those of the vertical flow relationship. The reach transmissivity coefficient (Γ_r) is calculated from the channel top width, the distance from the channel to the point of aquifer head measurement, and the aquifer transmissivity; the first two of these parameters are simply distances that are easy to obtain and calculate. The transmissivity of the aquifer is generally available from published sources or pump tests, particularly in areas where previous groundwater research has been conducted. In some cases, the aquifer transmissivity may not be constant over the distance between the channel and the point of aquifer head measurement. This condition is commonly a result of clogging of the aquifer by deposition of channel sediments (Chin, 1991; Genereux and Guardiario, 1998). In such a case, it is possible to calculate an effective reach transmissivity that takes variations in aquifer transmissivity into account (Chin, 1991).

In contrast to the reach transmissivity relationship, use of the vertical flow relationship requires determination of the hydraulic conductivity and thickness of a low permeability layer at the channel bed. These data are not as readily obtainable and sometimes cannot be known without collecting samples of the channel bed. Furthermore, determination of these parameters becomes even more problematic if the leakage does not actually conform to the vertical flow conceptualization (i.e. all the leakage does not occur through a clearly definable, constant-thickness, low permeability layer at the boundary of the channel). This condition can occur in unlined artificial channels that have been excavated from the aquifer material, resulting in the absence of a low permeability bed layer. Also, the vertical flow relationship may not be an accurate conceptualization of flow in natural channels where the bed material is more coarse and permeable than the soil material in the surrounding aquifer.

When the vertical flow relationship does not represent the actual leakage mechanisms, the channel bed thickness and hydraulic conductivity become calibration parameters that are developed by trial and error, rather than direct measurement. As a result, the bed thickness (b') becomes defined mathematically as the distance from the point in the channel where the stage is Z to the point in the model cell where the head is h (Swain et al., 1996). A numerical model employing the vertical flow relationship requires input of the lumped parameter k'/b'. If a low permeability layer at the channel bed does not exist, leakage occurs directly into the aquifer material, so k' becomes the hydraulic conductivity of the whole aquifer, rather than a discrete bed layer. By definition, b' must be smaller than the largest dimension of each model cell; it is apparent that b' must decrease as the cell size decreases. The aquifer's hydraulic conductivity is essentially independent of cell size, however. As a result, the parameter k'/b' will increase as the cell size decreases, meaning that the calibrated leakage parameter in the vertical flow relationship is dependent on the model grid spacing. This phenomenon can also be explained by considering the relationship between cell size and groundwater heads. In reality, the groundwater head is equal to the channel stage at the bank of the channel and slopes continuously toward the ambient head; the influence of the channel on the groundwater head decreases as distance from the channel increases. If the model cells are very large, the average groundwater head in each cell will be only slightly affected by the channel stage. However, when the cell spacing is small, the presence of the channel will significantly affect the groundwater head in cells near the channel; the average head

(h) in the cell containing the channel becomes dependent on the size of the model grid. Obviously, this situation is not desirable, because it means that leakage parameters calibrated for one grid spacing cannot be applied to any other grid spacing. The reach transmissivity relationship is not subject to this problem. The reach transmissivity equation is only applicable when the Dupuit–Forcheimer assumption is valid; the distance L must be sufficiently large that the groundwater head used as an input to the equation is not significantly affected by the presence of the channel. As long as this condition is maintained, the input parameters of the reach transmissivity relationship (aside from the distance L, which is apparent once the grid size is determined and reference cells are chosen) are not dependent on model grid spacing.

The reach transmissivity relationship offers significant benefits over the vertical flow leakage relationship. The input parameters required for use of the reach transmissivity relationship are more easily obtained a priori and, in many cases, have a more legitimate physical basis. As a result, the amount of model calibration by trial and error is significantly reduced. Additionally, the input parameters of the vertical flow relationship are dependent on the model grid spacing; this dependence is particularly significant when leakage does not actually conform to the vertical flow conceptualization. In the reach transmissivity relationship, only the distance to the groundwater head reference cell is dependent on grid spacing; the other input parameters are not affected by it.

6. Summary and conclusions

A method to quantify the exchange of water between surface water channels and the groundwater aquifer based on the concept of reach transmissivity was evaluated for use under transient conditions. This method of calculating leakage is highly applicable to use in numerical models. Linking groundwater and surface water models to each other is frequently problematic because the two models use different sets of governing equations; additionally, the time-scale of interest is often significantly longer for groundwater modeling than for surface water modeling. The reach transmissivity relationship provides a means of coupling groundwater and surface water models by calculating leakage between them.

Currently, the formulation of the most common method used for leakage calculations assumes that flow of water occurs through a discrete, low permeability layer at the boundary of the surface water channel. Differences between the vertical flow and reach transmissivity relationships were examined. The input parameters required for the reach transmissivity relationship are easier to obtain from published sources than those required for the vertical flow relationship, which must be established through model calibration, drawdown tests, or site-specific sampling. The reach transmissivity relationship also calculates leakage to each side of the channel separately, and its parameters are less dependent on model grid spacing.

The derivation of a form of the reach transmissivity relationship that is suitable for use in numerical modeling relies on the following assumptions: steady state conditions, full penetration of the aquifer by the channel, and the Dupuit–Forcheimer assumption. These assumptions may preclude use of the reach transmissivity relationship in certain conditions, such as when very short time steps are used in a numerical model. Furthermore, the validity of the Dupuit–Forcheimer assumption requires that groundwater heads used as input to leakage calculations be obtained from locations a significant distance from the edge of the surface water channel. Despite these restrictions, the reach transmissivity leakage relationship is applicable to a wide range of conditions, and was successfully tested in analytical and numerical models of field problems. A method was developed to quantify the error associated with use of the reach transmissivity relationship to simulate transient conditions; this method can be used to evaluate the suitability of the reach transmissivity relationship for a particular application.

Acknowledgements

This research project was funded, in part, through US Geological Survey Cooperative Agreement No. 1434-HQ-97-AG-01825.

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