

Joint two-dimensional cross-gradient imaging of magnetotelluric and seismic traveltimes data for structural and lithological classification

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Accepted 2007 February 6. Received 2007 February 1; in original form 2005 October 13

SUMMARY

Collocated magnetotelluric (MT) and seismic profiling is emerging as a necessary combined approach for deep and near-surface imaging but the resulting experimental data are typically interpreted separately since no production programs exist for multidimensional joint inversion of MT and seismic data. We present a joint 2-D inversion approach for imaging collocated MT and seismic refraction data with cross-gradient structural constraints. We describe the main features of the algorithm and first apply it to synthetic data generated for a hypothetical complex geological model. For the synthetic data, we find that the scheme leads to models with remarkable structural resemblance and improved estimates of electrical resistivity and seismic velocity. We apply the scheme to near-surface field data to test the consistency of a previously suggested resistivity–velocity interrelationship and its potential use for subsurface lithofacies discrimination or structural classification. The MT-seismic relationship is found to be in excellent accord with that derived previously for DC resistivity and seismic data set at the test site. Our results suggest that joint MT-seismic cross-gradient imaging leads to improved characterization of heterogeneous geological targets at near-surface to mantle depths.

Key words: crustal and upper mantle structure, inversion, magnetotellurics, refraction seismology, structure classification.

1 INTRODUCTION

Collocated magnetotelluric (MT) and seismic imaging experiments have provided valuable information on fundamental processes within the earth. Both methods are proving to be an essential combination for imaging deep crustal and upper mantle targets in complex geological terrains (e.g. Nelson *et al.* 1996; Jones 1998; Mechie *et al.* 2004; Unsworth *et al.* 2005). With the availability of improved portable audiofrequency magnetotelluric (AMT) field instruments, the combined approach involving coincident seismic refraction and AMT surveying is also becoming increasingly popular in studies of the near surface (e.g. Meju *et al.* 2003). The results from collocated seismic velocity and MT resistivity imaging experiments are typically presented as uncorrelated or separately constrained models (see e.g. Meju *et al.* 2003; Mechie *et al.* 2004; Unsworth *et al.* 2005) and sometimes as cooperatively reconstructed multidimensional models of the subsurface physical property distribution from which geological deductions are made. It may be argued that the resulting models with concordant features should permit geological associations or lithofacies classification. However, due to geological heterogeneity and model non-uniqueness, it is not uncommon to find significant differences between the separately or cooperatively reconstructed MT and seismic models making it necessary to use independent information for geological discrimination. In particular,

not all the features that appeared in the MT and seismic images are effectively supported by both data sets. We posit that much of the discrepancies faced by the separate MT and seismic models can be significantly reduced using an appropriately coupled or joint multidimensional inversion of MT impedance and seismic traveltimes data. In this paper, we are interested in the reconciliation of 2-D seismic velocity and MT resistivity images of the subsurface (from near-surface to mantle depths) using seismic first-arrival and MT data from collocated experiments.

Until recently, the development of joint 2-D MT and seismic traveltimes inversion was hindered by a poor knowledge of how to relate both methods and, to our knowledge, MT and seismic data have only been jointly inverted for 1-D layered-earth structures (Manglik & Verma 1998). It is highly desirable to advance MT-seismic inversion one step further by developing 2-D joint inversion so as to yield consistent models that will make for improved geological understanding of the earth's deep crust and upper mantle. Past approaches to joint inversion include those that depend on either direct relationships among physical parameters (e.g. Berge *et al.* 2000; Tillmann & Stocker 2000; Roecker *et al.* 2004) or predefined structures with exactly coincident borders (e.g. Lines *et al.* 1988; Afnimar *et al.* 2002; Ditmar 2002; Gallardo *et al.* 2005b); however, their usefulness is limited in heterogeneous models. Other approaches have shown that the field of the physical property gradients

(or curvature) can characterize the geometrical features of an image and might be used as a linkage for two disparate data sets (Haber & Oldenburg 1997; Droske & Rumpf 2003; Gallardo & Meju 2003; Saunders *et al.* 2005). In particular, it was shown that using the cross-products of the resistivity and seismic velocity gradients as constraints provides an effective structural linkage between seismic refraction and DC resistivity models in what was termed cross-gradient joint inversion (Gallardo & Meju 2003, 2004; Gallardo *et al.* 2005a). The cross-gradient technique was shown to lead to accurate models, and to improve the structural conformity between the geophysical images without forcing or assuming the form of the relationship between them. Moreover, the resulting geophysical relationships are thought to also have petrophysical significance (Gallardo & Meju 2003; Linde *et al.* 2006; Tryggvason & Linde 2006).

The cross-gradient structural approach lends itself to the inverse problem of joint 2-D MT and seismic inversion and we, therefore, envisage that it will furnish a method for characterizing geological heterogeneity or petrophysical relationships at different crustal or lithospheric scales. The fundamental question here is therefore: Can we obtain accurate 2-D joint inversion models that will facilitate the identification of the structural and/or lithological architecture of the earth's crust and upper mantle using only MT and seismic first-arrival data? Finding an answer to this question forms the main thrust of our paper. It is desirable to be able to construct structurally consistent MT-seismic models that satisfy our collocated data profiles and improve our understanding of the resistivity-velocity interrelationship in heterogeneous subsurface materials.

Here, we present a cross-gradient imaging algorithm for joint 2-D inversion of collocated MT and seismic refraction data profiles. We describe in detail the objective function for joint MT-seismic imaging and the methodologies of forward and derivative computation of MT and seismic refraction data. We use a particularly challenging hypothetical model of the subsurface to gauge the performance of the algorithm and test its accuracy. Given the difficulty of finding ground-truthing data for deep crustal or lithospheric MT-seismic models, we base the field validation test of the inversion algorithm on shallow-depth imaging data from a test site studied by Meju *et al.* (2003), where there are alternative geophysical measurements interpreted in terms of both separate and joint inversions. We demonstrate in this paper that the cross-gradient joint inversion of MT impedance and seismic traveltime profile data produces structurally consistent models which are more accurate than those of separate 2-D inversions and provides a potential tool for classifying 2-D MT resistivity and seismic velocity interrelationships at different spatial scales.

2 METHODOLOGY OF JOINT INVERSION OF MT AND SEISMIC REFRACTION DATA

2.1 The objective function and its solution

A key difficulty in the joint inversion of two or more seemingly disparate geophysical data sets is how to couple the model parameters. For instance, there are joint inversion formulations based on direct probabilistic relationships among model parameters which are lithologically constrained (Bosch & McLaughy 2001). A different philosophy is the use of common geometrical constraints. For example, the analysis of the vector field of the gradients as a geometrical indicator has found some applications in medical image registration (e.g. Droske & Rumpf 2003 and references therein). A

major convenience for a purely geometrical approach is the total independence of any correlation that may exist between the parameters themselves. This geometrical approach requires the definition of an objective measure of the direction of the changes or the normalization of the vector field of the gradients to enhance only the direction dependent correlation. Examples of such normalizations can be found in Zhang & Morgan (1996), Haber & Oldenburg (1997), Droske & Rumpf (2003) or Saunders *et al.* (2005). A convenient un-normalized way to measure the geometrical similarity of the models is through the use of the cross-gradient function (Gallardo & Meju 2003; Gallardo & Meju 2004) given by

$$\vec{\tau}(x, y, z) = \nabla m_{MT}(x, y, z) \times \nabla m_S(x, y, z), \quad (1)$$

where the null values of this function determine either the full collinearity or the negligibility of the gradients of the models (m_{MT} and m_S) at corresponding positions and, therefore, can be used to constrain the models. While avoiding normalization of the cross-gradients, we favour stable iterative search procedures for this second order function (Gallardo & Meju 2004) and encourage the dominance of the largest gradient, required by either data set, in saddle points.

For the present application $m_{MT}(x, z)$ (logarithm of resistivity) and $m_S(x, z)$ (slowness) are the 2-D MT resistivity and the seismic velocity images, respectively. The x -direction runs horizontally across the model and z points downward. In this case, the cross-gradient vector reduces to the y -component noted here as the scalar, $\tau_y(x, z)$. Following Gallardo & Meju (2004), the 2-D model of the subsurface is discretized into a group of nc rectangular blocks, each block (hereafter referred to as a cell) having a constant resistivity and seismic velocity (see Fig. 1). The set of parameters are jointly estimated by defining an objective function that incorporates the following criteria:

- (i) The MT and seismic models should satisfactorily reproduce their respective experimental data in a least-squares sense by simultaneously minimizing

$$\varphi_{MT} = \sum_i^{nrho} \left\{ \frac{\log(\rho_{ai}^{obs}) - \log[\rho_{ai}^{calc}(m_{MT})]}{\sigma_{\rho i}} \right\}^2 + \sum_j^{nph} \left[\frac{\phi_j^{obs} - \phi_j^{calc}(m_{MT})}{\sigma_{\phi j}} \right]^2 \quad (2)$$

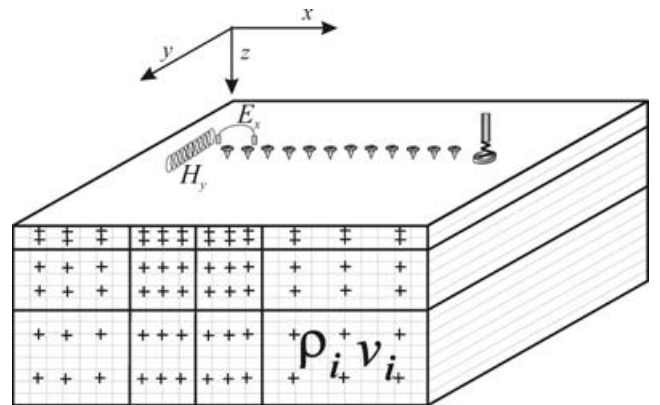


Figure 1. Discretization of the model for inversion into elongated blocks. The finer gridding (crosses) correspond to that used for the forward computation of the MT data and the smaller blocks to the elementary bodies used to compute the seismic traveltimes. The field surveys illustrate collocated seismic refraction and MT (TM-mode) arrays used for 2-D profiling.

and

$$\varphi_S = \sum_i^{nT} \left[\frac{t_i^{\text{obs}} - t_i^{\text{calc}}(m_S)}{\sigma_{ti}} \right]^2. \quad (3)$$

In these expressions ρ_a and ϑ refer to the *nrho* apparent resistivities and *nph* phases available for all measurement frequencies in both TE and TM modes at the MT sites; these are weighted by their corresponding uncorrelated data errors with standard deviations σ_ρ and σ_ϑ . Similarly, t refers to the seismic traveltimes with standard deviation σ_t . The superscript indices *obs* and *calc* stand for observed and calculated values, respectively.

(ii) The MT and seismic models should be structurally identical in terms of the cross-gradient function, that is, satisfy

$$\tau_{yi} = \nabla_i m_{\text{MT}} \times \nabla_i m_S = 0, \quad i = 1, \dots, nc. \quad (4)$$

Gallardo *et al.* (2005a) showed that the conditions (2)–(4) are not always sufficient to fully constrain near-surface electrical resistivity and seismic models and that they are still dependent to a great extent on the coverage of geophysical data themselves. Because of this, areas covered by only one data set may yet find multiple models that equally satisfy the data and cross-gradient constraints (e.g. any constant or parallel image). Thus, the selection of any of such ambiguous solutions will still require *a priori* information. As a partial remedy to this problem, two regularizing conditions were implemented in the present development as described below.

(iii) The models should be as simple as possible, providing they still justify their data and the cross-gradient constraint. This is achieved using smoothness (Tikhonov & Arsenin 1977; Twomey 1977; de Groot-Hedlin & Constable 1990) and ridge regression-type constraints (Hoerl & Kennard 1970; Marquardt 1970). This last constraint serves for maintaining closeness to an *a priori* model and it is particularly useful for assigning resistivity or velocity values in areas without data coverage or where directly sampled parameters values (e.g. from rock outcrop characterization or borehole logging) are available. The corresponding term of the objective function is:

$$\varphi_{\text{apr}} = \sum_i^{nc} \left[\alpha_{\text{MT}}^2 \nabla_i^2 m_{\text{MT}} + \alpha_S^2 \nabla_i^2 m_S + \left(\frac{m_{\text{MT}i} - m_{\text{MT}i}^{\text{apr}}}{\sigma_{\text{MT}i}^{\text{apr}}} \right)^2 + \left(\frac{m_{Si} - m_{Si}^{\text{apr}}}{\sigma_{Si}^{\text{apr}}} \right)^2 \right]. \quad (5)$$

In this expression, ∇_i^2 is a discrete version of the Laplacian, which is based on a five-cell stencil centred on the i th cell weighted by the values α_{MT} and α_S that define the level of smoothness required in the models. $m_{\text{MT}}^{\text{apr}}$ and m_S^{apr} are the parameter values for any *a priori* MT and seismic models with standard deviations $\sigma_{\text{MT}}^{\text{apr}}$ and σ_S^{apr} , respectively.

The composite objective function for the cross gradient joint inversion of MT and seismic data is then stated as:

$$\min\{\varphi_{\text{MT}} + \varphi_S + \varphi_{\text{apr}}\} \quad \text{subject to } \tau_{yi} = 0, i = 1, \dots, nc. \quad (6)$$

This objective function is solved iteratively using the respective Taylor's series expansion of the non-linear functions involved in (2)–(4). In order to ensure the convergence of the algorithm, it is required to develop a balanced gradual process that finds geometrically similar images which fit the MT (apparent resistivity and phase) and seismic traveltime data. Previous studies show that including the cross-gradient functions as equality constraints provided a more stable search than when it is directly incorporated in the objective function. It also makes the search process more robust against saddle

points where it avoids a simple averaging of incompatible property gradients. The details of this minimization process and the relevant mathematical equations can be found in Gallardo & Meju (2004) and Gallardo *et al.* (2005a).

2.2 MT and seismic refraction forward and derivative computation approaches

A necessary practical consideration in this joint MT-seismic cross-gradient inversion approach is the use of fast forward and derivative computation schemes. The progressive finite difference method of Vidale (1988) is widely used for the computation of traveltimes of first arrivals and wide-angle seismic reflections and is known to be computationally efficient (e.g. Hole & Zelt 1995; Zelt *et al.* 1996; Zelt & Barton 1998). It is therefore, adopted here for the computation of the seismic traveltimes and is based on the propagation of a wave front in 2-D media with seismic velocity $v(x, z)$ discretized into a regular grid. The propagation of the wave front is governed by the Eikonal equation for 2-D media given by:

$$\left[\frac{\partial t(x, z)}{\partial x} \right]^2 + \left[\frac{\partial t(x, z)}{\partial z} \right]^2 = \frac{1}{v^2(x, z)}. \quad (7)$$

The traveltime computation starts from a source position and propagates in rectangular fronts in the 2D model grid using the finite difference stencils given by Vidale (1988). For the approach adopted here, Gallardo (2004) incorporated some of the modifications suggested by Hole & Zelt (1995) to account for guided waves and causality violations and slightly modified the original stencils to work with discrete velocity in the cells rather than in the nodes.

The derivatives required for the linearized scheme were computed using the seismic ray paths. The rays are traced using the gradient of the time field for each shot point starting each ray from the receivers as suggested by Vidale (1988). For a given ray, the traveltime can be computed as

$$t^{\text{calc}} = \sum_j l_j m_{Sj}, \quad (8)$$

where l_j denotes the ray length in the j th cell and thus provides the required derivatives. As shown in Gallardo & Meju (2004), the combination of these forward and derivative computation schemes is fast and sufficiently accurate for the purposes of joint inversion.

Similarly, the requirement for a rapid scheme for MT forward and derivative computations led us to adopt the method developed by Smith & Booker (1991). In this approach, the forward computation is performed through a finite difference algorithm that solves the pair of equations that result from decoupling Maxwell Equations for 2-D models (e.g. Rodi 1976). We consider the model orientation so as to follow the TM and TE modes commonly assumed in MT 2-D work. For the TE mode, the algorithm of Smith & Booker (1991) computes the transverse electric field (E_y) for a rectangular grid by solving the equation

$$\frac{\partial^2 E_y(x, z)}{\partial z^2} + \frac{\partial^2 E_y(x, z)}{\partial x^2} = \frac{i\omega\mu_0 E_y(x, z)}{\rho(x, z)}. \quad (9)$$

This equation is valid for an harmonic electromagnetic wave of frequency ω travelling in a medium of electrical resistivity $\rho(x, z)$ and constant magnetic permeability $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$.

To solve this equation the subsurface model of Fig. 1 is subdivided into a finer grid for improved accuracy. The grid is more finely discretized for the near-surface than in deeper cells to take advantage of the MT data sensitivity. The equation is discretized using a finite difference scheme based on a five-point centred stencil for

the internal nodes and adding appropriate boundary conditions. The top border of the grid is padded with some nodes as air layers to account for the natural decay of the field. To compute the observed data at each MT site, the electrical field is interpolated under the site at position x_s and the corresponding perpendicular magnetic field (H_x) on the surface is computed using the equation

$$\frac{\partial E_y(x_s, z)}{\partial z} = i\omega\mu_0 H_x(x_s, z). \quad (10)$$

The appropriate MT model responses are computed using the equations

$$\rho_{aTE} = \frac{1}{\omega\mu_0} \left| \frac{E_y(x_s, 0)}{H_x(x_s, 0)} \right| \quad (11)$$

and

$$\phi_{TE} = \arg \left(\frac{E_y(x_s, 0)}{H_x(x_s, 0)} \right). \quad (12)$$

The corresponding formulation for the TM mode is similar. The interested reader is referred to the original work of Smith & Booker (1991) for further details of their forward computation scheme.

For the partial derivative calculation, the reciprocity principle (e.g. Tripp *et al.* 1980; Mackie & Madden 1993) can provide accurate derivatives; however, it requires extra forward modelling solutions that may increase significantly the processing time. As a fast alternative, the derivative computation approach proposed by Smith & Booker (1991) is preferred here. The pseudo-2-D approach of Smith & Booker (1991) supposes that the horizontal gradients of the E_y and H_y fields are smaller than their corresponding vertical gradients and uses the horizontal gradients of the previous iterate to eliminate the x 's dependence and simplify the corresponding equations. The corresponding formulations for the derivatives for the TE mode as adapted here are given by:

$$\frac{\partial \log_{10}(\rho_a)}{\partial m_{MTi}} = \text{real} \left[\frac{\sigma_{0i}}{E_{y0}(x_s, 0)H_{x0}(x_s, 0)} \int_{z_{i-1}}^{z_i} E_{y0}^2(x_s, z) dz \right] \quad (13)$$

$$\frac{\partial \phi}{\partial m_{MTi}} = \frac{\log_e(10)}{2} \text{imag} \left[\frac{\sigma_{0i}}{E_{y0}(x_s, 0)H_{x0}(x_s, 0)} \int_{z_{i-1}}^{z_i} E_{y0}^2(x_s, z) dz \right]. \quad (14)$$

In this expression the 0 subindexed variables refer to values computed from a reference resistivity model (previous iterate), and z_i is the depth to the bottom of the i th analysed cell. The corresponding derivative expressions for the TM mode result from substituting E_y by E_x and H_y by H_x in eqs (13) and (14). Note that (13) and (14) define the change in the observable data due to a change in the resistivity model, which is estimated from the computed electric fields at practically no time cost.

Although the derivative approach of Smith & Booker (1991) is fast and takes horizontal derivatives into account, it may lose accuracy in sharp 2-D structures. However, this implemented version of the Smith & Booker (1991) scheme does not include any interpolation of conductivity between sounding sites, that is, any feature between two consecutive MT sites must agree with the seismic model in terms of the cross-gradient constraint. We also expect that the derivative inaccuracies lie within the range of the actual MT data errors and that it will suffice for demonstrating the feasibility of 2-D MT-seismic cross-gradient inversion for structural and lithological classification.

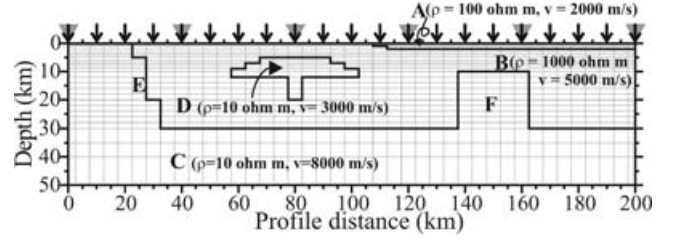


Figure 2. Synthetic homogeneous test model and grid design. The numbers in the model are the seismic P -wave velocities and MT-resistivities for the corresponding structural units (A–F) outlined by thick lines. The arrows on the surface denote the locations of the MT soundings and the inverted triangles mark the locations of the seismic shotpoints.

3 INVERSION OF SYNTHETIC DATA

3.1 Homogeneous test model

The test model is shown in Fig. 2. It includes homogeneous units separated by sharp boundaries that pose major challenges to both MT and seismic refraction imaging approaches. These hypothetical units also resemble some geological features that constitute typical targets for deep MT and seismic exploration, namely a sedimentary basin (unit A), undifferentiated crust (unit B), upper mantle (unit C), isolated crustal reservoir or magma chamber (unit D), steep contact fault (feature E) and basement arch or mantle wedge (feature F).

We generated synthetic data for 21 MT sounding positions and for six seismic shot points equally spaced along the profile as depicted in Fig. 2. The MT sounding data consist of apparent resistivities and phases for both transverse electric (TE) and transverse magnetic (TM) modes for 22 frequencies in the range 0.01–150 Hz. The synthetic MT data were computed using the 2-D MT forward code of R. Mackie (Rodi & Mackie 2001). The seismic traveltimes were generated using the scheme of Vidale (1988), for which we used 101 surface receivers (geophones) located every two kilometres. The coverage of our hypothetical structures is depicted by the corresponding ray tracing in Fig. 3. In this figure, note the amount of diffracted rays and the absence of rays in the low velocity unit (D); these features make this a somewhat complicated model for inverse modelling of the seismic traveltimes. We added Gaussian distributed noise to all the synthetic data ($\sigma_\rho = 5\%$, $\sigma_\phi = 2.5^\circ$ and $\sigma_t = 0.1$ s).

The noise-degraded data were then used to perform two comparative inversion experiments. First, the MT and seismic data were inverted simultaneously without cross-gradient constraints resembling the conventional separate inversion approach. Then the same data were inverted using the full cross-gradient joint inversion. In both experiments the sought normalized rms misfit (Gallardo & Meju 2004; eqs 15 and 16) was set to decrease logarithmically from 31.6 to 1 in seven iterative steps to control the convergence of the process and avoid local minima. The initial models for both inversion

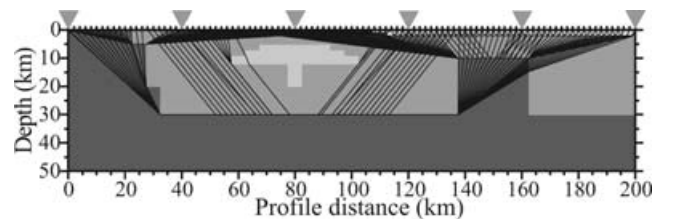


Figure 3. Seismic ray coverage for the synthetic test model of Fig. 2. The inverted triangles mark the locations of the seismic shotpoints.

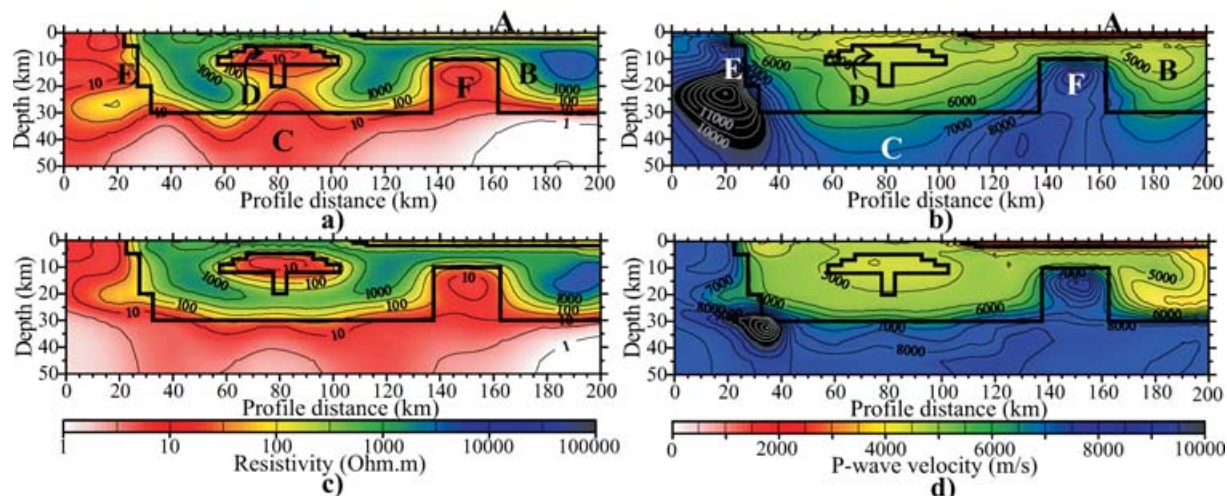


Figure 4. (a) MT-resistivity model recovered by separate inversion using the formulation of Smith & Booker (1991) implemented in this study. (b) Seismic model obtained by separate inversion of the seismic data using the scheme implemented by Gallardo (2004). (c) MT-resistivity and (d) seismic velocity models obtained by joint inversion using cross-gradient constraints. The thick lines outline the test structures that are the exploration targets (units A–F).

tests consisted of homogenous half spaces of resistivity of 10 ohm m and seismic velocity of 8000 m s^{-1} corresponding to the basal structural units in the test model (Fig. 2). In both experiments, the accuracy of the forward computations is increased by finely resampling the cells used for inversion as shown in Fig. 1.

3.2 Results from separate inversion

The deep resistivity and velocity models resulting from separate inversion (i.e. without cross-gradient constraints) of the synthetic test data are presented in Figs 4(a) and (b), respectively. The model responses match the inverted data to normalized rms misfits of 1.72 for MT and 1.75 for the seismic traveltimes. Attempts to further improve the respective fits led to models with inconsistent and hence undesirable structural features.

The resistivity and seismic velocity models (Figs 4a and b) recovered many of the structural features of the test model as permitted by the data coverage and the accuracy of the forward and derivative computation methods employed. The upper part of the conductive unit D and the shallowest flat layer (unit A) were well resolved by the MT inversion. However, the resistivity structure of the deep part of the model (*cf.* units B, C and D) is poorly resolved; note that the conductor D would have been erroneously interpreted as being connected to the basal conductor (unit C). Also, the anomalous high resistivity zone at 30 km depth at profile position 5–30 km in the model is an artefact caused by the low penetration depths of the soundings located in that conductive area and possibly to the limitations in the derivative approach adopted here. Note that, whereas the MT sounding data can be affected by the strong lateral resistivity contrast of this feature, their derivatives can only modify the resistivity of the cells located exactly below the sounding site.

In the seismic model (Fig. 4b), the shallowest flat feature (unit A) and the top surface of the deep basement arch (feature F) were well resolved. The top part of the high velocity layer (unit C) was also well delineated but its deep structure is poorly resolved especially near the steep contacts (features E and F). The model shows an anomalous high velocity zone around 20–40 km depth at profile positions 5–35 km (left of feature E). This high velocity zone provides the necessary acute bending of the seismic rays in this part of the model

(*cf.* Fig. 3). The low velocity feature at 10 km depth (unit D) was barely imaged.

3.3 Results from joint inversion with cross-gradient constraint

The results of joint inversion of the MT and seismic data with cross-gradient constraints, using the same initial models, damping factors and number of iteration as for the separate inversions, are shown in Figs 4c and d. The normalized rms data misfit achieved was not significantly different from that of separate inversion, being 1.72 and 1.64 for the MT and seismic data, respectively. The apparent resistivity and phase data and their corresponding residuals are shown in the form of pseudo-sections in Figs 5(a)–(h). We also plotted the computed traveltimes and their residuals in Figs 6(a) and (b).

Both the MT and seismic models (Figs 4c and d) show improved features when compared to the results of separate inversion (*cf.* Figs 4a and b). The structure of the upper part of unit C is better delineated and so also are the steep features (D, E and F) in the joint inversion models. Notably, the isolated conductive and low velocity body at 10 km depth (unit D) is now better resolved in both the MT and seismic models. The resistivity and velocity values of the various units of the test model (e.g. units B, C and D) are also better determined in the joint inversion models. The anomalous high-velocity and high-resistivity values centred at 30 km depth at profile distance 5–30 km are reduced despite insufficient data coverage in this area. However, while the results are in remarkable accord, there are some undesirable features in areas without common data coverage as shown in the right hand side of the unit B of the seismic model. This would suggest the need for adequate data coverage in collocated MT-seismic regional experiments. Note also that most of the deficiency of this particular seismic data coverage could have been solved by extending the seismic profile length or by incorporating seismic reflection data.

To assess the accuracy of the final models, we plotted the differences between the synthetic test model parameters and those obtained by joint and separate inversion in Figs 7(a)–(d). As expected, the resistivity and seismic velocity values are better estimated for areas with both MT and seismic data coverage, generally the top 30 km. The maximum differences occurred in areas without data

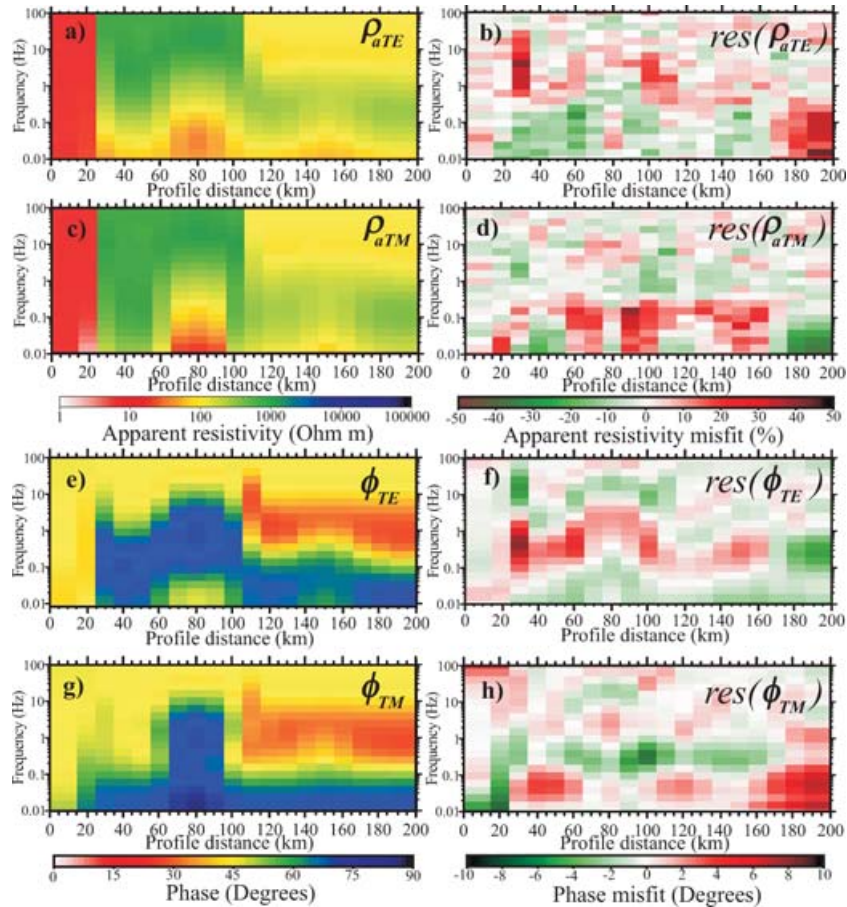


Figure 5. MT apparent resistivity (a and c) and phase (e and g) pseudo-sections for the synthetic homogeneous test model of Fig. 2. The depicted blocks denote the actual datum. The corresponding residuals for the jointly inverted models of Figs 4(c) and (d) are shown in the same pseudo-section format in (b, d, f and h). Note that despite some large individual residuals, the global rms normalized misfit achieved is 1.75.

coverage and notably along the sharp boundaries of the units. This includes areas below the conductive materials (units D and C) in the MT images and the margins of the seismic model. Note also that although the geometrical aspect of the hypothetical low-velocity feature (unit D) was better resolved by joint inversion (Fig. 4d), there is only a marginal improvement in the accuracy of the estimated velocity for this unit. This perhaps highlights the importance of data sensitivity for accurately resolving resistivity and seismic parameters.

Figs 7(e) and (f) demonstrate the general improvement of the geometrical resemblance in terms of the cross-gradient values of the separately and jointly reconstructed models. Overall, the synthetic analyses summarized in Fig. 7 show that joint MT-seismic inversion with cross-gradient constraint permits superior inferences of subsurface structure and physical properties without the need to specify *ab initio* any resistivity–velocity relationship. A pertinent question here is: Can these structurally improved images serve for lithological and/or structural classifications?

3.4 Lithological classification in a homogeneous model

It can be argued that the structural improvements in the models obtained from joint inversion have implications for lithofacies classification. This is supported by the fact that neither the resistivity nor the seismic velocity is predefined or biased towards a specific value; the objective function (6) merely encourages the models to be linked

by the orientation of their first order derivatives. We have plotted the resistivity and seismic velocity values for every cell of the models of Fig. 4 in the subsurface in Fig. 8 (*cf.* Bosch & McGaughey 2001; Gallardo & Meju 2003; Bedrosian *et al.* 2004). These resistivity–velocity cross-plots show that the resistivity and seismic velocity values of the jointly inverted models are less dispersed than those obtained from the separately inverted models. The resistivities and velocities in the homogeneous subsurface units with good data coverage are clustered around the true unit values, whereas the parameter estimates from areas without complete data coverage or with severe non-uniqueness problems are clustered around a biased value (e.g. the low velocity feature of unit D). In spite of this, it is remarkable that all the clusters (biased or not) can be distinguished, that is, classified in terms of their geophysical values and their subsurface distribution. The cross-plots also reflect a noticeable dispersion of the resistivity and velocity values in systematic trends defined by the smoothed boundaries, a collateral effect induced by the smoothness constraint.

3.5 Heterogeneous test model

The above experiment with a homogenous test model shows that subsurface classification is feasible even though some boundaries may be smeared by the smoothing assumptions and that achieving the true property values is conditional on the available geophysical information. We go one step further and explore whether the

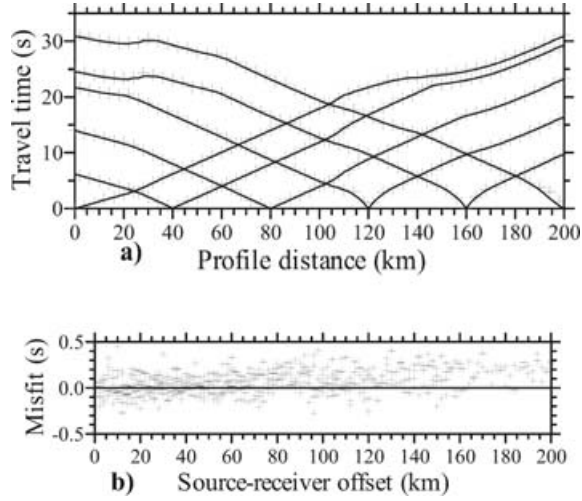


Figure 6. (a) Synthetic response of the velocity model of Fig. 4d (solid line) and inverted traveltimes test data (crosses). (b) Corresponding traveltimes residuals as a function of source-receiver offset. Although, the systematic trend of the residuals for the largest offsets are caused by the spatial smearing of the sharp boundary separating features (C and D), the average normalized misfit remains within tolerable values (1.64).

classification is achievable in heterogeneous units where the physical values may change gradually, and whether the MT and seismic joint inversion models provide useful information for studying the resistivity-velocity interrelationships at the depth ranges covered by both methods. For this experiment, we used the same synthetic model of Fig. 2, incorporating a differentiated crust (Unit B). The unit B is now defined by a seismic velocity that increase linearly with

depth and an electrical resistivity that increases logarithmically with depth as illustrated in Figs 9(a) and (b). Similar to the previous experiment, we generated synthetic data using the same alternative modelling programs, and the same shot-geophone spread and frequencies illustrated in Figs 2 and 3. The data were also corrupted with noise and then inverted. The models obtained from separate inversion of these new data are shown in Figs 9(c) and (d) while the models obtained from joint inversion of the data are shown in Figs 9(e) and (f).

The models reproduce the data satisfactorily (rms misfits 1.47 and 1.32 for the MT and seismic data, respectively). The jointly inverted models define, as in the previous experiment, the vertical features better than the separately estimated models and also define the structures underlying feature F (Magma Chamber). It is noted that the geometrical resemblance of the heterogeneous models is also achieved as required by the cross-gradient constraint. At this point it is important to remark that even though the MT derivative approach adopted here is not exact, it lead to acceptable models with sharp boundaries and satisfactory data fit. We expect that exact derivative approaches like those of Rodi & Mackie (2001) or Siripunvaraporn & Egbert (2000) can produce models with better data fit but may also increase the processing time. A convenient approach would be to use such exact derivative approaches only during the final iterations, when the data misfit is significantly reduced. This could lessen undesirable features and increase marginally the processing time.

To examine further the possibilities of classification of the recovered units through their resistivity-velocity associations, we compare in Fig. 10, the resistivity-velocity cross-plots obtained from the reconstructed heterogeneous models. Similar to the previous case, the estimated values are dispersed around the true synthetic values for separate inversion, but are less dispersed in the cross-plots

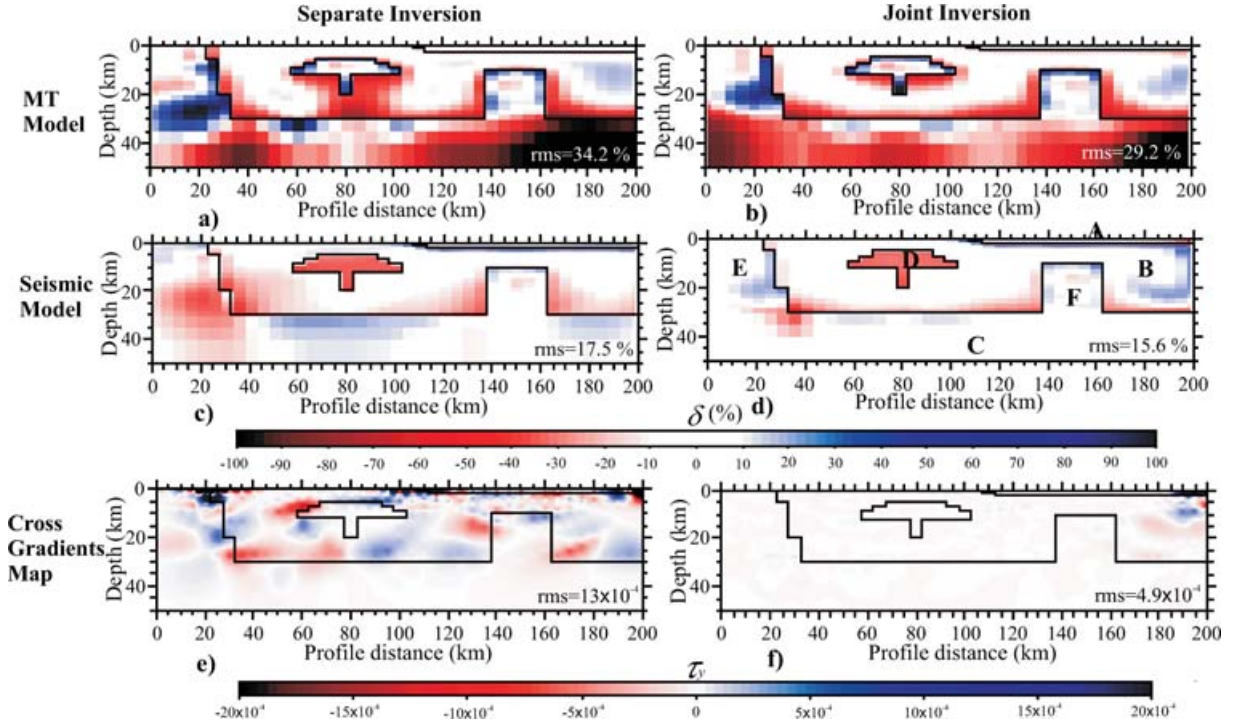


Figure 7. Differences between the synthetic MT model parameters with those obtained by (a) the separate inversion experiment and (b) the joint inversion experiment. (c) and (d) show the corresponding differences between the seismic test models and those obtained by separate inversion and joint inversion, respectively. (e) and (f) show the structural similarity (measured by τ_γ) between the MT and seismic models obtained by separate inversion (e) and joint inversion (f). Note the improved accuracy of the MT resistivity and seismic velocity parameters and the significant improvement in structural similarity.

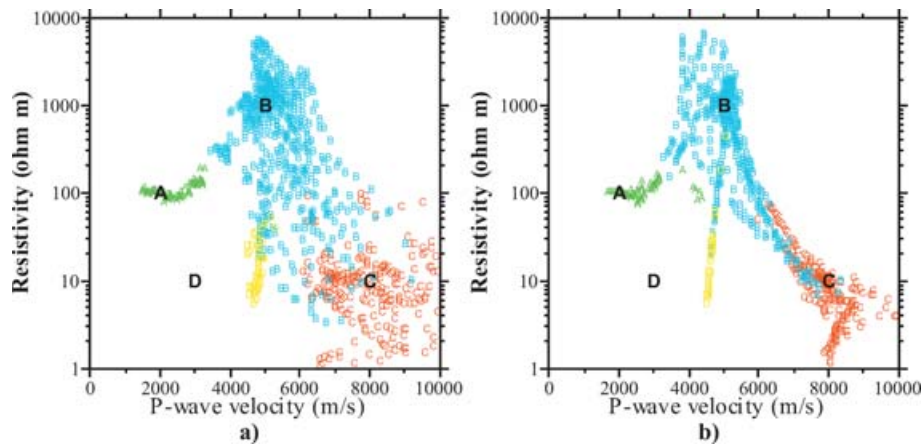


Figure 8. Resistivity velocity cross-plots derived from the separate (a) and joint (b) inversion of the MT and seismic data computed from a homogeneous synthetic model. The colours are directly associated to the actual zones defined in Fig. 2. The pairs of resistivity–velocity values for the theoretical homogeneous model are marked with the letters A–D and are named according to the corresponding units of Fig. 2.

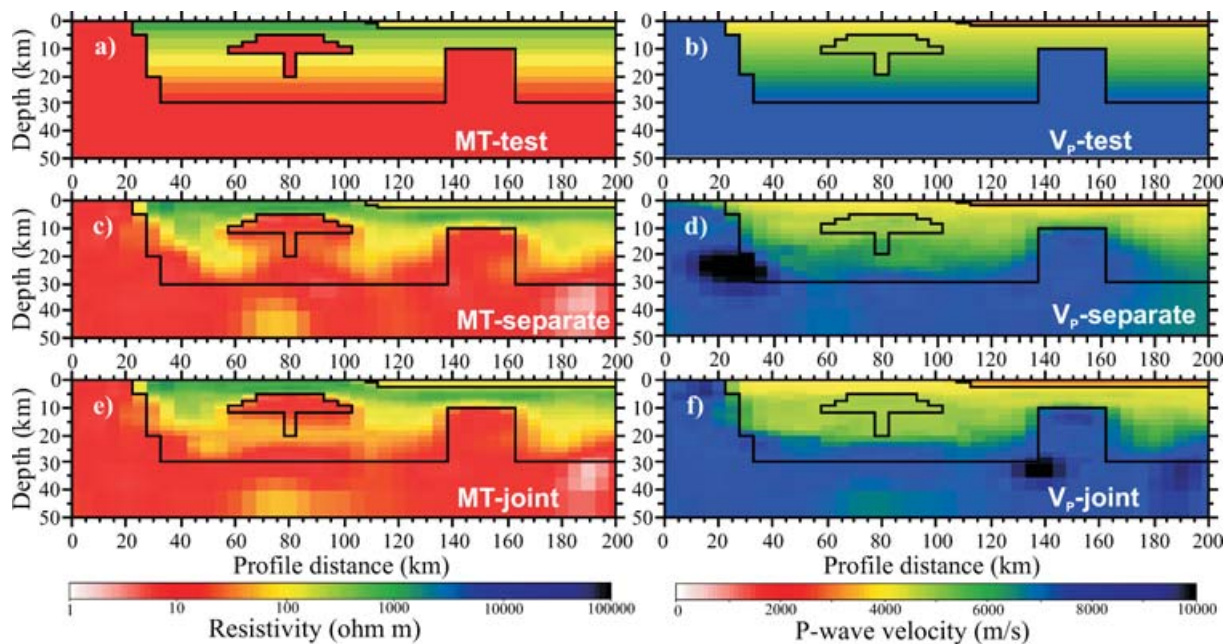


Figure 9. Synthetic heterogeneous resistivity (a) and seismic velocity (b) test models. (c) Resistivity model and (d) velocity model recovered by separate inversion of MT and seismic traveltime data. (e) Resistivity model and (f) velocity model recovered by joint inversion of the same data. The thick lines outline the test structures defined in Fig. 2.

derived from the jointly inverted models. Note that the resistivity–velocity trend estimated for unit B matches the theoretical trend defined for the subsurface unit and, therefore, has a physical meaning. We note that this cluster not only follows the predefined resistivity–velocity trend but also deviates from that resulting from pure interpolation as shown in the homogeneous model (*cf.* Fig. 8b). It is also remarkable that the systematic trends associated with the smoothed boundaries of homogeneous units remain, and will make any classification difficult. Fig. 10 shows that the cross-gradient joint inversion of MT and seismic refraction data accounts for heterogeneous units and should facilitate the deduction of physical property variations necessary to determine the underlying common features. However, the classification of systematic trends must be performed carefully. In this hypothetical example, the heterogeneity has a linear resistivity–velocity relationship and was isolated together with units of homogeneous resistivity and velocity. A more

challenging task is gauging the occurrence of such systematic trends in true geological environments or determining whether they can be accurately detected using joint inversion of MT and traveltime data.

4 JOINT INVERSION OF FIELD DATA

Proper field validation of joint MT-seismic inversion requires a site where adequate borehole data (ground-truth) or well-controlled alternative geophysical imaging measurements are available for comparison. We selected a field site where collocated audiofrequency magnetotelluric (AMT), DC resistivity, transient electromagnetic and seismic refraction experimental data are available (Meju *et al.* 2003) and for which DC resistivity and seismic data have been jointly inverted (Gallardo & Meju 2003). Since the joint inversion

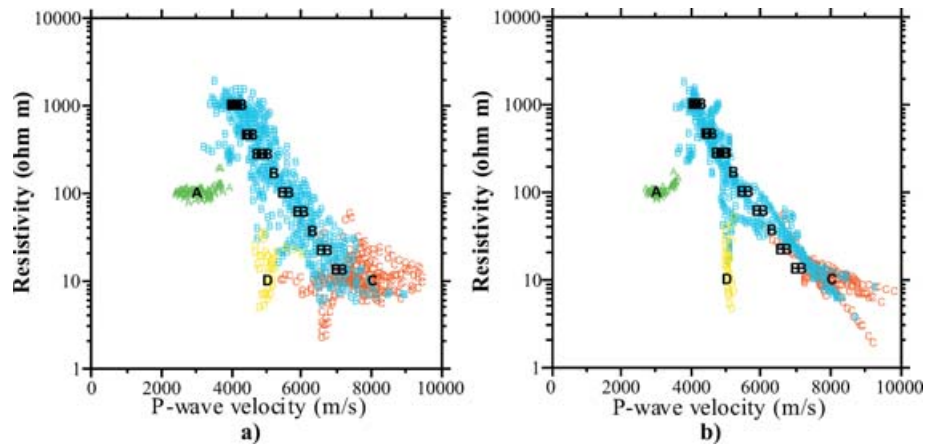


Figure 10. Resistivity velocity cross-plots derived from the separate (a) and joint (b) inversion of the MT and seismic data computed from a heterogeneous synthetic model. The colours are directly associated to the actual zones defined in Fig. 2. The pairs of resistivity–velocity values for the theoretical model are marked with the letters A–D and are named according to the corresponding units of Fig. 2.

of collocated seismic refraction and either electrical or electromagnetic data sets (providing they have similar depth of penetration and lateral resolution) should ideally yield the same results, the existing joint 2D DC resistivity and seismic models should provide an additional validation test for the joint MT–seismic inversion and its use for structural or lithological classification as proposed in Gallardo & Meju (2003).

4.1 The field measurements

We adopted the AMT and seismic refraction data from the Quorn field site studied by Meju *et al.* (2003). This site is characterized by a highly fractured granodioritic basement and heterogeneous sedimentary cover consisting of mudstone and glacial till. The seismic survey employed 2 m geophone spacing and four shot points along a 165 m transect that is orthogonal to the known local geological strike. The AMT data of interest in this study were recorded at 15 m station intervals using 15-m-long electric dipoles aligned in the seismic profiling direction and a magnetic induction coil placed in the orthogonal direction. A total of 14 AMT stations were recorded in the frequency range 80 KHz to 1 Hz over a distance of 195 m on the seismic line. These AMT data correspond to the TM mode measurements. The apparent resistivity data show errors ranging from 1 to 10 per cent and are deemed acceptable but the phase information was much noisier and hence has not been considered in the present study. All the AMT apparent resistivity data were corrected for static shift using coincident transient electromagnetic soundings thus providing good control of data quality.

4.2 Joint inversion of AMT and seismic refraction field data and subsurface classification

Each AMT datum was assigned the corresponding observational error from the field survey and the seismic data were assigned a standard deviation of 1 ms. The subsurface model was discretized into rectangular cells exactly coincident to those used in Gallardo & Meju (2003) to permit a direct comparison. To reduce the number of steps required in the iterative process the AMT–seismic inversion process was started from layered-earth models with velocity and resistivity increasing with depth. The smoothing factors were selected

after some experimentation and we found that stable results were obtained using $\alpha_{MT} = \alpha_S = 100$.

The final models obtained are shown in Fig. 11. These models matched the seismic data well (to normalized rms misfit of 0.97) but less satisfactorily for the AMT data which have a misfit of 4.104. The distributions of the normalized residuals for AMT and seismic data are presented, respectively, in Figs 12 and 13. The recovered models show some refinements when compared to the results of separate AMT and seismic inversion given in Meju *et al.* (2003). Figs 14(a) and (b) show the improvement in the geometrical similarity as judged by the computed rms value of the cross-gradient function which is five times smaller than that of the separately inverted models. As required by the geophysical data, the cross-gradient constraint permitted that significant changes in the electrical resistivity were unaccompanied by consequential changes in the seismic velocity, something common in natural earth materials. This is noticeable

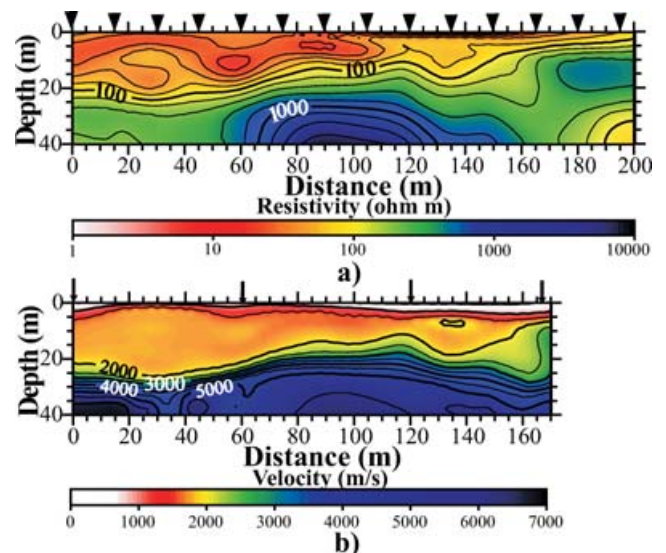


Figure 11. (a) AMT resistivity model and (b) seismic velocity model for the Quorn data profile obtained by joint inversion of the field data. The inverted triangles denote the sounding positions and the arrows the position of the seismic sources.

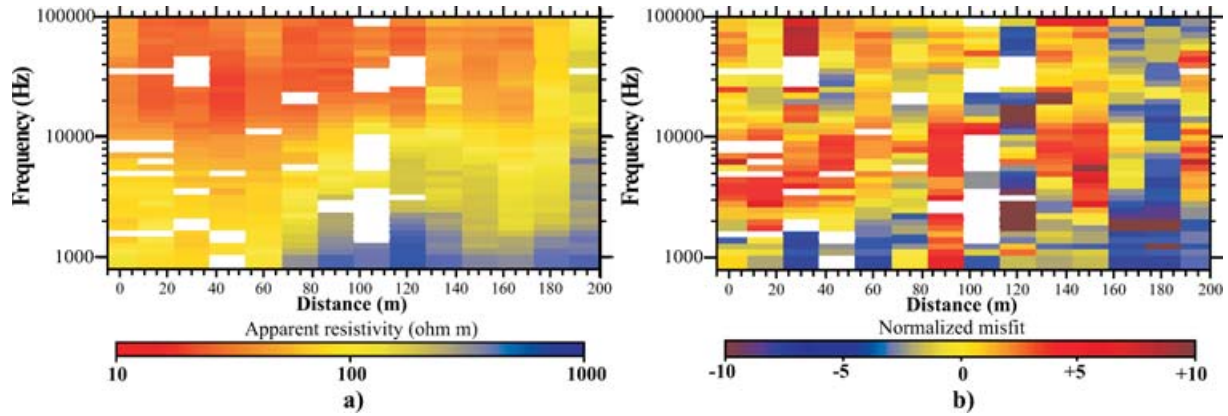


Figure 12. (a) Image of the measured AMT apparent resistivity data used for joint inversion in pseudo-section format. The plotted cells denote the actual TM data. (b) Image of the normalized residuals achieved after the joint inversion process plotted in their corresponding pseudo-section position.

in the subsurface model of Fig. 11 at profile positions 0–80 m and depths ranging from 5 to 20 m.

To gauge the accuracy and contribution of the AMT data to joint inversion, it is instructive to compare the AMT resistivity model of Fig. 11(a) with the DC resistivity model obtained by Gallardo & Meju (2003, Fig. 3) given that the same seismic data were used in both joint inversion experiments and that both DC and AMT should ideally result in identical resistivity images. The computed differences (in percent) between both resistivity models are shown in Fig. 15(a). This figure shows that there are only minor differences between them in the depth interval 5–20 m, which is relatively well sampled by both the DC resistivity and AMT methods. There is a notable difference in the resistivity models below 30 m depth (e.g. see profile positions 20 and 80 m) but these areas are not constrained by the DC resistivity data; similarly, some differences at depths shallower than about 3 m correspond to areas not constrained by the present AMT data. Both resistivity images are structurally similar as evaluated by the cross-gradient function in Fig. 15(b) and the maximum geometrical differences are coincident with the largest differences seen in Fig. 15(a). Based on these differences, the evaluation

of AMT-seismic interrelationships is reduced to the depth interval sampled by both DC resistivity and AMT methods but we suggest that the analysis can be generalized to greater depths.

In Fig. 16 is shown the AMT resistivity and seismic velocity interrelationships as deduced from the joint inversion models (Fig. 11). Note the correlatable trends and the excellent agreement with those previously determined by Gallardo & Meju (2003) based on joint DC resistivity-seismic inversion. In Fig. 16, the AMT resistivity–velocity trend (dark crosses) is associated with the deepest structural features shown in the corresponding section (see inset in Fig. 16). Thus, the AMT-seismic joint inversion results not only confirm the existence of such trends but also demonstrate the consistency that can be achieved in the geometrical structure, parameter accuracy and resistivity–velocity interrelationships from joint inversion of AMT and seismic refraction data. It can be expected that the same deductions would apply to models from deep regional MT and seismic surveys.

5 CONCLUSIONS

This study shows that the cross-gradient technique can be successfully adapted to the problem of joint 2-D inversion of collocated

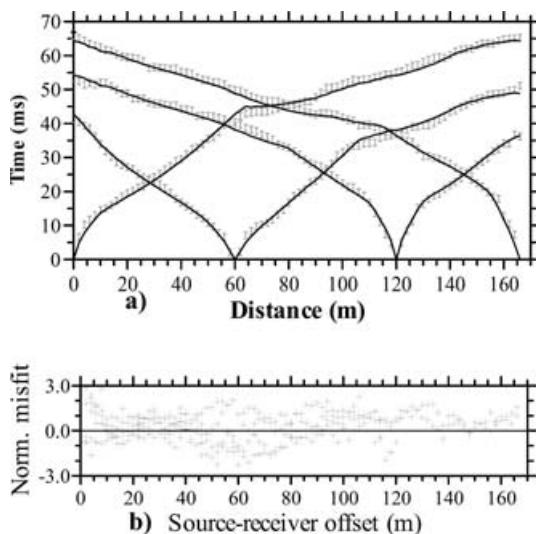


Figure 13. (a) Observed traveltimes data (Grey bars) versus travel times computed using the model of Fig. 11(b) (Solid lines). The bars show the assumed standard deviation of the data. (b) Corresponding normalised traveltimes residuals arranged according to source–receiver offset.

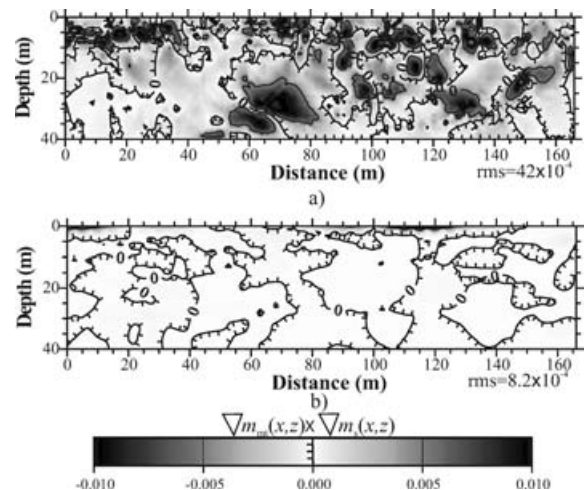


Figure 14. Cross-gradient values computed from the MT and seismic models for Quorn profile. (a) Using models obtained from separate inversion. (b) Using the models obtained from joint inversion of the same data sets.

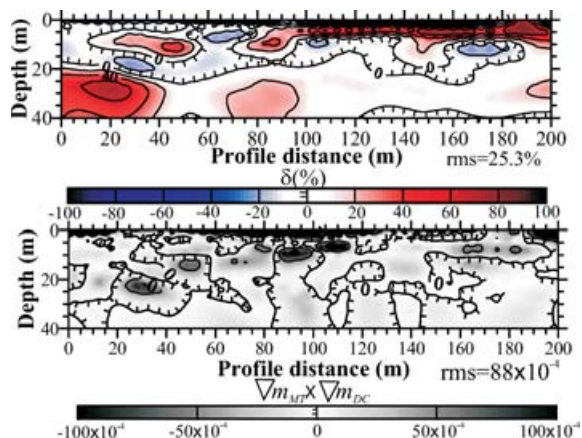


Figure 15. (a) Differences between the DC resistivity and AMT resistivity values obtained from joint inversion of DC and traveltime data (Gallardo & Meju 2003) and joint inversion of AMT and traveltime data (Fig. 11). Most of the large differences are coincident with areas without simultaneous coverage from both methods being considered. (b) Cross-gradient values computed for the same resistivity images used in (a).

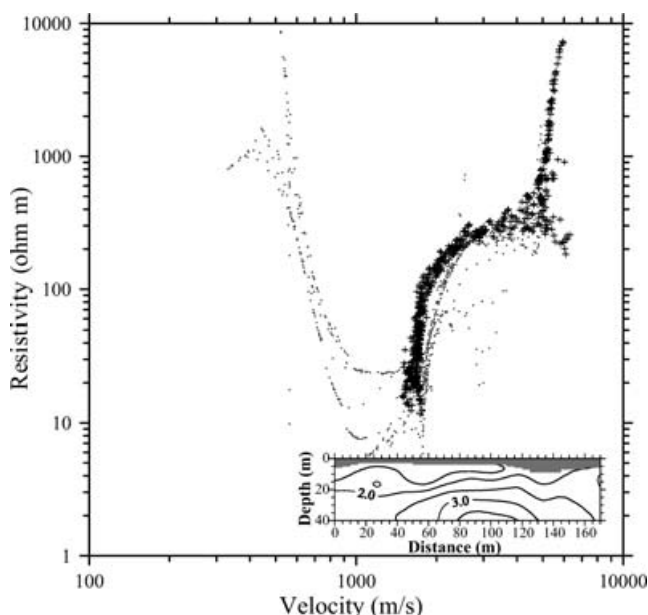


Figure 16. Comparison of resistivity and seismic velocity cross-plots for the Quorn profile. The grey dots correspond to those from DC-seismic joint inversion (Gallardo & Meju 2003). The black crosses are from the present AMT-seismic joint inversion results. The inset shows the actual subsurface distribution of the model cells (pixels) corresponding to the AMT-seismic cross-plot. Note the coincidence of the AMT and DC trends for velocities greater than 1500 m s^{-1} .

MT and seismic data from measurements at different spatial scales. Comparative tests with synthetic data show that joint MT-seismic inversion with cross-gradient constraints leads to models that are in superior structural accord and closer to the ground-truth than those derived from separate inversion of MT and seismic data. The joint inversion models recovered the salient features of hypothetical electrically conductive, low-velocity zones and steep features that posed significant imaging problems to the individual methods. Notably, the resistivity and seismic velocity estimates provided improved information that can facilitate the lithotype and structural classification of

the subsurface. Application of the algorithm to field data sets from a test site where comparative geophysical imaging measurements are available yielded AMT and seismic models that are in excellent geometrical accord and remarkable consistency with those obtained independently from joint 2-D inversion of collocated DC resistivity and seismic data. The new AMT-seismic models thus confirmed the existence of resistivity–velocity trends at the field site that were interpreted in a previous study to have petrophysical significance. We suggest that joint inversion of collocated MT-seismic profiles and the use of the results in structural or lithological classification will lead to improved subsurface characterization in complicated geological terrains and should be seen as the way forward in deep subsurface studies.

ACKNOWLEDGMENTS

We thank J. T. Smith and R. L. Mackie for making available their forward modeling codes. We appreciate the constructive comments of Martyn Unsworth, Paul Bedrosian and anonymous reviewers which greatly improved the manuscript.

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