

Feasibility of using full tensor gradient (FTG) data for detection of local lateral density contrasts during reservoir monitoring

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SUMMARY

Three-Dimensional Full Tensor Gradiometry (3-D FTG) acquires ultrasensitive measurements of the Earth's (vector) gravity gradient field. Departures from simple weakening of the field in the vertical direction are due to subsurface variations in density. We have undertaken a numerical examination of the feasibility of using this system for detecting time variations in local lateral density contrasts in subsurface layers during reservoir production monitoring. Our gravity modelling focuses on the value added by taking account of the horizontal components of gravity gradient in imaging local targets. We have studied the sensitivity of these components to model and acquisition parameters. Iterative regularized inversion algorithms that can explain the behaviour of a hydrocarbon reservoir in terms of time-lapse density changes have been described. These algorithms are based on the homotopy method that utilizes the solution space topology. To stabilize the inversion, we have introduced the maximum compactness of anomalous sources along one or more directions. This allows the use of *a priori* information natural to time-lapse monitoring about the arbitrary shape and spatial locations of the sources. The time-evolving density contrast within the local target zone is recovered by means of an iterative procedure. The treatment of either correlated or random noise fits naturally into this procedure. The final scheme has been illustrated by several examples using synthetic FTG data generated from various density distributions of practical interest. We have considered the effect of model and data uncertainties on the inversion solution. It appears that the iterative process is effective, numerically stable and rapidly convergent in the presence of both random and structural noise. Results show that it is relatively insensitive to the choice of starting model. Appropriate applications include direct monitoring of gas–oil contact and temperature front expansion during steam injection in heavy-oil reservoirs at shallow or moderate depths.

Key words: FTG, regularized inversion, reservoir monitoring.

INTRODUCTION

Qualitative and quantitative information derived from potential field data can be used to reduce uncertainties of reservoir history matching. On scales of some hundreds of metres, exploration geophysicists attempt to use these data to directly delineate earth heterogeneities associated with various recovery mechanisms within the reservoir. It is well known that the detection of local lateral density contrasts in subsurface layers of some tens of metres thickness presents a challenge for conventional geophysical inversion methods based on gravity measurements or geoid height observations converted to gravity (e.g. Blakely 1996). This occurs because, as distance from the causative mass anomaly increases, shorter wavelengths are relatively more attenuated than longer wavelengths. By exploiting high-resolution full tensor gradient (FTG) measurements

along with robust inversion methods, it is envisaged that weak lateral density contrasts can be better delineated.

FTG has attracted the interest of exploration geophysicists since the mid-1990s. To our knowledge, Bell *et al.* (1997) were the first to write about potential applications of FTG, and one of the applications they discussed in that paper was time-lapse FTG monitoring of reservoirs. Robin Bell also published a good introductory paper on FTG in *Scientific American* in the following year (Bell 1998). In the United States, the FTG technique has frequently been applied in subsalt exploration in the Gulf of Mexico. Coburn (2002) gave some case studies that described the value of using FTG and 3-D seismic imaging in mapping the base of allochthonous salt sheets. He performed iterative forward modelling of FTG to constrain the density contrast between the salt bodies and the surrounding sediments. The technique has also been successfully applied to image both

structural and lithological contacts in several challenging areas of the North Sea (Murphy *et al.* 2002). Over the course of these studies, FTG processing tools have been refined and upgraded to extract more geological signal than was possible previously.

In this paper, we study the hydrocarbon production processes through changes in the density of reservoir fluids. Our objective is to determine the feasibility of using ultrasensitive real-time FTG measurements for mapping relatively small ('4-D') density changes caused by changes in fluid or gas saturation during production. We attempt to answer the following question: can differences between the baseline and monitored gravity anomalies be used to monitor fluid or gas distribution changes in the reservoir? To answer this question positively, we develop special forward modelling and regularized inversion schemes that enhance production-related density anomalies of the reservoir. The components of the gradient tensor are used to constrain the reservoir model on a local level by defining lateral density contrasts of the model that might otherwise be masked by acquisition and processing artefacts. We illustrate the method performance by applying it to synthetic FTG data for various density models occupying the half-space $Z > 0$, where the Z -axis points vertically downwards. The observations are measured on plane $Z = 0$ (ground-based acquisition).

BASIC PRINCIPLES OF FTG TECHNOLOGY

The gravity vector field is expressed as the spatial derivative of gravity potential g , viz., $G_i = \partial g / \partial x_i$. Traditional gravimeters measure the vertical component of gradient G_Z , and spatial variations in G_Z may be used to infer vertical and horizontal gradients G_{ZZ} , G_{ZX} and G_{ZY} that give low-resolution information on the subsurface mass distribution. 3-D FTG is a high precision multiple accelerometer system that is composed of three gravity gradient instruments (GGIs). As well as recording the magnitude of the gravity vector G_i this system measures the tensor field $G_{ij} = \partial^2 g / \partial x_i \partial x_j$ at prospect level resolution comparable to that of seismic reflection method (J. Macfarlane 2002, private communication). In principle, nine tensor components mathematically arranged in a 3×3 matrix are measured. Since four tensor components are redundant, only five independent tensor measurements are recorded. These data are taken as a part of the routine data gathering process. Positioning is achieved using standard differential GPS techniques with accuracy of down to 1 m.

Fig. 1a shows the vector gravity gradient components G_X , G_Y and G_Z in the right-handed coordinate system $\{X = x_1, Y = x_2, Z = x_3\}$, where the X -axis points to the north and the Y -axis points to the east. The 3-D FTG system uses these components to measure spatial rates of change in the gravity field represented by the gradient tensor (Fig. 1b)

$$\mathbf{G} = \nabla \nabla g = \begin{pmatrix} G_{XX} & G_{XY} & G_{XZ} \\ G_{YX} & G_{YY} & G_{YZ} \\ G_{ZX} & G_{ZY} & G_{ZZ} \end{pmatrix}, \quad (1)$$

where the spatial gradient operator ∇ is applied to observation points $\mathbf{x} = \{x_i\} (i = 1, 2, 3)$ and g is the gravitational potential. Since the gravity field is derived from a potential, this tensor is symmetric

$$\mathbf{G} = \mathbf{G}^T \quad (2)$$

and traceless

$$\text{tr} \mathbf{G} = 0 \quad (3)$$

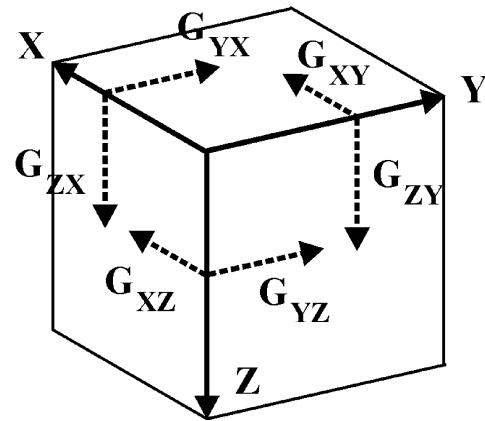
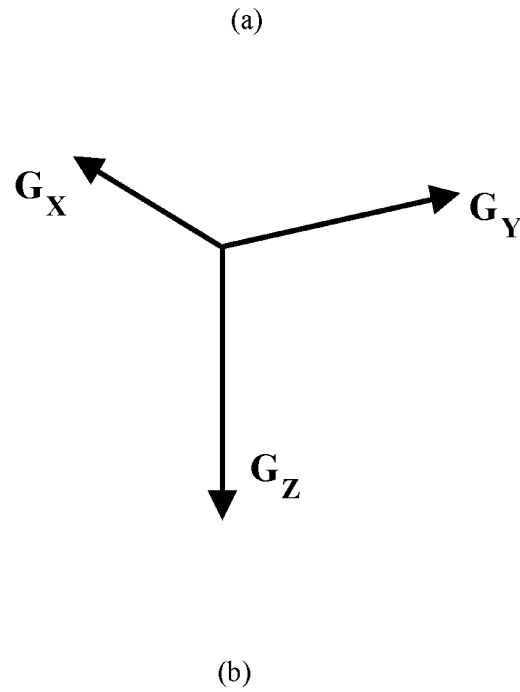


Figure 1. Schematic illustration of FTG measurements: (a) three components of gravity vector recorded by GGI and (b) nine components of gravity tensor. Conventional gravimeters measure the G_Z component only. The 3-D FTG system uses five independent components of the tensor (1) recorded by three GGIs.

leaving only five independent components, say, $\tilde{G}_{XX} = G_{XX} - G_{YY}$, G_{ZZ} , G_{XZ} , G_{YZ} and G_{XY} . The component G_{ZZ} is the vertical gradient. The components G_{XY} and $\tilde{G}_{XX}$ are referred to as the curvature gravity gradient. They represent the deviation from spherical symmetry of the curvature of the equipotential. The components G_{XZ} and G_{YZ} form a vector called the horizontal gradient. Conventionally, the amplitude and directions of the horizontal gravity gradient and of the curvature gradient are expressed as (after Eötvös)

$G_H = \sqrt{G_{XZ}^2 + G_{YZ}^2}$, $G_C = \sqrt{4G_{XY}^2 + \tilde{G}_{XX}^2}$, $\alpha_H = \tan^{-1}(-G_{XZ}/G_{YZ})$ and $\alpha_C = (1/2)\tan^{-1}(-2G_{XY}/\tilde{G}_{XX})$. These quantities are referred to as the FTG field gradient and curvature phase and amplitude attributes. Finally, we note that the commonly used units for gravity are the mGal ($10 \mu\text{ms}^{-2}$). For FTG, the appropriate

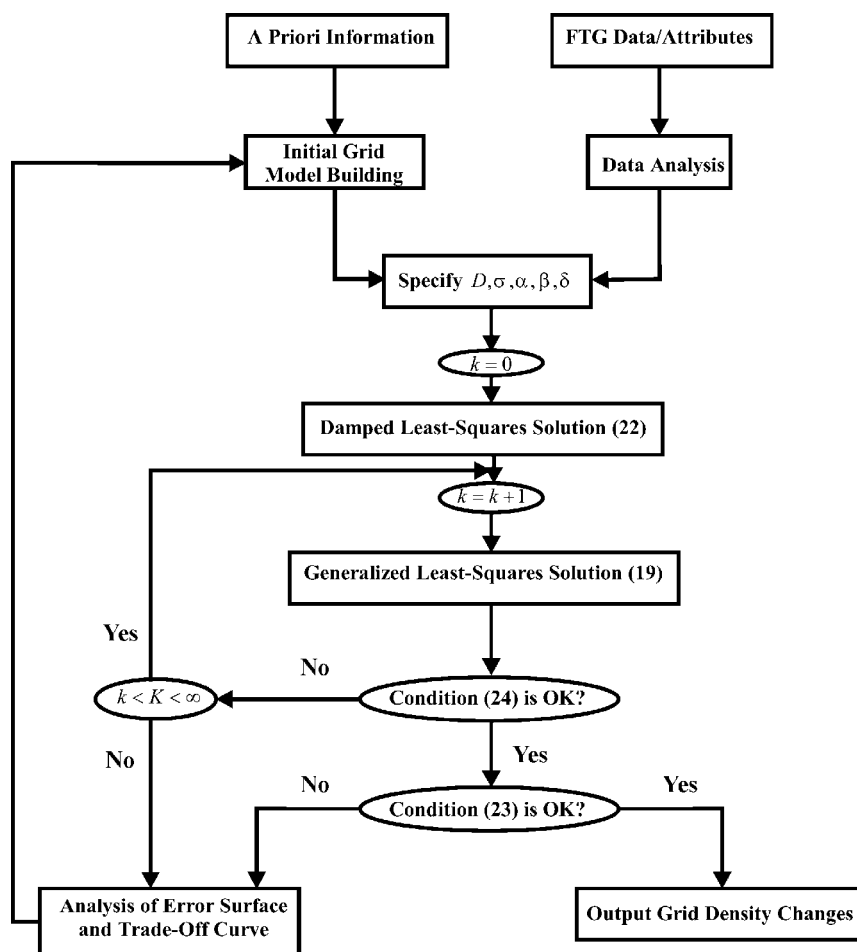


Figure 2. Flow diagram of the regularized 4-D FTG inversion algorithm to detect local lateral density changes within the domain D .

unit is the Eötvös (Eö) (0.1 mGal km^{-1} or $100 \mu\text{Gal km}^{-1}$) (Bell *et al.* 1997). For example, the Earth's gravity gradient at the surface is approximately 3080 Eö.

KEY ADVANTAGES OF FTG ACQUISITION

The Bell's FTG system is derived from a moving-platform gradiometer developed in the 1970s by the US Government and Lockheed Martin as a 'stealth' navigation system for Trident class nuclear-powered ballistic-missile submarines. The Bell system is able to measure changes in the Earth's gravity field simultaneously at two points (using a series of spinning GGIs). The Bell instrument measures the total gravity field in all directions (nine component tensor (1)), whereas the rival BHP Billiton's Falcon system measures only the gravity changes in the vertical direction (three component tensor).

FTG measurements differ in many aspects from conventional high-resolution gravity data. One important difference is in resolution. The Bell gravity gradiometers are capable of producing data appropriate for exploration geophysics with accuracies of better than 1 Eö and resolutions of tens of metres (C. Murphy 2003, private communication). Fine resolution enables excellent lateral definition of the edges of stratigraphic or structural units of different densities, or the imaging of vertical or subvertical structures and fault planes. FTG resolution is high at the shallow section making it ideal for

near surface and seabed mapping. In contrast, traditional airborne gravimetry has produced data with accuracies of about 3 mGal, resolved to wavelengths of 20 km or shorter at an altitude of 0.6 km. Detailed ground gravimetry can provide gravity gradient information with a noise level of about 10 Eö. This performance is at a resolution and precision more suitable for large-scale regional studies (Blakely 1996). The greatly increased sensitivity of FTG data allows the retention of the high spatial frequency, short wavelength signals generated by shallow to intermediate geological features, which are not retained in conventional high resolution gravimeter data. As well as the ability to provide better sensitivity and resolution, FTG does not require corrections equivalent to the latitude and Bouguer corrections necessary for standard gravimetry (e.g. Dransfield 1994).

FORWARD FTG MODELLING

In this section the emphasis is on the forward gravity gradient solution (Dransfield 1994) applicable to the FTG fields, using both simple and arbitrary density models. This solution may be considered as a forward magnetic problem with the known magnetization direction.

Let the point mass (monopole) or a sphere of mass Π be located at the origin. The gravity gradient and FTG responses to this source

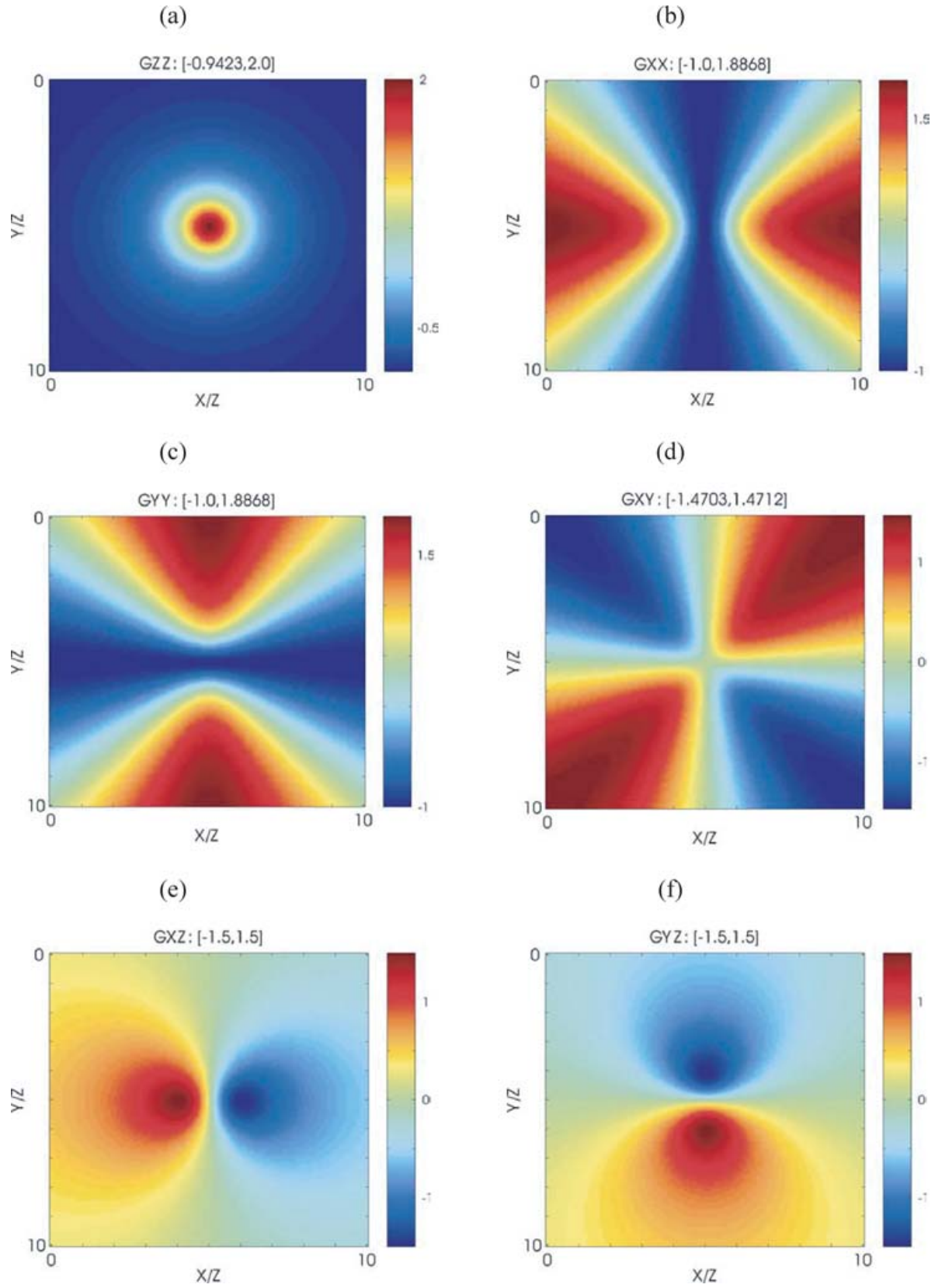


Figure 3. Normalized FTG response to a monopole given by eq. (5): (a) $\bar{G}_{33}$, (b) $\bar{G}_{11}$, (c) $\bar{G}_{22}$, (d) $\bar{G}_{12}$, (e) $\bar{G}_{13}$ and (f) $\bar{G}_{23}$.

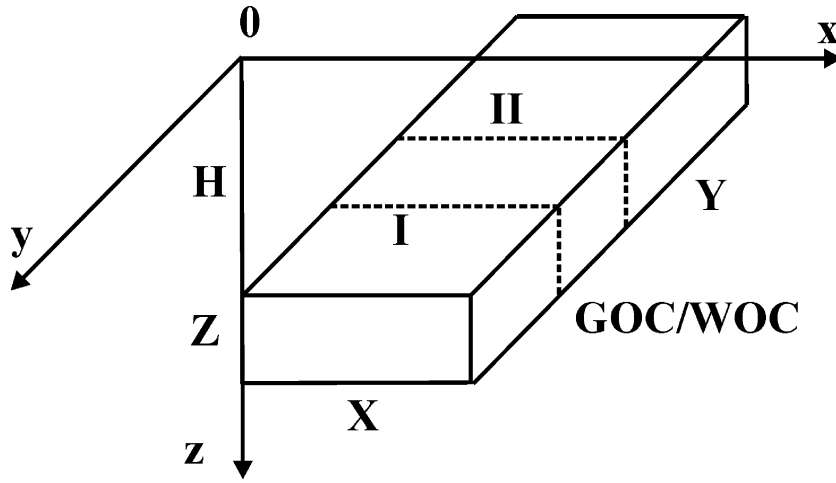


Figure 4. Local geometry of models A and B with saturation values listed in Table 1.

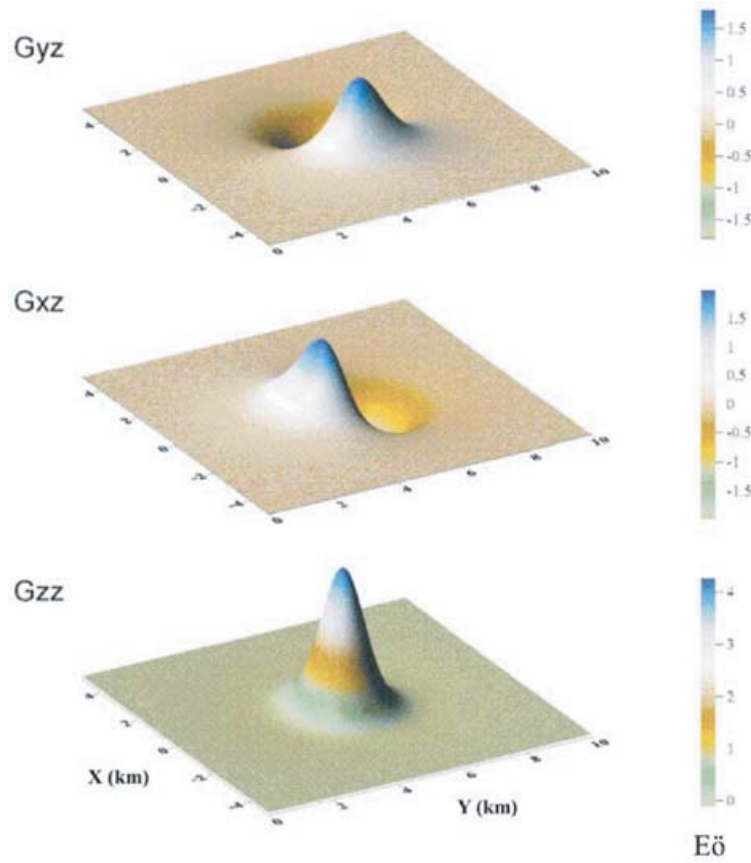


Figure 5. 3-D FTG response ($E\ddot{o}$) to gas-oil replacement in model B (Table 1) with $N = 10^4$ (100×100 grid over the area of 100 km^2) and $M = 1$.

are as follows

$$G_i(\mathbf{x}) = -\frac{\gamma \Pi}{r^2} \frac{x_i}{r} \quad (4)$$

and

$$\begin{aligned} \bar{G}_{ij}(\mathbf{x}) &= G_{ij}(\mathbf{x}) / \bar{G}_3(\mathbf{x}) \\ &= \begin{cases} (3x_i^2 - r^2)/r^2 & i = j \\ 3x_i x_j / r^2 & i \neq j \end{cases}, \quad i, j = 1, 2, 3, \end{aligned} \quad (5)$$

where $\bar{G}_3 = G_3/x_3 = -\gamma \Pi / r^3$, G_{ij} are the components of the tensor (1), γ is the gravitational constant ($\gamma = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$) and $r = |\mathbf{r}|$ is the distance between the point mass and the observation point.

Next, we evaluate the gravity gradient tensor (1) at a point $\mathbf{x}$ external to a homogeneous rectangular prism of the uniform density ρ . The prism is, without loss of generality, assumed to lie with its centre at the origin of the coordinate system and its edges aligned

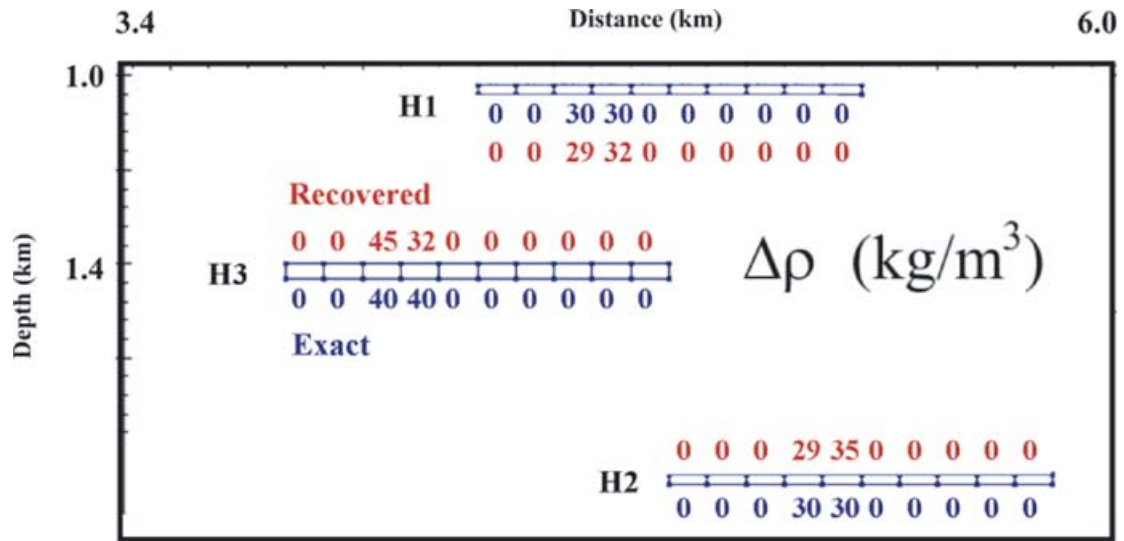


Figure 6. Regularized FTG inversion of G_{XZ} and G_{ZZ} gradients ($N = 100$ and $M = 30$): exact and recovered density changes between surveys I and II (in kg m^{-3}).

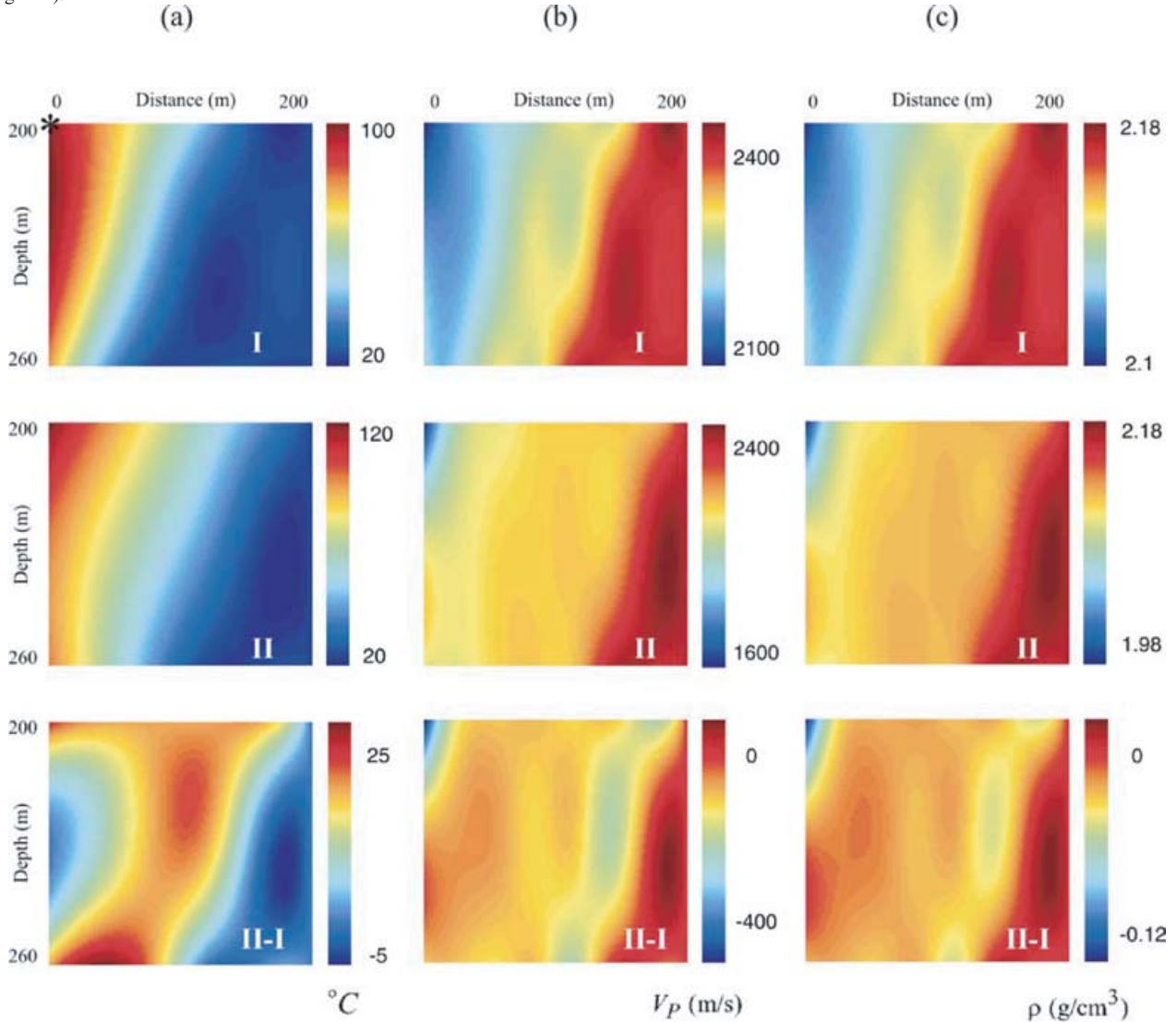


Figure 7. (a) Temperature ($^{\circ}\text{C}$), (b) P -wave velocity (in m s^{-1}) and (c) density (in g cm^{-3}) distributions (phases I and II) and their difference (II-I) as functions of distance from the injection well and measured depth (in metres). The star symbol indicates the injection point located at the depth of 200 m.

Table 1. Saturation values versus predicted maximum 4-D density and corresponding G_{ZZ} changes in models A and B.

Model	S_O	S_G	S_W	$\max(\Delta\rho)$ (g cm ⁻³)	$\max(\Delta G_{ZZ})$ (Eö)
A (WOC)	0.78	0.00	0.22	0.04	0.86
B (GOC)	0.62	0.04	0.34	0.25	6.0

with the coordinate axes. The prism has sides of length $2X$ in the x_1 , $2Y$ in the x_2 and $2Z$ in the x_3 direction. In this case, the tensor (1) is given by

$$\begin{aligned}
 -(\gamma\rho)^{-1}\mathbf{G}(\mathbf{x}) = & \mathbf{H}(x_1 - X, x_2 - Y, x_3 - Z) \\
 & - \mathbf{H}(x_1 + X, x_2 - Y, x_3 - Z) - \mathbf{H}(x_1 - X, x_2 + Y, x_3 - Z) \\
 & - \mathbf{H}(x_1 - X, x_2 - Y, x_3 + Z) + \mathbf{H}(x_1 + X, x_2 + Y, x_3 - Z) \\
 & + \mathbf{H}(x_1 + X, x_2 - Y, x_3 + Z) + \mathbf{H}(x_1 - X, x_2 + Y, x_3 + Z) \\
 & - \mathbf{H}(x_1 + X, x_2 + Y, x_3 + Z)
 \end{aligned} \quad (6)$$

with the coefficients

$$\begin{aligned}
 H_{II} = & -\tan^{-1}\left(\frac{x_J x_K}{x_I r}\right) + \frac{x_I x_J x_K r}{x_I^2 r^2 + x_J^2 x_K^2} + \frac{x_J^3 x_I x_K}{x_I^2 r^3 + x_J^2 x_K^2 r} \\
 & + \frac{x_I x_J}{x_K r + r^2} + \frac{x_I x_K}{x_J r + r^2}
 \end{aligned} \quad (7)$$

and

$$H_{IJ} = \ln(x_K + r), \quad (8)$$

where $r = \sqrt{x_I^2 + x_J^2 + x_K^2}$ ($I, J, K = 1, 2, 3, I \neq J \neq K$). Since (6)–(8) are valid for any of the x_1, x_2 or x_3 coordinates, the coordinate variables are subscripted by I, J or K .

A homogeneous prism is a reasonable local approximation to geological structures since more complex density models can be built from sets of rectangular prisms, each assigned a fixed density. Let the subsurface volume be broken down into a discrete 3-D partition of M prismatic cells, each cell having a fixed density contrast ρ_m ($m = 1, \dots, M$) with respect to some constant background value ρ_0 . In practice, the cell sizes should be made sufficiently small so that the gravity gradient due to the change in density across a cell boundary will be significantly less than 1 Eö. Following the classical Newtonian gravity theory (Blakely 1996), the FTG field value at the n th observation point

$$\mathbf{G} = \nabla \nabla g_n, \quad n = 1, \dots, N, \quad (9)$$

may be expressed in terms of the gravity potential

$$g_n = \gamma \sum_{m=1}^M a_{nm} \rho_m, \quad (10)$$

where the matrix elements $a_{nm} = |\mathbf{r}_m - \mathbf{r}_n|^{-1}$ describe the influence of the m th prism on the n th gravity value. Here, $\mathbf{r}_n = (x_{1n}, x_{2n}, x_{3n})$ and $\mathbf{r}_m = (X_m, Y_m, Z_m)$ are, respectively, the positions of the observation point and the source point (centre of mass). Substituting for g_n from (10) into (9) and taking into account (6), we obtain the FTG response to the arbitrary density distribution

$$\mathbf{G}(\mathbf{r}_n) = -\gamma \sum_{m=1}^M \mathbf{B}_{nm} \rho_m, \quad (11)$$

$$\begin{aligned}
 \mathbf{B}_{nm} = & \mathbf{H}(x_{1n} - X_m, x_{2n} - Y_m, x_{3n} - Z_m) \\
 & - \mathbf{H}(x_{1n} + X_m, x_{2n} - Y_m, x_{3n} - Z_m) \\
 & - \mathbf{H}(x_{1n} - X_m, x_{2n} + Y_m, x_{3n} - Z_m) \\
 & - \mathbf{H}(x_{1n} - X_m, x_{2n} - Y_m, x_{3n} + Z_m) \\
 & + \mathbf{H}(x_{1n} + X_m, x_{2n} + Y_m, x_{3n} - Z_m) \\
 & + \mathbf{H}(x_{1n} + X_m, x_{2n} - Y_m, x_{3n} + Z_m) \\
 & + \mathbf{H}(x_{1n} - X_m, x_{2n} + Y_m, x_{3n} + Z_m) \\
 & - \mathbf{H}(x_{1n} + X_m, x_{2n} + Y_m, x_{3n} + Z_m)
 \end{aligned}$$

with the tensor $\mathbf{H}$ whose components are defined by expressions (7) and (8). The forward modelling thus becomes the evaluation of the matrix–vector product in eq. (11).

According to eq. (11), the FTG response to the test mass $\Pi = \rho D$ is associated with a tensor field related to the given density distribution ρ within the target volume D . For the cases of interest here, the density is chosen to represent the bulk density of a fluid-saturated porous medium, that is (Wang 2001)

$$\rho = (1 - f)\rho_s + f\rho_f, \quad (12)$$

where f denotes the porosity, $\rho_d = (1 - f)\rho_s$ the dry density of the rock, ρ_s the matrix (grain) density and ρ_f the density of a three-phase pore fluid given in turn by

$$\rho_f = S_O \rho_O + S_W \rho_W + S_G \rho_G, \quad (13)$$

S_O, S_W and S_G being saturation values for the oil (O), water (W) and gas (G) constituents expressed as volume fractions of the pore space, $S_O + S_W + S_G = 1$. As seen in eqs (12) and (13), several input parameters are required to calculate fluid effects on the density distribution in and around the reservoir. The grain density, porosity and densities of the oil, gas and water constituents can be estimated through lab measurements, well logs or empirical relations over an extensive temperature and a certain pressure range, as described by Batzle & Wang (1992). Quantitative stochastic estimates of saturation values can be obtained if monthly production rates of oil, water and gas are available for an ensemble of history matching reservoir models (Christie *et al.* 2002).

REGULARIZED FTG INVERSION

This section presents a method for inverting FTG data to determine local distributions of lateral density contrasts aimed at reducing uncertainties of reservoir history matching within the union of pre-established volumes of bodies denoted by D . The goal is to relate two high-resolution measurements of the tensor (1) at different times so as to characterize the density change of the geological medium during this time interval. To distinguish between two surveys, we use the superscript ‘ I ’ to denote the quantities at the reference time (base or legacy survey), and the superscript ‘ II ’ to denote the quantities at the monitoring time (repeat survey). The 4-D inverse FTG problem of estimating the density change

$$\Delta\rho(\mathbf{x}) = \begin{cases} \rho^{II} - \rho^I & \mathbf{x} \in D \\ 0 & \mathbf{x} \notin D \end{cases} \quad (14)$$

from the observed FTG difference, $\Delta\mathbf{G}_{\text{obs}} = \mathbf{G}^{II} - \mathbf{G}^I$, may be formulated using the homotopy method (Everett 1996), which permits the incorporation of *a priori* information about the maximum compactness of anomalous sources along several axes (Last & Kubik

1983; Barbosa & Silva 1994). Specifically, we minimize the following parametric functional of the densities and errors

$$P_\alpha(\Delta\rho) = (1 - \alpha) \|\mathbf{A} \cdot \Delta\rho - \Delta\mathbf{G}_{\text{obs}}\|^2 + \alpha\Omega(\Delta\rho) \rightarrow \min, \quad \alpha \in [0, 1], \quad (15)$$

while requiring that the forward solution (11) written in the matrix form

$$\Delta\mathbf{G} = \mathbf{A} \cdot \Delta\rho \quad (16)$$

be satisfied. Here,

$$\Delta\mathbf{G} = \text{col}\{\Delta\mathbf{G}(\mathbf{r}_i)\}, \Delta\rho = \text{col}\{\Delta\rho_j\}, \mathbf{A} = -\gamma\{\mathbf{a}_{ij}\}, \mathbf{a}_{ij} = \text{col}\{\tilde{\mathbf{H}}_{ij}\},$$

and the elements

$$\begin{aligned} \tilde{\mathbf{H}}_{ij} = & \mathbf{H}(x_{1i} - X_j, x_{2i} - Y_j, x_{3i} - Z_j) \\ & - \mathbf{H}(x_{1i} + X_j, x_{2i} - Y_j, x_{3i} - Z_j) \\ & - \mathbf{H}(x_{1i} - X_j, x_{2i} + Y_j, x_{3i} - Z_j) \\ & - \mathbf{H}(x_{1i} - X_j, x_{2i} - Y_j, x_{3i} + Z_j) \\ & + \mathbf{H}(x_{1i} + X_j, x_{2i} + Y_j, x_{3i} - Z_j) \\ & + \mathbf{H}(x_{1i} + X_j, x_{2i} - Y_j, x_{3i} + Z_j) \\ & + \mathbf{H}(x_{1i} - X_j, x_{2i} + Y_j, x_{3i} + Z_j) \\ & - \mathbf{H}(x_{1i} + X_j, x_{2i} + Y_j, x_{3i} + Z_j) \end{aligned}$$

depend on the discretization geometry only. In eq. (15), $\|\cdot\|$ is the appropriate norm in the space L_2 of square integrable functions (this means that the solution converges to the desired one rms), α is the scalar homotopy parameter, and Ω is a functional carrying the *a priori* information about the model parameters. The optimization search (16) performed over a range of α values incorporates the condition (14) into the inversion process by minimizing the functional $P_\alpha(\Delta\rho)$ subject to eq. (16), within a tolerance imposed by the *a priori* estimate of signal-to-noise ratio (SNR). With the aid of numerical optimization methods (e.g. Cheney & Kincaid 1999), a trajectory in $(\Delta\rho, \alpha)$ space is mapped out along which the Jacobian derivative $\partial P_\alpha(\Delta\rho)/\partial\rho$ vanishes.

Typically, the minimization problem (15) is overdetermined, that is, $N \gg M$. It has a simple appearance, but its numerical implementation is not trivial. In the presence of random or *structural* (correlated) noise a range of solutions may be examined in a least-squares sense (Meju 1994). To minimize the cost function efficiently, it is necessary to have its gradients readily available. Therefore, derivative-based optimization schemes could be applied only after spatial (e.g. B-spline) interpolation of the original grid (Cheney & Kincaid 1999). Our numerical tests show that conventional maximum smoothness stabilizers without a compactness criterion (overviewed by Portniaguine & Zhdanov 1999) are not suitable for 4-D FTG inversion. In our approach, we add the second term in eq. (15) to minimize a localized reservoir volume. In this case a compactness criterion conforms to our expectation of the physical process of fluid. This step is equivalent to the so-called minimum support compactness functional representing a definition of area for our model (Last & Kubik 1983)

$$\Omega(\rho) = \left\| \frac{\rho}{\sqrt{\rho^2 + \beta^2}} \right\|^2, \quad (17)$$

where the parameter β should be chosen as small as possible without causing numerical instability.

The iterative least-squares optimization search (15) is summarized in Fig. 2. The k th step can be represented in matrix notation

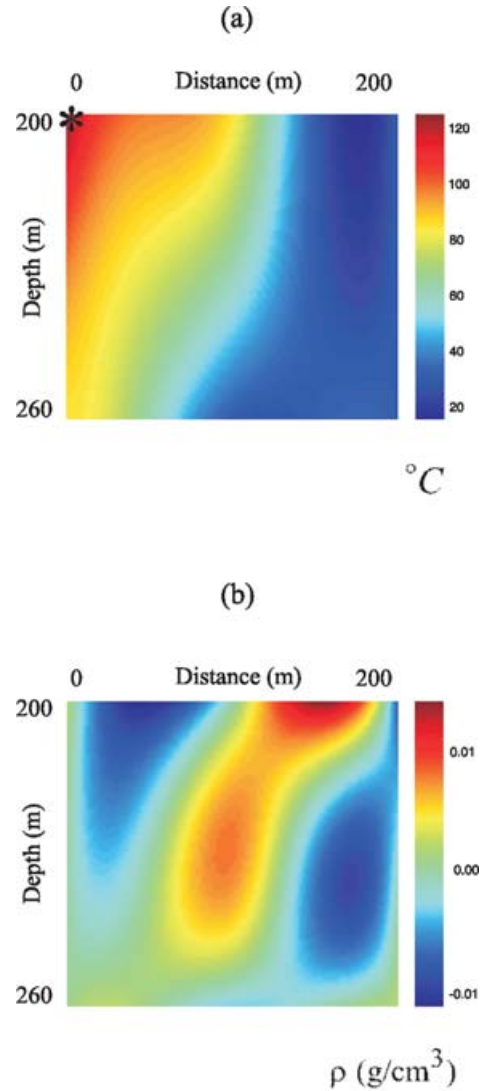


Figure 8. (a) Temperature distribution ($^{\circ}\text{C}$) as a result of density-to-temperature conversion of the recovered density ρ^{II} (compare with Fig. 7a, phase II) and (b) difference between exact and recovered density changes $\Delta\rho = \rho^{II} - \rho^I$ and $\Delta\rho_{\text{inv}}^{II} = \rho_{\text{inv}}^{II} - \rho^I$ (g cm^{-3}) (see model in Fig. 7), where $\rho_{\text{inv}}^{II} = \rho_{\text{inv}}^{II(15)}$ is the recovered density after the 15th iteration. The reference density is ρ^I (Fig. 7c, phase I). Exact density changes $\Delta\rho$ are shown in Fig. 7c (II-I).

as (Last & Kubik 1983)

$$\Delta\rho^{(k)} = [\mathbf{W}_\rho^{(k-1)}]^{-1} \mathbf{A}^T \{ \mathbf{A} [\mathbf{W}_\rho^{(k-1)}]^{-1} \mathbf{A}^T + [\mathbf{W}_e^{(k-1)}]^{-1} \}^{-1} \Delta\mathbf{G}_{\text{obs}} \quad (18)$$

with the following matrix weighting coefficients

$$[\mathbf{W}_e^{(k-1)}]^{-1} = \sigma^2 \text{diag}\{ \mathbf{A} [\mathbf{W}_\rho^{(k-1)}]^{-1} \mathbf{A}^T \} \quad (19)$$

and

$$\mathbf{W}_\rho^{(k-1)} = \text{diag}\{ [\Delta\rho_j^{(k-1)}]^2 + \beta^2 \}^{-1}, \quad j = 1, \dots, M, \quad (20)$$

where σ^2 is the *a priori* estimated SNR. The first iteration may start from the damped least-squares solution (Meju 1994; Menke 1989)

$$\Delta\rho^{(0)} = \{ \mathbf{A}^T \mathbf{A} + \alpha \mathbf{I} \}^{-1} \mathbf{A}^T \Delta\mathbf{G}_{\text{obs}}, \quad (21)$$

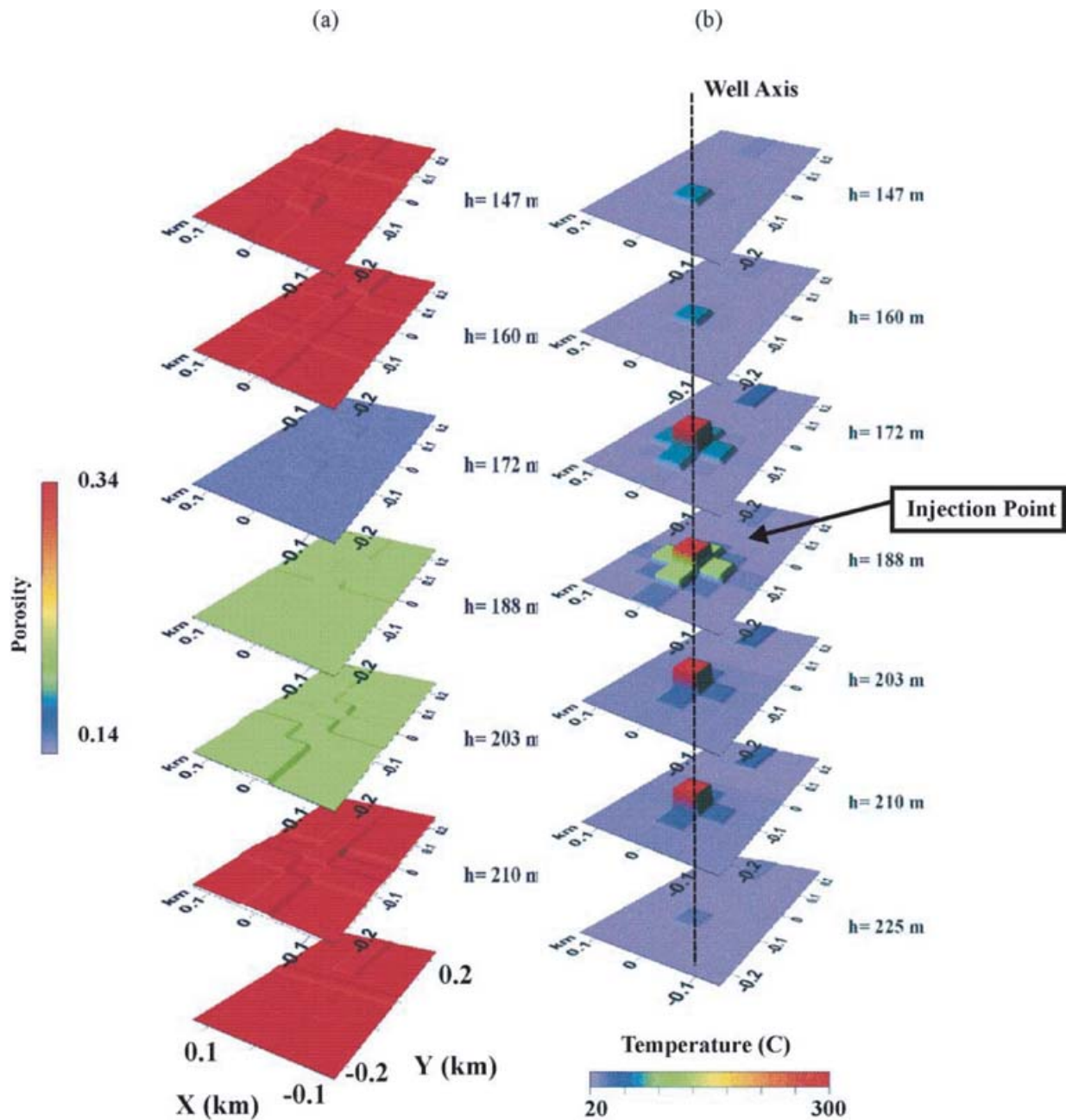


Figure 9. Output of thermal reservoir simulator in the vicinity of the vertical injection well: (a) porosity map and (b) temperature distribution after 212 d of continuous steam injection (horizontal slices $Z = h$).

where $\mathbf{I}$ is the identity matrix. Iterations (typically fewer than 30) increase the compactness of the solution until the required minimum area is reached (Fig. 2).

In 4-D FTG inversion, we usually search for solutions in the vicinity of major producing or injecting well(s). In many cases, concrete and precise *a priori* knowledge about the reservoir geometry and depth may be provided by calibrated log and seismic data to reduce the solution instability. The automatic derivation of depth solutions from FTG has also been discussed by Huang & Versnel (2000). This makes it possible to introduce directional and global convexity based on the information about a part of a localized reservoir volume which is associated with the largest lateral spread of mass (Barbosa & Silva 1994). We take advantage of the two types of bias, such as

adding the α term in eq. (21), equivalent to applying a smoothness measure (Portniaguine & Zhdanov 1999), and minimization of the volume of the anomaly source.

The problem of using smoothness as a prior condition in inverting gravity anomalies has been pointed out by some previous researchers who attempted to constrain the geometry of salt bodies. For instance, Nagihara & Hall (2001) proposed a gravity inversion technique based on the simulated annealing optimization algorithm, in part to avoid this problem. In principle, the present inversion technique could use their algorithm to avoid some of the problems we described. That is, however, a topic for separate paper.

The essential steps of our inversion algorithm are as follows (Fig. 2)

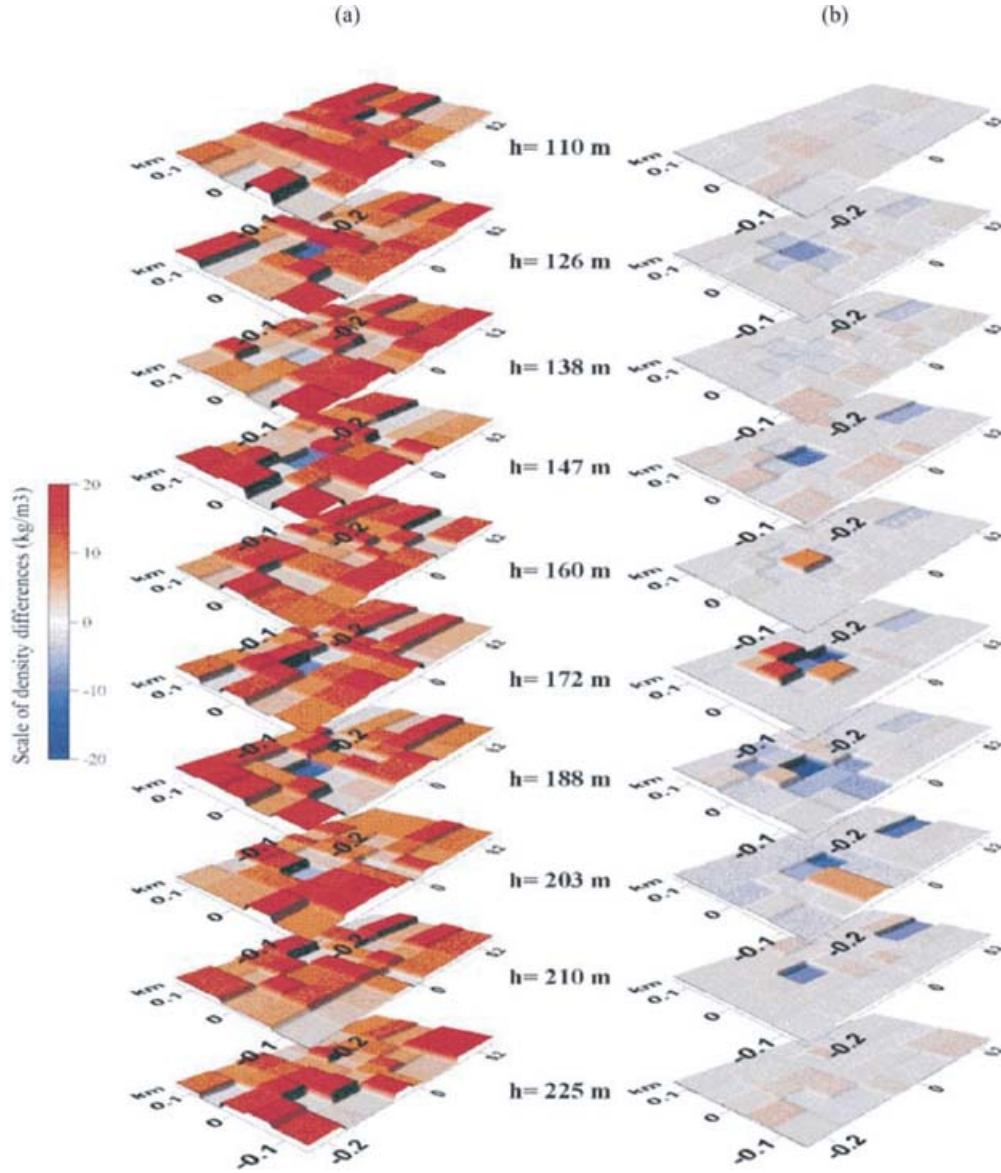


Figure 10. Model in Fig. 9: (a) difference between exact and reference densities $d\rho^0 = \rho^{II} - \rho^0$ and (b) difference between exact and recovered densities $d\rho = \rho^{II} - \rho_{\text{inv}}^{II}$, where the reference density $\rho^0 = \Delta\rho^{(0)}$ is defined by eq. (24) and $\rho_{\text{inv}}^{II} = \rho_{\text{inv}}^{(5)}$ with $\alpha = 0.1$ (densities in kg m^{-3}).

1. Input the initial density grid (model building) and FTG data or attributes.
2. Define the domain D and the parameters σ (noise), α (compactness weight 1) and β (compactness weight 2).
3. Assign $k = 0$ and compute the damped least-squares inverse (21).
4. Assign $k \rightarrow k + 1$ and compute the generalized least-squares inverse (18).
5. Check the convergence criteria

$$\|\Delta\rho^{(k)} - \Delta\rho^{(k-1)}\| \leq \delta/R \quad (22)$$

and

$$\|\mathbf{A} \cdot (\Delta\rho^{(k)} - \Delta\rho^{(k-1)})\| \leq \delta, \quad (23)$$

where δ is some prescribed value such as the random error of observations and $R = 1/\|\mathbf{A}^{-1}\|$ is the scalar measure of resolution of the

method (Menke 1989). Actually, the criterion (23) is sufficient to terminate the iteration procedure if there is no further improvement in fit to the data regardless of the model error given by (22).

The iteration procedure is repeated until either the convergence criterion (22) or (23) is satisfied. If either (22) or (23) is violated as $k \rightarrow \infty$ (say, $k \geq K$ with $K = 100$), then we examine the error surface $E_\alpha(\Delta\rho^{(k)}) = \|\mathbf{A} \cdot \Delta\rho^{(k)} - \Delta\mathbf{G}_{\text{obs}}\|^2$ and a plot of $E_\alpha(\Delta\rho^{(k)})$ versus $\Omega(\Delta\rho^{(k)})$ for different values of α being referred to as the trade-off curve. If possible, the error surface and the trade-off curve are plotted to adjust the initial model (step 1) and other input quantities (step 2). The operator adjusted iteration process is continued until either the requirement (22) or (23) is fulfilled (Menke 1989).

When we use eqs (18) and (21) directly, the inverse of $\mathbf{A}[\mathbf{W}_\rho^{(k-1)}]^{-1} \mathbf{A}^T + [\mathbf{W}_e^{(k-1)}]^{-1}$ and $\mathbf{A}^T \mathbf{A} + \alpha \mathbf{I}$ is calculated using the singular value decomposition (SVD) method well known in geophysical literature. A lucid introduction to the SVD and its properties is given by Cheney & Kincaid (1999). Calculating the SVD consists

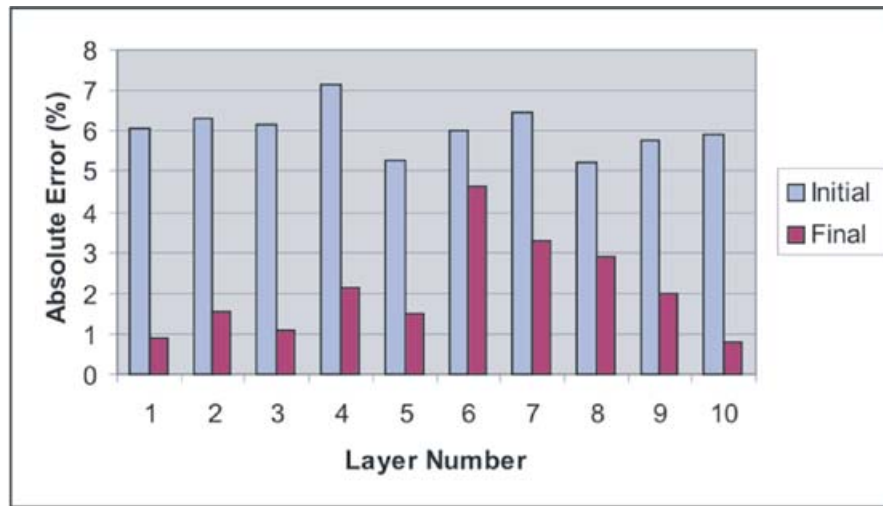


Figure 11. Absolute error of inversion in Fig. 10 as a function of grid layer (row) number: $d\rho^0$ ('Initial') versus $d\rho$ ('Final'). Layer 6 is the centre of mass.

of finding eigenvalues and eigenvectors. To exclude very small eigenvalues arising from round-off error in the computer, we apply a cut-off analysis for SVD eigenvalues, setting to zero values smaller than the cut-off value. Note that null spaces can be identified through the SVD of the matrix $\mathbf{A}$ (data kernel) in eq. (16). In addition, the local condition (14) has the advantage of leading to the sparse data kernel, thus simplifying the computations.

NUMERICAL TESTS WITH SIMPLE MODELS

Forward modelling

The simplest example is an isotropic sphere which produces the FTG field of a mass monopole described by the dimensionless analytic formula (5). Fig. 3 displays plan views of the vertical component of the gradient (4) and the components $\tilde{G}_{ij}$ plotted against horizontal distances, $0 \leq X/Z \leq 10$ and $0 \leq Y/Z \leq 10$, scaled by the depth to the monopole centre of mass ($Z = 1$ km). Fig. 3a shows that the vertical gravity gradient $\tilde{G}_{33}$ is invariant to rotation about the x_3 axis as is well known. In Fig. 3a, the minimum value is -94 per cent and the maximum value is 200 per cent of $\tilde{g}_3(0)$, the normalized vertical gravity field recorded at the distance of 1 km from mass Π . In Figs 3b–f, one can see that components G_{XX} , G_{YY} and G_{XY} are symmetric and components G_{XZ} and G_{YZ} are antisymmetric under rotation about the x_3 axis by π . Further, the component $\tilde{G}_{22}$ or G_{YY} is the component $\tilde{G}_{11}$ or G_{XX} rotated by $\pi/2$ (Figs 3b and c), the component $\tilde{G}_{13}$ or G_{XZ} is the component $\tilde{G}_{23}$ or G_{YZ} rotated by $\pi/2$ (Figs 3e and f). In Figs 3b and c, the minimum value is -100 per cent and the maximum value is 188 per cent of $\tilde{g}_3(0)$. In Fig. 3(d), the minimax value is ± 147 per cent of $\tilde{g}_3(0)$. In Figs 3(e) and (f), the minimax value is ± 150 per cent of $\tilde{g}_3(0)$. Consideration of Fig. 3 thus reveals that the joint analysis of G_{XX} , G_{ZZ} , G_{XZ} and G_{XY} far more closely defines gravity sources compared with classic vertical-component gravity data (after Bell 1998).

A second simple geometric source form is the FTG response to a rectangular prism useful for modelling geological layers. Here, eq. (6) was used to calculate 4-D anomalies of the vector $\{G_{XZ}, G_{YZ}, G_{ZZ}\}$ in the prototype Teal South (Gulf of Mexico) reservoir models A and B represented by local lateral movements of water–oil

contact (WOC) and gas–oil contact (GOC), respectively (Fig. 4). It is assumed that these movements are due to production-related changes in fluid saturation and pressure between exploration phases I (1997) and II (1999), as reported by Pennington *et al.* (2001). The depth H of the $X \times Y \times Z$ thin reservoir slab was set to be 1200 m. Other model parameters are as follows: $X = 200$ m, $Y = 1500$ m, $Z = 10$ m, $f = 0.29$, $\rho_s = 1.44$, $\rho_w = 1.0$, $\rho_o = 0.84$ and $\rho_G = 0.001$ (in g cm^{-3}). The saturation values are presented in Table 1. In both models it was assumed that 4-D local gravity effects are due to the 200 m lateral movement of WOC (model A) or GOC (model B). Fig. 5 shows plots of the 3-D FTG field which have rotational similarities with the corresponding maps in Fig. 3. The gradients G_{XZ} and G_{YZ} highlight the north–south and east–west edges of the anomaly. Note both positive and negative values of these gradients. The G_{ZZ} anomaly is always positive and emphasizes the centre of mass (Fig. 3a). As indicated in Table 1 and Fig. 5 only model B produces a surface signal whose maximum magnitude exceeds 1 Eö. Greater accuracy of FTG instrumentation would be required to map the WOC movement in model A. The sensitivity of existing GGIs is, however, appropriate for monitoring the lateral GOC movement in model B.

Regularized inversion

Fig. 6 illustrates spatial resolution in face of structural noise. The 2-D model consists of a target horizon 200×40 (m) with density change $H3$ of 40 kg m^{-3} ('signal') at the depth of 1.4 km. Two additional 200×30 (m) density anomalies $H1$ and $H2$ of 30 kg m^{-3} ('correlated noise') are situated above and below the target ($z = 1.00$ and 1.85 km, respectively). The challenge is to carry out multiparameter inversion within the volume $D = H1 \cap H2 \cap H3$ without the isolation or separation of the 'signal' and 'correlated noise' by means of a series of corrections to observed data. This aspect of field separation is often a subjective process and ultimately a matter of resolution. We still consider the linear case, eq. (11), where the source volume is divided into rectangular cells and the density change of each cell is a parameter to be determined by the present inversion method. The input synthetic FTG data are composed of components G_{XZ} and G_{ZZ} with the addition of 4 per cent random Gaussian noise simulating rms noise of FTG instruments. Based on the example in Fig. 5, we

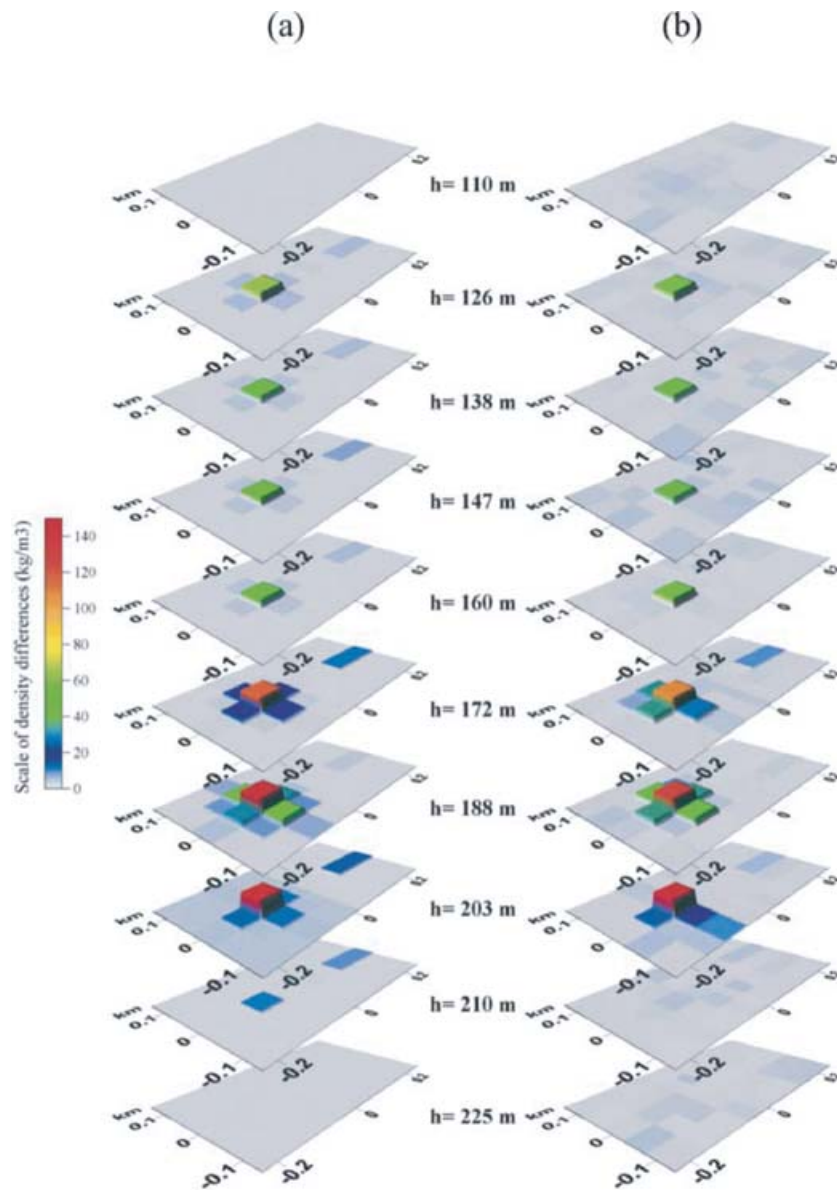


Figure 12. Model in Fig. 9: (a) exact versus (b) recovered 4-D density anomalies $\Delta\rho = \rho^{II} - \rho^I$ and $\Delta\rho_{\text{inv}}^{II} = \rho_{\text{inv}}^{II} - \rho^I$ due to steam injected into the reservoir, where ρ^I is the density distribution prior to steam injection.

believe that components G_{xz} and G_{zz} are sufficient for carrying out 2-D FTG inversions. As explained in Fig. 2, iterations start from the solution (22) with $\alpha \rightarrow 0$ and proceed by increasing the compactness of the model, choosing values of σ , α and β that are appropriate for the noise level in the data and the initial density model. At each step the matrix weights (20) and (21) are defined. The solution (18) is then constructed with the application of the spatial localization condition (14). Approximating the prism by 2-D rectangular cells was sufficient to reduce the CPU time t_0 to a few seconds with the computer available (Sun Blade 150). After four iterations with $\alpha = 0.1$, $\sigma^2 = 0.04$, $\delta = 0.2$ Eö and $\beta = 5 \text{ kg m}^{-3}$, the relative errors become 3–6 per cent ($1\text{--}2 \text{ kg m}^{-3}$), 12–20 per cent ($5\text{--}7 \text{ kg m}^{-3}$) and 3–16 per cent ($1\text{--}5 \text{ kg m}^{-3}$) for the top, middle and bottom anomalies, respectively (Fig. 6). It is seen that all density anomalies are adequately retrieved without *ad hoc* separation of ‘signal’ and ‘correlated noise’.

CASE EXAMPLES WITH COMPLEX MODELS

Heavy oil reservoir monitoring

In the first case numerical example involved real data, we simulate density changes associated with the lateral movement of injected steam in Athabasca tar sands (Paulsson *et al.* 1994). The site, situated about 60 km northeast of Fort McMurray, Alberta, is part of a heated-steam injection project, HASDrive, to enhance heavy oil secondary recovery from an unconsolidated sand reservoir located between 200 and 260 m below the surface. Our objective is to examine the feasibility of monitoring HASDrive enhanced oil recovery with FTG methods.

To create a 4-D reservoir model, we specify simplified but realistic temperature distributions after 6 and 12 months of continuous

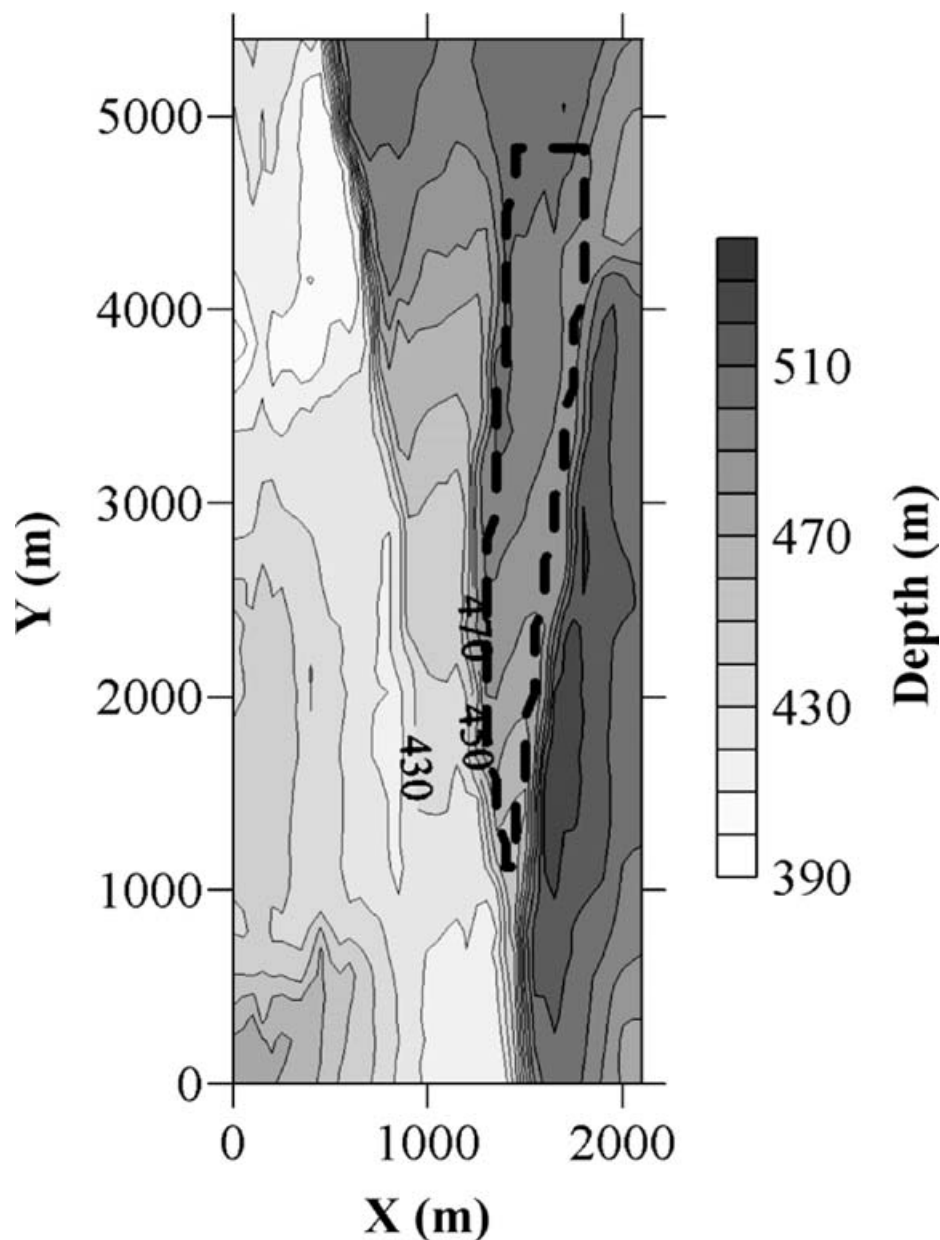


Figure 13. Top target horizon map derived from eq. (28) with $h_1 = 0.4$ km. This corresponds to the geometry of the Teal South producing reservoir (Pennington *et al.* 2001) rescaled to the depth range from about 0.4 to 0.5 km below MSL.

steam injection, referred to as the phases I and II (Fig. 7a). The distributions are consistent with actual temperature monitoring data (Lowe & Donovan 1989). In particular, the triangular distribution is likely due to the resistance of the cold viscous oil rim in the fractures preventing the steam from displacing the oil downwards. The feasibility of monitoring heavy oil reservoirs with seismic methods is based on the dramatic decrease in the P -wave velocity with temperature (Paulsson *et al.* 1994). Therefore, it is sensible to begin with velocity distributions using the pressure-volume-temperature relationship of Batzle & Wang (1992). Fig. 7b shows the temperature-to-velocity conversion. The subsequent velocity-to-density conversion with Gardner's formula (e.g. Barnola & White 2001) yields the 4-D density forward model in Fig. 7(c) used to generate synthetic 2-D FTG data.

The data inversion steps to calculate the model temperature distribution are as follows:

- (1) A trial model consisting of $30 \times 20 \times 20$ m blocks is constructed. The initial density anomaly (iteration 0) is shown in Fig. 7c (phase I). The forward solution (16) due to all blocks is calculated at each observation point. There are a total of 60 data points.
- (2) The residuals between the observed and calculated values of the gradient are found at each observation point.
- (3) The iterative solution (18) is constructed to solve the problem (15) for the optimum α value. The density contrast within blocks is kept constant, and the aperture is not extended, since edge effects are negligible.

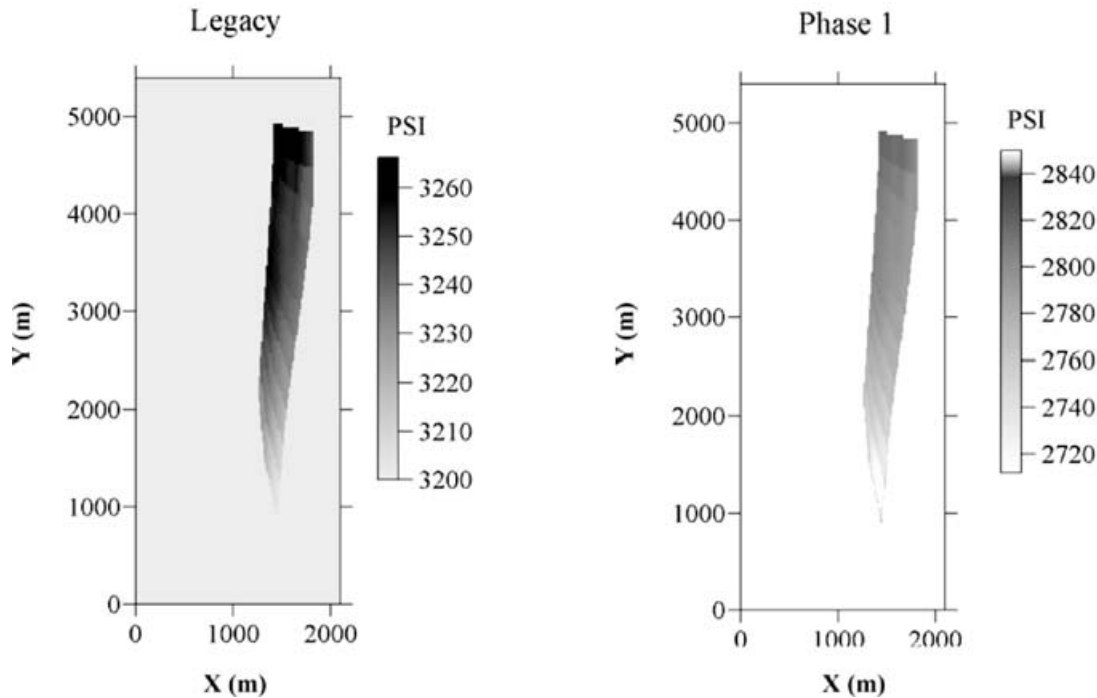


Figure 14. Output of reservoir simulation (Christie *et al.* 2002) for the target area in Fig. 13 enclosed by the dashed line: (a) pressure and (b) saturation maps before production (legacy) and during phase I of reservoir monitoring. The reservoir was initially overpressured at 3096 psi. The model is set up on an 11×11 grid.

These three steps are repeated until the user is satisfied that convergence is met, as illustrated in Fig. 2. After the iterative process, the final model will be constructed by adding to the reference density values and additional density contrast constrained by the compactness condition of Last & Kubik (1983). Location of the optimum α value is crucial in uncertainty estimation issues. Typically, for low α value, a better fit is obtained. When we use small α , the misfit decreases as the model improves with successive iterations.

In the present example, we illustrate the inversion of synthetic G_{XZ} and G_{ZZ} data contaminated by the 4 per cent random noise added at observation points. Fig. 8 shows the results of 15 iterations with the same α , σ , δ and β values as in the previous example. Computations took about $t_0 \approx 2$ min on a Sun Blade 150. The recovered temperature is shown in Fig. 8a, which compares well with the true model after spatial B-spline interpolation (Fig. 7a, phase II). In Fig. 8b, model errors are within the range $[-0.002, 0.007]$ g cm $^{-3}$ (i.e. the error of recovered density changes) that is sufficiently small with respect to the true 4-D density anomaly $\Delta\rho_{\text{inv}}^{\text{II}} \in [0.025, 0.075]$ g cm $^{-3}$ due to temperature gradient (Fig. 7c, II-I). Interestingly, this error distribution correlates with the true temperature changes in Fig. 7a (II-I).

Thus, we conclude that the FTG inversion has performed well in this case. When saturation changes as a high-temperature steam zone invades, density changes give a sufficiently detailed map of the heat movement.

Thermal reservoir simulation

To investigate potential 3-D applications of FTG monitoring to the steam pilot, a fifth reservoir model was created using the thermal multiblock reservoir simulator STARS 2001.10 of the Computer Modelling Group Ltd. (Calgary, Canada). The $417 \times 234 \times 120$ (m) model was represented by a 3-D irregular grid. The grid size varies from $23 \times 40 \times 5$ (m) to $100 \times 74 \times 20$ (m). For reservoir modelling,

two injector wells 1 and 2 and one producing well between injectors 1 and 2 were specified. The model was created for the analysis of the areal advance of the steam zone due to injector 1 only (Fig. 9b), given the porosity map in Fig. 9(a) and the constant initial temperature 20 °C.

The temperature distribution in Fig. 9b was converted to a density model ρ^{II} as in the previous example. The source volume D consists of 10 layers, each of which consists of 35 (5×7)–40 (5×8) blocks ($M = 400$). Each source block is 200×400 m wide. Thickness of each layer varies from 7 to 16 m. High-porosity layers are thin so they can characterize the reservoir geometry in some detail.

In Fig. 10(a), the initial density change is given by

$$\Delta\rho^{(0)} = 0.8\Delta\rho + \delta\rho, \quad (24)$$

where $\delta\rho = \xi \max(\rho^I)/4$ and ξ is a random variable in the range $[0, 1]$. This yields the FTG response

$$\Delta G^0 = \mathbf{A} \cdot \Delta\rho^0 = 0.8\Delta\mathbf{G} + \delta\mathbf{G}, \quad (25)$$

where the exact FTG response $\Delta\mathbf{G}$ is determined from (16) and $\delta\mathbf{G} = \mathbf{A} \cdot \delta\rho$ is the artificial field error caused by model uncertainties at gridpoints. Here, our objective is to test the performance of the inversion scheme in Fig. 2 with the erroneous starting model (24) that may simulate the initial least-squares solution (21).

The observation points are distributed as a square grid of 1×1 km with a constant 0.01 km interval. The total number of observation points is $N = 10^4$. The inverse problem is, therefore, grossly over-determined. The calculated FTG changes ΔG_{XZ} , ΔG_{YZ} and ΔG_{ZZ} were inverted by the method proposed, with the initial density (24). The iterative process took five iterations to converge using the same α , σ and δ values as in the previous example. The total CPU cost t_0 on a Sun Blade 150 was about 50 min. It has been found that the value $\beta = 30$ kg m $^{-3}$ yields good results in trying to reduce the ambiguity of the estimated density distributions $\Delta\rho \in [0, 150]$ (in

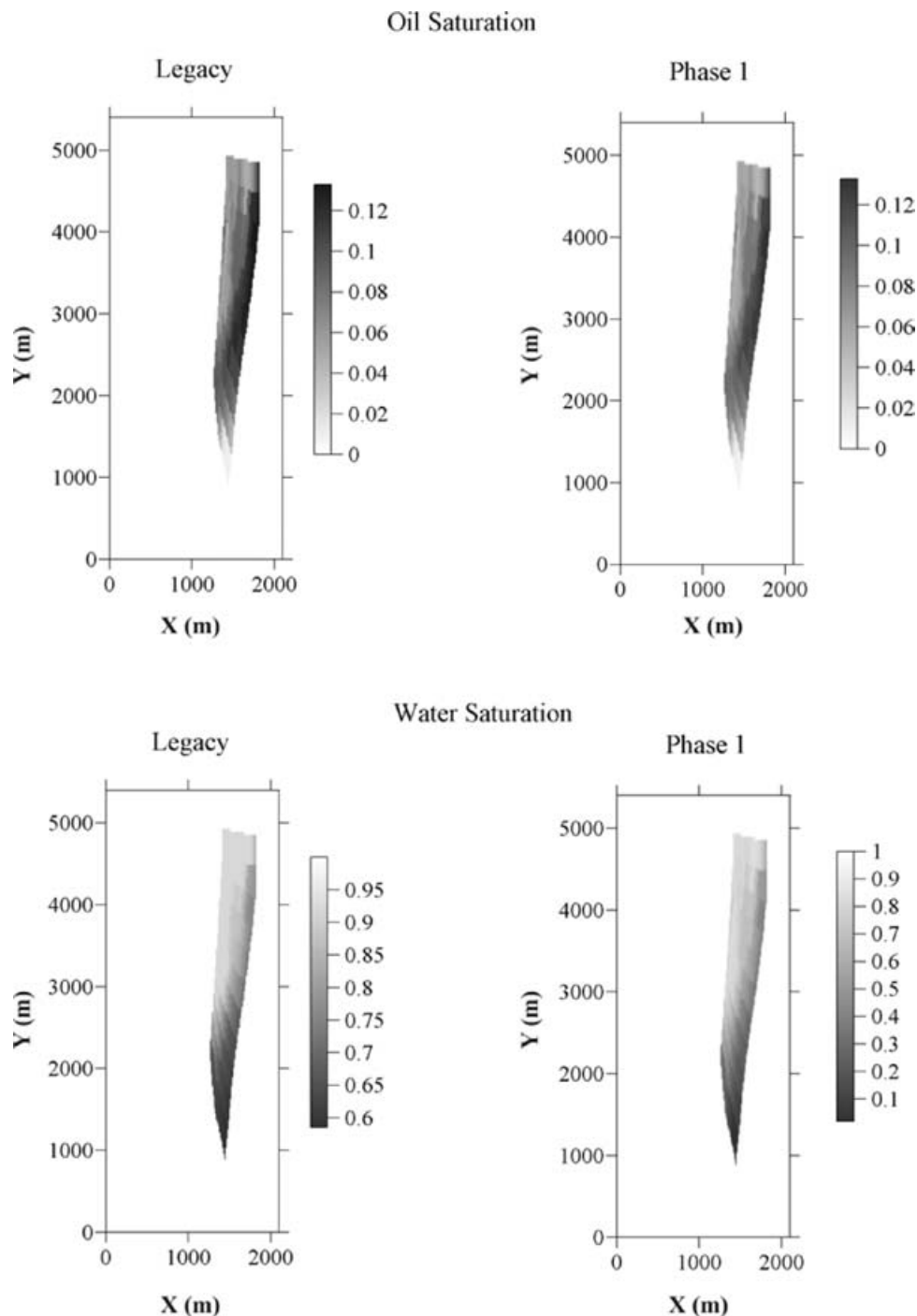


Figure 14. (Continued.)

kg m^{-3}). Absolute differences between the data calculated for the true and recovered models are less than 0.09 Eö (18 per cent of G_{YZ}^H) and 0.02 Eö (1.8 per cent of G_{ZZ}^H) for off-diagonal and vertical FTG components, respectively.

According to Fig. 9(b), the steam chamber around the well axis is subdivided into three zones: the heated zone, transition zone and cold zone. Figs 10(b) and 11 indicate that areas most affected by the uncertainties in the *a priori* information are mainly located directly at the injection point where $\Delta\rho \rightarrow \max$ (layer 6 in Fig. 11) and towards the grid sides where the compactness

condition is violated. In Figs 10(b) and 11, the maximum, arithmetic mean and standard deviation (*SD*) values of model residuals $d\rho^0$ and $d\rho$ are $\max(d\rho^0, d\rho) \approx (20.0, 13.2)$, $\text{mean}(d\rho^0, d\rho) \approx (9.4, 0.28)$ and $\text{SD}(d\rho^0, d\rho) \approx (6.0, 2.4)$ (in kg m^{-3}). From these numerical details and a close inspection of Figs 10–12, it is clear that the inversion is stable and provides an improved estimate of density changes, with an average error of less than 0.3 kg m^{-3} and a largest error of only 13.2 kg m^{-3} . Despite a partially incorrect initial guess, it achieves a solution sufficiently close to the exact density anomaly (*cf.* Figs 12a and b).

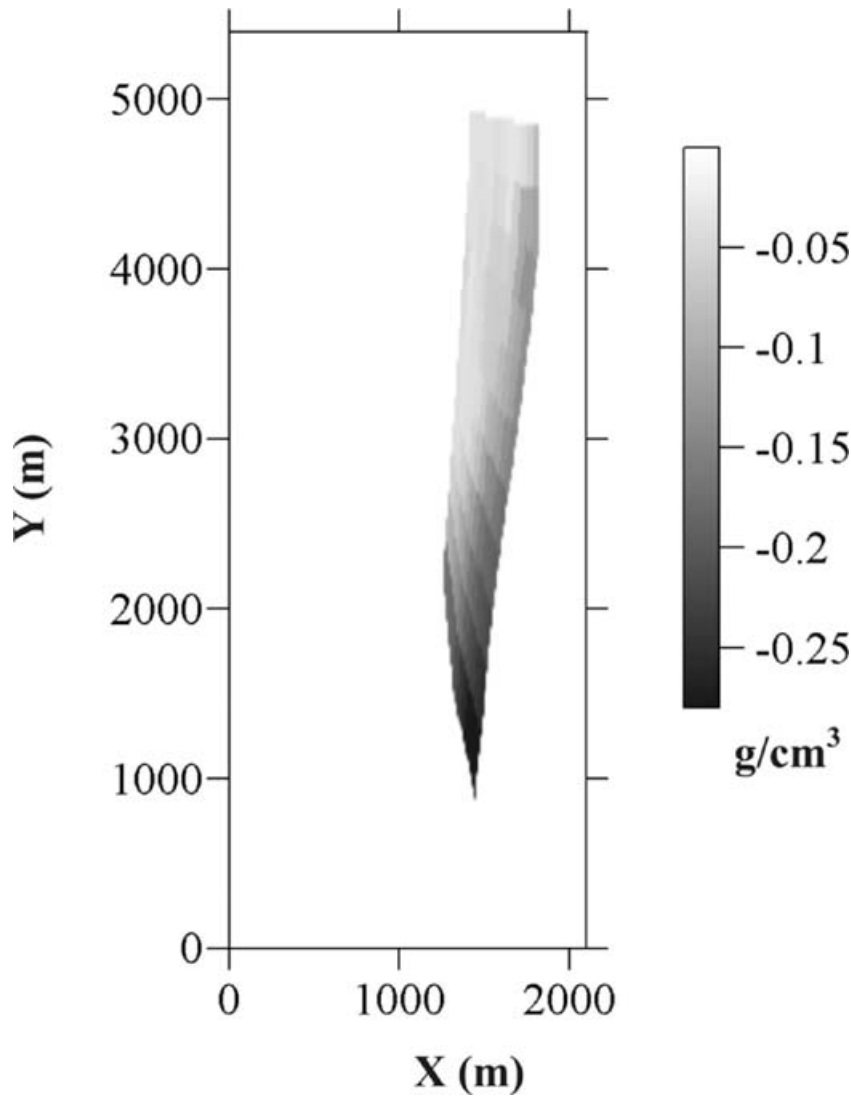


Figure 15. Model in Figs 13 and 14: density difference between reference (legacy) and monitor (phase I) density maps due to pressure and saturation changes in Fig. 14. The target domain D is within a rectangular area of 1×4 km.

Teal South reservoir

Next, we turn our attention to the Teal South (Gulf of Mexico) 4-D test study of a low-cost permanent reservoir monitoring system (Ebrom *et al.* 1998). This study has provided time-lapse 3-D seismic data sets prior to production ('legacy' data acquired in 1995) and at two times during production (phases I and II). The focus is on the so-called '4500-ft' sand reservoir, which was near the bubble point at discovery. Pennington *et al.* (2001) summarized the production history of this reservoir.

The depth-converted top-structure map $z(x, y)$ of the 4500-ft sand may be represented as

$$z(x, y) = z_0 + \delta z(x, y), \quad (26)$$

where $z_0 = 1.37$ km is the datum level below MSL and $\delta z(x, y)$ is the horizon depth in km measured from this level (courtesy of ERCH). In our model, the 3-D target domain D is confined by the following boundaries

$$z_1(x, y) = h_1 + \delta z(x, y) \quad (27)$$

and

$$z_2(x, y) = h_2 + \delta z(x, y), \quad (28)$$

where $h_2 = h_1 + \Delta h$, h_1 is the arbitrary datum level, $\Delta h = 0.01$ km is the average reservoir thickness, and the residual depth $\delta z(x, y)$ is determined from (26). The top of the reservoir after depth rescaling is mapped in Fig. 13. The dashed line encloses the target area sealed by faults.

Christie *et al.* (2002) used a 3-D reservoir simulator to generate pressure and saturation maps for production history matching. Figs 14(a) and (b) show the pore pressure and oil-water saturation distributions for the target zone in Fig. 13 (phase I only). Both pressure and saturation changes in Fig. 14 were transformed into corresponding density changes in Fig. 15 using eqs (12) and (13) as well as empirical relations between densities and pore pressure (Batzle & Wang 1992) with the following input parameters: $f = 0.29$, $\rho_s = 2.03$ g cm⁻³, $\rho_w = 0.99$ g cm⁻³, $\rho_o = 1.02$ g cm⁻³ (API = 7).

In Fig. 16, we calculate the 4-D FTG response to the model (27) with $h_1 = 0.4 - 1.0$ km. The calculated data consist of the two diagonal components, ΔG_{ZZ} and $\Delta \tilde{G}_{XX}$, and the two off-diagonal

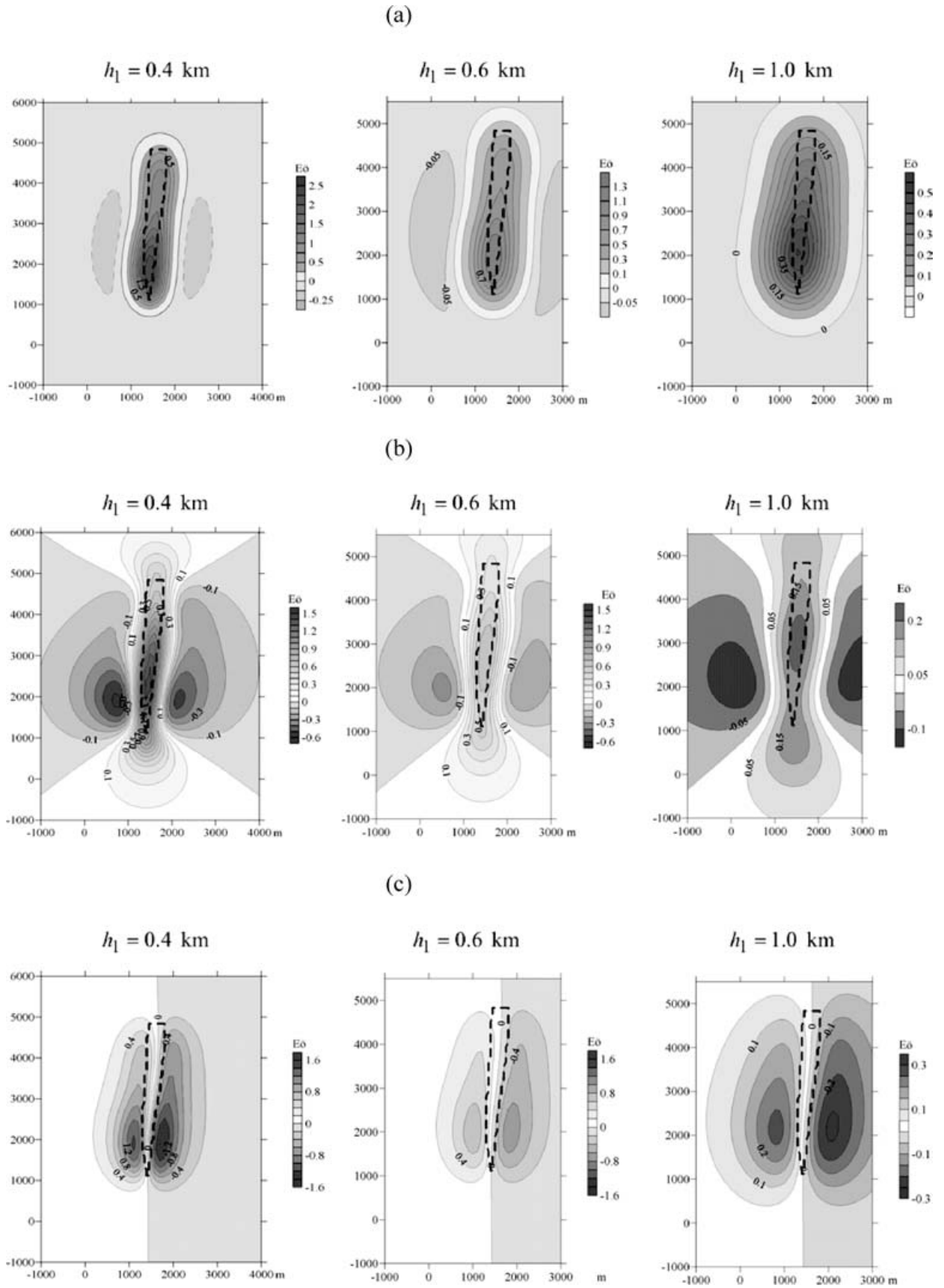


Figure 16. 4-D noise-free FTG field with $h_1 = 0.4$ – 1.0 km (see model in Figs 13 and 14): (a) ΔG_{zz} , (b) $\Delta \tilde{G}_{xx}$, (c) ΔG_{xz} and (d) ΔG_{yz} components of the gradient tensor (16).

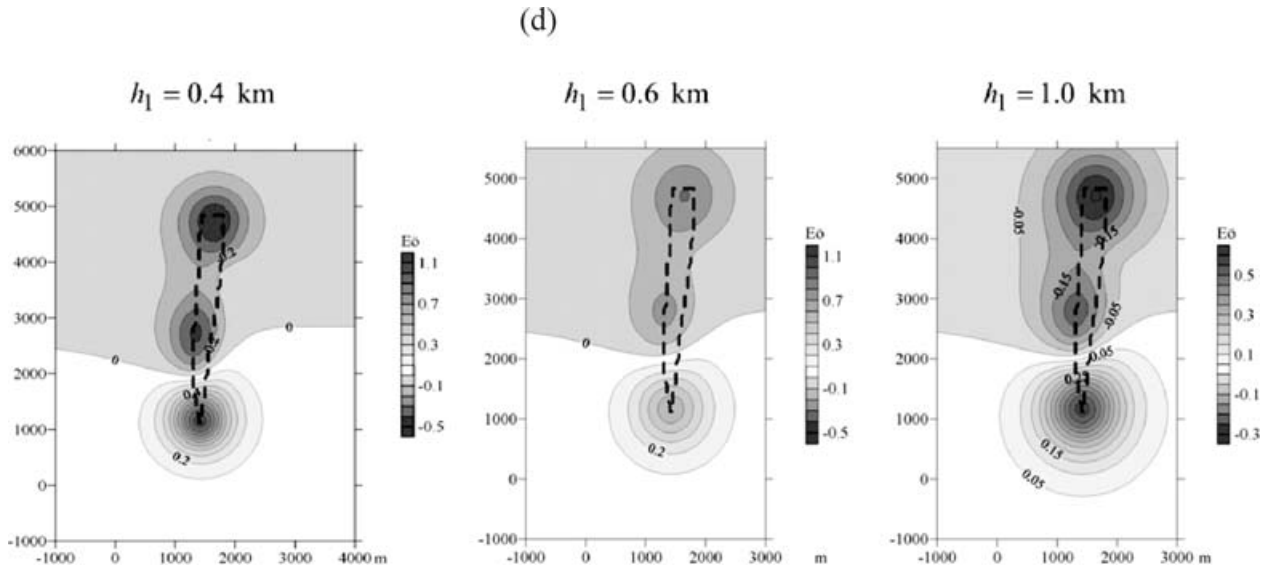


Figure 16. (Continued.)

components, ΔG_{XZ} and ΔG_{YZ} , of the gradient tensor (16). The observation points are distributed as a rectangular grid of 5×7 km with a constant 0.1 km interval. For this initial experiment, we assume the data are free from any errors associated with measurements and model parameters. Later, we discuss the effects of noisy data.

As expected, the vertical gradient component produces the most useful positive lobe that picks out the density anomaly very clearly (Fig. 16a). Maps of the $\Delta \tilde{G}_{XX}$ and ΔG_{XZ} gradient components are somewhat more complex: note how the ‘dipolar’ and ‘minus-plus-minus’ signals produce a sharper response to the fault lines (*cf.* Figs 16b and c). Additionally, the ΔG_{YZ} component (Fig. 16d) shows distinctive circular anomalies associated with corners. While it is true that maps generated by the non-vertical components of the gravity gradient tensor have relatively weak amplitudes that may be difficult to interpret, it is also evident that their use in inversion may be valuable in discriminating local lateral density variations around sealing or leaking faults, as illustrated in Fig. 16. Unfortunately, the attenuation of gravity gradient components with distance from source as shown in Fig. 16 indicates that FTG inversion may be insufficiently accurate at $h_1 \geq 0.6$ km, where the data are likely to be too highly attenuated to provide useful lateral resolution of inversion results.

For the rest of this section, we present a detailed feasibility analysis of our inversion scheme designed to be robust to local minima and to solve problems without a close initial guess. As a first realistic inversion test we estimate the density anomaly in Fig. 15 from the noisy FTG data in Fig. 17 (which may be compared against the corresponding noise-free data in Fig. 16 to see the impact of random noise) within the domain D shown in Fig. 13 ($h_1 = 0.4$ km).

Fig. 18 displays the results of incorporating somewhat arbitrary starting models with or without random fluctuations $\delta\rho$ assigned to the blocks (‘geological noise’). In Fig. 18, one can compute the goodness of the iterative solution as the rms misfit between FTG responses to the true and recovered models (curve 1), the rms error of the recovered density anomaly $\rho^{II} - \rho^I$ (curve 2) or the rms misfit between density changes $\Delta\rho^{(k)}$ and $\Delta\rho^{(k-1)}$ (curve 3) versus iteration number k . The inversions were performed with $\sigma^2 = 0.1$, $M = 121$ and $N = 3500$. That is, the iterative algorithm (Fig. 2) was applied for a series of trial values of α and the above rms misfits

(curves 1–3) for each α were evaluated. Here, we fixed the values of other parameters at their optimal values while iterating for α . Even though the accuracy of the generalized inverse (18) is strongly dependent upon the optimal value of α due to the ill-posed nature of the problem, the results (Fig. 18) indicate that the iterative process converges as $\alpha \rightarrow \alpha_0$ and $k \rightarrow k_0$. The approximate α_0 and k_0 values may be found by visual inspection of curves 1–3 in Fig. 18. In Figs 18(a–c), the α_0 values are on the orders of 10^{-2} ($k_0 \approx 12$ and $t_0 \approx 1$ hr), 10^{-1} ($k_0 \approx 3$ and $t_0 \approx 15$ min) and 10^{-4} ($k_0 \approx 6$ and $t_0 \approx 30$ min), respectively.

Once the appropriate α_0 and β values have been estimated, greater accuracy in the initial guess model parameters is not required, as demonstrated by the example in Fig. 18. Moreover, the inverse solution also shows little effect of the noise in the FTG data and the total number of observation points provided that the problem is overdetermined. In general, the more data points the higher the accuracy if the measurements fully describe the target. However, Fig. 19 shows that the accuracy of the solution, despite being unique and stable, decreases as $h_1 \rightarrow \infty$. In Fig. 19, four-percent random noise is added to the synthetic data prior to inversion. As in Fig. 18(a), the inversion is started with an initial model of uniform density changes throughout the whole domain D . The progress of the inversion is shown in Figs 19a–c, which gives the maximum difference in true and calculated density changes of less than 3, 12 and 17 per cent corresponding to the average depth values of 0.4, 0.6 and 1.0 km, respectively. This means that the solutions are highly dependent on the depth h_1 that is not a desirable feature. The reason is that the sensitivity of the FTG data decays as inverse distance cubed. Hence, an additional depth weighting of the data designed to counteract the decay of sensitivities is required, as suggested by Li (2001).

Finally, we consider again the example with noisy data in Fig. 17 produced by the model in Fig. 13 with $h_1 = 0.4$ km. To examine the robustness of the algorithm, the geological noise $\delta\rho$ is also added to the initial guess $\Delta\rho^{(0)}$. The inversion results will be somewhat sensitive to the erroneous starting model because, as pointed out before, the choice of parameter α is critical to obtain meaningful results after a reasonable number of iterations (Fig. 19). Figs 20(a) and (b) show a comparison between the estimated density changes, which were

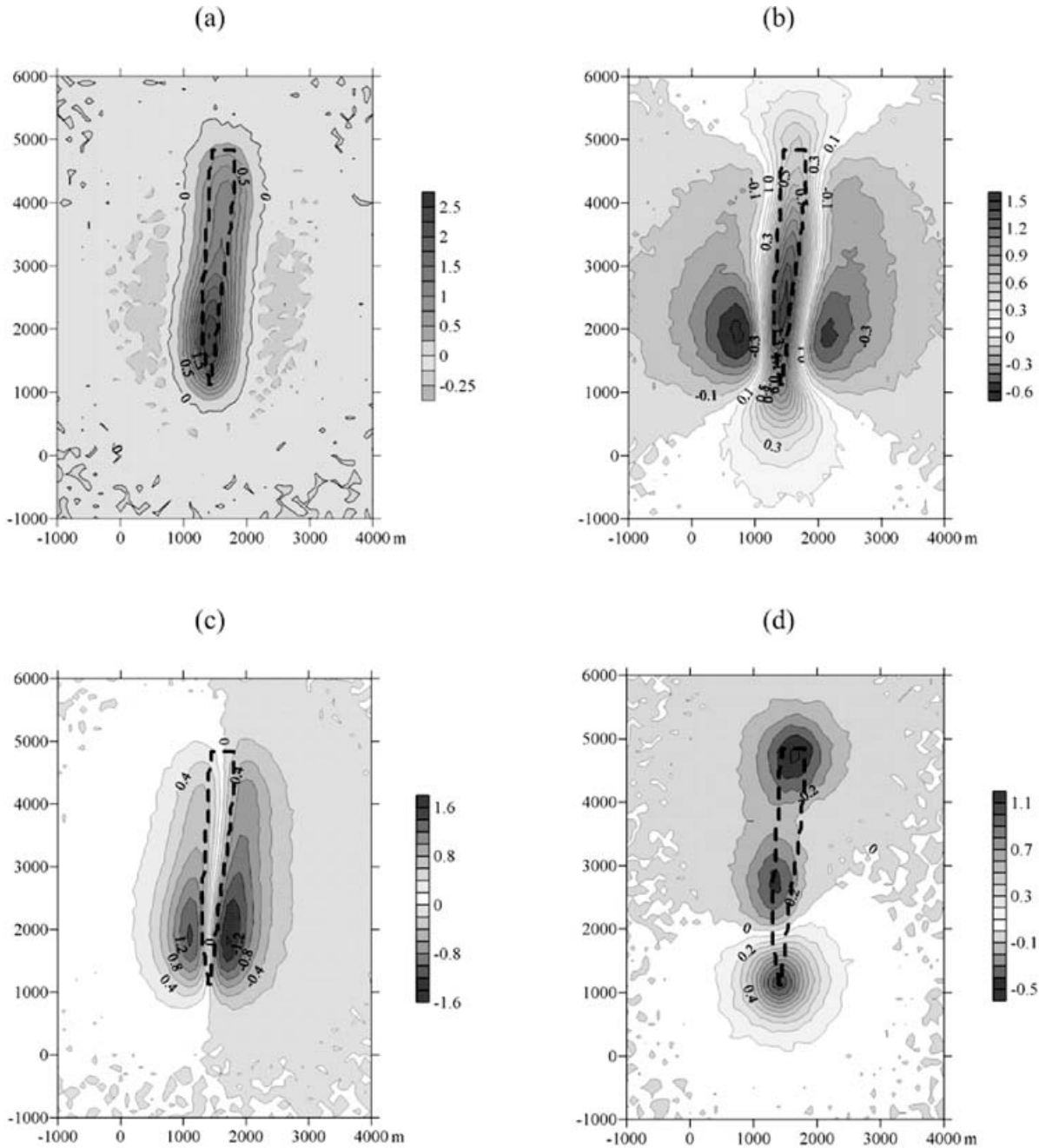


Figure 17. 4-D FTG response to the density anomaly in Fig. 15 ($h_1 = 0.4$ km) with 10 per cent random noise added at observation points: (a) ΔG_{ZZ} , (b) $\Delta \tilde{G}_{XX}$, (c) ΔG_{XZ} and (d) ΔG_{YZ} components of the gradient tensor (16).

predicted by the inverse solution (18) with $\Delta\rho^{(0)} = \Delta\bar{\rho} + \delta\rho$ (uniform density changes with geological noise) and $\Delta\rho^{(0)} = \Delta\rho/2 + \delta\rho$ (50 per cent model error and geological noise), where $\Delta\rho$ is the exact density change and $\Delta\bar{\rho}$ is the average density change. One can see in Fig. 20 how the difference between the calculated and true density changes decreases with increasing number of iterations regardless of the initial guess. The addition of observation errors and geological noise leads to an earlier stop of the iteration process, although at a larger error value. After seven iterations, the residual rms error is negligible, and both inversion results in Fig. 20 fit the true model equally well.

DISCUSSION

In principle, quantitative information derived from independent components of the gradient tensor (1), its invariants or gradient-curvature attributes can be utilized to reduce uncertainties of dynamic reservoir monitoring if the reservoir is shallow enough and the density contrasts are large enough. The basic concept of time-lapse or 4-D FTG methodology may be understood as follows: since different FTG components are measuring a common underlying '4-D signal' during real-time reservoir monitoring, we should be able to use the inherent compact (spatially localized) relationships among the

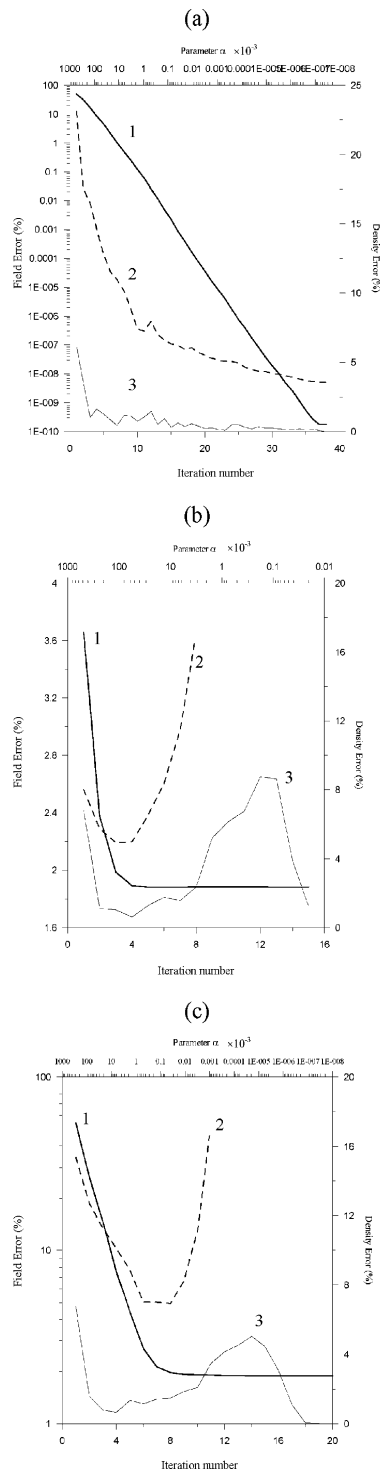


Figure 18. Regularized inversion of noisy 4-D FTG data in Fig. 17 ($h_1 = 0.4$ km): percentages of maximum FTG field error (curve 1), average error of the recovered density anomaly $\rho^{II} - \rho^I$ (curve 2) and misfit between density changes $\Delta\rho^{(k)}$ and $\Delta\rho^{(k-1)}$ (curve 3) versus iteration number k for various values of the regularization parameter $\alpha \in (0, 1]$. Density solution sensitivity to uncertainty in the *a priori* information and/or error of the initial least-squares solution (21) considered as geological noise added at gridpoints: (a) $\Delta\rho^{(0)} = \Delta\bar{\rho}$ ($\beta = 0.1$ g cm $^{-3}$), (b) $\Delta\rho^{(0)} = \delta\rho$ ($\beta = 0.25$ g cm $^{-3}$) and (c) $\Delta\rho^{(0)} = \Delta\bar{\rho}/2 + \delta\rho$ ($\beta = 0.12$ g cm $^{-3}$), where $\Delta\bar{\rho}$ is the exact density change, $\Delta\bar{\rho} = -0.15$ g cm $^{-3}$ is the average density change (8.67 per cent of the average reservoir density of 1.73 g cm $^{-3}$ prior to production), and $\delta\rho$ is the artificial random model error defined in (24).

components, based on the forward solution (11), to extract this signal in the presence of distributed ‘4-D noise’ that does not obey these relationships.

Following the recent success of a ground-based FTG acquisition and processing methods (C. Murphy 2002, personal communication), we have done a feasibility risk assessment and identified potential applications of FTG to 4-D studies. In doing so, we have looked briefly at a variety of important problems such as the limited sensitivity of GGI instruments, optimal acquisition parameters, appropriate depth range, bandwidth and resolution issues, various types and levels of noise, the inability to define an appropriate starting model to guide an inversion process, possible pre-processing errors and numerical errors of modelling and inversion operators.

From the point of view of classical inverse theory (e.g. Menke 1989), any inverse FTG solution that estimates density contrasts and their spatial locations by minimizing differences between observed and predicted field quantities without *ad hoc* constraints is ambiguous, unstable and error prone. We have attempted to solve this problem based on regularization algorithms, which permit applying a semi-automatic selection of the proper solution using *a priori* information. We believe regularization corresponds to physical conditions in reservoir production.

Unfortunately, most global regularization methods (Meju 1994) require that the initial guess must be close enough to the correct answer and that the input data values must contain only small random errors. To handle various noise properties and minimize the dependency on the initial guess, we formulate the local problem of estimating lateral density changes within the given target domains under the assumption that information about spatial locations of gravity sources is available prior to inversion.

An additional important constraint imposes a minimum mass solution that guarantees compactness and stability (Last & Kubik 1983). Starting from the initial model (iteration 0), the method iteratively concentrates the mass into smaller volumes by minimizing the number of blocks with non-null density changes. In other words, the method estimates compact sources concentrated about a geometric element (injection point, well axis or reservoir horizon) whose spatial position is known. If the density of a given block violates the boundaries of a prescribed interval, it is dumped by means of the functional (17). Portniaguine & Zhdanov (1999) have shown that this is a stabilizer in Tikhonov’s regularization method, so the solution is stable.

The above two assumptions are referred to as strong constraints because the solution will be stable but biased towards a wrong result if *a priori* information is not precise (Barbosa & Silva 1994). Fortunately, in most real data 4-D situations, the reservoir geometry is well enough known that neither the first nor the second condition is a problem.

Imposing compactness can be likened to assuming that reservoir flow units are defined by discrete geological formations characterized by block-like geometry disturbed by relatively small fluctuations about a robust mean (note that typical deviations from the RMS in block properties may be on the order of 20–50 per cent of the mean). Hence, our regularization may be regarded as an inversion tactic to try to guide the inversion process by locating its ‘signal’ in as compact a volume as possible and neglecting the mismatches distributed outside the target domains. While experience has shown that oil field production can effect earth volume surrounding production zones, clearly the compact production zones should be the target of data inversions.

In summary, the present 4-D FTG inversion scheme may work well in practice if the following recommendations are considered

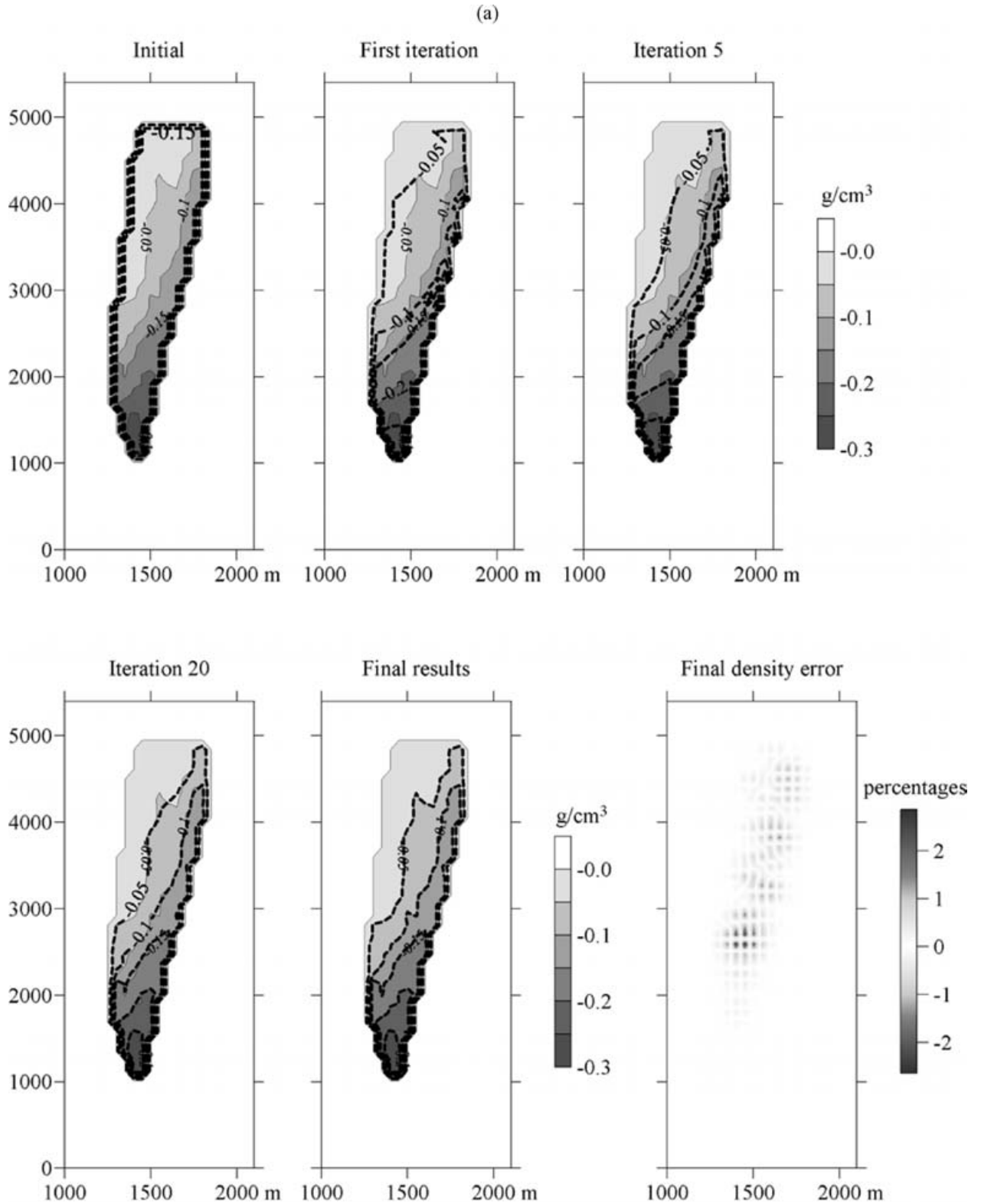


Figure 19. Model in Fig. 13 with (a) $h_1 = 0.4$, (b) $h_1 = 0.6$ and (c) $h_1 = 1.0$ (in km). Regularized inversion of FTG data in Fig. 16, with the addition of 4 per cent random noise at observation points. Final results were obtained after 30 iterations ($t_0 \approx 2.5$ h). Dashed contour lines are recovered density anomalies $\Delta\rho^{(k)}$ ($k = 0, 1, 2, \dots$), where $\Delta\rho^{(0)} = \Delta\bar{\rho} = -0.15 \text{ g cm}^{-3}$ ($M = 121$, $N = 3500$, $\sigma^2 = 0.04$, $\beta = 0.1 \text{ g cm}^{-3}$ and $\alpha_0 = 10^{-2}$). For comparison, exact density changes $\Delta\rho$ (solid contour lines) are shown.

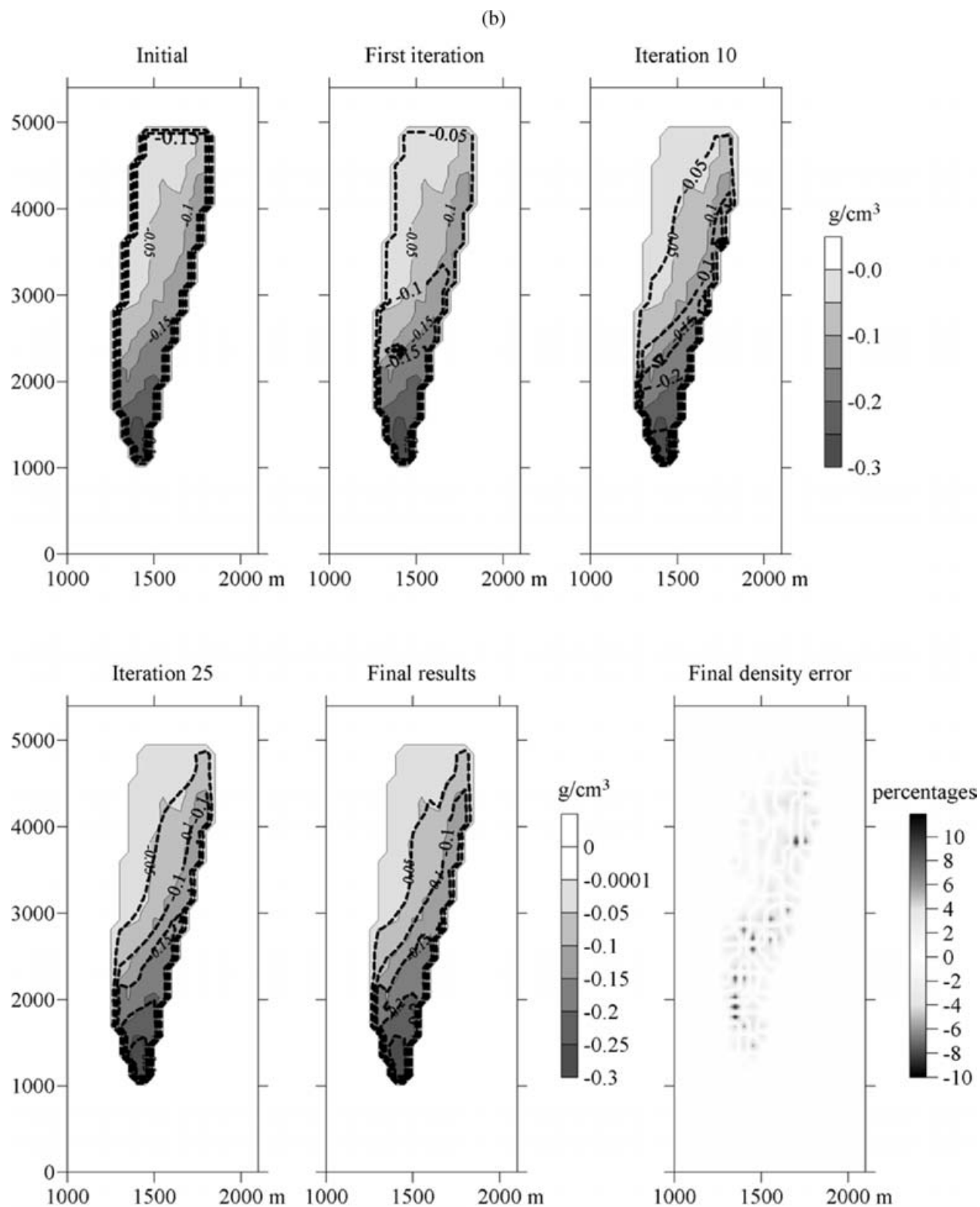


Figure 19. (Continued.)

1. It is preferable to use high-precision FTG field measurements (either independent gradient components or tensor invariants) with RMS errors less than 1 Eö and resolutions of tens of metres than the single vertical gravity gradient or geoid height measurements converted to gravity.

2. Extreme care should be exercised when choosing an optimal signal enhancement sequence during the data pre-processing stage. One problem with conventional methods (spectral low-pass approximations, F-K analysis, SVD-type filtering, etc.) is that they may destroy the FTG response to weak 4-D density changes, producing

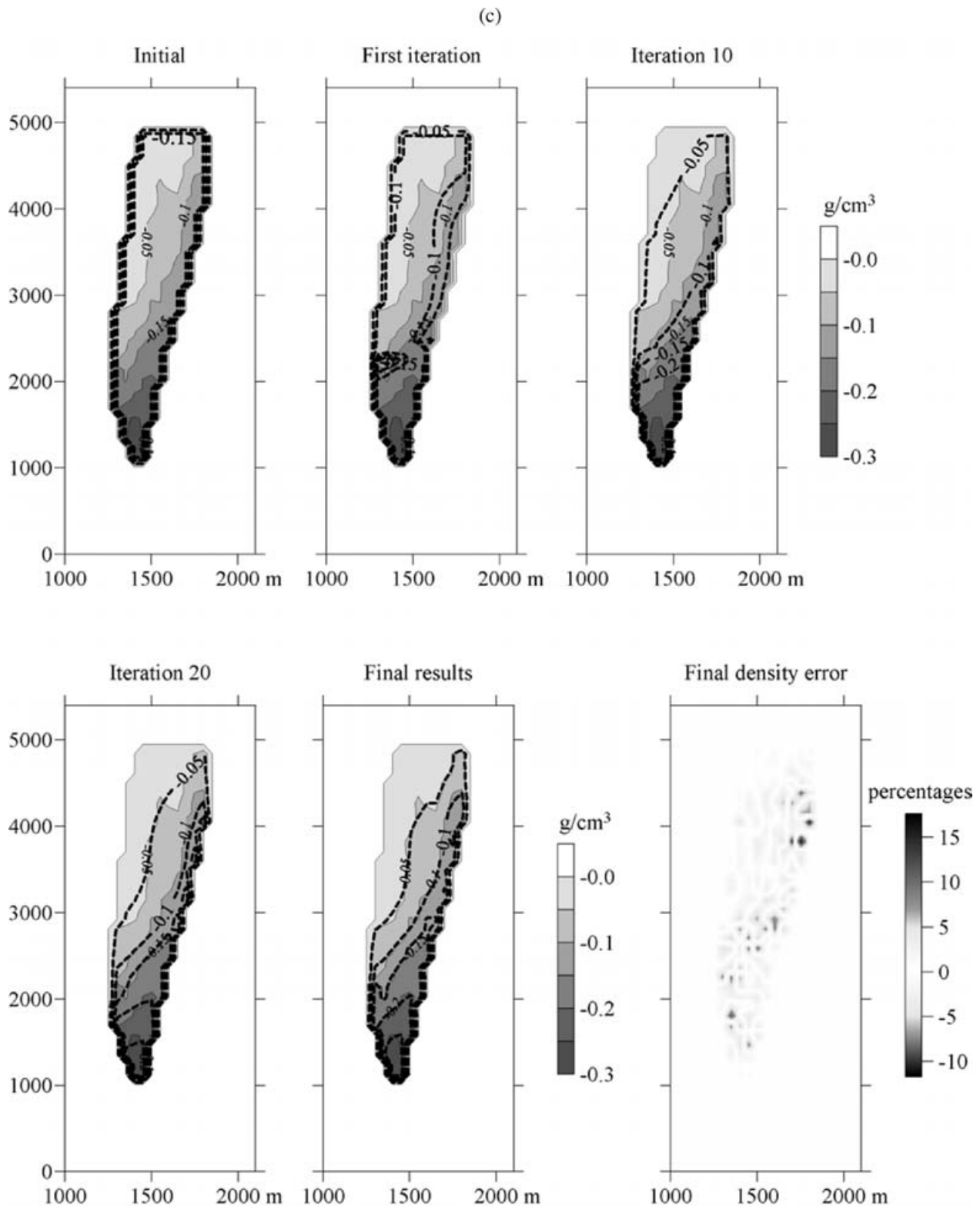


Figure 19. (Continued.)

residuals contaminated by false anomalies. Our treatment of noise in the data does not require special pre-processing as it fits naturally into the regularization scheme.

3. Typically, we consider overdetermined problems that have much more data than unknowns. The lateral extent of the model

is constrained by choosing blocks that lie within the target volume D . In a situation with an adequate number of observation points, and with a local model grid that has enough detail, it is possible to resolve a trade-off between resolution and stability of inversion. Here, we take the advantage gained from an extensive sampling of

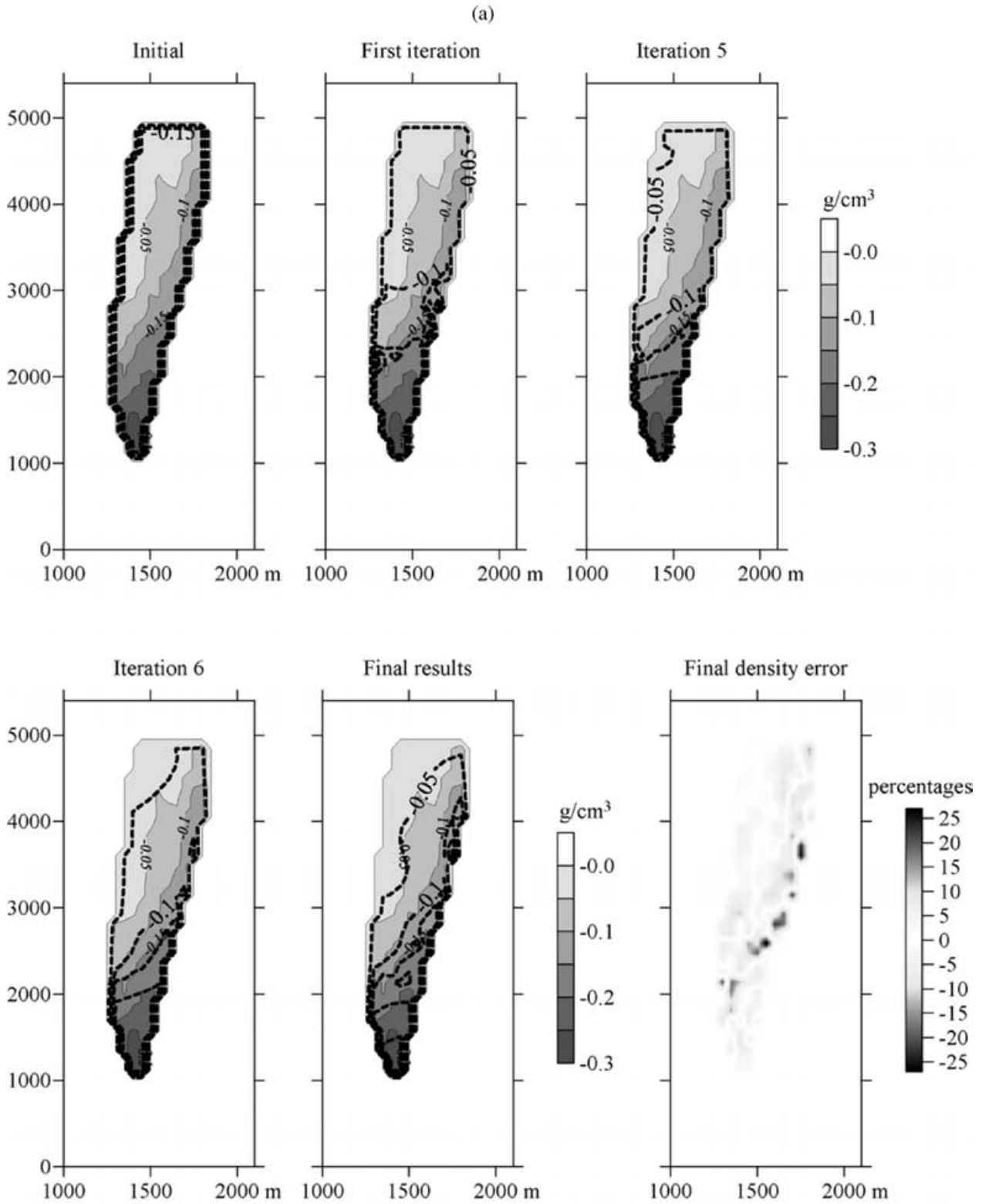


Figure 20. Regularized inversion of noisy 4-D FTG data in Fig. 17 ($M = 121$, $N = 3500$, $\sigma^2 = 0.1$ and $h_1 = 0.4$ km): (a) $\Delta\rho^{(0)} = \Delta\bar{\rho} + \delta\rho$ ($\alpha_0 = 10^{-2}$ and $\beta = 0.1 \text{ g cm}^{-3}$) and (b) $\Delta\rho^{(0)} = \Delta\rho/2 + \delta\rho$ ($\alpha_0 = 10^{-4}$ and $\beta = 0.12 \text{ g cm}^{-3}$), where $\Delta\bar{\rho} = -0.15 \text{ g cm}^{-3}$ and $\delta\rho$ is the artificial random model error defined in (24). Dashed contour lines are recovered density anomalies $\Delta\rho^{(k)}$ ($k = 0, 1, 2, \dots$). For comparison, exact density changes $\Delta\rho$ (solid contour lines) are shown.

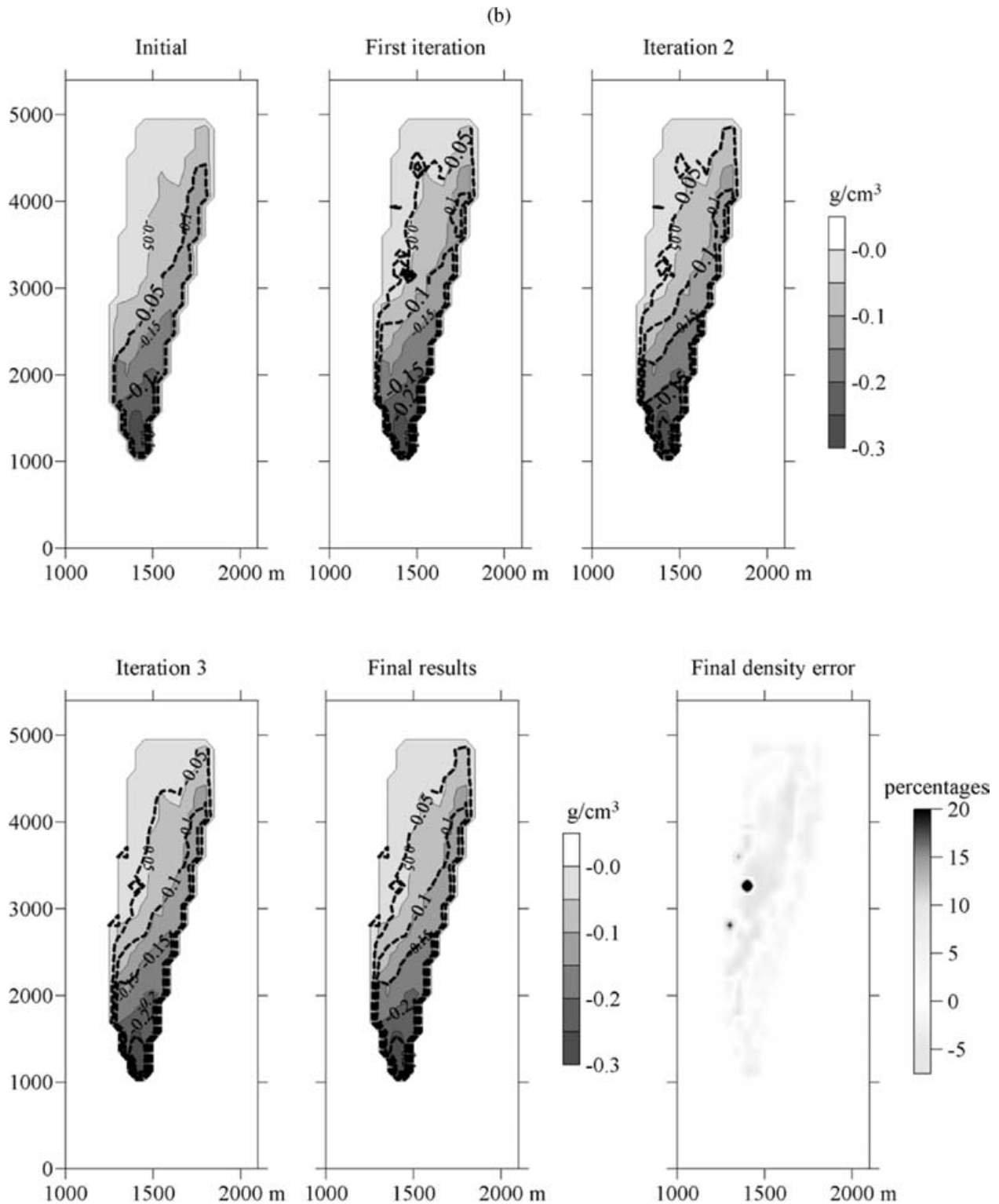


Figure 20. (Continued.)

the 3-D FTG response that delineates 3-D lateral density changes sufficiently well. This also allows us to facilitate signal and noise separation implicit in the least-squares formulation.

4. Our regularization method introduces *a priori* information about the geometric shape, spatial locations of the sources and the maximum size of the parameter corrections. In addition, we stabilize the problem by imposing a certain degree of smoothness on the ini-

tial guess using damping, see eq. (21). Because the solution (21) is used as a reference model, subsequent model corrections estimated by the solution (18) will lead to greater relaxation of the smoothness constraint and produce a better resolution.

5. When we use eqs (18) and (21) directly, the matrix inverse is performed with the SVD method. The algorithm is sufficiently fast because very small eigenvalues can be truncated. The advantage

of using SVD is that it accounts for round-off errors and helps to identify null subspaces. In addition, the matrix–vector multiplications can be sped up by means of well-known wavelet transform-based compression algorithms.

6. The 4-D FTG inversion of field data would yield the correct results if the GGI instruments could detect the smallest possible signal. Our tests show that the technique may be applied to problems involving GOC or temperature front monitoring at shallow or moderate depths (unconsolidated porous rocks with high gas-to-oil ratio, heavy oil reservoirs, etc.). However, the method is technically challenging in applications involving WOC monitoring and/or large target depths. In particular, if source regions are not shallow, the current inversion scheme may introduce an excessive bias and fail to delineate 4-D density anomalies. Our tentative experiments with synthetic data illustrate the practical importance of depth weighting (Li 2001) or robust downward continuation methods that compensate for the decay of FTG sensitivities with depth.

7. Our examples account for the suspected correlated or uncorrelated observational noise as well as the damaging effects of model uncertainties represented as geological noise. Despite the irregular shape of production volumes, the inversion produces a solution very close to the exact model even in the presence of noise.

8. Current research and the road ahead involve tests on field data and joint inversion in ways that integrate FTG data with other geophysical and reservoir data types such as 4-D seismic data, well logs, flow simulations, core measurements, production data, etc. It is a subject of another investigation to compare the implications of the above strong constraints for inverting or calibrating the large-scale non-compact reservoir heterogeneity based on the statistical fluctuations of well logs, for example (P. Leary 2005, private communication).

CONCLUSIONS

Good-quality 3-D FTG data can be acquired during reservoir monitoring, and may be used for reducing uncertainties of production history matching. Interpretation of these data requires reliable and efficient inversion methods which adequately incorporate prior information. We have developed a regularized compact FTG inversion method that achieves stable solutions by introducing *a priori* information about mass concentrations via the compactness (minimum volume) condition on the model. We define the block-like model geometry and estimate the 4-D density changes of the blocks. In 4-D FTG inversion, we worked with gradient anomalies and density contrasts instead of gravity fields and densities. This poses no restrictions on the method as long as the compactness requirement for source regions is satisfied. The accuracy of the solution is demonstrated by synthetic data examples. First, we considered rather simple models to evaluate the overall time-lapse monitoring capability of FTG. At this stage, we focused on issues of optimal FTG acquisition design and resolution of the computed estimates. Secondly, more complex models were involved to analyse the sensitivity of inverse solutions to the initial guess under reasonably high noise in observations and model parameters. Results demonstrate that the scheme is not sensitive to uncertainties in the *a priori* information. Lateral density contrasts are also well determined by the method when input data contain correlated or uncorrelated noise. Potential 4-D applications include CO₂/steam injection, heavy oil reservoir recovery, gas cap expansion and temperature monitoring at shallow or moderate depths.

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