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Fuzzy and Multiple Regression Modelling for Evaluation of Intact Rock Strength Based on Point Load, Schmidt Hammer and Sonic Velocity

By

M. Karakus and **B. Tutmez**

Mining Engineering Department, Inonu University, Malatya, Turkey

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Summary

Uniaxial Compressive Strength (UCS), considered to be one of the most useful rock properties for mining and civil engineering applications, has been estimated from some index test results by fuzzy and multiple regression modelling. Laboratory investigations including Uniaxial Compressive Strength (UCS), Point Load Index test (PL), Schmidt Hammer Hardness test (SHR) and Sonic velocity (V_p) test have been carried out on nine different rock types yielding to 305 tested specimens in total. Average values along with the standard deviations (Stdev) as well as Coefficients of variation (CoV) have been calculated for each rock type. Having constructed the Mamdani Fuzzy algorithm, UCS of intact rock samples was then predicted using a data driven fuzzy model. The predicted values derived from fuzzy model were compared with multi-linear statistical model. Comparison proved that the best model predictions have been achieved by fuzzy modelling in contrast to multi-linear statistical modelling. As a result, the developed fuzzy model based on point load, Schmidt hammer and sonic velocity can be used as a tool to predict UCS of intact rocks.

Keywords: Fuzzy and multiple regression modelling, UCS, P-wave, Schmidt Hammer, Point load.

1. Introduction

Recent trend on estimating UCS from simple laboratory index tests has gained acceleration. Many researchers have been conducting investigations aimed at predicting UCS from non-destructive testing methods such as sound velocity test, porosity, and density. A great number of attempts have been made to predict uniaxial compressive strength of intact rocks (Kahraman, 2001; Katz et al., 2000; Grima and Babuska, 1999; Koncagul and Santi, 1999; Chau and Wong, 1996; Singh and Singh, 1993; Xu et al., 1990; Vallejo et al., 1989). Most of these studies have been dealt with simple models relating UCS to SHR, UCS to V_p, UCS to porosity n, UCS to PL and so on. Within this study, 9 different rock types have been tested according to ISRM suggested methods. These tests consist of Schmidt Hammer Hardness test, Point Load Index test, Sound Velocity and Unconfined Compressive Strength test. Each rock type has been subjected to the aforementioned four tests and their average values, standard deviations and Coefficients of Variation (CoV) from each test were calculated accordingly. Coefficient of variation has been calculated by dividing standard deviations by average values.

Fuzzy rule based modelling and multi-linear regression have been employed to estimate intact rock strength. In addition, not only efficient computation but also flexible modelling, which will be well suited to human input, have been investigated. Some predictive models taking into consideration simple, multiple regression and fuzzy modelling were developed by many researchers using index parameters such as Schmidt hammer, point load and physical and petrographical parameters (Grima, 1999; Kayabasi et al., 2002; Gokceoglu and Zorlu, 2003). Sonmez et al. (2003) reported that fuzzy set theory may be a useful tool for rock engineers and engineering geologists who study complex rock mass conditions.

Grima (2000) stated that in rule-based modelling, fuzzy logic deals with linguistic expressions, also called fuzzy propositions. The main reason is that most natural language involves vague imprecise terms, that is, fuzzy terms, like friction in mechanics. For example, a statement describing the performance of a rock excavation machine such as 'high excavation rate' can be regarded as a fuzzy proposition. Likewise, such statements can be represented by means of if-then rules. As a result, a number of fuzzy ''if-then'' rules were generated and UCS of intact rock specimens was appraised.

2. Experimental Studies

Rock blocks have been collected in different locations in Malatya and Elazig regions in Eastern Anatolia. Rock blocks mainly consisted of marble, limestone, and Dacite. Optical microscopy study has been conducted to define composition, and mineralogical and textural characteristics of the rock samples to be tested. Thin section analysis results are given in Table 1. Using core drilling machines in the laboratory, NX size core samples have been taken from the blocks. ELE Core trimmer and cut-off machine has been used to supply correct size and smooth ends. The standard laboratory test procedures have been employed during core sample preparations (ISRM, 1981).

2.1 Point Load Test

The point load strength test is an index test to determine the strength of intact rocks (Broch and Franklin, 1972; Broch, 1983). It is carried out on practically all major rock construction sites. The test provides a basis for a mechanical classification of the rock as well as an estimate of the uniaxial compressive strength of intact rock.

In the point load strength test, a piece of rock is taken from the core boxes and loaded between two hardened steel points of the point load tester. The system consists of a small hydraulic pump, a hydraulic jack, a pressure gauge and interchangeable

Table 1. Thin section analysis results

Sample ID	Texture	Composition	Rock name
D	Hypocrystalline porphyric texture, volcanic glass and feldspar, microlite	Plagioclase, quartz, Sanidine, Biotite	Dacite
BL	Clastic texture, sparitic cement	Calcite, Fossil, very little quartz in microcracks	Biosparitic limestone
YB	Nematoblastic texture	Hornblende, Epidote-(pistasite), Plagioclase, Opaque mineral (in a little amount)	Epidote-Amphibolite
LU	Clastic texture, sparitic cement	Calcite, very little Fe-Oxide in microcracks	Sparitic crystallised limestone
KF	Clastic texture, Micritic cement, sparite in pores	Calcite, Fossil	Biomicritic limestone
EM*	Not available	Not available	Elazıg marble
MK	Clastic texture, Sparitic cement	Calcite, Quartz, rock fragments (volcanic and radiolarite)	Sandy sparitic limestone
LW	Breccia texture, Sieve texture	Calcite, Serpentine, Fe-Oxide	Listwanite
TL*	Not available	Not available	Highly porous limestone

* Thin section analysis was not conducted for these rock specimens.

testing frame of very high transverse stiffness. The two steel points are of standard dimension (Broch and Franklin, 1972).

In the test, the rock sample is slowly loaded by activating the hand pump until failure of the sample. Failure load is then read from the dial gauge. The test can be applied to rock samples with irregular or regular shapes. Three test methods viz. diametrical test, axial test, and block tests are available.

In this work, axial test was conducted. For each rock type 10 NX drill cores with approximately 1 height/diameter ratio have been prepared and then samples were subjected to the test. Maximum capacity of machine is 56 kN. Failure of the rock samples was achieved within the 10–60 seconds (ISRM, 1978; ISRM, 1985).

The Point Load Strength Index I_s was calculated and corrected to the 50 mm standard core diameter. Table 2 shows mean values of the corrected point load index and their standard deviation along with CoV.

Table 2. Results of point load tests, standard deviation and coefficient of variation

Rock type	$I_{s(50)}$ (MPa)	Stdev (MPa)	CoV (%)
Dacite	7.45	1.23	16.5
Biosparitic limestone	4.91	0.25	5.09
Epidote-amphibolite	6.56	0.45	6.86
Sparitic crystallised limestone	4.36	0.67	15.37
Biomicritic limestone	4.95	0.7	14.14
Elazıg marble	5.09	0.47	9.23
Sandy sparitic limestone	3.72	0.29	7.80
Listwanite	3.61	0.51	14.13
High porous limestone	3.75	0.81	21.60

2.2 Schmidt Hammer Test

The Schmidt hammer test method is routinely used to test the strength and the quality of rock and hardened concrete. However, application of the Schmidt hammer test on very soft and extremely hard rocks is not recommended (Xu et al., 1990). There is a strong relation between Schmidt Hammer Rebound value (SHR) and Unconfined Compressive Strength of rock. ISRM (1978) suggested a rock hardness description based on SHR values. In this description, rebound values range from 0–10 to >60 corresponding to soft to extremely strong rocks.

The plunger of the hammer is placed against the specimen and is depressed into the hammer by pushing the hammer against the specimen. Energy is stored in a spring, which automatically releases at a prescribed energy level and impacts a mass against the plunger. The height of rebound of the mass is measured on a scale and is taken as a measure of hardness. The device is portable and may be used both in the laboratory and the field. SHR models are available in different levels of impact energy. L-Type Schmidt hammer having 0.74 Nm impact energy is preferred for rock samples (ISRM, 1981).

For the present work model L-Type Schmidt hammer was used to measure rebound values of the rock core samples having 54 mm diameter and 100 mm height. Minimum 10 core samples from each rock sample tested and 20 readings were taken for each core sample yielding to 180 readings for each rock types. In order to avoid material deflecting Schmidt hammer cradle was used. Average values of these readings and their standard deviation were calculated as shown in Table 3.

2.3 Sound Velocity Test

This test is designed for measuring dynamic properties of rocks and concrete. Portable Ultrasonic Non destructive Digital Indicating Tester, PUNDIT 6 Model PC1000 was used in all tests. As a non-destructive testing method, pulse velocity measurements can be conducted in three ways viz. direct transmissions with transducers on opposite faces of the sample, semi-direct method with transducers on adjacent faces, and indirect method or surface transmissions with the transducers on the same faces. The first transmission is the most sensitive method and thus has been used for all rocks in the present study. 5 NX drill cores with height/diameter ratio 2 were prepared for each rock types. Both faces of drill cores were trimmed and smoothened in order

Table 3. Average values of Schmidt hammer rebounds (SHR)

Rock type	SHR	Stdev	CoV (%)
Dacite	43.75	1.16	2.65
Biosparitic limestone	48.83	1.36	2.79
Epidote-amphibolite	50.79	2.48	4.88
Sparitic crystallised limestone	39.95	2.88	7.21
Biomicritic limestone	42.2	2.4	5.69
Elazıg marble	37.47	1.32	3.52
Sandy sparitic limestone	33.6	1.60	4.76
Listwanite	40.8	4.07	9.98
High porous limestone	20.58	3.26	15.84

Table 4. P-wave (V_p) velocity test results

Rock type	V_p (km/s)	Stdev (km/s)	CoV (%)
Dacite	3.5	0.31	8.86
Biosparitic limestone	5.78	0.09	1.56
Epidote-amphibolite	6.16	0.26	4.22
Sparitic crystallised limestone	5.75	0.46	8.00
Biomicritic limestone	5.13	0.36	7.02
Elazıg marble	4.88	0.02	0.41
Sandy sparitic limestone	3.52	0.47	13.35
Listwanite	5.3	0.69	13.02
High porous limestone	3.74	0.27	7.22

Table 5. Results of uniaxial compressive strength test

Rock type	UCS (MPa)	Stdev (MPa)	CoV (%)
Dacite	90.24	13.96	15.47
Biosparitic limestone	46.32	4.64	10.02
Epidote-amphibolite	89.1	16.57	18.60
Sparitic crystallised limestone	58.02	15.61	26.90
Biomicritic limestone	51.2	5.03	9.82
Elazıg marble	75.05	5.94	7.91
Sandy sparitic limestone	50.33	11.24	22.33
Listwanite	32.81	6.59	20.09
High porous limestone	11.5	2.93	25.48

for the receiver and emitter to adhere to core faces. Average P-wave velocity values (V_p), standard deviation and CoV are given in Table 4.

2.4 Uniaxial Compressive Strength Test

As pointed out earlier one of the most important parameters of rocks is their compressive strength. Thus, many researches have been undertaken to examine uniaxial compressive strength of rocks. Some of these researches have been involved with direct laboratory testing investigations and some others have been dealing with empirical and statistical works. Both statistical and real laboratory works have been included in the present work.

Uniaxial compressive strength tests have been carried out according to the ISRM suggested methods (ISRM, 1979). NX size drill core samples were prepared with 2–2.5 height/diameter ratio. End of core specimens were flattened within 0.02 mm and perpendicularity were kept within 0.05 mm so that load could be applied uniformly. Pace rate was kept at 0.5 MPa/s. For each rock type, 10 specimens yielding to 90 tests in total have been conducted and average values were calculated accordingly. Tests results are given in Table 5.

3. Fuzzy Modelling Algorithm

This section was organized in three parts. In the first part, the membership functions and derivation of fuzzy sets from statistical data were explained. In the second part, inference rules to combine fuzzy descriptors for the point load, Schmidt hammer and sonic velocity tests were given. As the last part, the defuzzification was introduced.

The concept of linguistic fuzzy models imitating the human way of thinking was elaborated by Zadeh in his pioneer works (1965). Fuzzy models can cope with the complexity of complex and ill-defined systems in a flexible and consistent way (Grima and Babuska, 2000). In the linguistic fuzzy model, also called the Mamdani Model, both the antecedent and consequent are fuzzy propositions (Babuska, 1998).

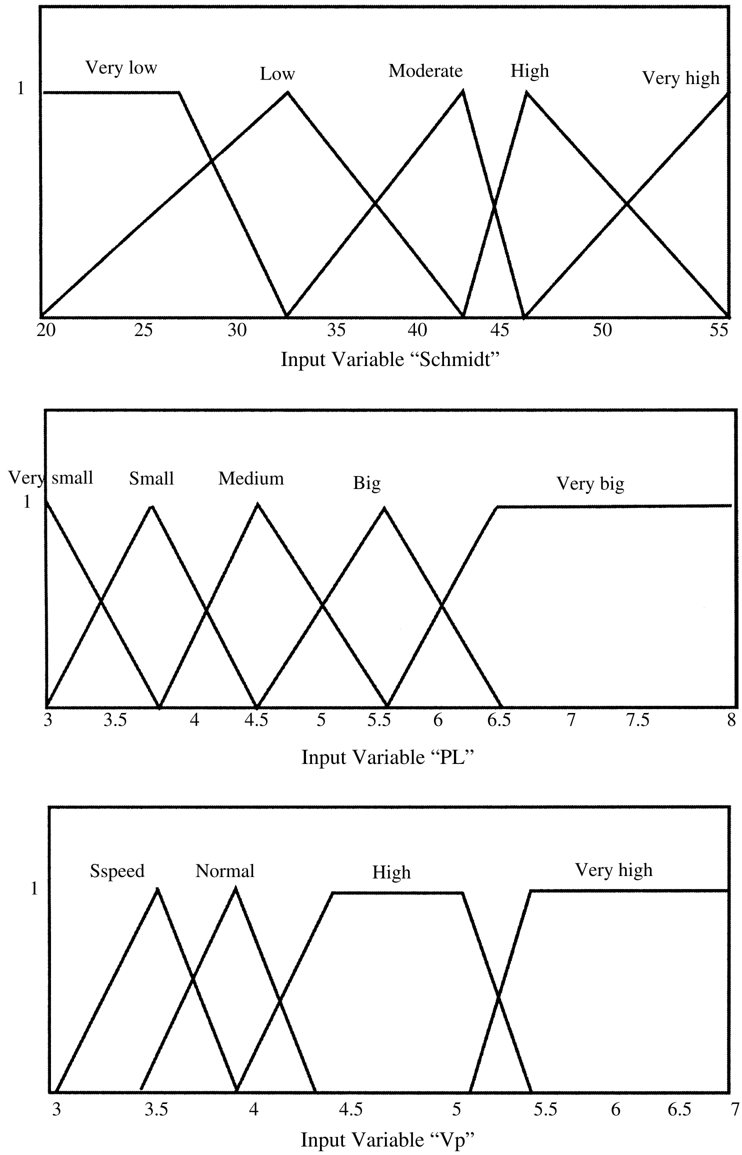

Fig. 1. Input membership functions for Schmidt hardness, point load (PL) and V_p

In this study, a rule base model was constructed, then UCS of intact rock was estimated from some index parameters using a modelling (inference) mechanism.

3.1 Membership Functions

The membership functions for fuzzy sets can have many different shapes, depending on definition. They can be represented either in a continuous domain or in a discrete domain. In the beginning, membership grades of the numeric values of the model inputs in fuzzy sets were calculated. Suitable linear membership functions of the mechanism are depicted in Figs. 1 and 2. The membership functions of antecedents and consequent are designed by data file.

To calculate membership grades in particular fuzzy sets, their membership functions must be precisely defined in respect of quality (function type) and of quantity (function parameters). Both the parameters and shape of the membership functions strongly influence the model accuracy (Baglio, 1994).

3.2 Fuzzy Inference System (FIS)

Constructing an output subset from input subsets using relationships expressed by if-then statements was achieved by inferencing. The inference operation involves the following steps (Piegat, 2001):

- evaluation of fulfillment degrees of particular rules (to be precise, premises of the rules),
- determination of activated membership functions of conclusions of particular rules,
- determination of the resulting membership function of the conclusion of all rules in the rule base.

A general form of linguistic fuzzy if-then rules is:

R_i: **IF** the value of variable x_1 is A_i **AND** .... **THEN** the result y is B_i

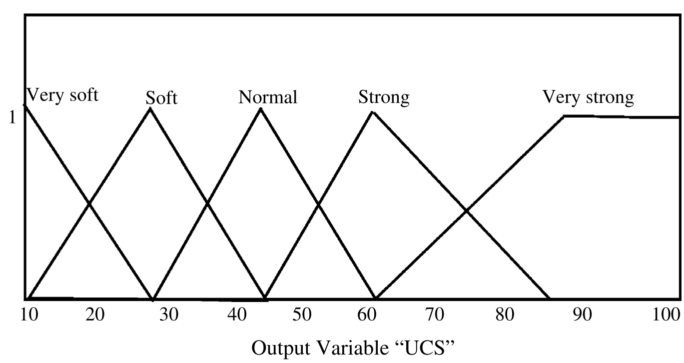

Fig. 2. Output membership function for UCS

Table 6. Selected ‘‘If-then’’ rules for the UCS fuzzy inference from rule base mechanism

Rule no.	Definition of if-then rules
1	If (Schmidt is high) and (PL is very big) and (V_p is speed[a]) then (UCS is very strong)
2	If (Schmidt is high) and (PL is medium) and (V_p is very high) then (UCS is normal)
3	If (Schmidt is high) and (PL is very big) and (V_p is very high) then (UCS is very strong)
4	If (Schmidt is mod[b]) and (PL is medium) and (V_p is very high) then (UCS is strong)
5	If (Schmidt is mod) and (PL is medium) and (V_p is high) then (UCS is normal)
6	If (Schmidt is low) and (PL is big) and (V_p is high) then (UCS is strong)
7	If (Schmidt is low) and (PL is small) and (V_p is speed) then (UCS is normal)
8	If (Schmidt is mod) and (PL is small) and (V_p is very high) then (UCS is soft)
9	If (Schmidt is very low) and (PL is small) and (V_p is normal) then (UCS is very soft)
10	If (Schmidt is mod) and (PL is big) and (V_p is normal) then (UCS is strong)
11	If (Schmidt is mod) and (PL is big) and (V_p is high) then (UCS is strong)
12	If (Schmidt is very low) and (PL is very small) and (V_p is speed) then (UCS is very strong)
13	If (Schmidt is low) and (PL is big) and (V_p is high) then (UCS is very strong)
14	If (Schmidt is mod) and (PL is very big) and (V_p is speed) then (UCS is very strong)

[a] Slow speed; [b] Moderate.

where x_1 is the input (antecedent) variable, and A_i are the antecedent linguistic terms (constants). Similarly, y is the output (consequent) linguistic variable and B_i are the consequent linguistic terms (Babuska, 1998).

Due to an increasing interest in obtaining fuzzy models from measured data (Setnes et al., 1998) the data-driven if-then rule mechanism was constructed and, as shown in Table 6, a total of 108 ‘‘if-then’’ rules were extracted from the data to predict UCS of intact rock samples. Because of the nature of the rock, some impossible rules are ignored in the extraction of the ‘‘if-then’’ mechanism. Some possible rules from this rule base mechanism were selected and the variations employed in this system are illustrated in Table 6.

3.3 Defuzzification

The desired output of a fuzzy model is often not the fuzzy subset for y, but a single element within the subset. To obtain a crisp value, the output subset is defuzzified.

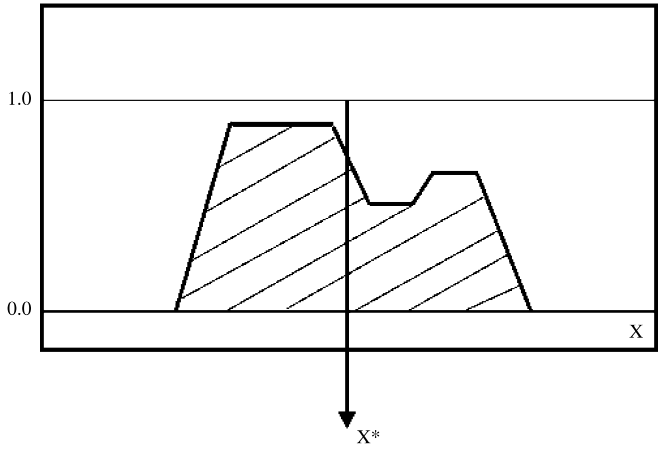

Fig. 3. Centre-of-gravity method for defuzzification

Amongst the defuzzification methods, recently proposed seven defuzzification methods are considered to be popular for defuzzifying fuzzy output functions (Hellendoorn and Thomas, 1993). In this study, the Centre of Gravity method (COG) for the defuzzification with the Mamdani Inference scheme was considered. An example of the COG is shown in Fig. 3. The advantage of COG method is that all activated membership functions of the conclusions (all active rules) take part in the defuzzification process (Piegat, 2001).

4. Multiple Regression Analysis

Regression analysis is the statistical methodology for predicting values of one or more dependent variables from a collection of (independent) variable values. As the second part of the work, a multiple linear regression model for the prediction of UCS of rock has been conducted. In general, multiple regression models are given by the following equation:

$$y = \beta_0 + \beta_1 x_1 + \ldots\ldots\ldots \beta_k x_k + e$$

where y is called the response variable whose outcome depends on the k predictor variables $x_1, x_2, \ldots\ldots x_k$, chosen by the experimenter, and $\beta_0, \ldots \beta_k$, unknown, called regression parameters (Srivastava, 2002). In this connection, we define the predicted value of y, denoted by y^*. It is given by

$$y^* = x(x'x)^{-1}x'$$

The residuals are aso defined by

$$e = y - y^*$$

For the prediction, we have seen how the problem of fitting a straight line by least squares can be handled through the use of matrices. Determining the statistical significance of the regression model under consideration at each step, we used the F-statistics.

Table 7. Coefficients and summary of some parameters affecting the quality of model for dependant variable UCS

Predictor	Coefficient	Stdev
Constant	−35.89	33.64
Schmidt	0.890	1.232
PL ($I_{s(50)}$)	13.123	6.298
V_p	−1.682	8.589

Stdev: Standard deviation.

Table 8. Analysis of variance

Model		Sum of squares	Degree of freedom	Mean square	F
1	Regression	4075.7	3	1358.591	5.540
	Residual	1226.0	5	245.216	
	Total	5301.8	8		

Table 9. Model summaries for dependant variable (UCS)

R	R-square	Adjusted R-square	Standard error of the estimate
0.8769	0.769	0.630	15.66

Some of test statistics and the Analysis of Variance table for this regression problem are shown in Tables 7–8. Regression model summary is also given in Table 9.

The multiple regression (MR) equation for average values found is as follows:

$$\mathrm{UCS} = -35.9 + 0.89\ \mathrm{Schmidt} + 13.1\ \mathrm{PL} - 1.68\ \mathrm{V_p} \tag{1}$$

5. Appraisal of FIS and MR Modelling Performances

Results of the two methods are compared in terms of the estimation performances. Figures 4 and 5 indicate that R-squared value of FIS is bigger than value of MR.

To assess the performance of Mamdani fuzzy model and multiple regression model, two effective performance indexes were used. First one was the Root Mean Square Error (RMSE) and second was Variance Account for (VAF):

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - y_i^*)}$$

$$\mathrm{VAF} = \left(1 - \frac{\mathrm{var}(y - y^*)}{\mathrm{var}(y)}\right) * 100$$

where y_i is the measured value, $y_i{}^*$ is the estimated value and N is the number of samples. As can be seen from Table 11, the prediction achievement in terms of both RMSE and VAF is better for the fuzzy modelling algorithm than for the multiple

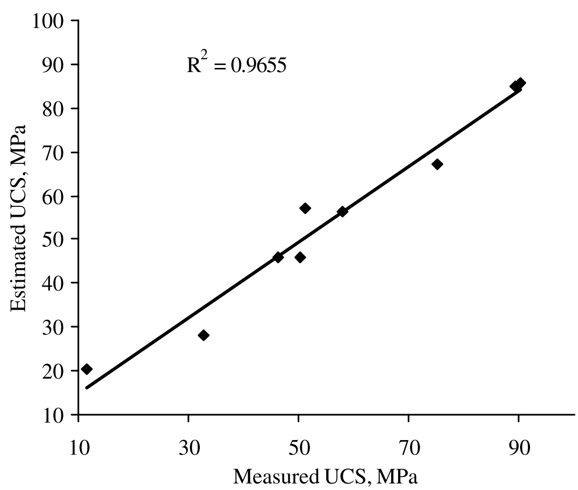

Fig. 4. Estimation performance for fuzzy inference

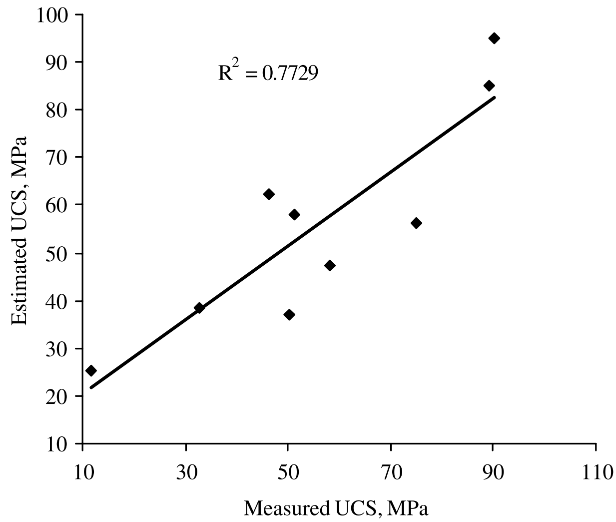

Fig. 5. Estimation performance for multiple regression

Table 10. Comparison of FIS and MR predictions with measured data

Measurements	Predictions by FIS	Predictions by MR
90.24	85.9	94.9
46.32	46	62.2
89.1	84.8	85.1
58.02	56.4	47.2
51.2	57.1	57.9
75.05	67.3	56.3
50.33	46	37.0
32.81	28	38.5
11.5	20.6	25.3

Table 11. The Root Mean Square Errors (RMSE) and Variance Account For (VAF) for both model predictions

Prediction models	RMSE	VAF (%)
Fuzzy Inference System (FIS)	4.36	89.4
Multiple Regression (MR)	11.38	79.2

regression modelling, which shows a good generalization capability of the Mamdani fuzzy model.

6. Conclusions

Rule based fuzzy model combines good quantitative properties and qualitative results for evaluating vague systems. Reason to choose fuzzy modelling instead of other modelling techniques like multiple regression modelling, is mainly the possibility of integrating logical information processing with the attractive mathematical properties of general function approximations. Comparing the RMSE and VAF of linear regression model with the results of the fuzzy model, it can be seen that performance of the fuzzy technique to predict UCS of intact rock is much better. The results also proved that the fuzzy rule based approach is a good and flexible tool to evaluate uncertainties in rock properties. Thus, this approach can easily be extended to other mining based domains such as modelling tunnelling operations in highly weathered rock mass conditions, support design, blasting optimisations and similar complex structural designs.

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Author's address: Assistant Prof. Murat Karakus, Mining Engineering Department, Engineering Faculty, Inonu University, 44280 Malatya, Turkey. e-mail: mkarakus@inonu.edu.tr