

Estimating vertical stochastic scale parameters from seismic reflection data: deconvolution with non-white reflectivity

Christian Poppeliers

Department of Physics, Angelo State University, San Angelo, TX, USA. E-mail: christian.poppeliers@angelo.edu

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SUMMARY

Seismic wavefields reflected from the Earth's crystalline crust often appear quite complex due to the non-horizontally layered, seemingly random spatial patterns of velocity structure. Deterministic mapping of such a velocity field is often impractical, therefore, we rely on stochastic descriptions to quantify textural aspects of a given Earth model. This paper introduces an iterative method to invert (in 1-D) a reflected seismic wavefield for vertical stochastic parameters. In a previous method, standard deconvolution is used to form an estimate of the Earth's reflectivity. Numerical integration of the resulting reflectivity is an estimate of the binary velocity perturbation model to which we can fit a theoretical von Kármán stochastic autocorrelation function. In this paper, an improved iterative model-based approach is introduced. The algorithm estimates the vertical von Kármán stochastic parameters by iteratively modifying a deconvolution filter based on previously estimated von Kármán model parameters. By inverting synthetic seismic data, it is shown that this iterative approach is more stable numerically than an earlier algorithm, and is able to recover both the vertical characteristic length and the Hurst exponent. The method is sensitive to noise in the data because the noise has a stochastic behaviour similar to that of the data. This new method is applied to a single common shot record that was recorded as part of the PASSCAL Basin and Range experiment and recovers values of stochastic parameters that are consistent with observations of exposed crustal sections.

Key words: stochastic inversion, von Kármán, crustal scattering.

INTRODUCTION

Examinations of middle and deep crustal rocks have shown that parts of the Earth's crystalline crust can be approximated as discrete, binary or ternary velocity distributions with self-affine, stochastic fabrics (e.g. Holliger & Levander 1992; Holliger *et al.* 1994). The success of this stochastic approach to characterizing the crust is evidenced by the similarity of synthetic seismograms to recorded field data (Holliger & Levander 1994; Holliger *et al.* 1994; Levander *et al.* 1994a,b; Hurich 1996; Hurich & Kocurko 1999) and the explanation of regional distance scattered seismic phases (e.g. P_n) from this type of lower crustal structure (Nielsen *et al.* 2003). The strength of using statistical models to describe the texture of lithology is that, in certain cases, the rock mass is so complex that it may be more convenient to represent a rock body as an equivalent statistical model (Fig. 1).

Although it is useful to model the Earth's crust stochastically for forward modelling of seismic wavefields, we often wish to go in the opposite direction; given a seismic wavefield from a geologically complex area we would like to estimate the stochastic parameters of the velocity field. This inverse approach may form a useful interpretation tool, and the development of a reliable and robust tool is the primary motivation for the work presented in this paper. Previous

work on estimating *horizontal* stochastic parameters can be found in Hurich (1996) and Hurich & Kocurko (1999). However, this paper presents a tool to extract *vertical* stochastic parameters from seismic reflection data. The formulation of the method presented here is based on an assumption of a binary velocity model that can be statistically modelled with a discrete von Kármán autocorrelation model (see Goff *et al.* 1994, for details), where the distribution of spatial structures is proportional to the spatial distribution of the lithology and velocity structures, and are self-affine in nature (e.g. Holliger *et al.* 1994). Additional assumptions concern the seismic data itself. First, we assume that the data is singly scattered, near-vertical incidence, with no predominant multiples or interbed multiples. The method presented in this paper builds on previous work of estimating vertical stochastic parameters (Poppeliers & Levander 2004) based on an estimate of the reflectivity using a standard method of deconvolution. However, in the work presented in Poppeliers & Levander (2004), the algorithm was only able to produce estimates of vertical characteristic length, not the Hurst exponent. The algorithm presented here accounts for the non-white reflectivity spectrum of a self-affine velocity model. This improvement allows for more stable estimates of the vertical characteristic length and estimation of the Hurst exponent, which was not possible in the formulation presented in Poppeliers & Levander (2004).

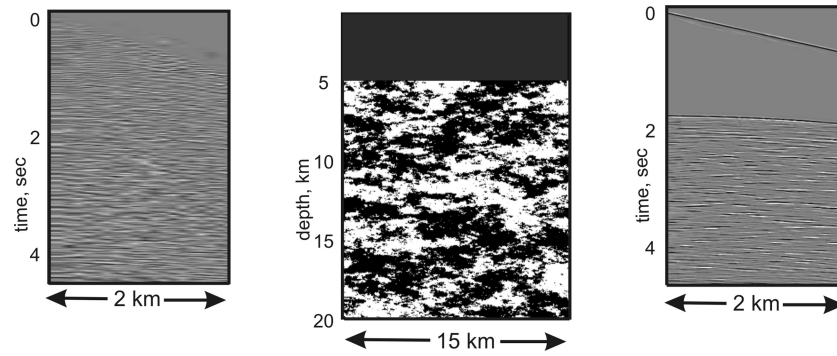


Figure 1. Comparison of field data to synthetic data generated from a synthetic stochastic velocity model. The panel on the left is data recorded as part of the 1986 PASSCAL Basin and Range Experiment (PASSCAL Working Group 1988; Holbrook 1990). The upper 4 s of this data set contain complex reflections that can be explained as arising from a stochastic velocity field. For comparison, the middle panel shows a stochastic velocity model, where the black portions indicate a P -wave velocity of 6.0 km s^{-1} and the white portions indicate a P -wave velocity of 6.3 km s^{-1} . The synthetic velocity model contains von Kármán statistics, where $a_z = 300 \text{ m}$, $a_x = 1000 \text{ m}$, and $\nu = 0.3$. The panel on the right shows synthetic acoustic data that was generated via finite differences using the synthetic velocity model. Attempting to interpret discrete interfaces based on the reflection data would be nearly impossible in this case; recovering stochastic parameters that describe the statistics (i.e. the texture) of the velocity model may prove more useful.

To demonstrate the utility of the algorithm, results from numerical experiments on synthetic data are shown. The tests show that the inversion is able to recover model parameters when the synthetic data are noise-free. However, additional tests show that the inversion is sensitive to coloured noise in the data due to the stochastic character of the noise. As a final example, the inversion is applied to data collected for the PASSCAL Basin and Range Experiment (Benz *et al.* 1990; Holbrook 1990). For the field data, recovered stochastic parameters are similar to observations of other exposed crustal sections.

METHOD

Forward model

We start by characterizing the forward model. Assume that the velocity perturbations are self-affine, with a 1-D von Kármán power spectrum given by

$$P_{\delta v}(k) = \frac{\Gamma(2\nu)a_z}{2^{2\nu-2}\Gamma^2(\nu)(1+k^2a_z^2)^{\nu+1/2}}, \quad (1)$$

where k is the wavenumber, Γ is a gamma function, ν is the Hurst exponent, and a_z is the characteristic length of the velocity perturbations in the z direction (Goff *et al.* 1994). The parameter ν describes the model's power-law decay, and is related to fractal dimension D by

$$D = E + 1 - \nu, \quad (2)$$

where E is the Euclidean dimension (Goff & Jordan 1988). We also assume that the velocity perturbations $\delta v(\mathbf{x})$ are much smaller than the background velocity $v_0(\mathbf{x})$,

$$\frac{\delta v(\mathbf{x})}{v_0(\mathbf{x})} \ll 1, \quad (3)$$

and that $\delta v(\mathbf{x})$ is a zero mean and locally stationary field. The variable $\mathbf{x}$ indicates spatial position in a, for example, Cartesian coordinate system. The velocity model can be expressed as

$$v(\mathbf{x}) = v_0(\mathbf{x}) + \delta v(\mathbf{x}) \quad (4)$$

(e.g. Holliger & Levander 1992, 1993, 1994). Because $v_0(\mathbf{x})$ varies smoothly and is regional in scale, it can be determined by seismic

tomography or CMP velocity analysis. However, $\delta v(\mathbf{x})$ is smaller scaled and ultimately responsible for the complex response we see in a typical seismic reflection section. In other words, $\delta v(\mathbf{x})$ contains the high wavenumber components of the velocity field, and $v_0(\mathbf{x})$ contains the low wavenumber components of the velocity field. Also, we assume that the data to be inverted are near offset so that P to S conversions are not significant. Therefore, the data to be used for inversion are primary, compressional wave reflections from high-wavenumber velocity perturbations.

We wish to relate the velocity perturbations to the reflectivity. Therefore, we make the approximation that the physical structure and distribution of the impedance field is proportional to the velocity field as well as the density (e.g. Christensen & Mooney 1995) which is true for many, but not all, crustal rocks (Milkereit & Eaton 1998). That is,

$$p(\mathbf{x}) = p_0(\mathbf{x}) + \delta p(\mathbf{x}), \quad (5)$$

where $p(\mathbf{x})$ is the impedance. The velocity and density may not be strictly proportional for all velocities. However, assume that the impedance contrasts that they produce will be roughly similar for different average velocities, that is, the power spectrum of the velocity and density will have the same form, with similar corner wavenumbers and power-law decays. Implicit in these assumptions is

$$\frac{\delta p(\mathbf{x})}{p_0(\mathbf{x})} \ll 1, \quad (6)$$

similar to eq. (3).

If eqs (3) and (6) hold, then the seismic data are only singly, or weakly, scattered (see Flatte *et al.* 1979, for details on scattering regimes), a common assumption in seismic imaging techniques. In other words, the seismic section is modelled as a simple convolution of a source wavelet with the reflectivity of a velocity field. Also, explicit in this assumption is that there are no multiple reflections present and that mode conversions (e.g. P to S conversions) are absent. If multiple reflections are present in the data, they must be removed prior to the application of this algorithm. Because the data is nearly vertically incident, P to S conversions are minimized.

To relate the reflectivity to the velocity, assume that the acoustic impedance p is given by

$$p(t) = \rho(t)v(t), \quad (7)$$

where ρ is density, v is the velocity, and t is time. Although the velocity and density are usually given in the space domain, mapping into the time domain is accomplished by the application of the appropriate, regional-scale velocity model. Eq. (7) is presented in the time domain as it makes the following equations clearer. Pullammanappallil *et al.* (1997) showed that the reflectivity is the first derivative of the velocity, or impedance, and that reflectivity in time $r(t)$ can be related to the impedance $p(t)$ by

$$r(t) = \frac{1}{v_0} \frac{d}{dt} \delta p(t), \quad (8)$$

where we rescale the impedance from depth to time by $t = \frac{z}{v_0}$, the δp term indicates that we are considering the stochastic portions of the impedance field, and v_0 is the average, background velocity. If density is nearly constant, we can substitute eqs (7) into (8) to give

$$r(t) = \frac{1}{v_0} \frac{d}{dt} [\delta v(t) \rho(t)], \quad (9)$$

where rescaling to the time domain is implied. Although density is not strictly constant in the Earth's crust, the impedance contrasts are controlled mostly by the velocity perturbations, as the magnitude of the velocity variations are significantly larger than the magnitude of the density variations. Fixing the density allows a simpler derivation without straying too far from reality. With this approximation, eq. (9) becomes

$$r(t) \approx \frac{1}{v_0} \frac{d}{dt} \delta v(t). \quad (10)$$

Using the convolutional model, the seismic data is $s(t)$ expressed in the time domain as a convolution of a source wavelet $w(t)$ with the stochastic reflectivity $r(t)$;

$$s(t) = \int_{-\infty}^{\infty} w(t - \tau) r(\tau) d\tau. \quad (11)$$

Substituting eqs (10) into (11) gives

$$s(t) = \int_{-\infty}^{\infty} \left\{ [w(t - \tau)] \left[\frac{1}{v_0} \frac{d}{dt} \delta v(t) \right] \right\} d\tau. \quad (12)$$

Fourier transforming eq. (12) over time yields

$$S(f) = W(f) \frac{i2\pi f}{V_0} \delta V(f), \quad (13)$$

where f is frequency and $i = \sqrt{-1}$. The reflectivity for nearly vertical incident wavefields can be expressed in the frequency domain in terms of the (frequency domain) velocity model;

$$R(f) = \frac{i2\pi f}{V_0} \delta V(f), \quad (14)$$

where $\delta V(f) = \delta V(f, a_z, \nu)$ is the stochastic velocity function and a_z and ν are the parameters of the von Kármán model. Because we assume that the autocorrelation of the velocity perturbations is similar to a von Kármán autocorrelation, the power spectrum of the velocity perturbations can be described by a von Kármán power-spectrum model, described by eq. (1). The point of eq. (14) is that the reflectivity is stochastic. The power spectrum of the stochastic reflectivity $R(f)$ is given by squaring both sides of eq. (14):

$$P_R(f) = \frac{-4\pi^2 f^2}{V_0^2} |\delta V(f)|^2. \quad (15)$$

The parameters rms variation and the Hurst exponent are not dependent on orientation; however, the characteristic length is. For the work here, the concern is to estimate vertical characteristic length, denoted as a_z . Vertical heterogeneities larger than the characteristic

length are spectrally white, whereas vertical heterogeneities smaller than the characteristic length are self-affine. For a continuous velocity field, the rms velocity variation is second order, which is defined as the difference between the high-wavenumber binary velocity values.

The von Kármán model can be used to describe the autocovariance of a variety of fields. The functional form of the von Kármán autocorrelation is given by

$$C_{hh}(\tau) = \frac{H^2 G_\nu[r(\tau)]}{G_\nu(0)}, \quad (16)$$

where τ is lag, H is the rms variation of the velocity, and G_ν is defined by

$$G_\nu(r) = r^\nu K_\nu(r), \quad 0 \leq r < \infty, \quad \nu \in [0, 1]. \quad (17)$$

K_ν is the modified Bessel function of the second kind and order ν . The parameter ν is the Hurst exponent and is related to the fractal dimension of a continuous field given by eq. (2). Goff *et al.* (1994) presented a method to map a continuous field with a von Kármán covariance to a corresponding discrete field by partitioning the continuous velocity values into 'high' and 'low' binary values depending on whether they fell above or below a cut-off value (e.g. Figs 2a and b). This method is necessary for generating a synthetic discrete field of specific spatial properties. They showed that there exists a relationship between the Hurst exponent of a continuous field ν_c and the Hurst exponent of a discrete field ν ;

$$\nu \approx \frac{\nu_c}{2}, \quad \nu_c < 0.5. \quad (18)$$

Also, they show that the corner wavenumber of the binary field is related to the corner wavenumber of the continuous field. These respective Hurst exponents are scaled by a constant that depends on the Hurst exponent and the relative proportion of the two discrete velocities in a given field (see table 1 of Goff *et al.* 1994, for details). For this work we use a scaling constant of

$$k_0 \approx 0.87 k_{0,c}, \quad (19)$$

where $k_{0,c}$ is the corner wavenumber ($a = 1/k_0$) for a continuous field and k_0 is the corner wavenumber for a binary field. The scaling relationship shown in eq. (19) is for a continuous-field Hurst exponent of $\nu_c = 0.3$ and a 50–50 binary velocity distribution.

Although the scaling constant changes slightly for different values of both the Hurst exponent and the relative proportion of the velocity distribution, the value in eq. (19) is chosen because $\nu = 0.3$ is fairly typical for crustal igneous rocks (e.g. Holliger & Levander 1994; Holliger *et al.* 1994). Furthermore, a 50–50 distribution of velocities is used as it represents an 'average' value of velocity distributions. Different values of the continuous-field Hurst exponent and of the relative proportions of the velocity distribution will change the scaling constant in eq. (19). However, for realistic ranges of these two parameters, the change will only be a few per cent. Therefore, using the constant in eq. (19) will introduce a small systematic error of a few per cent in the final result if the velocity distribution is not truly 50–50.

Estimation of vertical stochastic parameters

In the inversion outlined here, the data are assumed to be near-vertical incidence. Therefore, the data are assumed to contain P -wave energy derived from reflections from the small-scale velocity perturbations and nearly devoid of converted S -wave arrivals. This inversion is not applicable for far-offset traces. For typical Earth

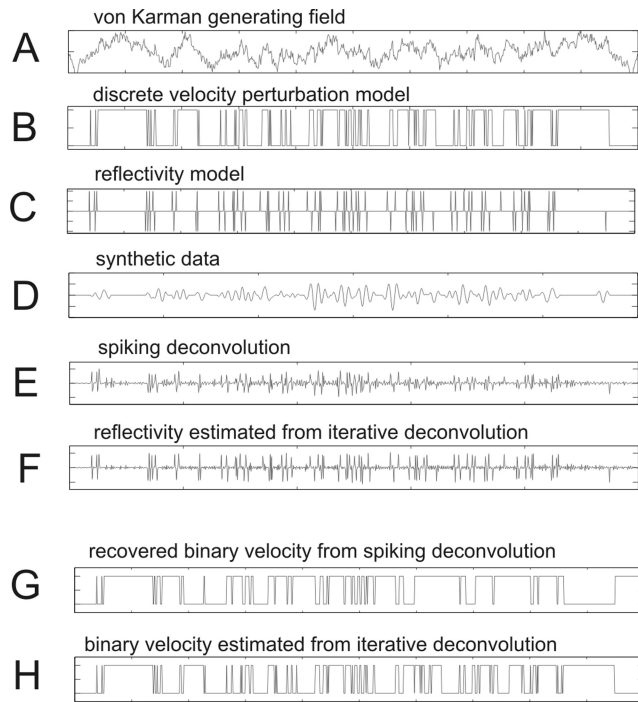


Figure 2. The difference between the spiking deconvolution with the iterative, model-based deconvolution. Panel A shows a 1-D synthetic stochastic generating field. The stochastic parameters of the continuous generating field are chosen so that when its partitioned into a binary velocity field (Panel B), the stochastic parameters of the binary field are $a = 300$ m and $v = 0.3$. Panel C is the reflectivity of the discrete, binary velocity perturbation model shown in Panel B, and Panel D shows synthetic data generated by convolving a zero phase Ricker wavelet ($f_c \approx 20$ Hz) with the reflectivity. In Panel E, spiking deconvolution is applied to the synthetic data to recover an estimate of the reflectivity. Panel F shows the recovered reflectivity estimate after applying the model-based algorithm presented in this paper. Panels G and H show the results of thresholding and numerically integrating the recovered reflectivity estimates. Specifically, Panel G shows the velocity perturbation model that is recovered from the reflectivity estimate in Panel E, the spiking deconvolution. Panel H shows the velocity perturbation model obtained from the estimated reflectivity shown in Panel F, the iterative model-based deconvolution. The recovered velocity perturbation shown in Panel H more closely matches the original binary velocity given in Panel B.

materials and a wavefield with a central frequency of 20 Hz, roughly the first 4–6 s of the data adheres to the single scattering assumption, if multiples are not an issue (see Flatte *et al.* 1979; Hurich 1996). Therefore, for this work, approximately 5 s of each seismogram are selected after the first arrival.

Because the method presented here is built upon previous work (specifically, Poppeliers & Levander 2004), a brief review of this previous algorithm is shown here. The algorithm presented by Poppeliers & Levander (2004) relied on deconvolution to provide an estimate of the complex reflectivity of a binary velocity field with an assumed von Kármán autocorrelation. The modification presented here is to iteratively update the spectrum of the deconvolution filter by the spectrum of the previously estimated, non-white, von Kármán model.

To estimate a_z and v , the first step is to obtain an estimate of the reflectivity. Using the convolutional model, the seismic data $S(f)$ is

$$S(f) = R(f)W(f), \quad (20)$$

where $R(f)$ is the frequency domain reflectivity and $W(f)$ is the source wavelet. By rearranging the terms in eq. (20) the reflectivity

can be estimated by

$$\hat{R}(f) \approx S(f)\hat{W}(f)^{-1}, \quad (21)$$

where $\hat{R}(f)$ is an approximation of the true reflectivity $R(f)$ and $\hat{W}(f)^{-1}$ is the deconvolution filter designed from an estimate of the illuminating wavelet. Eq. (21) is essentially spiking deconvolution, which assumes white reflectivity, such that $|R(f)|^2 = 1$. The operation in eq. (21) is numerically unstable, so a small amount of white noise is commonly added to the denominator (Yilmaz 1987).

To estimate vertical stochastic parameters, convert the data to depth using a background average (P -wave) velocity v_0 . This depth conversion is actually a rescaling of the data from time to depth according to an approximate 1-D velocity estimate and can introduce a small error in the estimated scales of the velocity heterogeneities. However, this error will be relatively small so we neglect this effect. Take the inverse Fourier transform of eq. (21), and apply a thresholding criterion

$$\hat{r}_T(z) = \begin{cases} 1, & \hat{r}(z) > T_R \\ 0, & T_R \geq \hat{r}(z) \geq -T_R \\ -1, & \hat{r}(z) < -T_R, \end{cases} \quad (22)$$

where z is depth and T_R is some arbitrary fraction of the maximum amplitude of $\hat{r}(z)$ (typically, 50 per cent). The binary thresholding constraint is imposed in recognition of the (assumed) binary velocity distribution.

The amplitudes of the reflections that are thresholded by eq. (22) are related to the rms velocity distribution of the binary velocity field. In noise-free data, thresholding essentially separates out any information on rms velocity so the inversion can focus on the spatial correlation properties. This method does not invert for rms velocity variation; thus when the data contain noise, the amplitude information arising from reflections are competing with the amplitude of the noise. Therefore, when the data contain noise, the thresholding operation can erroneously emphasize amplitudes that do not arise from reflections. The implication is that velocity fields with higher rms velocity distribution give rise to data that will provide more robust results with this inversion in the presence of noise. Velocity fields with small rms velocity variation will produce reflections with amplitudes that can approach those of the noise, rendering unreliable results.

If we assume that $\hat{r}_T(z) \approx r(z)$, then we can combine with eq. (8) to give

$$\frac{d}{dz}\delta v(z) = v_0\hat{r}_T(z), \quad (23)$$

where v_0 is the average background velocity. Integrating with respect to z yields an estimate of the binary velocity perturbation:

$$\delta v(z) = v_0 \int_0^z \hat{r}_T(z) dz. \quad (24)$$

To obtain an estimate of the von Kármán stochastic parameters for a given trace at location x_0 , we first compute the autocorrelation of $\delta\hat{v}(z)$, given by

$$C_{\delta v}(\tau, x_0) = \int_{\tau=0}^N \delta v(z - \tau)\delta v(z) d\tau, \quad (25)$$

where τ is the lag of a function N samples long. Then minimize the function

$$E_i(x : a_z, v) = \frac{1}{N} \left\{ \sum_{\tau=1}^N [C_{\delta v}(\tau, x_0) - C_{hh}(\tau : a_z, v)]^2 \right\}, \quad (26)$$

where $C_{hh}(\tau : a_z, \nu)$ is a theoretical von Kármán autocorrelation function parametrized by a_z and ν , and N is the number of points in the autocorrelation function. Eq. (26) is minimized by performing a grid search over reasonable values of a_z and ν , where N is at least 100 samples. Where E_i is a minimum, a_z and ν are recorded. Although a grid search is not optimum in terms of computational performance, it is robust and relatively straightforward to implement.

The deconvolution in eq. (21) operates under the assumptions of white reflectivity and a minimum phase illuminating wavelet (Ziolkowski 1984; Yilmaz 1987). Converting data to minimum phase is a relatively straightforward process via the Hilbert transform, but reflectivity in the Earth is not necessarily white (Kelemen *et al.* 2000; Braun & Kelemen 2002). For reflections from a velocity field with von Kármán spatial statistics, the reflectivity power spectrum $|R(f)|^2$ is modulated by a power-law decay of the stochastic velocity model. Therefore, we need a different approach to account for a coloured velocity/reflectivity spectrum of the Earth. An iterative approach to modify the power spectrum of the deconvolution filter by the recovered reflectivity model is outlined in the following paragraph.

Because this method is inverting for spatial statistics, we switch from the frequency (f) to the wavenumber (k) domain. Therefore, the seismic data are rescaled to depth ($s(z)$) prior to the application of this method. Eq. (20) can then be restated as

$$S(k) = W_0(k)R(k), \quad (27)$$

with a corresponding power spectrum

$$|S(k)|^2 = |W_0(k)|^2 |R(k)|^2. \quad (28)$$

The initial assumption of white reflectivity means that $|R(k)|^2 = 1$; thus

$$|S(k)|^2 = |W_0(k)|^2. \quad (29)$$

The power spectrum of $|W_0(k)|^2$ is stored for construction of the deconvolution filter. By taking the inverse Fourier transform of $|W_0(k)|^{-1}$ we obtain an operator $\hat{w}_0^{-1}(z)$ that, when applied to the seismogram, recovers the reflectivity (Ziolkowski 1984; Yilmaz 1987). The operator $\hat{w}_0^{-1}(z)$ is commonly referred to as a deconvolution filter, or an inverse filter. A minimum phase condition is imposed upon the operator $\hat{w}_0^{-1}(z)$, which is the only phase information contained in $\hat{w}_0^{-1}(z)$. The first estimate of the reflectivity $\hat{r}_0(z)$ is given by convolving $\hat{w}_0^{-1}(z)$ with the data $s(z)$;

$$\hat{r}_0(z) = \int_{-\infty}^{\infty} \hat{w}_0^{-1}(z - \tau) s(\tau) d\tau. \quad (30)$$

After computing $\hat{r}_0(z)$, the thresholding (eq. 22) and numerical integration (eq. 24) operations are applied to obtain an estimate of a binary velocity perturbation model $\delta\hat{v}(z)$. From $\delta\hat{v}(z)$, estimates of a_z and ν are obtained using equation 26 and the scaling constants for a binary velocity field. From the estimates of a_z and ν , a new reflectivity power-spectrum model $|R_1(k)|^2$ is formed which is given by

$$|R_1(k)|^2 = \frac{-4\pi^2 k^2}{\nu_0^2} \delta v(k, a_z, \nu), \quad (31)$$

where δv is a velocity field with von Kármán statistics parametrized by a_z and ν . $|R_1(k)|^2$ is used to modify the spectrum of the inverse operator

$$|W_1(k)|^2 = \frac{|W_0(k)|^2}{|R_1(k)|^2}. \quad (32)$$

Thus, the new inverse operator is

$$w_0^{-1}(z) = F^{-1} \left[\frac{1}{W_1(k)} \right], \quad (33)$$

where the notation $F^{-1}[\cdot]$ is shorthand for the inverse Fourier transform. By iterating through eqs (31)–(34), the spectrum of the deconvolution filter is successively modified to account for the non-white reflectivity spectrum. Numerical tests show that the estimates of a_z and ν typically stabilize to within a few per cent in three to five iterations.

RESULTS

Synthetic tests

In the first test (Fig. 2) a comparison is made between standard spiking deconvolution and the iterative, model-based approach in their ability to recover a 1-D synthetic (von Kármán) reflectivity model. Although the test in Fig. 2 is an idealized example, it serves to illustrate the methodology and validity of the new algorithm presented here. In Fig. 2(a), a 1-D synthetic stochastic generating field is constructed. The stochastic parameters of the continuous generating field are chosen so that when its partitioned into a binary velocity field (Fig. 2b), the stochastic parameters of the binary field are $a = 300$ m and $\nu = 0.3$. The amplitude of the binary velocity field is arbitrary. For details of constructing the binary field from the stochastic field, see Goff *et al.* (1994). The reflectivity is computed (Fig. 2c) and convolved with a zero phase Ricker wavelet (with a central frequency $f_c \approx 20$ Hz) to generate a synthetic seismogram (Fig. 2d). The synthetic seismogram shown in Fig. 2(d) is used to test two cases. In the first case, standard spiking deconvolution is used to estimate the reflectivity (Fig. 2e). For the second case, the iterative method is used to estimate reflectivity (Fig. 2f). The most obvious difference between the result obtained in panel 2E and that obtained in panel 2F is that the iterative, model-based approach more accurately recover the amplitudes of the original reflectivity model; ideally, the recovered reflectivity amplitudes will be a consistent, arbitrary, signed value. This is in contrast to the reflectivity amplitudes estimated from the spiking deconvolution approach, which are both over and underestimated. Both of the estimates of the reflectivity are thresholded and numerically integrated to produce the results seen in Figs 2(g) and (h). Ideally, the recovered binary velocity should be identical to the original velocity model (Fig. 2b). A comparison of the normalized autocorrelations of the estimated binary velocity models with the normalized autocorrelation of the actual velocity model is shown in Fig. 3. This comparison shows that the iterative method more accurately reproduces the autocorrelation of the actual velocity model. The autocorrelation of the binary velocity model estimated from the spiking deconvolution approach shows an autocorrelation that is not as wide (at small lags) as the autocorrelation of the actual model. This corresponds to a binary velocity model estimate that is biased towards smaller values of a_z .

Figs 2 and 3 illustrate that the relative amplitudes of the recovered reflectivity can effect the form of the estimated binary velocity profile, from which a_z and ν are subsequently determined. This is important and illustrates a potential weakness in the method presented here. The thresholding operation is highly sensitive to the relative amplitude of individual reflection events. Although the argument here is that the iterative method provides more consistent estimates of reflectivity, the presence of noise will greatly influence this. The rms velocity variation of the model will determine the amplitude of the reflected arrivals recorded in the seismogram. In the

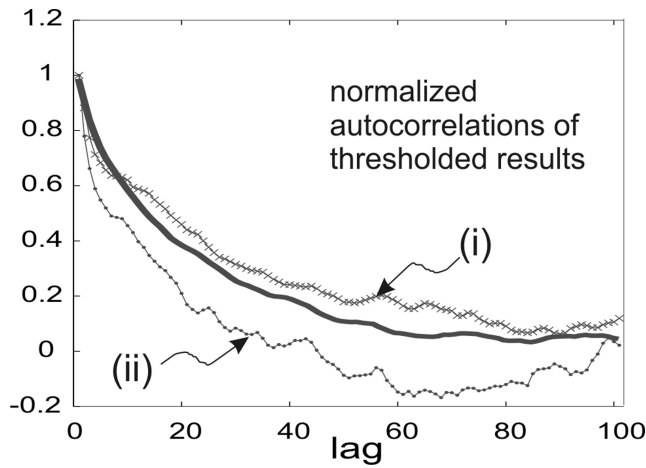


Figure 3. The normalized autocorrelations of the recovered velocity perturbations from Fig. 2. The curve labelled (i) is the autocorrelation of the binary velocity estimated from the new, iterative approach (Fig. 2, Panel H) and (ii) is the autocorrelation of the binary velocity estimated from the spiking deconvolution (Fig. 2, Panel G), and the heavy curve is the autocorrelation of the original binary velocity model shown in Fig. 2, Panel B. The iterative approach of recovering the binary velocity perturbations more accurately reproduces the form of the autocorrelation of the original, binary velocity model.

presence of noise, the relative amplitude of a given reflection will be stronger when the rms velocity variation is greater. Therefore, the method presented here will be sensitive to the rms velocity variations of the model only in that the rms variation of velocity will control the amplitude of the reflections relative to the amplitude of the background noise in the seismogram. Specifically, a greater rms velocity variation will produce stronger reflection events in terms of absolute amplitudes, which will better overcome the effects of noise. Conversely, a small rms velocity variation will produce reflection amplitudes that will more closely match the amplitude of the noise, in which case the data will contain a_z of both the reflections and the a_z of the noise.

More realistic tests are performed by constructing a 2-D velocity model with von Kármán statistics. This velocity model could represent a section of the Earth's crystalline crust and employs a 2-D von Kármán power spectrum, given by

$$P_{\delta v}(\mathbf{k}) = \frac{4\pi\nu}{K_\nu(0)} \frac{a_x a_z}{(1 + \mathbf{k})^{\nu+1}}, \quad (34)$$

where the boldface variable $\mathbf{k}$ indicates that the wavenumber is 2-D and K_ν is the modified Bessel function of the second kind and order ν . For further details on computing a von Kármán binary field, the reader is referred to Goff *et al.* (1994) and Holliger & Levander (1993). The model (Fig. 4) had dimensions of 30 km wide by 25 km deep, with a discrete grid spacing of 25 m. The top 5 km of the model was a homogeneous 6 km s⁻¹ and the stochastic velocity portion of the model (5–25 km depth) was characterized by a binary velocity field with a von Kármán autocorrelation with stochastic parameters of $a_x = 1000$ m, $a_z = 300$ m and a Hurst exponent of $\nu = 0.3$. These stochastic parameters refer directly to the binary field. Velocities of 6.0 and 6.3 km s⁻¹ were assigned, which are values typical for the crystalline crust (Holliger *et al.* 1994; Holliger & Levander 1994; Levander *et al.* 1994a,b). Although a binary velocity distribution is not geologically realistic everywhere, it allows for convenient model construction, reduces the number of free parameters that can affect the inversion, and faithfully represents a number of lower crustal ex-

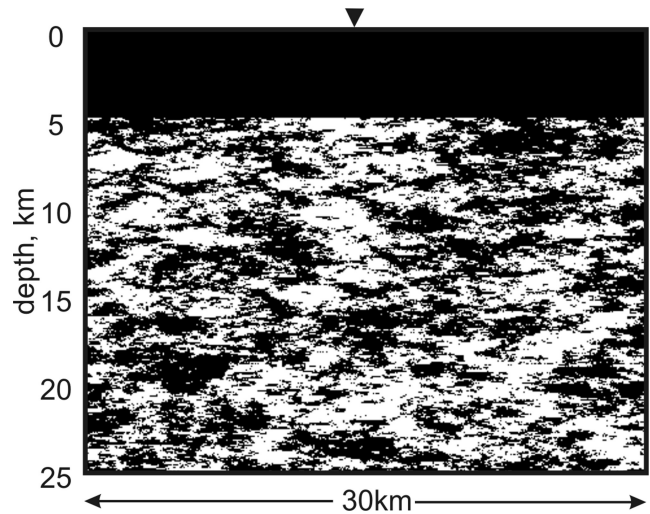


Figure 4. The velocity model used to produce synthetic data for the 2-D tests. The black represents areas with a velocity of 6.0 km s⁻¹, and the white represents areas with a velocity of 6.3 km s⁻¹. The velocity field contains a discrete distribution with von Kármán spatial statistics, where $a_x = 1000$ m, $a_z = 300$ m, and $\nu = 0.3$. The location of the single shot is indicated by the inverted triangle.

posures (Holliger & Levander 1992; Holliger *et al.* 1994; Levander *et al.* 1994a,b). Although the model used to generate the synthetic data is 2-D, the inversion is still only 1-D. In a sense, inverting for a_z and ν is similar to deconvolution of seismic data; even though the model is multidimensional, a given seismogram is treated as a 1-D time-series and the algorithm works on the seismogram in this single dimension. The synthetic data for this test are generated via a 2-D finite difference approach to more closely simulate actual data that would be collected as part of a field survey.

A finite difference solution to the acoustic wave equation (second order in time and fourth order in space) is used to generate a synthetic shot record; the source was a zero phase Ricker wavelet with a central frequency of 20 Hz located at the surface ($z = 0$) of the model. A 20 Hz wavelet was used as it contains wavenumbers that are also present in the statistics of the velocity model. The reflections were recorded at the surface by 80 receivers located 25 m apart where the offsets ranged from 0 to 2 km. The finite difference model employed absorbing boundaries on all sides, thus minimizing the effects of free-surface multiples. Multiple reflections must either be removed from the data or the inversion presented here is inappropriate. After the synthetic data are computed, an automatic gain control (AGC) operator is applied, an all-pass minimum-phase filter is applied, and the data are re-scaled to depth using an average 1-D background velocity. The AGC operator helps spiking deconvolution to be more numerically stable as well as helping to mitigate the effects of geometrical spreading on the wavelet's shape (Ziolkowski 1984). By applying an AGC operator, we are essentially separating out the amplitude of the local velocity fluctuations, and as a result, the rms velocity variation. Estimates of a_z and ν are obtained on a trace-by-trace basis using windowed trace lengths of 3.0 km. The resulting estimates of a_z and ν are recorded in their relative spatial positions and presented in Fig. 5. Although the inversion is inherently 1-D, the results for many traces are shown in Fig. 5 to illustrate the degree of variability of the results.

Fig. 5 shows results from two tests. First, a_z is estimated without any iterative modification of the deconvolution operator (i.e. zero iterations), with average estimated values of $a_z = 251.8 \pm 151.0$ m

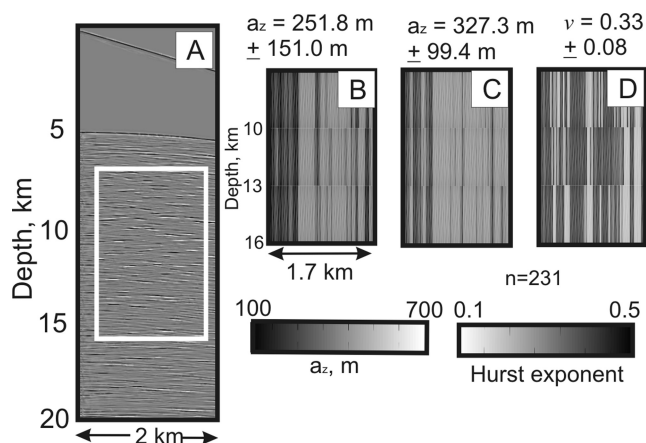


Figure 5. A test on finite difference data. Panel A shows a single synthetic shot produced for the velocity model shown in Fig. 4. The data were produced by solving a finite difference approximation (second order in time and fourth order in space) of the acoustic wave equation. The data are shown with AGC applied and the time axis is re-scaled to depth using the (known) 1-D average velocity profile. The heavy white box indicates the data used in the inversion. Panel B shows the results of estimating a_z without any iterative modifications to the deconvolution filter (i.e. zero iterations), without using the known wavelet to deconvolve the data. The average estimate of a_z , as well as the variability (defined as one standard deviation about the mean of $n = 231$ estimates of a_z per panel) is shown immediately above the panel. Panel C shows the result of estimating a_z using the iterative method presented in this paper. Panel D shows the estimated values of the Hurst exponent using the iterative method.

(Fig. 4b). The variability (the plus-or-minus term) is defined as one standard deviation about the mean of 231 estimates of a_z . For this case, stable estimates of v were not obtained and are not shown. The second test shows the results of estimating a_z with the new iterative approach, with average estimates of $a_z = 327.3 \pm 99.4$ m and $v = 0.33 \pm 0.08$ (Figs 4c and d). This test shows that the results obtained from the iterative approach are more accurate, with a lower degree of variability than for the non-iterative approach.

The algorithm presented here makes an explicit assumption of vertically incident reflections, however with a 2-D seismic survey this assumption is violated, especially for large offsets. Fig. 5 shows that as the offset of the shot-receiver pair is increased, the estimated value of a_z increases and the estimated value of v decreases. The seismograms at far offsets have a greater degree of attenuation of high frequencies than seismograms at near offsets. This will effect the estimation of both a_z and v , as the estimated corner wavenumber is explicitly linked to the high frequency components of the seismogram. An additional complication not considered in this test (because this synthetic data is acoustic) is that seismograms at far offsets can contain P to S conversions. However, this algorithm is not designed to recover stochastic parameters of far-offset data and illustrates an important limitation of this algorithm: only near-offset data should be used.

In the absence of noise and interbed multiples in the data, the returned estimates of a_z and v will be identical for various values of the rms velocity variation. The thresholding operation assumes a binary field and will return the same estimate of the stochastic velocity perturbation regardless of the rms variation of the binary velocity. Therefore, for noise free data applying an AGC operator will not effect the results of the returned velocity perturbation estimate. However, in the presence of noise, the rms velocity variation can have a significant effect on the final results, as the amplitude

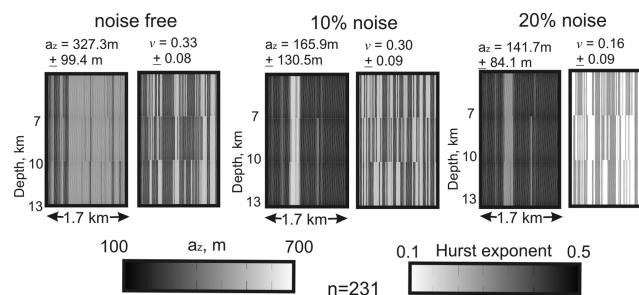


Figure 6. Noise tests on the data presented in Fig. 5. The two panels on the left are reproduced from Fig. 5, and show the recovered values of a_z and v for noise free synthetic data. The two panels in the middle show the recovered values of a_z and v for the same synthetic data where noise with an rms amplitude of 10 per cent of the data's normalized rms amplitude has been added. For each panel, 231 individual estimates of a_z and v were obtained. When the data contains 10 per cent noise, the recovered value of a_z is significantly reduced, whereas the recovered value of v is only slightly affected. The two panels on the right show the recovered values of a_z and v when 20 per cent noise is added. These simple noise tests show that for noise that has a similar frequency content to that of the data, the recovered value of a_z is reduced, and if enough noise is present, the recovered value of the Hurst exponent is also reduced.

of the reflections relative to the amplitude of the noise will be a function of the rms velocity variation in the binary field.

To illustrate the affects of noise on this inversion, noise was added to the shot record shown in Fig. 5. The noise was constructed by producing a random-phase time-series with a frequency range of five to twenty Hertz (which is the frequency range of the synthetic data), with a relatively flat spectrum between these two frequencies. The rms amplitude of the noise was scaled to be 10 per cent of the rms amplitude of the data for the first test, and 20 per cent of the data in a second test. The iterative algorithm was used to estimate a_z and v for the noisy synthetic data in the same manner as in the previous test (Fig. 6). Predictably, the results of the inversion are degraded by the presence of noise, as the noise are being treated as data. As the relative amplitude of the noise rises, the estimated values of a_z and v are reduced. Therefore, for this type of test, noise in the data causes the inversion to recover stochastic parameters which correspond to a rougher velocity model with a smaller characteristic length. The inversion will treat the noise as data, so the final estimate of a_z and v are a combination of the a_z and v of the noise and the a_z and v of the data.

The presence of noise in the data has a large effect on the recovered value of a_z however, the recovered value of the Hurst exponent is not as sensitive. The characteristic length is likely to be more sensitive to noise due to the form of the power spectrum of a von Kármán model. The characteristic length of a von Kármán model corresponds roughly to a corner wavenumber, which is defined by $k = 1/a$. The addition of noise in these tests shifts the corner wavenumber of the recovered velocity model's power spectrum to the right on a power-spectrum plot, or towards higher wavenumbers, which correspond to estimates of smaller characteristic length. In other words, as more noise is added to the data, the power spectrum of the noisy data becomes flatter. When the power spectrum of the noisy data is perfectly flat, the data become white (and a approaches zero). Because the noise spectrum is relatively flat in the frequency range of the data, the noise only slightly alters the slope of the roll-off in the spectrum of the recovered velocity model: for this test, the noise is relatively devoid of wavenumber components in the non-white portion of the data. However, for large amounts of noise, the

recovered Hurst exponent is biased towards higher values, corresponding to rougher models.

The noise tests serve to illustrate how the data can be affected by the amplitude of reflections relative to the amplitude of noise. In a sense, the noise tests presented in Fig. 6 are analogous to testing the effect of differing rms velocity variation. However, this algorithm does not directly estimate rms velocity variation. Rather, it is affected by the amplitude of reflections relative to the amplitude of background noise, which is a function of rms velocity variation. Where the velocity model has a relatively small rms velocity variation, the absolute amplitude of the reflections will be low relative to the amplitude of the noise, which corresponds to data with a low signal-to-noise ratio (SNR). The results of inverting such data can produce erroneous estimates of a_z and ν . A velocity model with a larger rms velocity variation will yield high-amplitude reflections, which for a given amplitude of noise, will mean that the data will have a higher SNR so the inversion will return estimates of stochastic parameters closer to that of the actual velocity model. The results shown in Fig. 6 indicate that the iterative approach presented in this paper is best used on data with a relatively high SNR or with data that is obtained in areas of high rms velocity variation.

An additional factor to consider is that for excessively high velocity contrasts, internal reverberations in the data can become problematic. For the test cases presented here, the binary velocity values of 6.0–6.3 km s⁻¹ were similar enough to minimize interbed multiples. However, where there exist high-amplitude reverberations, the inversion presented here can return erroneous results because of the interference from reverberations altering the data's spectrum. In this case the application of predictive deconvolution can be helpful; however, fully testing this is beyond the scope of this paper.

For the tests in this paper, the value of the thresholding operator T_R was arbitrarily set at 50 per cent of the maximum normalized amplitude of a given trace and this accurately estimated the value of a_z . However, reducing the value of T_R will yield wider regions of just one binary value or the other, and the estimated a_z will be too high. Conversely, setting T_R to a large value will produce separated, narrow peaks yielding estimates of a_z that are too small. Thus, calibration in terms of the parameter T_R is required. For the idealized cases presented in this paper a T_R of 50 per cent accurately (and repeatably) estimated a_z and ν . However, the effects of the magnitude of T_R on the recovery of a_z and ν , especially in the presence of noise, is the subject of future research.

APPLICATION TO FIELD DATA

To demonstrate the applicability of this method to field data, I use a single shot record (shot SP4) collected in the 1986 PASSCAL Basin and Range deep crustal Experiment (see fig. 2 of Benz *et al.* 1990, for the location of the shot) (PASSCAL Working Group 1988; Holbrook 1990). The shot record contains 96 channels with a maximum offset of approximately 9 km with an average trace interval of approximately 100 m. The data recorded crustal reflections produced from a large (100s of kg) chemical explosion detonated near the surface. Although the crust in this area may not contain a strictly bimodal velocity distribution, it is mostly igneous and so this data set should make a good test as igneous crust often contains discrete velocity populations (Benz *et al.* 1990; Holliger & Levander 1992, 1993).

For estimating the vertical stochastic parameters, the data would ideally be vertical incidence. Therefore, traces that had offsets ranging only from 0.5 to 2.0 km were selected. The nearest offsets were

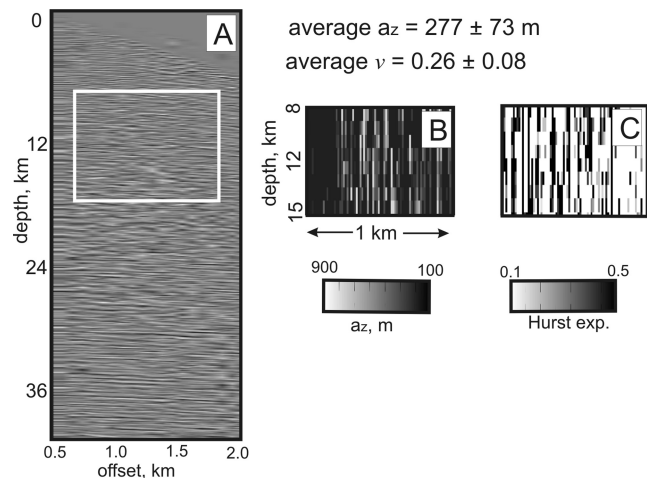


Figure 7. PASSCAL shot data and inversion results. Panel A shows the data from shot SP4, which was collected in Bueno Vista valley, Nevada (see Holbrook 1990, for details). Processing of this data includes AGC, filtering, and re-scaling to depth with 1-D velocity profile determined from velocity analysis. Normal moveout is not applied. The heavy white box shows the data analysis region. Panel B shows the results of estimating a_z with our method and Panel C shows the estimates of the Hurst exponent ν . The scale is similar for all three panels.

discarded to avoid complex arrivals arising from near-source effects. Both a_z and ν were estimated on a trace-by-trace basis, for windowed traces of 2.0 kilometers in length, where the windows overlapped by 0.25 km. The average estimated value of a_z for this data are $a_z = 277 \pm 73$ m (Fig. 7), which are comparable to the average, measured characteristic length of other observed crystalline crustal sections (Holliger & Levander 1994; Holliger *et al.* 1994; Levander *et al.* 1994a,b). The average estimated value of $\nu = 0.26 \pm 0.08$ appears to be slightly low when compared to those observed in crustal exposures, where the Hurst exponent was generally about 0.3.

Based on the noise tests shown in Fig. 5, the noise in the Buena Vista data will bias the estimates of a_z and ν . We can estimate the degree of noise in the data by comparing the average rms amplitude of the data prior to the first-arrival to the average rms amplitude of the data after the first arrival. Performing this computation shows that the average rms amplitude of a trace after the first arrival is about 15 times greater than the average rms amplitude prior to the first arrival. This corresponds roughly to data containing 6 per cent noise. In Fig. 5, we see that 10 per cent noise decreases the estimate of a_z by approximately a factor of two. If we adjust our estimate of a_z for the Buena Vista data to account for 6 per cent noise, and we assume that the bias of the amount of noise on the estimate of a_z and ν is roughly linear, we arrive at an estimate of a_z of approximately 550 m with a ν of approximately 0.25. These values of a_z and ν are reasonable for crystalline crust.

DISCUSSION AND CONCLUSIONS

This paper presents a new method to invert seismic reflection data for vertical stochastic parameters. In the first attempt at solving this problem (Poppeliers & Levander 2004), a standard deconvolution algorithm is used to estimate model reflectivity and then an amplitude thresholding constraint is imposed to recover an equivalent bimodal velocity perturbation model. However, this simple approach made the assumption of a white reflectivity spectrum, which is an

assumption made in many standard deconvolution recipes. The improvement presented in this paper is the iterative modification of the spectrum of the deconvolution filter by the spectrum of the recovered stochastic velocity perturbation model. The result of this model-based deconvolution is a more numerically stable estimate of the binary velocity perturbations, which leads to more consistent estimates of a_z and ν . However, we must make an assumption of a non-white, binary, statistical model of the velocity/reflectivity. In the event that the velocity spectrum is truly white, our approach here would likely not be any more beneficial than a strictly 'white' deconvolution approach.

Based on results from inverting synthetic data, the new iterative approach presented here produces estimates of a_z and ν that are more accurate and contain less scatter than the previous algorithm, which was presented in Poppeliers & Levander (2004). Additionally, the new method produces reliable estimates of vertical characteristic length a_z without a reliance on an accurate knowledge of the source wavelet. This is an improvement over the earlier algorithm, which was quite sensitive to the estimate of the source wavelet. Additionally, the new method is able to produce an estimate of ν , something the earlier method was unable to do reliably.

Numerical tests show that the inversion is quite sensitive to noise; when the rms amplitude of coloured noise is 10 per cent of the rms amplitude of the data, the estimates of a_z decrease by almost a factor of two, because the results become weighted by the a_z of the noise. The estimate of the Hurst exponent ν is affected much less; the estimate of ν only decreased by 1 per cent when 10 per cent noise was added to the synthetic data. When the rms amplitude of the noise is 20 per cent the rms amplitude of the data, the estimated value of a_z decreases by a factor greater than two, and the estimated value of ν is decreased significantly. The sensitivity of this inversion to noise suggests that it is most appropriate for high-quality data. A test of pre-first-arrival amplitudes to post-first-arrival amplitudes can give one a crude estimate of the relative amount of noise in the data. One can adjust the recovered values of the stochastic parameters based on the measured noise in the data, but this is a rather *ad hoc* method. As with any inverse method, one must make a judgment about the quality of the data and the resulting reliability of the results.

Noise in the data affects the relative amplitude of the recovered reflectivity, thus causing the thresholding operation to return inaccurate results. The SNR of the data is a function of the rms velocity variation. A high rms velocity variation will produce high amplitude reflections, and thus the SNR of the data will be higher than where the rms velocity variation is low. This algorithm is highly sensitive to noise in the data, with dubious results from data with a large amount of noise. Because this algorithm cannot estimate rms velocity variation, it is important to assess the SNR of the data a priori. Also, if the velocity model is not strictly binary, the data would show relative amplitude variations for individual reflection events, arising from changing impedance contrasts, in which case this inversion is not appropriate.

An implicit assumption that we make is that the data are only weakly scattered, with near-zero incidence angles. For example, data that are appropriate for the inversion would be of the same quality (and geometrical limitations) as data that are used for small-offset seismic reflection imaging, so long as the data is weakly scattered. For data that are strongly scattered, this method is not appropriate. Also, data with multiples are not appropriate for this inversion, as the single-scattering assumption is violated. Application of predictive deconvolution can be helpful to attenuate prominent, interbed multiples. At large source-receiver offsets, the increasing

strength of P to S conversions would alter the spectrum appreciably, degrading the estimates of a_z and ν . Finally, applying the inversion to field data returned stable estimates of vertical stochastic parameters, with magnitudes that are similar to those of observed crustal outcrops. This result provides promise for a new tool for interpreting seismic data that is collected over complex geological terrains.

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