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Performance of Different Regression Procedures on the Magnitude Conversion Problem

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Abstract Through an extensive set of simulations we investigate the performance of different linear regression procedures commonly used to convert magnitudes from one type into another one, an operation that also has strong influence on the slope of the frequency-magnitude (the *b*-value of the Gutenberg–Richter) distribution. It has already been demonstrated that a general orthogonal regression provides the most reliable results. However, questions arise when the ratio between the variances of the magnitudes to be related (the knowledge of which is required to apply the general orthogonal regression) cannot be computed.

We therefore systematically investigate the biases introduced by the classical standard least-squares regressions and the orthogonal regressions (or similar procedures) as a function of the true slope between magnitudes, of the ratio η between magnitude variances, and of the absolute variances of magnitudes. We compute such biases through simulations very close to the real cases inferred from the German and Chinese broadband networks.

We observe that for $0.7 < \sqrt{\eta_{\mathrm{true}}} < 1.8$ the orthogonal regression under the $\eta = 1$ assumption performs better than standard regressions. For values outside this interval neither procedure is capable of correct estimates. Therefore it is recommended to estimate the absolute errors and their ratio from empirical data and apply the general orthogonal regression. This requires that a seismological data center publish average estimates of event magnitudes and also their related standard deviations. Regrettably, this is not yet a common practice, thus impeding the derivation of optimal magnitude conversion relations.

Online material: Graphics illustrating the performance of different regression procedures on magnitude conversion.

Introduction

The large number of existing earthquake size estimators is due to the use of a variety of instruments, instrumental responses, and wave types. Presently there are at least four commonly used estimators: M_L, M_S, m_b, and M_w, besides a variety of other estimators often based on local procedures of computation. These estimators, although mutually related, are not identical and emphasize different aspects of the seismic source process. They are therefore complementary and all necessary for better understanding source complexity and differences in radiated seismic spectra. It follows that it is mandatory to keep all available magnitude estimators in seismic catalogs and to understand their mutual relationship.

On the other hand, when using seismic catalog data as input for probabilistic seismic hazard assessment, a single-magnitude value per event usually is employed to indicate the earthquake size. Presently, the most common indicator is the moment magnitude M_w, although it does not retain any direct information on frequency content, which would instead be essential for a realistic assessment of the hazard of structural damage due to ground shaking. The preference given to M_w is often due to the unjustified claim that it is the only physically well-defined magnitude. M_e (Choy and Boatwright, 1995) is equally well defined and scaled directly with seismic energy, which is highly relevant to the seismic-hazard assessment (Bormann, 2002, pp. 37–38, 50–57), whereas M_w is scaled with the earthquake size through the scalar seismic moment M_0. However, direct calculations of M_0, and thus M_w, are still exceptions, especially for earthquakes with magnitudes less than 6. In addition, M_w was introduced only in the 1980s. Therefore classical magnitudes, such as M_L, m_B, m_b, and M_s, are converted into M_w on the basis of empirical conversion relationships $M_w = M_x$.

This operation is of critical importance since biases introduced at conversion cause biases in the b-value of the Gutenberg–Richter frequency-magnitude distribution and thus in the hazard estimates (Castellaro *et al.*, 2006).

In converting magnitudes it is commonly assumed that the relation between magnitude data M_x and M_y is linear. This appears justified, as long as none of the magnitudes shows strong saturation. The latter would be the case, for example, with magnitudes determined in very different period ranges, such as m_b and M_S, with m_b showing strong saturation. If, however, both magnitudes are affected by saturation of the same order, then again their mutual relationship is expected to be linear within the limits of data variances and reasonable significance thresholds (Bormann *et al.*, 2007). When the assumption of linearity holds, the linear least-squares regression is still a widely used procedure to find the relationship between two magnitude types (Gutenberg and Richter, 1956; Gasperini and Ferrari, 2000; Gasperini, 2002; Bindi *et al.*, 2005; Braunmiller *et al.*, 2005). However, the application of the linear least-squares regression implies that error on the independent variable is negligible compared with the error on the dependent variable, but this is obviously not the case in magnitude conversions, because different magnitudes in general are affected by errors of comparable size.

Other regression procedures take into account the errors on both variables (Stromeyer *et al.*, 2002; Gutdeutsch *et al.*, 2005; Castellaro *et al.*, 2006). A main obstacle to their application is that they require the knowledge of the variance ratio (η) between the two variables, which is usually not known because this requires that several independent estimations of the size of a same earthquake, based on records at different stations, are available for both magnitude scales one wants to relate.

To overcome this problem η is traditionally set equal 1, relying on the assumption that error ratios of different magnitudes are approximately equal (Bormann and Khalturin, 1975; Bormann and Wylegalla, 1975; Ambraseys, 1990; Gusev, 1991; Panza *et al.*, 1993; Cavallini and Rebez, 1996; Kaverina *et al.*, 1996; Gutdeutsch *et al.*, 2002, 2005; Grünthal and Wahlström, 2003; Stromeyer *et al.*, 2004). However, it is a fact that when the calculation of η is possible, it may not be close to 1 and can easily be as low as 0.4 (Castellaro *et al.*, 2006; Bormann *et al.*, 2007).

Goal of the Present Article

The results of the erroneous use of the $\eta = 1$ assumption have been presented in Castellaro *et al.* (2006), as an application of the general case of datasets affected by errors and spanning ranges much larger than those encountered dealing with magnitudes and regression slopes $\beta_{\mathrm{true}} = 1$. In the present article we analyze the magnitude-regression problem in detail, applying the most common regression procedures to simulated datasets that respect the variable, error ranges, and distributions encountered in practice. We study the problem not only as a function of the variance ratio η but also as a function of the absolute size of the errors (σ_x and σ_y). This will help us in assessing the uncertainty threshold in the knowledge of η that can be afforded—as a function of the absolute error—to ensure that the common assumption $\eta = 1$ still yields acceptable magnitude conversion errors (<0.2 magnitude units [m.u.]). The applicability of the different regression procedures is explored in a wide range of possible slopes for real magnitude relations ($0.5 \leq \beta_{\mathrm{true}} \leq 2$). Finally, we outline the worst-case scenarios, that is how the different regression procedures may be affected when only small datasets are available.

In summary, this article aims at clearly defining the range of applicability of each regression procedure to minimize, especially at lower and higher magnitudes, the biases commonly introduced by misapplied regression relations. The theoretical treatment is omitted here and the reader should refer to Castellaro *et al.* (2006).

Ⓔ color figures available in the online edition of BSSA.

Regression Procedures

We will essentially consider two linear regression procedures (SR and GOR) and their two variants (ISR and OR) as defined in the following. We use the upper case letter (e.g., X, Y) to indicate the true value of a variable and the lower case letter (e.g., x, y) to indicate the value of the same variable affected by (measurement) errors. A computer program to compute SR, ISR, OR, and GOR has been made publicly available on Terraemoti (2007).

Standard Least-Squares Regression (SR)

This well-known linear regression procedure applies when the variable variances $\sigma_x^2 \to 0$ and $\sigma_y^2 > 0$. Under these assumptions, the line which best interpolates the experimental points is found by minimizing the square sum of the vertical distances between the experimental points (X_i, y_i) and their equivalents on the regression line (X_i, Y_i), that is,

$$\sum_{i=1}^{n} [y_i - \beta X_i - \alpha]^2 \tag{1}$$

in the unknowns, that is α and β. This leads to the prescription (Bormann *et al.*, 2007)

$$Y_i \leftarrow \beta_{\mathrm{SR}} X_i + \alpha_{\mathrm{SR}}, \tag{2}$$

where the left arrow indicates that equation (2) must be used only from right to left; that is, take a sample of X, multiply it by β_{SR}, add α_{SR}, and allocate it to Y. Taking a value of y to derive X is not allowed.

Inverted Standard Least-Squares Regression (ISR)

It is the equivalent of SR, which applies when $\sigma_y^2 \to 0$ and $\sigma_x^2 > 0$. Under these assumptions, the line that best interpolates the experimental points is found by minimizing the horizontal distance between the experimental points (x_i, Y_i) and their equivalents on the regression line (X_i, Y_i), that is,

$$\sum_{i=1}^{n}\left[x_i - \frac{Y_i - \alpha}{\beta}\right]^2 \quad (3)$$

in the unknowns, that is α and β. This leads to the prescription

$$Y_i \rightarrow \beta_{\mathrm{ISR}} X_i + \alpha_{\mathrm{ISR}}, \quad (4)$$

where the right arrow indicates that equation (4) must be used only from left to right. Taking a value of x to derive Y is not allowed.

General Orthogonal Regression (GOR)

This regression is designed to account for the effects of measurement errors on both linearly related variables. Let us define

$$\eta = \frac{\sigma_y^2}{\sigma_x^2}. \quad (5)$$

The general orthogonal regression estimator is the scaled squared Euclidean distance obtained by minimizing

$$\sum_{i=1}^{n}\left[\frac{(y_i - \alpha - \beta X_i)^2}{\eta} + (x_i - X_i)^2\right] \quad (6)$$

in the unknowns, that is α, β, X_i. This leads to the prescription

$$y_i = \beta_{\mathrm{GOR}}\, x_i + \alpha_{\mathrm{GOR}}, \quad (7)$$

which can be used in both directions, since GOR predicts Y from x and y.

Particular Orthogonal Regression (OR)

This is the special case of GOR that still takes into account the measurement errors on both variables but forces them to be equal, that is $\eta = 1$ in equation (5). The estimator to be minimized is the squared (not scaled) Euclidean distance of the point (x_i, y_i) from the line (X_i, $\alpha + \beta X_i$) (equation 6). This method is commonly used instead of GOR when true η is unknown or deliberately ignored.

Note on Terminology

The acronyms presented in this article follow the conventions used in Castellaro *et al.* (2006), of which this article represents the continuation. We note that in the literature SR and ISR are also called SR_1 and SR_2, respectively (e.g., Bormann *et al.*, 2007).

Note that in GOR the adjective "orthogonal" is somehow misleading. In fact, the distance of the experimental points (x_i, y_i) from the regression line which is minimized is the true orthogonal distance only when $\eta = 1$. In the other cases it is the Euclidean distance (when $\eta \neq 1$ axes are rescaled and equation 6 represents again the Euclidean distance). In the literature the orthogonal regression is sometimes also referred to as the method of moment estimators (Fuller, 1987).

Simulation of Magnitude Regressions

Since the performance of each regression procedure varies both with the ratio η between the variable variances and with their absolute values and, in addition, is a function of the slope β of the regression line, we run enough simulations to cover the ranges of

1. Slopes β,
2. Ratios η between variances,
3. Absolute values of errors σ_x and σ_y,

which may be encountered when converting magnitudes.

Generation of the Datasets

For each of the preceding items under consideration, we proceed as follows:

1. We generate 10^2 couples of magnitudes (M_x, M_y), with $3.5 \leq M_x, M_y \leq 9.5$. Magnitude data are sampled from exponential distributions with the B parameter (equation 2 in Castellaro *et al.*, 2006) set to $\log_e 10 = 2.3$. corresponding to $b \simeq 1$ in the Gutenberg–Richter relation. The choice of the exponential distribution derives from Utsu (1999), Kagan (2002a, 2002b, 2005), and Zaliapin *et al.* (2005).
2. We obtain the (m_x, m_y) by perturbing each (M_x, M_y) with a random error sampled from Gaussian distributions with deviations σ_x and σ_y. All magnitude data are rounded to a single decimal to be similar to published magnitude data (e.g., 4.3, 5.2).
3. We repeat items 1 and 2 10^3 times and perform the 10^3 SR, ISR, OR, and GOR regressions for each individual set of 10^2 (M_x, M_y). In this way we obtain the average β_{SR}, β_{ISR}, β_{OR}, and β_{GOR} as well as their standard deviations.

Figure 1 illustrates one of the 10^3 datasets produced for a single simulation. The 10^2 couples of magnitudes (M_x, M_y) sampled from the exponential distribution are represented by the stars. Data points after the Gaussian errors addition (m_x, m_y) are represented by the open circles. Results of each regression procedure are shown by the respective lines (see legend).

We chose to fix the number of couples for each realization to a relatively small value (10^2) to be close to the worst-case scenario. Sometimes many more couples of mag-

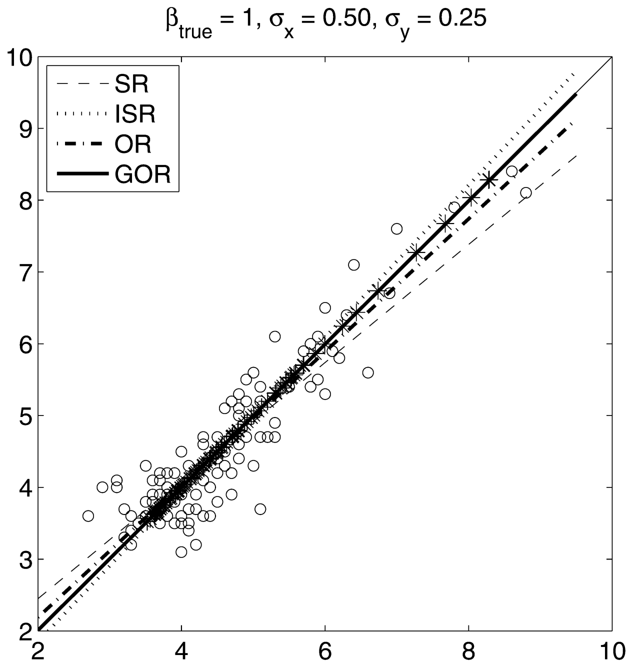

Figure 1. Each simulation starts from 100 couples of magnitudes (M_x, M_y) sampled from an exponential distribution with the B parameter set to $\log_e 10 = 2.3$ (equation 2 in Castellaro *et al.*, 2006) (stars). Gaussian errors are added to both M_x and M_y so that original data are shifted to the black open circles. In this example $\sigma_x = 0.50$, $\sigma_y = 0.25$. Results of each regression procedure are shown by the respective lines (see key). The average β and α values resulting from each regression procedure are derived from the average of 1000 simulations of which the one shown in this figure is only an example.

nitude data are available (e.g., Bormann *et al.*, 2007), but more frequently only a few (50–100) data are available to compute the regressions (e.g., Castellaro *et al.*, 2006). Thousand generations of each set of 10^2 couples ensure that we obtain a good estimation of the deviation of the average deviation of the regression parameters.

These average results would change very little by using larger datasets. The regression results from 10^3 realizations of 10^3 (m_x, m_y) points can be found in Ⓔ Figure 8 in the online edition of BSSA and in Terraemoti (2007). They can be compared with the figures we discuss subsequently. However, it can be seen that the standard deviation associated with the average values decreases considerably, as expected.

Determination of Realistic Regression Slopes

The regression slope between compatible types of magnitudes, determined from the same type of seismic waves with similar procedures and in comparable period and distance ranges, is expected to be approximately 1. But when comparing magnitudes calculated from different wave types (e.g., body and surface waves) at very different periods and with different saturation behavior at large magnitudes, then regression slopes may come close to 0.5 or 2 (compare Fig. 5 and 6 in Bormann *et al.*, 2007). Therefore we investigate the cases:

1. $\beta_{true} = 1$
2. $\beta_{true} = 0.5$
3. $\beta_{true} = 2$,

where $\beta_{true} = 1$ is the average condition and $\beta_{true} = 0.5$ and $\beta_{true} = 2$ are the end members.

Determination of Realistic Errors on Magnitudes

To define realistic magnitude errors and error ratios we studied two test datasets. The first was compiled by X. Ren from the China Earthquake Network Center (CENC) and the second one by S. Wendt from the Geophysical Observatory Collm (University Leipzig) using records of the 18 broadband stations of the German Regional Seismic Network (GRSN, 2007). These datasets comprise shallow earthquakes with magnitudes $4 < M_S < 9$ in a wide distance range between $18° < \Delta < 143°$. We determined the average teleseismic network magnitudes and the root-mean-square (rms) deviation of individual station magnitudes from the network averages. The rms values for individual earthquakes did not show clear magnitude or distance dependence. Therefore, for different types of magnitudes, we calculated the average rms (rms_{avg}) over the considered magnitude and distance range together with the variance of the individual event magnitude rms with respect to the rms_{avg}. We obtained the following results for the rms_{avg} for the following types of teleseismic body-wave and surface-wave magnitudes (cf., Bormann, 2007, for symbols meaning):

1. CENC: m_b, 0.35 ± 0.09; m_B, 0.28 ± 0.06; M_S, 0.23 ± 0.07; M_{S7}, 0.26 ± 0.10
2. GRSN: m_b, 0.19 ± 0.05; m_B, 0.13 ± 0.05; $M_{S(20)}$, 0.10 ± 0.03; $M_{S(BB)}$, 0.10 ± 0.04.

We observe that both the average error and the variance determined from different seismic networks may vary significantly even for the same type of magnitude. This can be due to differences in the instrumental response and measurement procedure, different accuracy in the instrumental calibration, and reliability in the application of measurement standards. Also different network size (local, regional, or global) and different topographic and geologic station site conditions may affect individual station magnitudes up to ±0.6 m.u.

In the specific case, the large differences between the Chinese and the German network magnitudes can be explained as follows (for details, see Bormann *et al.*, 2007): (1) GRSN data have been calculated by strictly applying the new standards defined by International Association of Seismology and Physics of the Earth's Interior (IASPEI) (2005), whereas the CENC data, collected earlier, were based on seismograph responses, measurement procedures, period, and time window ranges that deviate from these standards.

(2) Whereas all GRSN seismograph responses are known with high precision, the status of instrument calibration in the nationwide Chinese network was not yet comparably homogeneous and accurate in the years considered. (3) Although the GRSN data have been collected and measured at a central facility by a single analyst, the Chinese data have been determined by averaging station magnitude readings by many analysts. This also occurs with magnitudes published in regional or global earthquake catalogs by most data centers (e.g., NEIC and ISC). (4) The stations of GRSN cover a much smaller region than CENC, thus azimuthal variations of the amplitudes at GRSN should be smaller for a given earthquake. Therefore, we can assume that the data scattering described previously spans well the expected range of absolute errors and error ratios for standard types of magnitudes determined by nearly ideal networks (like GRSN) and under the more common average conditions still prevailing at many regional or global networks.

Accordingly, for each regression slope, in our simulations we fix:

1. σ_y to 0.05, 0.15, 0.25, and 0.50 and we study the performance of each regression procedure when $\sqrt{\eta} = \sigma_y/\sigma_x$ varies from 0.25 to 3.
2. σ_x to 0.05, 0.15, 0.25, and 0.50 and we study the performance of each regression procedure when $\sqrt{\eta} = \sigma_y/\sigma_x$ varies from 0.25 to 3.

Results

The final results of our simulations are shown in graphical form in Figures 2–4. For each simulated β_{true} ($\beta_{\mathrm{true}} = 1$, Fig. 2; $\beta_{\mathrm{true}} = 0.5$, Fig. 3; $\beta_{\mathrm{true}} = 2$, Fig. 4), the average β_{SR}, β_{ISR}, β_{OR}, and β_{GOR}, as well as its standard deviation, is shown for a number of σ_x, σ_y, and η. Other figures can be found in Ⓔ the online edition of BSSA or in Terraemoti (2007).

Discussion

The aim of this work is to provide quantitative guidelines for the choice of the best-fitting regression procedure in converting magnitudes. All our simulations clearly show, as predicted by theory, that GOR is always the best-fitting procedure, which estimates slopes very close to β_{true} even in

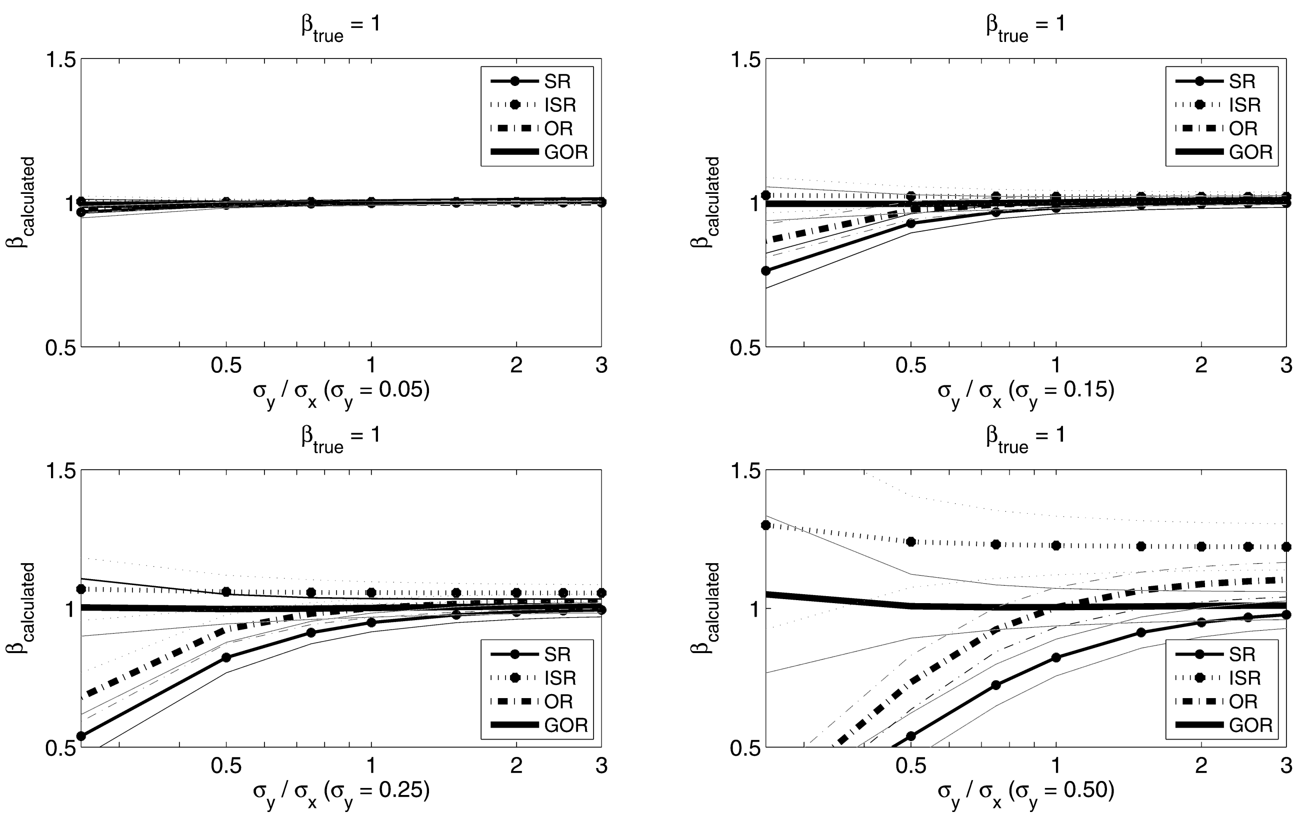

Figure 2. Comparison of the performance of the regression procedures described in the text (SR, ISR, OR, and GOR). A thousand datasets of 100 (m_x, m_y) data for each different σ_y/σ_x ratio were generated, keeping σ_y fixed to 0.05, 0.15, 0.25, and 0.50 (from top left to bottom right, 32 × 1000 realizations to obtain these four panels). The average β resulting from the different regression procedures are shown by the thick lines, their standard deviations are shown by the dotted lines. The simulation started from $\beta_{\mathrm{true}} = 1$. Note the logarithmic scale on the x-axis.

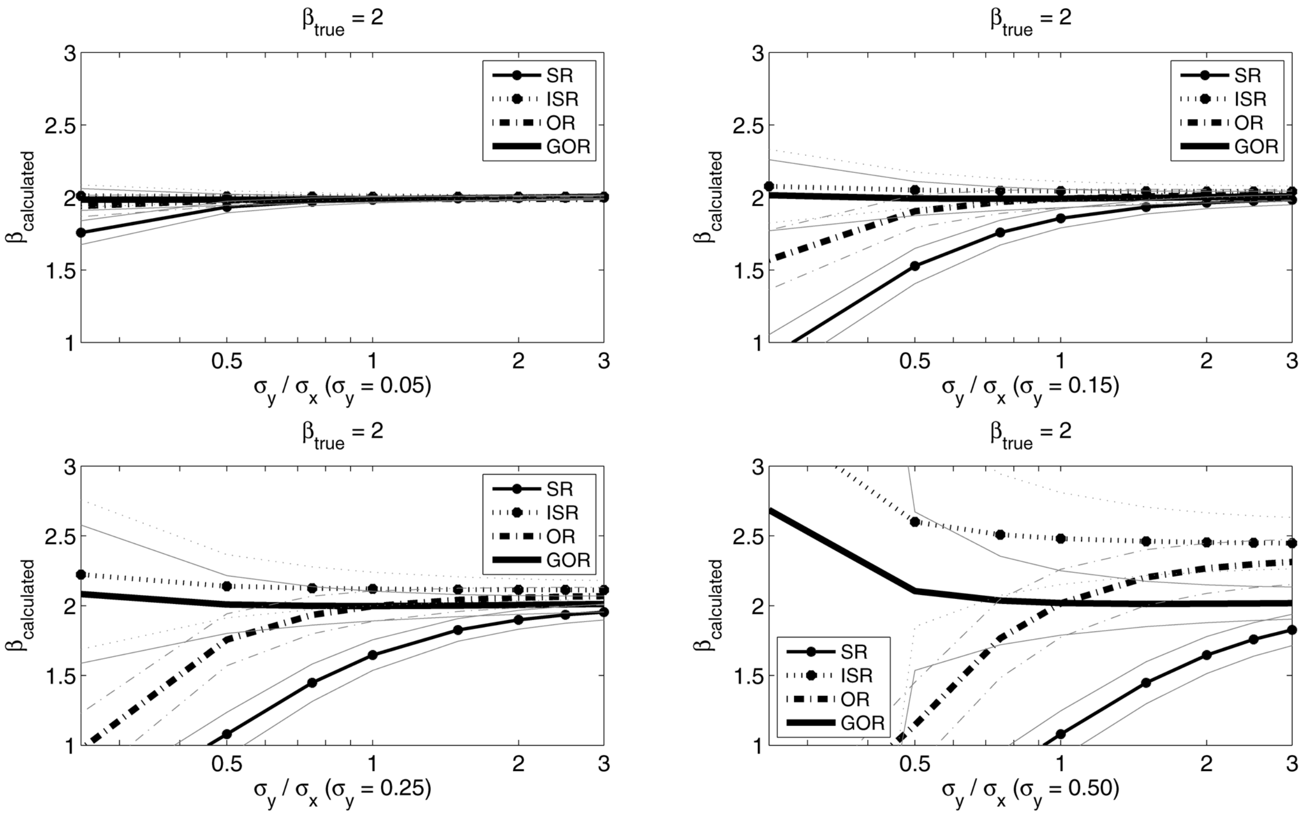

Figure 3. As in Figure 2 but with $\beta_{true} = 2$.

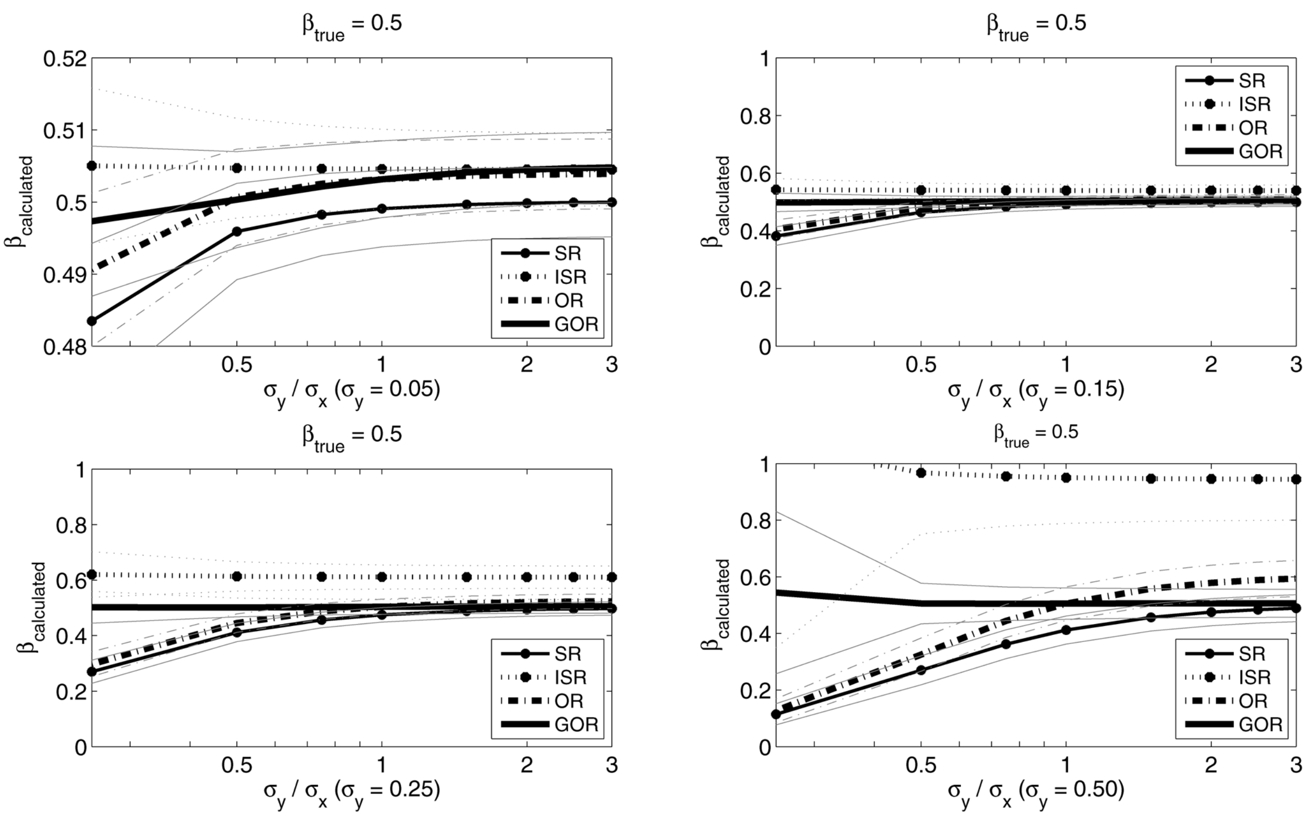

Figure 4. As in Figure 2 but with $\beta_{true} = 0.5$.

the presence of large σ_x and σ_y. However, the problem is that in most cases the variances of the magnitude values given in catalogs or compared in regression relations are not quantified or only roughly estimated. Therefore, it is necessary to state which type of regression among SR, ISR, and OR should be applied to minimize biases.

First, as stated by theory, we note that β_{OR} lies always between β_{SR} and β_{ISR} although this does not imply that β_{OR} is always closest to β_{true}.

The main results of our simulations are:

1. For very small σ_y and σ_x (less than 0.05–0.10) (top left panel in Figs. 2 and 5) all the regression procedures perform equally well within the range of acceptable conversion errors ($\leq$0.2 m.u.). A deterioration of the performances of either SR or ISR is expected if one of the two variances becomes much larger (twice) than the other. This effect appears earlier for $\beta_{true} \neq 1$.
2. For $\beta_{true} = 1$ and $\eta = 1$, then $\beta_{true} = \beta_{OR} = \beta_{GOR}$ and lies, for realistic magnitude errors, in the angular midst between β_{SR} and β_{ISR}. The angular separation between β_{SR} and β_{ISR} increases with the absolute values of σ_y and σ_x.
3. For equal variances on both axes ($\eta = 1$) but $\beta_{true} \neq 1$, the following holds: for $\beta_{true} < 1$, β_{SR} is closer than OR to β_{true} whereas for $\beta_{true} > 1$, β_{ISR} is closer to β_{true}.
4. For $\eta \neq 1$, β_{OR} is rotated away from β_{true} toward the axis with the larger variance (this is also clear in Fig. 1).

Considering magnitudes, apparently in most cases the regression slope spans the range $0.7 < \beta < 1.3$ (e.g., Console *et al.*, 1989; Gasperini, 2002; Castellaro *et al.*, 2006; Bormann *et al.*, 2007) and the magnitude errors span the range $0.1 < \sigma < 0.3$ m.u., which results in $0.3 < \sqrt{\eta} < 3$ (abscissa in Fig. 2–4, inverse of abscissa in Fig. 5).

From our simulations we observe that OR is the second best approximation for β_{true} (second only to β_{GOR}), leading to average conversion errors lower than 0.2 m.u. even for large magnitudes, provided that the error ratio between the variables is in the range $0.7 < \sqrt{\eta} < 1.8$ and the largest absolute standard deviation < 0.25 m.u. If $\sqrt{\eta} < 0.7$ and $\sqrt{\eta} > 1.8$, then ISR and SR, respectively, yield better conversion results than OR.

Deviations from the proposed range occur for larger and hardly realistic magnitude variances (e.g., $\sigma = 0.5$ and small or large β_{true}; e.g., bottom right panel in Fig. 3). In these cases OR performs better than SR or ISR for a wider range of η. The main conclusions are summarized in Table 1.

Worst-Case Scenario

The results discussed so far refer to the average of the many regression lines resulting from each set of simulations. This means that OR performs better than SR and ISR in the η and σ ranges given in the previous section but only when the α and β regression parameters of a considered (limited)

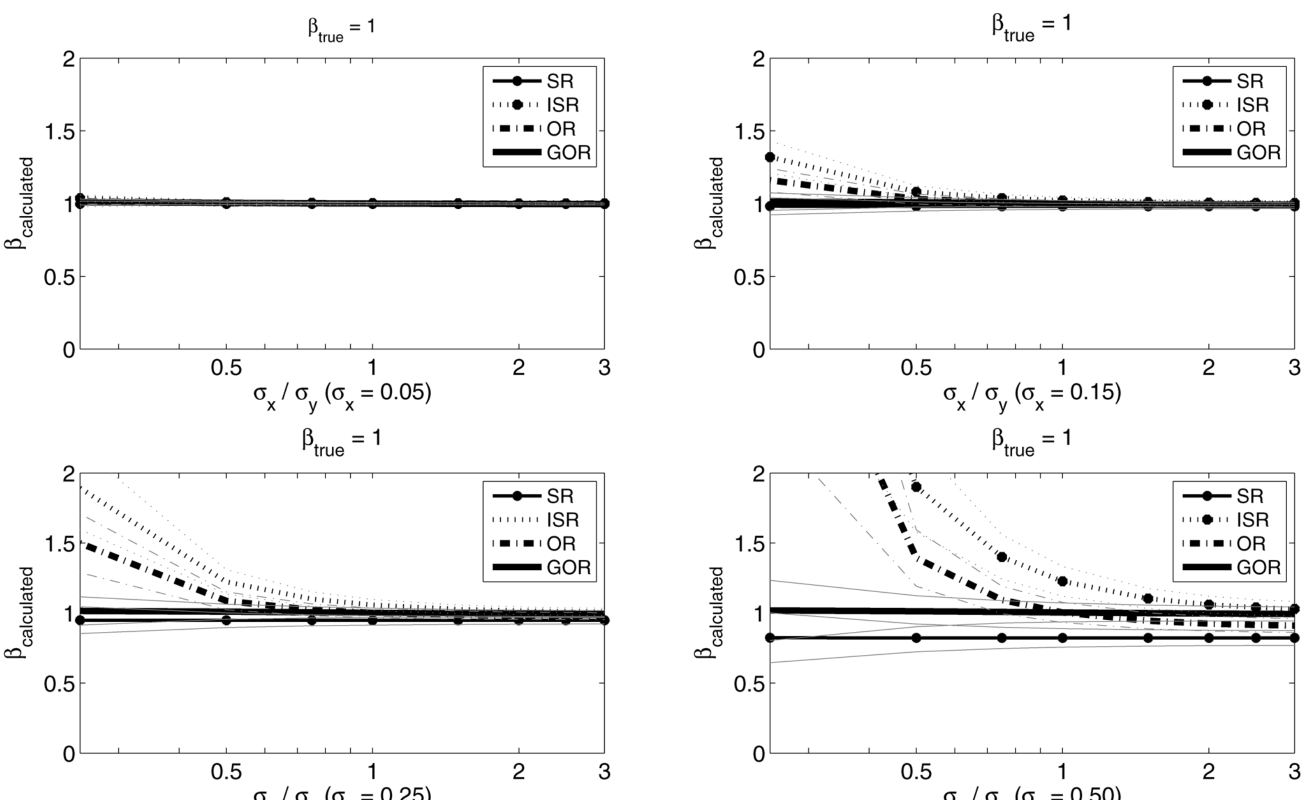

Figure 5. As in Figure 2 but with σ_x fixed to 0.05, 0.15, 0.25, and 0.50 (from top left to bottom right).

Table 1

Summary of the Best Regression Procedure for Different η and Largest Absolute Errors on the Variables in the Slope Range $0.5 \leq \beta_{\mathrm{true}} \leq 2$. The Maximum Average Conversion Errors Accepted Are $\leq$0.2 m.u.

Largest σ	$\sqrt{\eta} < 0.7$	$0.7 \leq \sqrt{\eta} \leq 1.8$	$\sqrt{\eta} > 1.8$
<0.1	SR $\simeq$ OR $\simeq$ ISR	SR $\simeq$ OR $\simeq$ ISR	SR $\simeq$ OR $\simeq$ ISR
0.1–0.25	ISR	OR	SR
0.25–0.5		← OR →	

Table 2

Conversion Errors of the Different Regression Procedures for the Extreme M 9 Value, When $\alpha_{\mathrm{calculated}}$ and $\beta_{\mathrm{calculated}}$ Fall at the Edge of the 2σ Interval of Values They Can Assume

	$\beta_{\mathrm{true}} = 1$ (I)	$\beta_{\mathrm{true}} = 1$ (II)	$\beta_{\mathrm{true}} = 0.5$ (III)	$\beta_{\mathrm{true}} = 0.5$ (IV)	$\beta_{\mathrm{true}} = 2$ (V)	$\beta_{\mathrm{true}} = 2$ (VI)
SR	0.35	0.35	0.30	0.25	0.17	0.21
ISR	0.41	0.37	0.51	0.33	0.18	0.21
OR	0.36	0.34	0.32	0.25	0.16	0.19
GOR	0.35	0.34	0.31	0.24	0.16	0.19

To illustrate the worst-case scenario, errors are inferred from 10^3 realizations of small datasets (10^2 couples (m_x, m_y)). I, $\sigma_y = 0.175$, $\sigma_x = 0.25$, $\sqrt{\eta} = 0.7$; II, $\sigma_y = 0.25$, $\sigma_x = 0.138$, $\sqrt{\eta} = 1.8$; III, $\sigma_y = 0.175$, $\sigma_x = 0.25$, $\sqrt{\eta} = 0.7$; IV, $\sigma_y = 0.25$, $\sigma_x = 0.138$, $\sqrt{\eta} = 1.8$; V, $\sigma_y = 0.175$, $\sigma_x = 0.25$, $\sqrt{\eta} = 0.7$; VI, $\sigma_y = 0.25$, $\sigma_x = 0.138$, $\sqrt{\eta} = 1.8$.

data sample fall within the average of their possible values. Figures 2–5 also show, together with the average β coefficients for each regression procedure (thick lines), their standard deviation (dashed lines), which increases with the absolute variable errors.

It is now useful to formulate the worst-case scenario, that is, to estimate the errors in terms of magnitude units that can be introduced by the different regressions when their parameters (α and β) fall at the edge of the respective 2σ interval. In Table 2 we summarize this scenario when the magnitude to be converted also has extreme values, that is, when M_x 9 for $\beta_{\mathrm{true}} = 1$ and 0.5, and M_y 9 for $\beta_{\mathrm{true}} = 2$ and $0.7 < \sqrt{\eta} < 1.8$. In the most unfavorable cases, the error introduced by OR can be even larger than 0.5 m.u., whereas that introduced by GOR will always be smaller. Errors introduced by SR and ISR, within these η and σ_x intervals, however, are comparable or larger than those introduced by OR.

Conclusions

We have presented, in graphical form, the biases introduced by different regression procedures commonly used in converting magnitudes from one scale to another, as a function of β_{true}, of the ratio η between variances, and of the absolute variances of magnitudes. Such biases have been computed through simulations very close to the real cases of common types of magnitude in a wide range of size ($3.5 \leq M \leq 9.5$) and variance, as determined by means of examples from the Chinese (CENC) and German (GRSN) broadband networks. We have avoided an analytical derivation because the choice of the best-fitting procedure to our simulated results would in turn introduce modeling problems and because, due to the number of variables present, it would not be so straightforward.

We have found that for most magnitude data, orthogonal regression relations (OR) that assume an error ratio $\eta = 1$ between the two magnitudes considered allow magnitude conversions with errors on average lower than 0.2 m.u. in the whole range of realistic magnitudes. Attention should be paid to the $m_{\mathrm{b}} - M_{\mathrm{S}}$ relationship since it is not linear (because of saturation of the short-period m_{b} for strong earthquakes), it has an error ratio of about 2 and usually a rather large absolute scatter in the m_{b} data.

Besides this specific conclusion, we derive some general guidelines for the choice of the best regression procedure for a real dataset as a function of the absolute errors and error ratios of the compared variables:

1. When the standard deviation of the magnitudes involved in the conversion is known, GOR is the best regression procedure.
2. When the ratio between the variances and/or the absolute values of each variance is not known, then it is suggested to compute from real data all three: β_{SR}, β_{ISR}, and β_{OR}. This can also reveal some properties of the variances involved:
   a. The closer β_{OR} is to β_{SR} with respect to β_{ISR}, the larger σ_x is, compared with σ_y (small η).
   b. On the contrary, the closer β_{OR} is to β_{ISR} with respect to β_{SR}, the larger σ_y is, compared with σ_x (large η).
   c. The larger the difference between β_{SR} and β_{ISR}, the larger the absolute variances are.
   d. If β_{OR} lies in the angular midst between β_{SR} and β_{ISR} and $\beta_{\mathrm{OR}} \simeq 1$, then $\eta = 1$ and β_{OR} is the best regression, because, in this case, $\beta_{\mathrm{OR}} = \beta_{\mathrm{GOR}}$.
3. In all the other cases, Figures 2–5 can help in inferring the best regression procedure for the real dataset as follows:
   a. If only σ_y is known, then one can enter one of Figures 2–4, according to the specific σ_y, and find from the computed β_{SR}, β_{ISR}, and β_{OR} the best matching x-coordinate, which allows an estimate of η_{true} and β_{true}.
   b. If only σ_x is known, then one can enter Figure 5 and those published complementary Ⓔ in the online edition of BSSA or in Terraemoti (2007), according to the specific σ_x, and find from the computed β_{SR}, β_{ISR}, and β_{OR} the matching x-coordinate, which allows one to estimate η_{true} and β_{true}.
   c. If none of the variances is known, then Figures 2–5 and those in the Ⓔ online edition of BSSA or Terraemoti (2007) should be analyzed in an attempt to infer information on η and the most suitable regression procedure.

Note that the standard deviation of the computed β increases (dashed lines in Figs. 2–5) as a function of the magnitude variances σ. OR seems nevertheless a better estimator than ISR and SR for most of the magnitudes and variances found in practice.

An incorrect regression procedure can lead to large magnitude conversion errors (much larger than 0.2 m.u.) and this may have serious consequences on the seismic-hazard estimates and the Gutenberg–Richter frequency-magnitude plots (for a quantification of misapplied regression procedures on true datasets see Castellaro *et al.*, 2006, in particular, Figures 10–12). Therefore, it is always recommended that seismological data centers also publish standard deviations for their average event magnitudes. Should these empirical average magnitude error estimates not be available, the guidelines provided in this article should be followed. A computer program to compute SR, ISR, OR, and GOR has been made available for download on Terraemoti (2007).

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