Geosciences Journal
Vol. 11, No. 4, p. 387 − 396, December 2007

Hydraulic conductivity and mechanical stiffness tensors for variably saturated true anisotropic intact rock matrices, joints, joint sets, and jointed rock masses

Jun-Mo Kim* *School of Earth and Environmental Sciences, Seoul National University, Seoul 151-742, Republic of Korea*

ABSTRACT: A series of global hydraulic conductivity and mechanical stiffness tensors for variably saturated true anisotropic intact rock matrices, joints, joint sets, and jointed rock masses is formulated to expand the fully coupled poroelastic governing equations presented by Kim (2004) for groundwater flow and solid skeleton deformation in porous geologic media to those for fractured and fractured porous geologic media. The global hydraulic conductivity tensors are derived from the local hydraulic conductivity tensors using coordinate transformation on the basis of the generalized Darcy's law. The global mechanical stiffness tensors are then derived from the local or global mechanical compliance tensors using coordinate transformation on the basis of the generalized Hooke's law.

Key words: intact rock matrix, joint, joint set, jointed rock mass, hydraulic conductivity tensor, mechanical stiffness tensor

1. INTRODUCTION

Since the pioneering work of Biot (1941), poroelastic governing equations for groundwater flow and solid skeleton deformation in variably saturated porous geologic media have been formulated and applied by many scientists (Biot, 1955; Verruijt, 1969; Safai and Pinder, 1979; Bear and Corapcioglu, 1981; Noorishad et al., 1982; Kim, 1996; Kim and Parizek, 1997; Kim and Parizek, 1999a; Kim and Parizek, 1999b; Kim, 2000; Kim, 2003; Kim, 2004; Kim, 2005a; Kim and Parizek, 2005). Kim et al. (1997) formulated fully, i.e., explicitly and implicitly coupled poroelastic governing equations for groundwater flow and solid skeleton deformation in fractured porous geologic media, which are hydraulically anisotropic and mechanically isotropic and applied them to simulation of fully coupled strata deformation and groundwater flow due to underground coal mining. However, they assumed that intact rock matrices are impermeable but deformable. In addition, fully coupled poroelastic governing equations for groundwater flow and solid skeleton deformation in joints (fractures) and joint sets (fracture sets) have recently been highly desirable in order to analyze more precisely and practically fully coupled groundwater flow and solid skeleton deformation in actual geologic systems due to various causes (e.g. pumping, loading, mining, tunneling). Thus a series of global hydraulic conductivity and mechanical stiffness tensors for variably saturated true anisotropic intact rock matrices (porous media), joints (discrete fracture networks), joint sets (fractured media), and jointed rock masses (fractured porous media) must be formulated to generalize the preexisting fully coupled poroelastic governing equations, which are mentioned above, for whole porous, fractured, and fractured porous geologic media.

The objective of this paper is to formulate a set of global hydraulic conductivity and mechanical stiffness tensors for variably saturated true anisotropic intact rock matrices, joints, joint sets, and jointed rock masses to expand the fully coupled poroelastic governing equations presented by Kim (2004) for groundwater flow and solid skeleton deformation in porous geologic media to those for fractured and fractured porous geologic media. The basic assumption underlying this study is that fractured and fractured porous geologic media behave as equivalent porous elastic continua with interconnected fractures dominating groundwater flow and solid skeleton deformation patterns (Oda, 1986; Zhu and Wang, 1993; Pariseau, 1993). The obvious advantage of treating fractured and fractured porous geologic media as equivalent continua is that the fully coupled poroelastic governing equations for groundwater flow and solid skeleton deformation, which have been developed for porous geologic media, can be applied directly to fractured and fractured porous geologic media, and no additional assumptions and parameters are involved. This equivalent porous elastic continuum representation is also convenient since it enables changes in porosity and saturated hydraulic conductivity tensors that result from solid skeleton deformation to be evaluated straightforwardly by using body strain tensors in porous, fractured, and fractured porous geologic media.

2. POROELASTIC GOVERNING EQUATIONS

A series of fully coupled poroelastic governing equations for groundwater flow and solid skeleton deformation in a variably saturated true anisotropic intact rock matrix (porous medium) *m*, joint (discrete fracture network) *jt*, joint set (fractured medium) *js*, or jointed rock mass (fractured porous medium) *rm* can be derived on the basis of the following basic assumptions (Kim, 1996; Kim and Parizek, 1997;

*Corresponding author: junmokim@snu.ac.kr

Kim et al., 1997; Kim and Parizek, 1999a; Kim and Parizek, 1999b; Kim, 2000; Kim, 2003; Kim, 2004; Kim, 2005a; Kim and Parizek, 2005): (1) the medium (i.e., solid skeleton) is elastic and structurally true anisotropic, while its solid constituent is less compressible and microscopically isotropic, (2) the water is slightly compressible, (3) variably saturated water flow follows the generalized Darcy's law (Kozeny, 1927; Carman, 1937; Carman, 1938; Carman, 1956; Parsons, 1966; Snow, 1968; Snow, 1969; Bear, 1972), (4) the inertial force is neglected in the force equilibrium equation, (5) the modified effective stress concept (Bishop and Blight, 1963; Cheng, 1997) is valid for the entire variably saturated flow regime, (6) the generalized Hooke's law (Love, 1944; Lekhnitskii, 1963; Goodman et al., 1968; Goodman and Dubois, 1972) holds for elastic deformation of the true anisotropic medium, and (7) the pore air pressure is equal to the atmospheric pressure (i.e., zero), which is taken as the reference pressure.

The governing equation for groundwater flow in a deforming unsaturated true anisotropic intact rock matrix, joint, joint set, or jointed rock mass can be derived from the conservation equations of water and solid masses using the above assumptions (1), (2), (3), (6), and (7) as follows (Kim, 1996; Kim and Parizek, 1997; Kim et al., 1997; Kim and Parizek, 1999a; Kim and Parizek, 1999b; Kim, 2000; Kim, 2003; Kim, 2004; Kim, 2005a; Kim and Parizek, 2005):

$$\nabla \cdot [-K_r \boldsymbol{K}_{sat} \cdot \nabla(h+h_e)] + \left(n\frac{dS_w}{dh} + nS_w\beta_w\gamma_w\right)\frac{\partial h}{\partial t} + S_w\alpha_{ij}\frac{\partial \varepsilon_{ij}}{\partial t} = q_w \qquad i,j = x,y,z \qquad (1)$$

where $K_r(h)$ is the relative hydraulic conductivity ($0 \le K_r \le 1$), $\boldsymbol{K}_{sat}(\varepsilon_{ij})$ is the global saturated hydraulic conductivity tensor ($K_{sat\,ij} = K_{sat\,ji}$), $h = P/\gamma_w$ is the pressure head, h_e is the elevation head equal to the global vertical axis z, $n(\varepsilon_{ij})$ is the porosity ($0 \le n \le 1$), $S_w(h)$ is the degree of water saturation ($0 \le S_w \le 1$), β_w is the compressibility of water, $\gamma_w = \rho_w g$ is the unit weight of water, $\alpha_{ij} = \delta_{ij} - D_{ijkl}\delta_{kl}/3K_s$ is Biot's hydromechanical coupling coefficient tensor or the effective stress coefficient tensor ($\alpha_{ij} = \alpha_{ji}$), $\varepsilon_{ij} = \partial u_i/\partial x_j + \partial u_j/\partial x_i(1-\delta_{ij})$ is the strain tensor (positive for tension), q_w is the water source/sink term, t is time, and x, y, and z are the global coordinate axes. Here $\boldsymbol{K} = K_r\,\boldsymbol{K}_{sat}$ is the global effective hydraulic conductivity tensor ($K_{ij} = K_{ji}$), P is the pore water pressure (positive for compression), ρ_w is the density of water, g is the gravitational acceleration constant, δ_{ij} and δ_{kl} are Kronecker's deltas, D_{ijkl} is the global mechanical stiffness (elastic modulus) tensor $\boldsymbol{D} = \boldsymbol{C}^{-1}$ ($D_{ijkl} = D_{jikl} = D_{ijlk} = D_{klij}$), and u_i is the component of the displacement vector $\boldsymbol{u}$ of solid in the i direction. In addition, $K_s = E_s/3(1\text{-}2\nu_s)$, E_s, and ν_s are the bulk modulus, Young's modulus, and Poisson's ratio, respectively, of the solid constituent, and $\boldsymbol{C}$ is the global mechanical (elastic) compliance tensor ($C_{ijkl} = C_{jikl} = C_{ijlk} = C_{klij}$). Note that $\phi = h + h_e$ is the hydraulic head, $\nabla(h + h_e)$ is the hydraulic gradient, $\boldsymbol{q} = -K_r\,\boldsymbol{K}_{sat} \cdot \nabla(h + h_e)$ is the Darcy velocity (flux) or the groundwater flow velocity implying the generalized Darcy's law, dS_w/dh is the specific saturation capacity, and $\theta_w = nS_w$ is the volumetric water content.

The governing equation for solid skeleton deformation of an unsaturated true anisotropic intact rock matrix, joint, joint set, or jointed rock mass can be derived from the force equilibrium equation using the above assumptions (1), (4), (5), (6), and (7) as follows (Kim, 1996; Kim and Parizek, 1997; Kim et al., 1997; Kim and Parizek, 1999a; Kim and Parizek, 1999b; Kim, 2000; Kim, 2003; Kim, 2004; Kim, 2005a; Kim and Parizek, 2005):

$$\frac{\partial}{\partial x_j}[(D_{ijkl}\varepsilon_{kl})^e - (S_w\alpha_{ij}\gamma_w h)^e] + [nS_w\rho_w + (1-n)\rho_s]^e g_i = 0 \qquad i,j = x,y,z \qquad (2)$$

where D_{ijkl} is the global mechanical stiffness (elastic modulus) tensor $\boldsymbol{D} = \boldsymbol{C}^{-1}$, $\varepsilon_{kl} = \partial u_k/\partial x_l + \partial u_l/\partial x_k(1-\delta_{kl})$ is the strain tensor (positive for tension), ρ_s is the density of the solid constituent, g_i is the component of the gravitational acceleration $\boldsymbol{g}$ in the i direction, and the superscripts e denote the incremental values of the physical quantities. For example, $h^e = h - h^o$ is the incremental pressure head in which h and h^o are the current and initial pressure heads, respectively. Here $\boldsymbol{C}$ is the global mechanical (elastic) compliance tensor, and u_k is the component of the displacement vector $\boldsymbol{u}$ of solid in the k direction. Note that $\sigma'_{ij} = D_{ijkl}\varepsilon_{kl}$ is the deformation-producing effective stress tensor (positive for tension) implying the generalized Hooke's law, $\sigma_{ij} = \sigma'_{ij} - S_w\alpha_{ij}\gamma_w h$ is the total stress tensor (positive for tension) implying the modified effective stress concept, $\rho_b = nS_w\rho_w + (1-n)\rho_s$ is the bulk density, and $f_i = \rho_b g_i$ is the component of the body force $\boldsymbol{f}$ in the i direction.

In equations (1) and (2), the shear components of the strain tensor ε_{ij} become effective when the geologic medium is mechanically true anisotropic, i.e., the off-diagonal terms of the global mechanical stiffness tensor D_{ijkl} and thus Biot's hydromechanical coupling coefficient tensor α_{ij} are not zero.

In summary, equations (1) and (2) constitute a set of four nonlinear partial differential equations with four dependent variables (i.e., h, u_x, u_y, u_z) in the global coordinates (x, y, z).

Figure 1 shows the spatial relationship between the global or geographical coordinate axes (x, y, z) and the local coordinate or material symmetry axes (x', y', z') for true anisotropic geologic structures. The geographical coordinate axes (x, y, z) are defined here as follows (Figure 1). The x axis is horizontal to the east, the y axis is horizontal to the north forming the xy plane parallel to the horizon, and the z axis is vertical to the zenith. The material symmetry axes (x', y', z') for true anisotropic (orthotropic) geologic structures are then defined here as follows (Figure 1). The x' axis is aligned with linear structures (e.g., mineral elongation, striation, joint intersection), the y' axis is 90° counter-clockwise (i.e., $+\pi/2$) from the x' axis forming the $x'y'$ plane parallel to planar structures (e.g. bedding, foliation, joint), and the z' axis is vertically upward from the $x'y'$ plane (i.e., pole to planar structures). Figure 2 represents a jointed rock mass composed of an intact rock matrix with bedding planes, individual joints, and joint sets, which are true anisotropic.

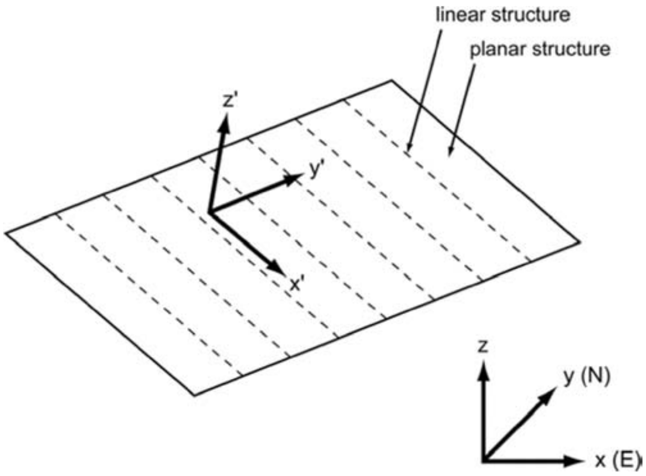

Fig. 1. Spatial relationship between the global or geographical coordinate axes (x, y, z) and the local coordinate or material symmetry axes (x', y', z') for true anisotropic geologic structures.

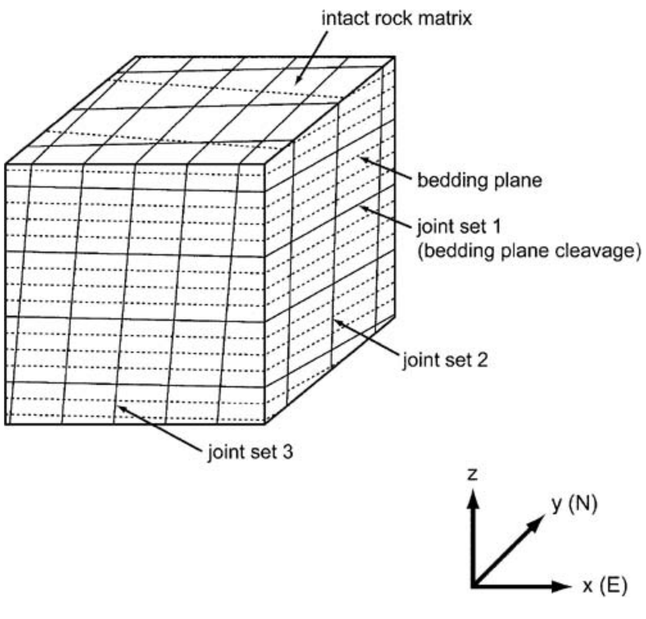

Fig. 2. Representation of a jointed rock mass composed of an intact rock matrix with bedding planes, individual joints, and joint sets, which are true anisotropic.

3. HYDRAULIC CONDUCTIVITY TENSORS

In this section, global hydraulic conductivity tensors in equation (1) for variably saturated true anisotropic intact rock matrices, joints, joint sets, and jointed rock masses are derived from local hydraulic conductivity tensors using coordinate transformation on the basis of the generalized Darcy's law (Kozeny, 1927; Carman, 1937; Carman, 1938; Carman, 1956; Parsons, 1966; Snow, 1968; Snow, 1969; Bear, 1972). Porosities in equations (1) and (2) for variably saturated true anisotropic intact rock matrices, joints, joint sets, and jointed rock masses are also derived.

3.1. Intact Rock Matrices

The Darcy velocity for a true anisotropic intact rock matrix in the global coordinates (x, y, z), i.e., in equation (1) can be expressed as follows:

$$\begin{aligned}\boldsymbol{q}_m &= -\boldsymbol{K}_m \cdot \nabla(h_m + h_e) = -K_{m\,r}\boldsymbol{K}_{m\,sat} \cdot \nabla(h_m + h_e)\\ &= -\boldsymbol{A}_m^T K_{m\,r}\boldsymbol{K}^L_{m\,sat}\boldsymbol{A}_m \cdot \nabla(h_m + h_e) = -\boldsymbol{A}_m^T\boldsymbol{K}^L_m\boldsymbol{A}_m \cdot \nabla(h_m + h_e)\end{aligned} \tag{3}$$

where the subscript m denotes the intact rock matrix, the superscript T denotes the matrix transpose, the superscript L denotes the local tensor in the local coordinates (x', y', z') (i.e., principal axes or directions), and the other terms correspond to their counterparts in equation (1). Here $\boldsymbol{A}_m$ is the coordinate transformation matrix for the global and local effective hydraulic conductivity tensors of the intact rock matrix (see Appendix A). In addition, the local saturated hydraulic conductivity tensor $\boldsymbol{K}^L_{m\,sat}$ of the intact rock matrix is defined as follows ($K^L_{m\,sat\,ij} = K^L_{m\,sat\,ji}$) (Kozeny, 1927; Carman, 1937; Carman, 1938; Carman, 1956):

$$\boldsymbol{K}^L_{m\,sat} = \frac{\rho_w g}{\mu_w}\boldsymbol{k}^L_{m\,sat} = \frac{\rho_w g}{\mu_w} f_{m\,n}\begin{bmatrix} f_{m\,s\,x'}d^2_{m\,x'} & 0 & 0 \\ 0 & f_{m\,s\,y'}d^2_{m\,y'} & 0 \\ 0 & 0 & f_{m\,s\,z'}d^2_{m\,z'} \end{bmatrix} \tag{4}$$

where μ_w is the dynamic viscosity of water, $\boldsymbol{k}^L_{m\,sat}$ is the local saturated intrinsic permeability tensor of the intact rock matrix ($k^L_{m\,sat\,ij} = k^L_{m\,sat\,ji}$), $f_{m\,n} = n_m^3/(1-n_m)^2$ is the porosity factor of the intact rock matrix, $f_{m\,s\,i} = 1/180$ is the shape factor of the intact rock matrix in the i direction, and $d_{m\,i}$ is the mean or effective diameter of the solid grains, which constitute the intact rock matrix, in the i direction. Here the solid grains are assumed to be ellipsoidal.

The global effective hydraulic conductivity tensor $\boldsymbol{K}_m$ of the intact rock matrix can thus be calculated from its local effective hydraulic conductivity tensor $\boldsymbol{K}^L_m$ using the coordinate transformation matrix $\boldsymbol{A}_m$ as follows:

$$\boldsymbol{K}_m = K_{m\,r}\boldsymbol{K}_{m\,sat} = \boldsymbol{A}_m^T K_{m\,r}\boldsymbol{K}^L_{m\,sat}\boldsymbol{A}_m = \boldsymbol{A}_m^T\boldsymbol{K}^L_m\boldsymbol{A}_m \tag{5}$$

On the other hand, the porosity n_m of the intact rock matrix in equations (1) and (2) can be expressed as follows (Kim and Parizek, 1999b):

$$n_m = 1 - \frac{1 - n_m^o}{1 + \varepsilon_{m\,v}} \tag{6}$$

where n_m^o is the initial porosity of the intact rock matrix prior to deformation, and $\varepsilon_{m\,v} = \varepsilon_{m\,xx} + \varepsilon_{m\,yy} + \varepsilon_{m\,zz}$ is the volumetric strain or volume dilation in the intact rock matrix.

3.2. Joints

The Darcy velocity for a true anisotropic joint in the global coordinates (x, y, z), i.e., in equation (1) can be expressed as follows:

$$\begin{aligned}\boldsymbol{q}_{jt} &= -\boldsymbol{K}_{jt} \cdot \nabla(h_{jt} + h_e) = -K_{jt\,r}\boldsymbol{K}_{jt\,sat}\nabla(h_{jt} + h_e) \\ &= -\boldsymbol{A}_{jt}^T K_{jt\,r}\boldsymbol{K}_{jt\,sat}^L \boldsymbol{A}_{jt} \cdot \nabla(h_{jt} + h_e) = -\boldsymbol{A}_{jt}^T \boldsymbol{K}_{jt}^L \boldsymbol{A}_{jt} \cdot \nabla(h_{jt} + h_e)\end{aligned} \tag{7}$$

where the subscript jt denotes the joint, and the other terms correspond to their counterparts in equations (1) and (3). Here $\boldsymbol{A}_{jt}$ is the coordinate transformation matrix for the global and local effective hydraulic conductivity tensors of the joint (see Appendix A). In addition, the local saturated hydraulic conductivity tensor $\boldsymbol{K}_{jt\,sat}^L$ of the joint is defined as follows ($K_{jt\,sat\,ij}^L = K_{jt\,sat\,ji}^L$) (Parsons, 1966; Snow, 1968; Snow, 1969):

$$\boldsymbol{K}_{jt\,sat}^L = \frac{\rho_w g}{\mu_w}\boldsymbol{k}_{jt\,sat}^L = \frac{\rho_w g}{\mu_w} f_{jt\,n}\begin{bmatrix} f_{jt\,s\,x'} b_{jt\,x'}^2 & 0 & 0 \\ 0 & f_{jt\,s\,y'} b_{jt\,y'}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{8}$$

where $\boldsymbol{k}_{jt\,sat}^L$ is the local saturated intrinsic permeability tensor of the joint ($k_{jt\,sat\,ij}^L = k_{jt\,sat\,ji}^L$), $f_{jt\,n} = 1$ is the porosity factor of the joint, $f_{jt\,s\,i} = 1/12$ is the shape factor of the joint in the i direction, and $b_{jt\,i}$ is the aperture of the joint in the i direction. Here the joint is assumed to be planar and associated with linear structures, and the aperture $b_{jt\,i}$ of the joint in the i direction can be expressed as follows:

$$b_{jt\,i}(\varepsilon_{jt\,ij}) = b_{jt\,i}^o(1 + \varepsilon_{jt\,z'z'}) \quad i = x', y', z' \tag{9}$$

where $b_{jt\,i}^o$ is the initial aperture of the joint in the i direction prior to deformation, and $\varepsilon_{jt\,z'z'}$ is the joint-normal strain in the joint, which can be obtained from equation (26) or (B3).

The global effective hydraulic conductivity tensor $\boldsymbol{K}_{jt}$ of the joint can thus be calculated from its local effective hydraulic conductivity tensor $\boldsymbol{K}_{jt}^L$ using the coordinate transformation matrix $\boldsymbol{A}_{jt}$ as follows:

$$\boldsymbol{K}_{jt} = K_{jt\,r}\boldsymbol{K}_{jt\,sat} = \boldsymbol{A}_{jt}^T K_{jt\,r}\boldsymbol{K}_{jt\,sat}^L \boldsymbol{A}_{jt} = \boldsymbol{A}_{jt}^T \boldsymbol{K}_{jt}^L \boldsymbol{A}_{jt} \tag{10}$$

On the other hand, the porosity n_{jt} of the joint in equations (1) and (2) can be expressed as follows:

$$n_{jt} = \frac{1}{2}\left(\frac{b_{jt\,x'}}{b_{jt\,x'}} + \frac{b_{jt\,y'}}{b_{jt\,y'}}\right) = 1 \tag{11}$$

3.3. Joint Sets

The Darcy velocity for a true anisotropic joint set in the global coordinates (x, y, z), i.e., in equation (1) can be expressed using equation (7) as follows:

$$\begin{aligned}\boldsymbol{q}_{js} &= -\boldsymbol{K}_{js} \cdot \nabla(h_{js} + h_e) = -K_{js\,r}\boldsymbol{K}_{js\,sat}\nabla(h_{js} + h_e) \\ &= -\boldsymbol{A}_{js}^T K_{js\,r}\boldsymbol{K}_{js\,sat}^L \boldsymbol{A}_{js} \cdot \nabla(h_{js} + h_e) = -\boldsymbol{A}_{js}^T \boldsymbol{K}_{js}^L \boldsymbol{A}_{js} \cdot \nabla(h_{js} + h_e) \\ &= -\boldsymbol{A}_{jt}^T K_{jt\,r}\boldsymbol{w}_{js}\boldsymbol{K}_{jt\,sat}^L \boldsymbol{A}_{jt} \cdot \nabla(h_{jt} + h_e) \\ &= -\boldsymbol{A}_{js}^T K_{js\,r}\boldsymbol{w}_{js}\boldsymbol{K}_{jt\,sat}^L \boldsymbol{A}_{js} \cdot \nabla(h_{js} + h_e)\end{aligned} \tag{12}$$

where the subscript js denotes the joint set, and the other terms correspond to their counterparts in equations (1), (3), and (7). Here $\boldsymbol{A}_{js}$ is the coordinate transformation matrix for the global and local effective hydraulic conductivity tensors of the joint set (see Appendix A), and $\boldsymbol{w}_{js}$ is the hydraulic fraction tensor of the joint set in a jointed rock mass as follows:

$$\begin{aligned}\boldsymbol{w}_{js} &= \begin{bmatrix} b_{js\,x'}/s_{js\,x'} & 0 & 0 \\ 0 & b_{js\,y'}/s_{js\,y'} & 0 \\ 0 & 0 & b_{js\,z'}/s_{js\,z'} \end{bmatrix} \\ &= \begin{bmatrix} b_{jt\,x'}/s_{js\,x'} & 0 & 0 \\ 0 & b_{jt\,y'}/s_{js\,y'} & 0 \\ 0 & 0 & b_{jt\,z'}/s_{js\,z'} \end{bmatrix}\end{aligned} \tag{13}$$

where $b_{js\,i}$ is the aperture of the joint set, which is equal to the aperture $b_{jt\,i}$ of the individual joints in the joint set, in the i direction, and $s_{js\,i}$ is the spacing of the joint set in the i direction. Here the aperture $b_{js\,i}$ and the spacing $s_{js\,i}$ of the joint set in the i direction can be expressed as follows:

$$b_{js\,i}(\varepsilon_{js\,ij}) = b_{js\,i}^o(1 + \varepsilon_{js\,z'z'}) \quad i = x', y', z' \tag{14}$$

$$s_{js\,i}(\varepsilon_{rm\,ij}) = s_{js\,i}^o(1 + \varepsilon_{rm\,z'z'}) \quad i = x', y', z' \tag{15}$$

where $b_{js\,i}^o$ is the initial aperture of the joint set in the i direction prior to deformation, $\varepsilon_{js\,z'z'}$ is the joint set-normal strain in the joint set, which can be obtained from equation (31) or (B3), $s_{js\,i}^o$ is the initial spacing of the joint set in the i direction prior to deformation, and $\varepsilon_{rm\,z'z'}$ is the joint set-normal strain in the jointed rock mass, which can be obtained from equation (38) or (B3). In addition, the local saturated hydraulic conductivity tensor $\boldsymbol{K}_{js\,sat}^L$ of the joint set is defined as follows ($K_{js\,sat\,ij}^L = K_{js\,sat\,ji}^L$) (Parsons, 1966; Snow, 1968; Snow, 1969):

$$\boldsymbol{K}^L_{js\,sat} = \frac{\rho_w g}{\mu_w}\boldsymbol{k}^L_{js\,sat} = \boldsymbol{w}_{js}\boldsymbol{K}^L_{jt\,sat} = \frac{\rho_w g}{\mu_w}\boldsymbol{w}_{js}\boldsymbol{k}^L_{jt\,sat}$$
$$= \frac{\rho_w g}{\mu_w}\begin{bmatrix} f_{jt\,s\,x'}b^3_{jt\,x'}/s_{js\,x'} & 0 & 0 \\ 0 & f_{jt\,s\,y'}b^3_{jt\,y'}/s_{js\,y'} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$= \frac{\rho_w g}{\mu_w}\begin{bmatrix} f_{js\,n\,x'}f_{js\,s\,x'}b^2_{js\,x'} & 0 & 0 \\ 0 & f_{js\,n\,y'}f_{js\,s\,y'}b^2_{js\,y'} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{16}$$

where $\boldsymbol{k}^L_{js\,sat}$ is the local saturated intrinsic permeability tensor of the joint set ($k^L_{js\,sat\,ij} = k^L_{js\,sat\,ji}$), $f_{js\,n\,i} = b_{js\,i}/s_{js\,i} = w_{js\,ii}$ is the porosity factor of the joint set in the i direction, and $f_{js\,s\,i} = 1/12$ is the shape factor of the joint set, which is equal to the shape factor $f_{jt\,s\,i} = 1/12$ of the individual joints in the joint set, in the i direction. Here the joints in the joint set are assumed to be parallel to each other and to be planar and associated with linear structures. It is also assumed that the hydraulic heads are in local equilibrium, i.e., $h_{js} = h_{jt}$.

The global effective hydraulic conductivity tensor $\boldsymbol{K}_{js}$ of the joint set can thus be calculated from its local effective hydraulic conductivity tensor $\boldsymbol{K}^L_{js}$ using the coordinate transformation matrix $\boldsymbol{A}_{js}$ as follows:

$$\boldsymbol{K}_{js} = K_{js\,r}\boldsymbol{K}_{js\,sat} = \boldsymbol{A}^T_{js}K_{js\,r}\boldsymbol{K}^L_{js\,sat}\boldsymbol{A}_{js} = \boldsymbol{A}^T_{js}\boldsymbol{K}^L_{js}\boldsymbol{A}_{js} \tag{17}$$

On the other hand, the porosity n_{js} of the joint set in equations (1) and (2) can be expressed as follows:

$$n_{js} = \frac{1}{2}(w_{js\,x'x'} + w_{js\,y'y'}) = \frac{1}{2}\left(\frac{b_{js\,x'}}{s_{js\,x'}} + \frac{b_{js\,y'}}{s_{js\,y'}}\right) \tag{18}$$

3.4. Jointed Rock Masses

The Darcy velocity for a true anisotropic jointed rock mass consisting of an intact rock matrix and joint sets in the global coordinates (x, y, z), i.e., in equation (1) can be expressed using equations (3) and (12) from the fact that the total water flux through the jointed rock mass is equal to the sum of individual water fluxes through the intact rock matrix and joint sets under the local hydraulic head equilibrium assumption (i.e., $h_{rm} = h_m = h_{js} = h_{jt}$) as follows:

$$\boldsymbol{q}_{rm} = -\boldsymbol{K}_{rm}\cdot\nabla(h_{rm}+h_e) = -K_{rm\,r}\boldsymbol{K}_{rm\,sat}\cdot\nabla(h_{rm}+h_e)$$
$$= -\boldsymbol{A}^T_{rm}K_{rm\,r}\boldsymbol{K}^L_{rm\,sat}\boldsymbol{A}_{rm}\cdot\nabla(h_{rm}+h_e)$$
$$= -\boldsymbol{A}^T_{rm}\boldsymbol{K}^L_{rm}\boldsymbol{A}_{rm}\cdot\nabla(h_{rm}+h_e) = \left(1-\sum_{js=1}^{nn}n_{js}\right)\boldsymbol{q}_m + \sum_{js=1}^{nn}\boldsymbol{q}_{js}$$
$$= -\left(1-\sum_{js=1}^{nn}n_{js}\right)K_{m\,r}\boldsymbol{K}_{m\,sat}\cdot\nabla(h_m+h_e)$$
$$-\sum_{js=1}^{nn}K_{js\,r}\boldsymbol{K}_{js\,sat}\cdot\nabla(h_{js}+h_e)$$
$$= -\left[\left(1-\sum_{js=1}^{nn}n_{js}\right)\boldsymbol{A}^T_m K_{m\,r}\boldsymbol{K}^L_{m\,sat}\boldsymbol{A}_m + \sum_{js=1}^{nn}\boldsymbol{A}^T_{js}K_{js\,r}\boldsymbol{K}^L_{js\,sat}\boldsymbol{A}_{js}\right]$$
$$\cdot\nabla(h_{rm}+h_e)$$
$$= -\left[\left(1-\sum_{js=1}^{nn}n_{js}\right)\boldsymbol{A}^T_m\boldsymbol{K}^L_m\boldsymbol{A}_m + \sum_{js=1}^{nn}\boldsymbol{A}^T_{js}\boldsymbol{K}^L_{js}\boldsymbol{A}_{js}\right]\cdot\nabla(h_{rm}+h_e) \tag{19}$$

where the subscript rm denotes the jointed rock mass, nn is the number of joint sets in the jointed rock mass, and the other terms correspond to their counterparts in equations (1), (3), (7), and (12). Here $\boldsymbol{A}_{rm}$ is the coordinate transformation matrix for the global and local effective hydraulic conductivity tensors of the jointed rock mass (see Appendix A).

The global effective hydraulic conductivity tensor $\boldsymbol{K}_{rm}$ of the jointed rock mass consisting of an intact rock matrix and joint sets can thus be calculated from the local effective hydraulic conductivity tensors $\boldsymbol{K}^L_m$ of the intact rock matrix and $\boldsymbol{K}^L_{js}$ of the joint sets using their coordinate transformation matrices $\boldsymbol{A}_m$ and $\boldsymbol{A}_{js}$ as follows:

$$\boldsymbol{K}_{rm} = K_{rm\,r}\boldsymbol{K}_{rm\,sat}$$
$$= \left(1-\sum_{js=1}^{nn}n_{js}\right)\boldsymbol{A}^T_m K_{m\,r}\boldsymbol{K}^L_{m\,sat}\boldsymbol{A}_m + \sum_{js=1}^{nn}\boldsymbol{A}^T_{js}K_{js\,r}\boldsymbol{K}^L_{js\,sat}\boldsymbol{A}_{js}$$
$$= \left(1-\sum_{js=1}^{nn}n_{js}\right)\boldsymbol{A}^T_m\boldsymbol{K}^L_m\boldsymbol{A}_m + \sum_{js=1}^{nn}\boldsymbol{A}^T_{js}\boldsymbol{K}^L_{js}\boldsymbol{A}_{js} \tag{20}$$

On the other hand, the porosity n_{rm} of the jointed rock mass consisting of an intact rock matrix and joint sets in equations (1) and (2) can be expressed as follows:

$$n_{rm} = n_m + n_{jst} = n_m + \sum_{js=1}^{nn}n_{js} \tag{21}$$

where n_{jst} is the porosity of nn joint sets, and it is assumed that joint intersection is negligible compared with the sum of porosities of nn joint sets. Note that the porosity n_m of the intact rock matrix can be expressed by equation (6), while the porosity n_{js} of each joint set can be expressed by equation (18).

4. MECHANICAL STIFFNESS TENSORS

In this section, global mechanical stiffness tensors in equations (1) and (2) for variably saturated true anisotropic intact rock matrices, joints, joint sets, and jointed rock masses are derived from local and global mechanical compliance tensors using coordinate transformation on the basis of the generalized Hooke's law (Love, 1944; Lekhnitskii, 1963; Goodman et al., 1968; Goodman and Dubois, 1972).

4.1. Intact Rock Matrices

The strain tensor for a true anisotropic intact rock matrix in the local coordinates (x', y', z') can be expressed as follows:

$$\{\varepsilon_m\}^L = C_m^L\{\sigma_m'\}^L = B_m C_m B_m^T\{\sigma_m'\}^L \tag{22}$$

where the subscript m denotes the intact rock matrix, the superscript L denotes the local tensor in the local coordinates (x', y', z') (i.e., principal axes or directions), the superscript T denotes the matrix transpose, and the other terms correspond to their counterparts in equation (2). Here B_m is the coordinate transformation matrix for the global and local mechanical stiffness tensors of the intact rock matrix (see Appendix B). In addition, the local mechanical compliance tensor $C_m^L = D_m^{L^{-1}}$ of the intact rock matrix is defined as follows ($C_{m\,ijkl}^L = C_{m\,jikl}^L = C_{m\,ijlk}^L = C_{m\,klij}^L$) (Love, 1944; Lekhnitskii, 1963):

$$C_m^L =$$

$$\begin{bmatrix} 1/E_{m\,x'} & -\nu_{m\,x'y'}/E_{m\,x'} & -\nu_{m\,x'z'}/E_{m\,x'} & 0 & 0 & 0 \\ -\nu_{m\,x'y'}/E_{m\,x'} & 1/E_{m\,y'} & -\nu_{m\,y'z'}/E_{m\,y'} & 0 & 0 & 0 \\ -\nu_{m\,x'z'}/E_{m\,x'} & -\nu_{m\,y'z'}/E_{m\,y'} & 1/E_{m\,z'} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{m\,x'y'} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{m\,y'z'} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{m\,z'x'} \end{bmatrix} \tag{23}$$

where D_m^L is the local mechanical stiffness tensor of the intact rock matrix ($D_{m\,ijkl}^L = D_{m\,jikl}^L = D_{m\,ijlk}^L = D_{m\,klij}^L$), $E_{m\,i}$ is Young's modulus (modulus of elasticity) of the intact rock matrix in the i direction, $\nu_{m\,ij}$ is Poisson's ratio for normal strain in the j direction due to effective normal stress of the intact rock matrix in the i direction, and $G_{m\,ij}$ is the shear modulus (modulus of rigidity) of the intact rock matrix in the ij plane.

The effective stress tensor for the intact rock matrix in the global coordinates (x, y, z), i.e., in equation (2) can also be expressed as follows:

$$\{\sigma_m'\} = D_m\{\varepsilon_m\} = C_m^{-1}\{\varepsilon_m\} = B_m^T D_m^L B_m\{\varepsilon_m\}$$

$$= B_m^T C_m^{L^{-1}} B_m\{\varepsilon_m\} \tag{24}$$

The global mechanical stiffness tensor D_m of the intact rock matrix can thus be calculated from its local mechanical compliance tensor C_m^L using the coordinate transformation matrix B_m as follows:

$$D_m = C_m^{-1} = B_m^T D_m^L B_m = B_m^T C_m^{L^{-1}} B_m \tag{25}$$

4.2. Joints

The strain tensor for a true anisotropic joint in the local coordinates (x', y', z') can be expressed as follows:

$$\{\varepsilon_{jt}\}^L = C_{jt}^L\{\sigma_{jt}'\}^L = B_{jt} C_{jt} B_{jt}^T\{\sigma_{jt}'\}^L \tag{26}$$

where the subscript jt denotes the joint, and the other terms correspond to their counterparts in equations (2) and (22). Here B_{jt} is the coordinate transformation matrix for the global and local mechanical stiffness tensors of the joint (see Appendix B), In addition, the local mechanical compliance tensor $C_{jt}^L = D_{jt}^{L^{-1}}$ of the joint is defined as follows ($C_{jt\,ijkl}^L = C_{jt\,jikl}^L = C_{jt\,ijlk}^L = C_{jt\,klij}^L$) (Goodman et al., 1968; Goodman and Dubois, 1972):

$$C_{jt}^L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/k_{jt\,z'}b_{jt\,z'} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/k_{jt\,y'}b_{jt\,y'} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/k_{jt\,x'}b_{jt\,x'} \end{bmatrix} \tag{27}$$

where D_{jt}^L is the local mechanical stiffness tensor of the joint ($D_{jt\,ijkl}^L = D_{jt\,jikl}^L = D_{jt\,ijlk}^L = D_{jt\,klij}^L$), $k_{jt\,i}$ is the stiffness of the joint in the i direction (normal stiffness if $i = z'$ and shear stiffness if $i \neq z'$), and $b_{jt\,i}$ is the aperture of the joint in the i direction. Here the joint is assumed to be planar and associated with linear structures, and the aperture $b_{jt\,i}$ of the joint in the i direction can be expressed as follows:

$$b_{jt\,i}(\varepsilon_{jt\,ij}) = b_{jt\,i}^o(1 + \varepsilon_{jt\,z'z'}) \qquad i = x', y', z' \tag{28}$$

where $b_{jt\,i}^o$ is the initial aperture of the joint in the i direction prior to deformation, and $\varepsilon_{jt\,z'z'}$ is the joint-normal strain in the joint, which can be obtained from equation (26) or (B3).

The effective stress tensor for the joint in the global coordinates (x, y, z), i.e., in equation (2) can also be expressed as follows:

$$\{\sigma_{jt}'\} = D_{jt}\{\varepsilon_{jt}\} = C_{jt}^{-1}\{\varepsilon_{jt}\} = B_{jt}^T D_{jt}^L B_{jt}\{\varepsilon_{jt}\} = B_{jt}^T C_{jt}^{L^{-1}} B_{jt}\{\varepsilon_{jt}\} \tag{29}$$

The global mechanical stiffness tensor D_{jt} of the joint can thus be calculated from its local mechanical compliance tensor C_{jt}^L using the coordinate transformation matrix B_{jt} as follows:

$$D_{jt} = C_{jt}^{-1} = B_{jt}^T D_{jt}^L B_{jt} = B_{jt}^T C_{jt}^{L^{-1}} B_{jt} \tag{30}$$

4.3. Joint Sets

The strain tensor for a true anisotropic joint set in the local coordinates (x', y', z') can be expressed using equation (26) as follows:

$$\begin{aligned}\{\varepsilon_{js}\}^L &= \boldsymbol{C}_{js}^L\{\sigma_{js}'\}^L = \boldsymbol{B}_{js}\boldsymbol{C}_{js}\boldsymbol{B}_{js}^T\{\sigma_{js}'\}^L \\ &= \boldsymbol{r}_{js}\{\varepsilon_{jt}\}^L = \boldsymbol{r}_{js}\boldsymbol{C}_{jt}^L\{\sigma_{jt}'\}^L = \boldsymbol{r}_{js}\boldsymbol{B}_{jt}\boldsymbol{C}_{jt}\boldsymbol{B}_{jt}^T\{\sigma_{jt}'\}^L \\ &= \boldsymbol{r}_{js}\boldsymbol{B}_{js}\boldsymbol{C}_{jt}\boldsymbol{B}_{js}^T\{\sigma_{js}'\}^L \end{aligned} \tag{31}$$

where the subscript js denotes the joint set, and the other terms correspond to their counterparts in equations (2), (22), and (26). Here $\boldsymbol{B}_{js}$ is the coordinate transformation matrix for the global and local mechanical stiffness tensors of the joint set (see Appendix B), and $\boldsymbol{r}_{js}$ is the mechanical fraction tensor of the joint set in a jointed rock mass as follows:

$$\boldsymbol{r}_{js} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{js\,z'}/s_{js\,z'} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{js\,y'}/s_{js\,y'} & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{js\,x'}/s_{js\,x'} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{jt\,z'}/s_{js\,z'} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{jt\,y'}/s_{js\,y'} & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{jt\,x'}/s_{js\,x'} \end{bmatrix} \tag{32}$$

where $b_{js\,i}$ is the aperture of the joint set, which is equal to the aperture $b_{jt\,i}$ of the individual joints in the joint set, in the i direction, and $s_{js\,i}$ is the spacing of the joint set in the i direction. Here the aperture $b_{js\,i}$ and the spacing $s_{js\,i}$ of the joint set in the i direction can be expressed as follows:

$$b_{js\,i}(\varepsilon_{js\,ij}) = b_{js\,i}^o(1+\varepsilon_{js\,z'z'}) \quad i = x', y', z' \tag{33}$$

$$s_{js\,i}(\varepsilon_{rm\,ij}) = s_{js\,i}^o(1+\varepsilon_{rm\,z'z'}) \quad i = x', y', z' \tag{34}$$

where $b_{js\,i}^o$ is the initial aperture of the joint set in the i direction prior to deformation, $\varepsilon_{js\,z'z'}$ is the joint set-normal strain in the joint set, which can be obtained from equation (31) or (B3), $s_{js\,i}^o$ is the initial spacing of the joint set in the i direction prior to deformation, and $\varepsilon_{rm\,z'z'}$ is the joint set-normal strain in the jointed rock mass, which can be obtained from equation (38) or (B3). In addition, the local mechanical compliance tensor $\boldsymbol{C}_{js}^L = \boldsymbol{D}_{js}^{L-1}$ of the joint set is defined as follows ($C_{js\,ijkl}^L = C_{js\,jikl}^L = C_{js\,ijlk}^L = C_{js\,klij}^L$) (Goodman et al., 1968; Goodman and Dubois, 1972):

$$\boldsymbol{C}_{js}^L = \boldsymbol{r}_{js}\boldsymbol{C}_{jt}^L$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/k_{js\,z'}s_{js\,z'} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/k_{js\,y'}s_{js\,y'} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/k_{js\,x'}s_{js\,x'} \end{bmatrix} \tag{35}$$

where $\boldsymbol{D}_{js}^L$ is the local mechanical stiffness tensor of the joint set ($D_{js\,ijkl}^L = D_{js\,jikl}^L = D_{js\,ijlk}^L = D_{js\,klij}^L$), $k_{js\,i}$ is the stiffness of the joint set, which is equal to the stiffness $k_{jt\,i}$ of the individual joints in the joint set, in the i direction (normal stiffness if $i = z'$ and shear stiffness if $i \neq z'$), and $s_{js\,i}$ is the spacing of the joint set in the i direction. Here the joints in the joint set are assumed to be parallel to each other and to be planar and associated with linear structures. It is also assumed that the effective stresses are in local equilibrium, i.e., $\sigma_{js}' = \sigma_{jt}'$.

The effective stress tensor for the joint set in the global coordinates (x, y, z), i.e., in equation (2) can also be expressed as follows:

$$\{\sigma_{js}'\} = \boldsymbol{D}_{js}\{\varepsilon_{js}\} = \boldsymbol{C}_{js}^{-1}\{\varepsilon_{js}\} = \boldsymbol{B}_{js}^T\boldsymbol{D}_{js}^L\boldsymbol{B}_{js}\{\varepsilon_{js}\} = \boldsymbol{B}_{js}^T\boldsymbol{C}_{js}^{L^{-1}}\boldsymbol{B}_{js}\{\varepsilon_{js}\} \tag{36}$$

The global mechanical stiffness tensor $\boldsymbol{D}_{js}$ of the joint set can thus be calculated from its local mechanical compliance tensor $\boldsymbol{C}_{js}^L$ using the coordinate transformation matrix $\boldsymbol{B}_{js}$ as follows:

$$\boldsymbol{D}_{js} = \boldsymbol{C}_{js}^{-1} = \boldsymbol{B}_{js}^T\boldsymbol{D}_{js}^L\boldsymbol{B}_{js} = \boldsymbol{B}_{js}^T\boldsymbol{C}_{js}^{L^{-1}}\boldsymbol{B}_{js} \tag{37}$$

4.4. Jointed Rock Masses

The strain tensor for a true anisotropic jointed rock mass consisting of an intact rock matrix and joint sets in the global coordinates (x, y, z), i.e., in equation (2) can be expressed using and inverting equations (24) and (36) from the fact that the total deformation (strain) within the jointed rock mass is equal to the sum of the individual deformations (strains) within the intact rock matrix and joint sets under the local effective stress equilibrium assumption (i.e., $\sigma_{rm}' = \sigma_m' = \sigma_{js}' = \sigma_{jt}'$) as follows:

$$\begin{aligned}\{\varepsilon_{rm}\} &= \boldsymbol{C}_{rm}\{\sigma_{rm}'\} = \boldsymbol{F}_{rm}^T\boldsymbol{C}_{rm}^L\boldsymbol{F}_{rm}\{\sigma_{rm}'\} \\ &= \left(1 - \sum_{js=1}^{nn} n_{js}\right)\{\varepsilon_m\} + \sum_{js=1}^{nn}\{\varepsilon_{js}\} \\ &= \left(1 - \sum_{js=1}^{nn} n_{js}\right)\boldsymbol{F}_m^T\boldsymbol{C}_m^L\boldsymbol{F}_m\{\sigma_m'\} + \sum_{js=1}^{nn}\boldsymbol{F}_{js}^T\boldsymbol{C}_{js}^L\boldsymbol{F}_{js}\{\sigma_{js}'\} \\ &= \left[\left(1 - \sum_{js=1}^{nn} n_{js}\right)\boldsymbol{F}_m^T\boldsymbol{C}_m^L\boldsymbol{F}_m + \sum_{js=1}^{nn}\boldsymbol{F}_{js}^T\boldsymbol{C}_{js}^L\boldsymbol{F}_{js}\right]\{\sigma_{rm}'\} \end{aligned} \tag{38}$$

where the subscript rm denotes the jointed rock mass, $\boldsymbol{C}^L_{rm} = \boldsymbol{D}^{L^{-1}}_{rm}$ is the local mechanical compliance tensor of the jointed rock mass ($C^L_{rm\,ijkl} = C^L_{rm\,jikl} = C^L_{rm\,ijlk} = C^L_{rm\,klij}$), nn is the number of joint sets in the jointed rock mass, and the other terms correspond to their counterparts in equations (2), (24), (29), and (36). Here $\boldsymbol{D}^L_{rm}$ is the local mechanical stiffness tensor of the jointed rock mass ($D^L_{rm\,ijkl} = D^L_{rm\,jikl} = D^L_{rm\,ijlk} = D^L_{rm\,klij}$). In addition, $\boldsymbol{F}_{rm}$, $\boldsymbol{F}_m$, and $\boldsymbol{F}_{js}$ are the coordinate transformation matrices for the global and local mechanical compliance tensors of the jointed rock mass, intact rock matrix, and joint set, respectively (see Appendix C), and n_{js} is the porosity of each joint set as follows:

$$n_{js} = \frac{1}{2}(w_{js\,x'x'} + w_{js\,y'y'}) = \frac{1}{2}\left(\frac{b_{js\,x'}}{s_{js\,x'}} + \frac{b_{js\,y'}}{s_{js\,y'}}\right) \tag{39}$$

The global mechanical compliance tensor $\boldsymbol{C}_{rm}$ of the jointed rock mass consisting of an intact rock matrix and joint sets can thus be calculated from the local mechanical compliance tensors $\boldsymbol{C}^L_m$ of the intact rock matrix and $\boldsymbol{C}^L_{js}$ of the joint sets using their coordinate transformation matrices $\boldsymbol{F}_m$ and $\boldsymbol{F}_{js}$ as follows:

$$\boldsymbol{C}_{rm} = \boldsymbol{F}^T_{rm}\boldsymbol{C}^L_{rm}\boldsymbol{F}_{rm} = \left(1 - \sum_{js=1}^{nn} n_{js}\right)\boldsymbol{F}^T_m\boldsymbol{C}^L_m\boldsymbol{F}_m + \sum_{js=1}^{nn}\boldsymbol{F}^T_{js}\boldsymbol{C}^L_{js}\boldsymbol{F}_{js} \tag{40}$$

The effective stress tensor for the jointed rock mass consisting of an intact rock matrix and joint sets in the global coordinates (x, y, z), i.e., in equation (2) can also be expressed as follows:

$$\begin{aligned}\{\sigma'_{rm}\} &= \boldsymbol{D}_{rm}\{\varepsilon_{rm}\} = \boldsymbol{B}^T_{rm}\boldsymbol{D}^L_{rm}\boldsymbol{B}_{rm}\{\varepsilon_{rm}\} = \boldsymbol{B}^T_{rm}\boldsymbol{C}^{L^{-1}}_{rm}\boldsymbol{B}_{rm}\{\varepsilon_{rm}\}\\ &= \boldsymbol{C}^{-1}_{rm}\{\varepsilon_{rm}\}\end{aligned} \tag{41}$$

where $\boldsymbol{B}_{rm}$ is the coordinate transformation matrix for the global and local mechanical stiffness tensors of the jointed rock mass (see Appendix B).

The global mechanical stiffness tensor $\boldsymbol{D}_{rm}$ of the jointed rock mass consisting of an intact rock matrix and joint sets can thus be calculated from its global mechanical compliance tensor $\boldsymbol{C}_{rm}$ by inverting as follows:

$$\boldsymbol{D}_{rm} = \boldsymbol{B}^T_{rm}\boldsymbol{D}^L_{rm}\boldsymbol{B}_{rm} = \boldsymbol{B}^T_{rm}\boldsymbol{C}^{L^{-1}}_{rm}\boldsymbol{B}_{rm} = \boldsymbol{C}^{-1}_{rm} \tag{42}$$

5. CONCLUSIONS

A series of global hydraulic conductivity and mechanical stiffness tensors for variably saturated true anisotropic intact rock matrices, joints, joint sets, and jointed rock masses was formulated to expand the fully coupled poroelastic governing equations presented by Kim (2004) for groundwater flow and solid skeleton deformation in porous geologic media to those for fractured and fractured porous geologic media. The global hydraulic conductivity tensors were derived from the local hydraulic conductivity tensors using coordinate transformation on the basis of the generalized Darcy's law. The global mechanical stiffness tensors were then derived from the local or global mechanical compliance tensors using coordinate transformation on the basis of the generalized Hooke's law. Although the global hydraulic conductivity and mechanical stiffness tensors formulated in this paper will not apply to all types of geologic media, they can find some useful applications in many problems associated with fully coupled groundwater flow and solid skeleton deformation in geologic media.

APPENDIX A: COORDINATE TRANSFORMATION TENSOR FOR HYDRAULIC CONDUCTIVITY TENSORS

In equations (3), (7), (12), and (19), $\boldsymbol{A}_m$, $\boldsymbol{A}_{jt}$, $\boldsymbol{A}_{js}$, and $\boldsymbol{A}_{rm}$ are the coordinate transformation matrices for the global and local effective hydraulic conductivity tensors of an intact rock matrix (m), joint (jt), joint set (js), and jointed rock mass (rm), respectively, as follows (Clebsch, 1994; Kim, 2003):

$$\boldsymbol{A}_I = \begin{bmatrix} l_{x'} & m_{x'} & n_{x'} \\ l_{y'} & m_{y'} & n_{y'} \\ l_{z'} & m_{z'} & n_{n'} \end{bmatrix}_I \qquad I = m,\ jt,\ js,\ rm \tag{A1}$$

where the nine components are the direction cosines between the local coordinates (x', y', z') (i.e., principal axes or directions) and the global coordinates (x, y, z), and thus they can be determined from geologic angle measurements of planar (x', y') structures (e.g., bedding, foliation, joint) and linear (x') structures (e.g., mineral elongation, striation, joint intersection) using the mathematical equations suggested by Kim (2005b). Here l, m, and n stand for x, y, and z, respectively. Note that $\boldsymbol{A}_{js} = \boldsymbol{A}_{jt}$.

APPENDIX B: COORDINATE TRANSFORMATION TENSOR FOR MECHANICAL STIFFNESS TENSORS

In equations (22), (26), (31), and (41), $\boldsymbol{B}_m$, $\boldsymbol{B}_{jt}$, $\boldsymbol{B}_{js}$, and $\boldsymbol{B}_{rm}$ are the coordinate transformation matrices for the global and local mechanical stiffness tensors of an intact rock matrix (m), joint (jt), joint set (js), and jointed rock mass (rm), respectively, as follows (Cook et al., 1989; Kim, 2003):

$$\boldsymbol{B}_I = \begin{bmatrix} l^2_{x'} & m^2_{x'} & n^2_{x'} & l_{x'}m_{x'} & m_{x'}n_{x'} & n_{x'}l_{x'} \\ l^2_{y'} & m^2_{y'} & n^2_{y'} & l_{y'}m_{y'} & m_{y'}n_{y'} & n_{y'}l_{y'} \\ l^2_{z'} & m^2_{z'} & n^2_{z'} & l_{z'}m_{z'} & m_{z'}n_{z'} & n_{z'}l_{z'} \\ 2l_{x'}l_{y'} & 2m_{x'}m_{y'} & 2n_{x'}n_{y'} & l_{x'}m_{y'} + l_{y'}m_{x'} & m_{x'}n_{y'} + m_{y'}n_{x'} & n_{x'}l_{y'} + n_{y'}l_{x'} \\ 2l_{y'}l_{z'} & 2m_{y'}m_{z'} & 2n_{y'}n_{z'} & l_{y'}m_{z'} + l_{z'}m_{y'} & m_{y'}n_{z'} + m_{z'}n_{y'} & n_{y'}l_{z'} + n_{z'}l_{y'} \\ 2l_{z'}l_{x'} & 2m_{z'}m_{x'} & 2n_{z'}n_{x'} & l_{z'}m_{x'} + l_{x'}m_{z'} & m_{z'}n_{x'} + m_{x'}n_{z'} & n_{z'}l_{x'} + n_{x'}l_{z'} \end{bmatrix}_I$$

$$I = m,\ jt,\ js,\ rm \tag{B1}$$

where the thirty-six components can be determined using the nine components in the corresponding coordinate transformation matrix $\boldsymbol{A}_I$ in equation (A1). Note that $\boldsymbol{B}_{js} = \boldsymbol{B}_{jt}$.

The global effective stress tensors of an intact rock matrix (m), joint (jt), joint set (js), and jointed rock mass (rm) can be obtained from the corresponding local effective stress tensors, respectively, using equation (B1) as follows:

$$\{\sigma'_I\} = \boldsymbol{B}_I^T\{\sigma'_I\}^L \quad I = m, jt, js, rm \tag{B2}$$

The local strain tensors of an intact rock matrix (m), joint (jt), joint set (js), and jointed rock mass (rm) can also be obtained from the corresponding global strain tensors, respectively, using equation (B1) as follows:

$$\{\varepsilon_I\}^L = \boldsymbol{B}_I\{\varepsilon_I\} \quad I = m, jt, js, rm \tag{B3}$$

APPENDIX C: COORDINATE TRANSFORMATION TENSOR FOR MECHANICAL COMPLIANCE TENSORS

In equation (38), $\boldsymbol{F}_m$, $\boldsymbol{F}_{jt}$, $\boldsymbol{F}_{js}$, and $\boldsymbol{F}_{rm}$ are the coordinate transformation matrices for the global and local mechanical compliance tensors of an intact rock matrix (m), joint (jt), joint set (js), and jointed rock mass (rm), respectively, as follows (Cook et al., 1989; Kim, 2003):

$$\boldsymbol{F}_I = \begin{bmatrix} l_{x'}^2 & m_{x'}^2 & n_{x'}^2 & 2l_{x'}m_{x'} & 2m_{x'}n_{x'} & 2n_{x'}l_{x'} \\ l_{y'}^2 & m_{y'}^2 & n_{y'}^2 & 2l_{y'}m_{y'} & 2m_{y'}n_{y'} & 2n_{y'}l_{y'} \\ l_{z'}^2 & m_{z'}^2 & n_{z'}^2 & 2l_{z'}m_{z'} & 2m_{z'}n_{z'} & 2n_{z'}l_{z'} \\ l_{x'}l_{y'} & m_{x'}m_{y'} & n_{x'}n_{y'} & l_{x'}m_{y'}+l_{y'}m_{x'} & m_{x'}n_{y'}+m_{y'}n_{x'} & n_{x'}l_{y'}+n_{y'}l_{x'} \\ l_{y'}l_{z'} & m_{y'}m_{z'} & n_{y'}n_{z'} & l_{y'}m_{z'}+l_{z'}m_{y'} & m_{y'}n_{z'}+m_{z'}n_{y'} & n_{y'}l_{z'}+n_{z'}l_{y'} \\ l_{z'}l_{x'} & m_{z'}m_{x'} & n_{z'}n_{x'} & l_{z'}m_{x'}+l_{x'}m_{z'} & m_{z'}n_{x'}+m_{x'}n_{z'} & n_{z'}l_{x'}+n_{x'}l_{z'} \end{bmatrix}_I$$

$$I = m, jt, js, rm \tag{C1}$$

where the thirty-six components can be determined using the nine components in the corresponding coordinate transformation matrix $\boldsymbol{A}_I$ in equation (A1). Note that $\boldsymbol{F}_{js} = \boldsymbol{F}_{jt}$.

The local effective stress tensors of an intact rock matrix (m), joint (jt), joint set (js), and jointed rock mass (rm) can be obtained from the corresponding global effective stress tensors, respectively, using equation (C1) as follows:

$$\{\sigma'_I\}^L = \boldsymbol{F}_I\{\sigma'_I\} \quad I = m, jt, js, rm \tag{C2}$$

The global strain tensors of an intact rock matrix (m), joint (jt), joint set (js), and jointed rock mass (rm) can also be obtained from the corresponding local strain tensors, respectively, using equation (C1) as follows:

$$\{\varepsilon_I\} = \boldsymbol{F}_I^T\{\varepsilon_I\}^L \quad I = m, jt, js, rm \tag{C3}$$

ACKNOWLEDGMENTS: This work was supported by the Carbon Dioxide Reduction and Sequestration Research and Development Center of the 21st Century Frontier Research and Development Program, Ministry of Science and Technology, Republic of Korea. This work was also supported in part by the Brain Korea 21 Project, Ministry of Education and Human Resources Development, Republic of Korea. The author would also like to thank Professors Chandong Chang and Youngseuk Keehm for their invaluable and constructive comments.

REFERENCES

Bear, J., 1972, Dynamics of Fluids in Porous Media, republication. American Elsevier Publishing Company, New York, 764 p.

Bear, J. and Corapcioglu, M.Y., 1981, Centrifugal filtration in deformable porous media. In: Bear, J. and Corapcioglu, M.Y. (eds.), A series of Four Papers on Water Flow in Deformable Porous Media. Technical Report UMR-0284, Department of Civil Engineering, University of Michigan, Ann Arbor, Michigan, Section I, 42 p.

Biot, M.A., 1941, General theory of three-dimensional consolidation. Journal of Applied Physics, 12, 155–164.

Biot, M.A., 1955, Theory of elasticity and consolidation of a porous anisotropic soil. Journal of Applied Physics, 26, 182–185.

Bishop, A.W. and Blight, G.E., 1963, Some aspects of effective stress in saturated and partly saturated soils. Géotechnique, 13, 177–197.

Carman, P.C., 1937, Fluid flow through granular beds. Transactions of the Institution of Chemical Engineers, 15, 150–166.

Carman, P.C., 1938, The determination of the specific surface of powders, I. Journal of the Society of Chemical Industry, 57, 225–234.

Carman, P.C., 1956, Flow of Gases through Porous Media. Butterworths Scientific Publications, London, 182 p.

Cheng, A.H.D., 1997, Material coefficients of anisotropic poroelasticity. International Journal of Rock Mechanics and Mining Sciences, 34, 199–205.

Clebsch, A. (ed.), 1994, Selected Contributions to Ground-Water Hydrology by C. V. Theis, and a Review of His Life and Work. U. S. Geological Survey Water-Supply Paper 2415, 70 p.

Cook, R.D., Malkus, D.S. and Plesha, M.E., 1989, Concepts and Applications of Finite Element Analysis, third edition. John Wiley and Sons, New York, 630 p.

Goodman, R.E. and Dubois, J., 1972, Duplication of dilatancy in analysis of jointed rock. Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, 98, 399–422.

Goodman, R.E., Taylor, R.L. and Brekke, T.L., 1968, A model for the mechanics of jointed rock. Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, 94, 637–659.

Kim, J.M., 1996, A Fully Coupled Model for Saturated-Unsaturated Fluid Flow in Deformable Porous and Fractured Media. Ph.D. thesis, Pennsylvania State University, University Park, Pennsylvania, 201 p.

Kim, J.M., 2000, A fully coupled finite element analysis of water-table fluctuation and land deformation in partially saturated soils due to surface loading. International Journal for Numerical Methods in Engineering, 49, 1101–1119.

Kim, J.M., 2003, Integrated Modeling for Water Level Fluctuation and Land Deformation Due to Groundwater Pumping. Final Report 5-3-1, Seoul National University, Seoul, Republic of Korea, 200 p. (in Korean with English summary and appendices)

Kim, J.M., 2004, Fully coupled poroelastic governing equations for groundwater flow and solid skeleton deformation in variably saturated true anisotropic porous geologic media. Geosciences Journal, 8, 291–300.

Kim, J.M., 2005a, Three-dimensional numerical simulation of fully coupled groundwater flow and land deformation in unsaturated true anisotropic aquifers due to groundwater pumping. Water Resources Research, 41, W01003, doi:10.1029/2003WR002941.

Kim, J.M., 2005b, Vectorial formulation of direction cosines for anisotropic geologic structures from their geologic angle measurements. Mathematical Geology, 37, 929–941.

Kim, J.M. and Parizek, R.R., 1997, Numerical simulation of the Noordbergum effect resulting from groundwater pumping in a layered aquifer system. Journal of Hydrology, 202, 231–243.

Kim, J.M. and Parizek, R.R., 1999a, Three-dimensional finite element modelling for consolidation due to groundwater withdrawal in a desaturating anisotropic aquifer system. International Journal for Numerical and Analytical Methods in Geomechanics, 23, 549–571.

Kim, J.M. and Parizek, R.R., 1999b, A mathematical model for the hydraulic properties of deforming porous media. Ground Water, 37, 546–554.

Kim, J.M. and Parizek, R.R., 2005, Numerical simulation of the Rhade effect in layered aquifer systems due to groundwater pumping shutoff. Advances in Water Resources, 28, 627–642.

Kim, J.M., Parizek, R.R. and Elsworth, D., 1997, Evaluation fully-coupled strata deformation and groundwater flow in response to longwall mining. International Journal of Rock Mechanics and Mining Sciences, 34, 1187–1199.

Kozeny, J., 1927, Über kapillare leitung des wassers im boden. Sitzungsberichte, Akademie der Wissenschaften, Wien, 136, 271–306.

Lekhnitskii, S.G., 1963, Theory of Elasticity of an Anisotropic Elastic Body. Holden-Day, San Francisco, 404 p.

Love, A.E.H., 1944, A Treatise on the Mathematical Theory of Elasticity, fourth edition, republication. Dover Publications, New York, 643 p.

Noorishad, J., Mehran, M. and Narasimhan, T.N., 1982, On the formulation of saturated-unsaturated fluid flow in deformable porous media. Advances in Water Resources, 5, 61–62.

Oda, M., 1986, An equivalent continuum model for coupled stress and fluid flow analysis in jointed rock masses. Water Resources Research, 22, 1845–1856.

Pariseau, W.G., 1993, Equivalent properties of a jointed Biot material. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 30, 1151–1157.

Parsons, R.W., 1966, Permeability of idealized fractured rock. Society of Petroleum Engineering Journal, 6, 126–136.

Safai, N.M. and Pinder, G.F., 1979, Vertical and horizontal land deformation in a desaturating porous medium. Advances in Water Resources, 2, 19–25.

Snow, D.T., 1968, Rock fracture spacings, openings, and porosities. Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, 94, 73–91.

Snow, D.T., 1969, Anisotropic permeability of fractured media. Water Resources Research, 5, 1273–1289.

Verruijt, A., 1969, Elastic storage of aquifers. In: De Wiest, R.J.M. (ed.), Flow through Porous Media. Academic Press, New York, p. 331–376.

Zhu, W. and Wang, P., 1993, Finite element analysis of jointed rock masses and engineering application. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 30, 537–544.

Manuscript received November 15, 2005
Manuscript accepted September 18, 2007