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GEOPHYSICS

Seismic Migration by Gaussian Beams Summation[1]

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Searching for composite traps of hydrocarbons is an actual problem of oil- and gas-geology. The development of fast and at the same time accurate methods for migration of surface-recorded seismic data for construction of an image of the geological boundaries in the subsurface remains one of the major directions of development in modern exploration geophysics.

The evolution of computers has enabled practical application of various expensive migration algorithms. For example, in seismic prospecting, pre-stack Kirchhoff migration is nowadays widely used.

Unfortunately, in the construction of seismic images of complex geology containing vertical and/or subvertical boundaries, and also erosional surfaces and layers that have pinch-outs, numerous caustics arise, and most migration methods face difficulties in these cases.

However, it is well-known ([1, 2]) that the Gaussian beams method reduces to an effective algorithm of wavefield calculation, which is independent of the presence of caustics and their geometrical structure. Therefore, it seems natural to use this method in problems of migration in composite heterogeneous media where the presence of caustics is a common phenomenon.

Apparently, the application of Gaussian beams to seismic migration problems was published for the first time in [3]. Typical features of the migration algorithm suggested in that paper are (1) expansion of seismograms in an integral on plane waves and (2) representation of each plane wave as a superposition of Gaussian beams and back-propagation of the wave into the migration domain with the help of Gaussian beams. However, we cannot consider this procedure to be adequate in the case of velocity models corresponding to complex geologic structures.

Recently, Gaussian beams began to be applied to imaging within the framework of a Born approximation [4]. However, in the geophysical context in which the Born approximation is used, see, e.g., [5], it contains, generally speaking, a mathematical inconsistency which concerns the following: for a compact domain of migration, the basic equation of the Born method is a weak singular integral equation of the Fredholm type. Iterations for this equation converge right up to the first zero of the Fredholm denominator, and that imposes a restriction from above on frequency ω. At the same time, replacement of the Green function by its high-frequency asymptotic form and the subsequent inversion of the elliptic Fourier integral operator in terms of pseudodifferential operators require large ω, formally $\omega \to \infty$. Therefore, the substantiation of this approach demands additional examinations and we cannot share the optimism of the authors of [4] concerning the construction of seismic images in the true amplitudes within the framework of this method.

In our approach we adhere to the classical plan of Kirchhoff migration in which the Green functions are replaced by their asymptotic representation in terms of Gaussian beams. The problem that we solve can be visually described as follows.

Let us assume that we have identified a reflected wave on the seismogram. Then, for a smoothed version of the subsurface velocity model, we can propagate this wave back in time and fix it in a position in the migration domain where it coincides in phase with the direct wave generated by a point source, placed on the earth's surface.

This position is determined by the maximum of coherence (in the frequency domain) between the direct and back-propagated wavefields. As the requirement of coherence of these fields on a reflecting boundary is necessary but not sufficient, a procedure of stacking is essential because it allows us to remove areas of a casual coherence.

We assume that a wavefield $U(\mathbf{x}, t)$, $\mathbf{x} = (x, y, z)$ satisfies the wave equation

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[1] This article was translated by the authors.

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$$LU \equiv \left(\Delta - \frac{1}{c^2(\mathbf{x})}\frac{\partial^2}{\partial t^2}\right)U = 0,$$

where Δ is the Laplace operator and $c(\mathbf{x})$ is the wave propagation velocity.

The wavefield recorded on the seismic boundary S is denoted by $U^{(0)}(\mathbf{x}, t) = U(x, y, 0, t)$, $t \in [0, T]$. Inverse propagation of the wavefield $U^{(0)}(\mathbf{x}, t)$ to the migration domain is carried out with the help of the Kirchhoff formula.

Let us designate through $G(\mathbf{x}, t; \mathbf{x}_0, t_0)$ the following Green function for the wave equation

$$LG(\mathbf{x}, t; \mathbf{x}_0, t_0) = \delta(t - t_0)\delta(\mathbf{x} - \mathbf{x}_0), \quad G|_{t < t_0} \equiv 0.$$

Then the Kirchhoff formula takes the form

$$U(\mathbf{x}_0, t_0) = \int_{t_0}^{T} dt \int\int_{\partial\Omega} dS_x \Big(U^{(0)}(\mathbf{x}, t)\frac{\partial}{\partial n_x} G(\mathbf{x}, t; \mathbf{x}_0, t_0) - G(\mathbf{x}, t; \mathbf{x}_0, t_0)\frac{\partial}{\partial n_x} U^{(0)}(\mathbf{x}, t)\Big),$$

in which, as usual, $\partial\Omega$ is the boundary of a closed domain Ω and $\frac{\partial}{\partial n_x}$ is the derivative along the exterior normal to Ω.

Next, we forcedly replace $\partial\Omega$ with the nonclosed seismic boundary S (in most cases S is a part of the plane $z = 0$). The Green function in the Kirchhoff formula is substituted by its asymptotic form in terms of Gaussian beams $G_{\mathrm{GB}}(\mathbf{x}, t; \mathbf{x}_0, t_0)$, which is built in such a way that it satisfies the condition $G|_{z=0} = 0$ on the seismic boundary in the Kirchhoff approximation. In outcome there is the following approximate formula for the back-propagated wavefield calculated at a point $\mathbf{x}_0$ of the migration domain at an instant $t_0 \in [0, T]$:

$$U(\mathbf{x}_0, t_0) \approx -2\int_{t_0}^{T} dt \int\int_{z=0} dx\, dy\, U^{(0)}(\mathbf{x}, t)\frac{\partial}{\partial z} G_{\mathrm{GB}}(\mathbf{x}, t; \mathbf{x}_0, t_0).$$

The direct wavefield $U_{x_s}^{(D)}(\mathbf{x}, t)$, generated by a point source placed at x_s, is the solution of the following problem

$$LU_{x_s}^{(D)}(\mathbf{x}, t) = f(t)\delta(\mathbf{x} - \mathbf{x}_s), \quad U_{x_s}^{(D)}\Big|_{t<0} \equiv 0,$$

where $f(t)$ is an initial wavelet (source wavelet), with $f(t)|_{t<0} = 0$. The asymptotic form of the Green function for calculation of the direct wavefield is also constructed with the help of Gaussian beams, so that we get the following approximate formula

$$U_{x_s}^{(D)}(\mathbf{x}, t) \approx \frac{1}{\pi}\mathrm{Re}\int_0^{\infty} d\omega\, e^{-i\omega t} f_F(\omega) G_{\mathrm{GB}}(\mathbf{x}, \mathbf{x}_s; \omega),$$

where $f_F(\omega)$ is the Fourier transform of the initial wavelet and $G_{\mathrm{GB}}(\mathbf{x}, \mathbf{x}_s; \omega)$ is the asymptotic form of the Green function of the Helmholtz equation represented in terms of Gaussian beams.

Note that for a special kind of initial wavelet $f(t)$, it is possible, with good precision, to avoid calculation of the integral over frequency by means of usage of space–time Gaussian beams (quasi-jets) or by application of the residue theorem (see for details [6], [7]).

Furthermore, an important stage in embodying the method is to build the asymptotic form of the Green function $G_{\mathrm{GB}}(\mathbf{x}, \mathbf{x}_s; \omega)$ for the Helmholtz equation in terms of Gaussian beams, as the following formula holds true

$$G_{\mathrm{GB}}(\mathbf{x}, t; \mathbf{x}_0, t_0) = \frac{1}{\pi}\mathrm{Re}\int_0^{\infty} d\omega\, e^{-i\omega(t - t_0)} G_{\mathrm{GB}}(\mathbf{x}, \mathbf{x}_0; \omega).$$

The solution of this problem was explained in detail in the paper [1] where the Gaussian beam method for calculations of wavefields in the high-frequency approximation was suggested for the first time.

Here, we specify only the basic steps of the method. The Green function $G_{\mathrm{GB}}(\mathbf{x}, \mathbf{x}_0; \omega)$ is associated with a field of central rays emanating from the point $\mathbf{x}_0$. The ray-parameters are angles ϑ and φ of the spherical coordinate system. For each ray a Gaussian beam U_{GB} in local ray-centered coordinates s, q_1, q_2, is built. Then, the Green function G_{GB} is defined by the formula

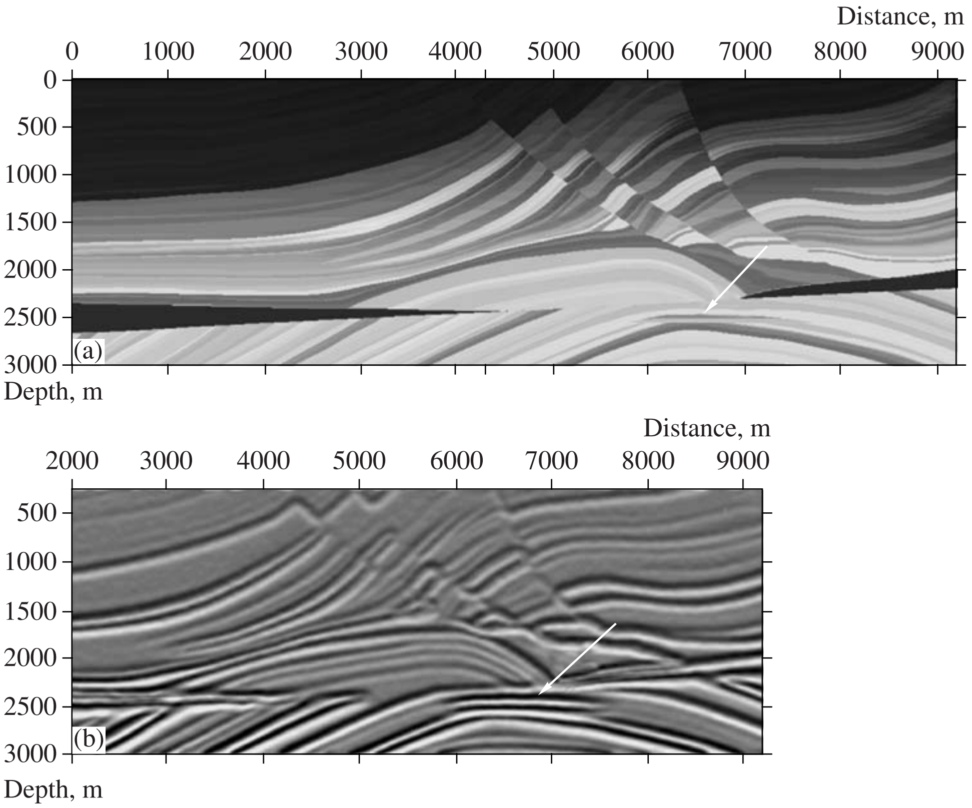
$$G_{\mathrm{GB}}(\mathbf{x}, \mathbf{x}_0; \omega) = \int d\vartheta \int d\varphi \Phi(\theta, \varphi; \omega) U_{\mathrm{GB}}(\mathbf{x}, \mathbf{x}_0; \omega),$$

where the initial amplitudes $\Phi(\theta, \varphi; \omega)$ of the Gaussian beams are found by matching this formula with the ray-asymptotic form of the Green function for the Helmholtz equation in the free-from-caustics vicinity of the source. The limits of integration over ϑ and φ remain parameters of the algorithm. Visualization of the reflecting boundaries in the migration domain is carried out by means of calculation of a correlation function (in the time-domain) between direct $U^{(D)}(\mathbf{x}_0, t_0)$ and back-propagated $U(\mathbf{x}_0, t_0)$ wavefields. The position of boundaries is determined by the maximum of the correlation function (compare, for example, with paper [8]).

The properties of the Gaussian beam method lead to the following advantages of our approach to migration problems: (1) the absence of problems with caustics and (2) no necessity in solving the two-point problem, specific for the ray method, and also in interpolation of wavefields.

Furthermore, the suggested method automatically takes into account in construction of the seismic images not only the first, but also late arrivals, and also it uses the full range of amplitudes of direct and backward propagated wavefields.

It is especially significant that a specific modification of the suggested method allows computation of the



Marmousi velocity model (a) and migration image (b).

true-amplitude images. This situation will be demonstrated in subsequent publications.

In order to be convinced of the practical applicability of the method, we carried out a pre-stack depth migration for the synthetic model MARMOUSI. This dataset consists of the velocity model presented in Fig. 1a, which corresponds to a composite thin-layer medium with numerous pinch-outs and fault-generated gas- and oil-traps, and also the set of seismograms generated by means of a finite-difference method and providing multifold pre-stack data.

Our purpose was the complete, as far as possible, reconstruction of the structure of this complex model that includes laminated sands, faults, pinch-outs, and gas- and oil-traps. There was particular interest in how the algorithm performed, in terms of accuracy of the migration image, in one of the structurally most complex parts of the MARMOUSI subsurface: the region of the hydrocarbon lens, which is shaded by a salt-cornice and a massive dome consisting of marl (in the figure these lenses are marked with an arrow).

For this purpose, a computer program was created for testing the suggested pre-stack depth migration method. It allows the user to compute an image of any amount of points in the migration- (viz. depth-) domain and also enables the user, in the case of limited availability of computing power or too great a volume of seismic data, to perform the migration only in those subsurface domains that are of interest at that particular moment. Especially for 3D-imaging this "target-oriented" imaging capability has large computational advantages.

Note that the first version of the program based on the mathematical approach described above was created in 2003 and was successfully applied to several model problems [9]. Later, the computing effectiveness of the method was considerably boosted by means of a rational algorithm of calculation of coordinates s, q_1, q_2, and also by grouping the points in the migration domain in clusters associated with the widths of the Gaussian beams.

It is known that the preliminary choice of the method of smoothing (and also its power) of the initial velocity model essentially influences the resolution of the migration image. For smoothing the velocity model, we used an algorithm based on the least squares method which permits us to conduct the degree of smoothing of not only the velocity, but also of its first and second derivatives. We also would like to emphasize that the rational accounting in the ray-tracing algorithm of wavefront curvature in a neighborhood of an exit point of the ray considerably influences the precision of the travel time calculation and also the quality of the migration image.

As the algorithms of the migration method described here for the most part have a parallel structure, the expandability under parallelizing in our case tends to a linear function of the number of processors.

The migration result for the MARMOUSI model obtained by means of our program on the cluster is represented in Fig. 1b (we should mention here that bandpass filtration was carried out for raising the clarity of the obtained image).

It is clear that our migration method allows us to spot correctly on the depth-image the position of closely spaced reflecting horizons and to localize precisely the boundaries of the gas- and oil-traps. The structure of the top- and bottom-boundaries of the hydrocarbon lens is clearly visible.

In summary, our migration method is capable of providing a high-quality, high-resolution, structural image of a structurally highly complex subsurface.

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