

Special Section — Marine Controlled-Source Electromagnetic Methods

1D inversion and resolution analysis of marine CSEM data

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ABSTRACT

We present results from an investigation into 1D inversion of controlled-source electromagnetic (CSEM) data. Based on inspection of a data set, we formulate a simple empirical noise model described by a few pragmatically determined parameters. We also investigate the effects of transmitter height above the seafloor and include the data uncertainty resulting from varying transmitter height in our noise model. Based on the noise model and assumptions about the available data, we analyze model parameter uncertainty estimates derived from the a posteriori model covariance matrix for a resistive layer buried at depth. We find that the layer parameters uncertainty primarily depends on the depth of burial and the thickness of the layer. We then formulate quantitative bounds for these parameters, within which we have a small uncertainty of the parameters of the resistive layer. The depth of burial and the transverse resistance of the layer become better determined the higher its resistance. We invert a field data set with multilayer and four-layer models and find a resistive layer at a depth of 1500–1600 m below the seafloor. Seismic results from the area indicate a depth of 2000 m.

INTRODUCTION

Controlled-source electromagnetic (CSEM) methods have become an important tool for investigations in the marine environment. The method has evolved, from the 1980s (Chave and Cox, 1982; Chave et al., 1991) to a point where it has found its role as a valued supplement to seismic methods in the evaluation of oil and gas prospects in the marine environment.

The specific contribution of marine controlled-source electromagnetic methods is their ability to map resistive layers at depth,

thereby indicating the possible presence of oil and gas; layers that can be difficult to discern from seismic records alone. Resistivity increases considerably when oil and gas displace salt water. Resistivity is mapped as a function of depth by measuring the electric and magnetic fields at the seafloor from an electric bipole source as a function of the transmitter-receiver (Tx-Rx) separation. Resistive layers appear as an increase in the electric field (Eidesmo et al., 2002).

Several companies now offer CSEM investigations and various types of data processing and inversion (MacGregor et al., 2001; Amundsen et al., 2004; Mittet et al., 2004; Johansen et al., 2005). Here, we present results from an investigation of the applicability of 1D inversion of CSEM data. A noise model for the data set is developed, we investigate the effects of oscillating source height, inversion is carried out with multilayer models (smooth models) and few-layer models, and we investigate the a posteriori model parameter uncertainty estimates for a buried resistive layer as a function of its resistivity, thickness, and depth of burial.

For an inversion to be meaningful and an assessment of parameter uncertainty to be possible, data noise must be estimated. By inspection of the field data set, a simple empirical noise model is developed to describe the data. Data are also influenced by the varying Tx height above the sea floor during CSEM data collection. We have estimated the maximum effect on the data and included it in the noise estimates.

Much effort is going into the development of 2D and 3D modeling codes (e.g., Newman and Alumbaugh, 1997; Newman and Hoversten, 2000; Hoversten et al., 2006) and inversion strategies (e.g., Carazzone et al., 2005; Zhdanov and Yoshioka, 2005; Abubakar et al., 2006) for CSEM data. However, 1D inversion of electromagnetic data is readily available, and though the models over which CSEM data are collected are not in general 1D, a 1D model may suffice where lateral changes are slow. 1D inversion of a joint CSEM and MT data set is presented in Tompkins et al. (2004). We have inverted our field data with multilayer (smooth) and few-layer 1D models to

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produce a model section by concatenation and compared the model section with a seismic section from the area.

The resolution capabilities of CSEM data have been studied by many authors. Edwards (1997) analyzes the resolution capability of transient electric dipole-dipole configurations, with special emphasis on the resolution of resistive gas hydrate layers relatively close to the seafloor. Constable and Weiss (2006) present an analysis of the relative sensitivity to a resistive layer in terms of the frequency and Tx-Rx separation for inline electric dipole-dipole measurements and relate the results to the noise floor of CSEM measurements. In this paper we shall present an analysis of the a posteriori model parameter uncertainty estimates of the CSEM method in relation to a buried resistive target layer, based on the a posteriori model covariance matrix of a least-squares inversion formulation. We have assumed a typical present day data set containing amplitude and phase data of the inline electric and perpendicular magnetic fields at the three frequencies 0.05, 0.15, and 0.25 Hz.

MODELING, INVERSION, AND ANALYSIS

The modeling, inversion, and analysis are performed using a subroutine specifically developed for CSEM responses and integrated in the program SELMA (Christensen and Auker, 1992) using standard expressions for the electric and magnetic fields (Sinha and Bhattacharya, 1967; Chlamtac and Abramovici, 1981). The fields are calculated in the frequency domain as Hankel transforms of wavenumber domain expressions (Christensen, 1990). The basic model is a 1D layered half-space consisting of homogeneous and transversely isotropic layers, the model parameters are thus the layer resistivities, the layer thicknesses, and the coefficient of anisotropy for each layer. However, we shall consider all layers to be isotropic. Transmitters (Tx) and receivers (Rx) can be situated anywhere in the first layer.

The inversion problem is solved using an iterative damped least-squares approach (Tarantola and Valette, 1982; Menke, 1989). The model update at the n th iteration is given by

$$\begin{aligned} \mathbf{m}_{n+1} = & \mathbf{m}_n + [\mathbf{G}_n^H \mathbf{C}_{\text{obs}}^{-1} \mathbf{G}_n + \mathbf{C}_{\text{prior}}^{-1} + \mathbf{R}^H \mathbf{C}_R^{-1} \mathbf{R} + \lambda \mathbf{I}]^{-1} \\ & \times [\mathbf{G}_n^H \mathbf{C}_{\text{obs}}^{-1} (\mathbf{d}_{\text{obs}} - \mathbf{g}(\mathbf{m}_n)) \\ & + \mathbf{C}_{\text{prior}}^{-1} (\mathbf{m}_{\text{prior}} - \mathbf{m}_n) + \mathbf{R}^H \mathbf{C}_R^{-1} \mathbf{R} \mathbf{m}_n], \end{aligned} \quad (1)$$

where $\mathbf{m}$ is the model vector consisting of the logarithm of the model parameters, $\mathbf{G}$ is the Jacobian matrix containing the derivatives of the data with respect to the logarithm of the model parameters, $\mathbf{C}_{\text{obs}}$ is the data error covariance matrix, $\mathbf{C}_{\text{prior}}$ is the covariance matrix for the prior model, $\mathbf{R}$ is the roughening matrix containing 1 and -1 s for the constrained parameters and 0s in all other places, $\mathbf{C}_R$ is a covariance matrix describing the strength of the constraints, λ is the Marquard damping factor, $\mathbf{I}$ is the identity matrix, $\mathbf{d}_{\text{obs}}$ is the field data vector, $\mathbf{g}(\mathbf{m})$ is the nonlinear forward response vector, and $\mathbf{m}_{\text{prior}}$ is the prior model vector. The noise on the data is assumed to be uncorrelated, so that $\mathbf{C}_{\text{obs}}$ is a diagonal matrix containing the data variances.

Both few-layer and multilayer inversions of CSEM data will be presented in the following sections. In the multilayer inversion, the layer boundaries are totally fixed and only the layer resistivities are free parameters. The multilayer inversion is regularized through vertical constraints ($\mathbf{R}$ in equation 1) imposing identity between the resistivity of neighboring layers within a given relative uncertainty. In the few-layer inversion, the layer resistivities and thicknesses are

free to vary and no equality constraints are applied to the model parameters, corresponding to excluding $\mathbf{R}$ in equation 1. Generally, the few-layer inversion aims at minimizing the data misfit using the smallest number of layers.

Besides the least-squares L_2 -norm optimization, inversion can also be carried out with L_1 -norm optimization using the algorithm of Madsen and Nielsen (1993). To obtain more blocky models, the optimization problem is solved with a L_1 -norm in the multilayer case (Farquharson and Oldenburg, 1998); whereas a traditional L_2 -norm is used in the few-layer case.

The analyses of the a posteriori model parameter uncertainty estimates rely on a linear approximation to the a posteriori model covariance matrix $\mathbf{C}_{\text{est}}$, given by

$$\mathbf{C}_{\text{est}} = [\mathbf{G}_n^H \mathbf{C}_{\text{obs}}^{-1} \mathbf{G}_n + \mathbf{C}_{\text{prior}}^{-1} + \mathbf{R}^H \mathbf{C}_R^{-1} \mathbf{R}]^{-1}, \quad (2)$$

where $\mathbf{G}_n$ is based on the final model (Inman, 1975). The a posteriori model parameter uncertainty estimates are obtained as the square root of the diagonal elements of $\mathbf{C}_{\text{est}}$.

CSEM DATA SET AND NOISE MODEL

Data set

Our CSEM line is in the Mauritanian region. A total of 37 receivers, Rx01–Rx37, was deployed on the seafloor along a straight profile with a distance of 750 m between receivers. A 270 m Tx dipole with an alternating direct current source of ≈ 1000 A was towed along the profile line at an elevation above the seabed of ≈ 40 m, current direction was switched every 10s. Electric and magnetic data recorded in the time domain were stacked and transformed to frequency domain by the contractor at 100 m intervals along the profile to give the three frequencies 0.05, 0.15, and 0.25 Hz.

The data set thus consists of inline electric and transverse magnetic field amplitude and phase for the intow (Tx going toward the receiver) and outtow (Tx going away from receiver) soundings for each of the 37 receivers at the frequency 0.05 Hz and its first two odd harmonics. Most of the sounding data appear to be of a good quality. Magnetic data are absent at five Rx units and are of an inferior quality for six Rx units.

Model for the data noise of CSEM data

The contractor has not provided any noise specifications for the data set. However, to perform a meaningful inversion and model parameter uncertainty analysis, the data noise must be estimated. By inspection of the data set we have developed a simple empirical noise model for the electric and magnetic amplitude and phase data in which the noise characteristics are described by a few pragmatically determined parameters.

The relative uncertainty of the electric and magnetic field amplitude data can be expressed as

$$\Delta_{\text{rel}} = \sqrt{b_a^2 + (s/F)^2}, \quad (3)$$

where b_a is a basic relative noise level under which the relative uncertainty cannot go, b_a was set to 0.02 for both electric and magnetic field amplitudes, s is an absolute noise level that must be found individually for each sounding through data inspection of both electric and magnetic field data, and F is the field amplitude.

The absolute uncertainty of the electric and magnetic phase data can be expressed as

$$\Delta_{abs} = \sqrt{b_p^2 + p^2}, \quad (4)$$

where b_p is a basic absolute noise level under which the absolute uncertainty cannot go, b_p was set to 0.035 (2°) for both electric and magnetic field data, and p is an absolute noise level that must be found individually for each sounding through data inspection of both electric and magnetic field data.

It is characteristic for the phase data at a certain Tx-Rx separation, R_0 , to suddenly deteriorate completely. From about half of the Tx-Rx separation the noise increases from practically zero to around 0.5 radians. With this in mind, the absolute noise level p can be given by

$$p = \begin{cases} 0 & \text{for } r < R_0/2 \\ r/R_0 - \frac{1}{2} & \text{for } R_0/2 < r < R_0 \\ \infty & \text{for } r > R_0 \end{cases} \quad (5)$$

and must be ascribed individually for each sounding by choosing R_0 appropriately.

Typical values of the noise parameters s_E and s_M for the electric and magnetic fields, respectively, and R_0 are given in Table 1 for the three frequencies 0.05, 0.15, and 0.25 Hz.

Figure 1 is a plot of the 0.05 Hz data from Rx09, an apparently good data set, with the ascribed noise. The amplitudes of the electric and magnetic fields normalized with the Tx moment have been plotted in a double logarithmic and the phases have been plotted in a double linear coordinate system. In this way, the data plots approximately straight lines, thus assisting the noise estimation.

Errors due to variations in source altitude

The length of the source dipole is 270 m. The height of the source above the seafloor varies between 35 and 50 m along the profile in an oscillatory manner with a period of a bit less than 700 m. We have analyzed the effect of varying Tx height by modeling the response of a two-layer model with a water layer with resistivity of 0.255 Ωm , a thickness of 500 m, and a subbottom resistivity of 1 Ωm for Tx heights 35 and 50 m for 0.05, 0.15, and 0.25 Hz. At Tx-Rx separations below 500 m the difference can be up to 34% for the E-field amplitude and 8% for the H-amplitude, for separations above 500 m the difference is typically smaller than 2% for the electric field and 3% for the magnetic field. The phases do not display particularly large differences at very short separations. The maximum difference is reached at a separation of several kilometers, a distance that decreases with increasing frequency. At very long distances, the difference goes toward a non-zero asymptotic value. In Table 2, the maximum of the ratio logarithm of the amplitudes and the maximum phase difference between 35 and 50 m Tx height for the electric and magnetic fields are shown together with the asymptotic values.

It is seen that the maximum difference for the electric field reaches 2.3% for 0.25 Hz at 6150 m Tx-Rx separation. For the magnetic field the maximum difference is 3.2% for 0.25 Hz at 4200 m Tx-Rx separation. The maximum phase-difference for the electric field is

Table 1. The parameters of equations 3-5 describing the noise on the electric and magnetic data. s_E and s_M are the absolute noise levels on the electric and magnetic field amplitudes, respectively, and R_0 defines the phase noise of both electric and magnetic fields.

Frequency (Hz)	s_E (Ωm^{-2})	s_M (m^{-2})	R_0 (m)
0.05	$3 \cdot 10^{-14}$	$5 \cdot 10^{-11}$	12,000
0.15	$0.7 \cdot 10^{-14}$	$1 \cdot 10^{-11}$	9000
0.25	$0.7 \cdot 10^{-14}$	$1 \cdot 10^{-11}$	8000

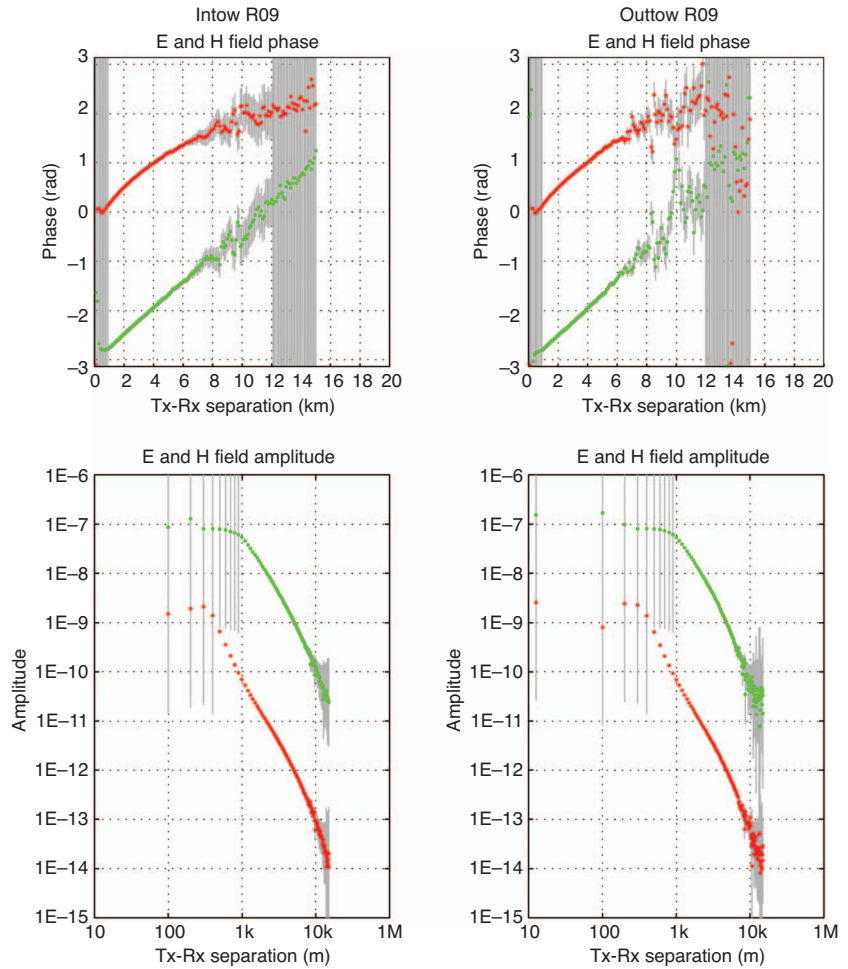


Figure 1. Data and noise plot for the frequency 0.05 Hz for the R09 receiver. Electric data are shown in red and magnetic data are shown in green. A data set of apparently good quality, the amplitudes of the electric and magnetic fields are normalized with the Tx moment and are in the units (Ωm^{-2}) and (m^{-2}), respectively.

0.042 rad and is 0.067 rad for the magnetic field. The asymptotic values are below 1% for the amplitudes of the electric and magnetic fields and below 0.016 rad for the phases.

Data have been stacked in 100 m intervals by the contractor, before inversion we bin the data in intervals according to the midpoint between Tx and Rx, so every sounding will contain data recorded with different Tx heights above the seafloor in an unpredictable manner. We have assumed a constant Tx height of 40 m for all inversions.

It is reasonable to assume that the data error and the errors caused by varying Tx height are independent, the combined influence is found by summing the variances. Table 2 lists the maximum errors between a Tx height of 35 and 50 m, the errors in assuming a Tx height of 40 m will typically be smaller. We have taken an average value of the Tx height variation error, the one for the frequency 0.15 Hz, and added it to the data error. We then obtain a basic relative error for the amplitudes of the electric and magnetic fields, b_e , of 0.03 and an absolute phase error, b_p , of 0.05 for the electric field and 0.06 for the magnetic field. These noise figures will be used in both the inversion and the analysis of the posterior model parameter uncertainty.

Possibly because of Rx amplifier saturation, we observed that the electric and magnetic data for Tx-Rx separations smaller than 500 m and 1 km, respectively, cannot be fitted with 1D models and they have been culled before inversion. We have thereby also excluded the data that are most error prone because of Tx height variations.

PARAMETER UNCERTAINTY ESTIMATES FOR THE CSEM METHOD

Galvanic and inductive aspects of the data

CSEM data are recorded in the time domain, but most often data are transformed to the frequency domain before interpretation. It is considerably faster to invert data in the frequency domain and becomes important for the 2D and 3D inversion efforts which are presently pursued with much fervor in the research community. The data sets we have analyzed were all frequency domain data already transformed by the contractor.

The horizontal electric source employed in CSEM surveys produces electric fields with both a galvanic and an inductive component (Sinha and Bhattacharya, 1967; Chlamtac and Abramovici, 1981) at any frequency. The galvanic part, corresponding to currents being injected from the electrodes, has a vertical component of the electric field and results in a charge buildup at conductivity discontinuities at the layer boundaries. The resolution of galvanic methods has a geometric relationship to distance: As a rule of thumb, if an

anomalous body is twice as far away, it has to be twice as big to produce the same measured anomaly. Galvanic data respond to resistivity contrasts and resolve resistive and conductive structures in much the same manner. Depth penetration is determined geometrically by the Tx-Rx separation and by the frequency. The sensitivity of galvanic measurements is concentrated around the electrodes. When inverting galvanic data, one of the well-known equivalencies is the high-resistivity equivalence, stating that for a thin resistive layer, neither resistivity nor thickness can be resolved. It affects the data through its resistance, the product of resistivity and thickness. Resistance is sometimes resolved even when the resistivity and thickness are not. This is particularly relevant for the main target of CSEM measurements: a thin resistive gas or oil bearing layer buried under the seafloor.

The relative importance of the inductive part of the data is determined through the unitless induction number $r\sqrt{\omega\mu\sigma}$, where r is the Tx-Rx separation, ω is the angular frequency, μ is the magnetic permeability, most often set equal to that of free space, $\mu_0 = 4\pi \cdot 10^{-7} \Omega\text{m}^{-1}$, and σ is the conductivity. For typical values of frequency (0.1 Hz) and conductivity (1 S/m) in CSEM measurements, the induction number reaches a value of 1 for $r \approx 1125$ m. Considering that Tx-Rx separations go up to 15 km, the induction number can get very high and the inductive part of the data becomes dominant. When the induction number becomes high, the attenuation of the electromagnetic fields becomes exponential with distance and depth penetration stifled. When a resistive layer is present, increasing the frequency will also increase the magnitude of the measured electric field up to a frequency where attenuation sets in, after which the field amplitude decreases rapidly (Constable and Weiss, 2006). The inductive element of CSEM data, corresponding to the inductive effects of the Tx dipole wire, has no vertical component of the electric field. It responds to the absolute value of the horizontal conductivity, high resistivities are poorly resolved. In the context of resolving thin resistive layers, the inductive element of CSEM data responds primarily to the well-conducting layers above and below the layer. By contributing to the determination of the depth to the good conductor below the resistive layer, the effect of the high-resistivity equivalence is reduced and the resistivity and thickness of the resistive layer can be partially or fully resolved.

Parameter uncertainty estimates for a resistive layer at depth

We analyzed the model parameter uncertainty estimates for the CSEM method for different data combinations and for a wide variety of models. Many of these results are no surprise and can be stated simply: Having amplitude and phase data is better than having only

Table 2. The maximum of the ratio logarithm of the electric and magnetic fields and the maximum phase difference for Tx heights of 35 and 50 m. The Tx-Rx separation, where the maximum occurs, is shown after the @-sign. The asymptotic value is listed after the slash.

Abs(max(value))/Frequency	0.05 Hz	0.15 Hz	0.25 Hz
$\log(E_{50}/E_{35})$	0.0095@12000m/0.0061	0.0173@7600m/0.0073	0.0232@6150m/0.0074
$\log(H_{50}/H_{35})$	0.0231@5530m /0.0065	0.0284@4500m/0.0106	0.0321@4200m/0.0114
$\phi_{50}^E - \phi_{35}^E$	0.0164@9500m /0.0074	0.0341@9500m/0.0133	0.0418@5000m/0.0164
$\phi_{50}^H - \phi_{35}^H$	0.0321@4600m /0.0077	0.0480@3600m/0.0136	0.0670@3300m/0.0142

amplitude data; having electric and magnetic data is better than having only electric data; having three frequencies is better than having only one frequency. For reasons of space, we will show only one of the many analyses we have made: How well is a thin resistive layer at depth determined? We consider the series of four-layer models seen in Figure 2.

We consider only analyses of few-layer models, in which case equation 2 becomes

$$\mathbf{C}_{\text{est}} = [\mathbf{G}^H \mathbf{C}_{\text{obs}}^{-1} \mathbf{G} + \mathbf{C}_{\text{prior}}^{-1}]^{-1}. \quad (6)$$

The analyses are based on an assumption about the available data, the data noise and the prior information, and they are carried out by computing the elements of the matrices in equation 6 and evaluating the expression. For a given model to be analyzed, theoretical data are generated in a large Tx-Rx separation interval by forward calculating the model response. When the response is known, noise is then ascribed to the theoretical data according to our noise model, whereby the elements of the data error covariance matrix, $\mathbf{C}_{\text{obs}}$, in equation 6 are found. We assume that data errors are uncorrelated, so $\mathbf{C}_{\text{obs}}$ becomes diagonal, containing the squares of the standard deviation of the data. For all model parameters and data points, the derivatives of the response with respect to the logarithm of the model parameters are calculated whereby the elements of the Jacobian matrix, $\mathbf{G}$, are defined. We consider only prior information on the model parameters themselves, so $\mathbf{C}_{\text{prior}}$ becomes diagonal, containing the squares of the standard deviations of the constrained model parameters.

We assume that we have inline data for the three frequencies 0.05, 0.15, and 0.25 Hz in the Tx-Rx separation interval from 500 m to 20 km. There is not a useful signal at a Tx-Rx separation of 20 km, however, the maximum separation has been chosen to ensure that all relevant data are included; the noise ascribed to the data will give a correct data weighting. Concerning the data noise, we assume that the values found in the data example mentioned above are representative. For the amplitudes of the electric and magnetic fields — normalized with respect to the Tx moment — we assume a basic relative noise level of 0.03 and an absolute noise of $3 \cdot 10^{-14} \Omega\text{m}^{-2}$ and $5 \cdot 10^{-11} \text{m}^{-2}$, respectively. For the phases, we assume a value of $R_0 = 12, 9, \text{ and } 8 \text{ km}$ for the frequencies 0.05, 0.15, and 0.25 Hz, respectively; with an absolute error of 0.05 and 0.06 rad for the electric and magnetic fields, respectively.

Concerning prior information, we assume that we have information on water depth and water resistivity with a relative uncertainty of 0.1. Such prior information would be available most often, and therefore it is relevant to include.

The analyses were carried out on the logarithm of the model parameters, $\log(\text{resistivity})$ and $\log(\text{thickness})$, and provide the absolute uncertainty estimates of the logarithms (i.e., we get the estimates [in a statistical sense])

$$\log(p) - \Delta \log(p) < \log(p) < \log(p) + \Delta \log(p), \quad (7)$$

or equivalently

$$p / \exp[\Delta \log(p)] < p < p \times \exp[\Delta \log(p)]. \quad (8)$$

Using $\Delta \log(p) = \Delta p/p$, we have for small $\Delta \log(p)$, approximately

$$p \times [1 - \Delta p/p] < p < p \times [1 + \Delta p/p], \quad (9)$$

it is seen that for small uncertainties the absolute uncertainty on the logarithm of the parameter is equal to the relative uncertainty on the

parameter itself. It must be remembered that the analyses were carried out on a linear approximation to the a posteriori model covariance matrix, meaning that the uncertainty estimates can be trusted quantitatively only when they are small. When they are large, they indicate that the parameter is undetermined.

Results of the analyses

The results of the analyses are seen in Figure 3 where the relative uncertainty of the resistivities, RHO2 and RHO3; thicknesses, THK2 and THK3; resistances, RES2 and RES3; and conductances, CON2 and CON3, of the second and third layers, respectively, are shown as color coded pixels in analysis templates. The relative uncertainty of each model parameter of the analyzed models is depicted in a square template as a function of 17 different thicknesses of the second layer along the abscissa and 17 different thicknesses of the third layer along the ordinate. For each template, the other parameters will have to be fixed. To investigate the dependence on the resistivity of the third layer, the analyses have been carried out for the five resistivities 8, 16, 32, 64, and 128 Ωm . Each of these analyses have been performed for five water depths: 500 m, 1 km, 2 km, 5 km, and 10 km. Computation time for all model parameters, for every choice of resistivity of the third layer and every choice of water depth for the 17×17 models, is about seven minutes (860 MHz CPU). In Figure 3, there are selected templates for a water depth of 500 m and those are selected templates for the three resistivities of the third layer of 8, 16, and 32 Ωm .

First, the analyses do not depend greatly on the water depth. This can most likely be attributed to the fact that we have assumed prior information on the water layer parameters and we show analyses for the water depth of only 500 m. To avoid confusion, a remark about the shallow-water problem of the CSEM method may be useful. All analyses depend critically on the noise model and the prior information applied in the analyses. For the analyses to be valid the assumptions must be valid. Our results concerning the relative independence on water depth are based on our specific noise model and the prior information on the water depth. This does not take into account the increasing data noise encountered in more shallow water. To analyze this effect, we would have to define a data noise as a function of water depth.

Second, there is little difference in the character of the uncertainty estimates of the parameters of the third layer when resistivities are above 32 Ωm , so we show only analyses for the values 8, 16, and 32 Ωm .

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RH01 = 0.25 $\Omega\text{m}$	Water depth	THK1 = 0.5, 1, 2, 5, and 10 km
RH02 = 1.00 $\Omega\text{m}$		THK2 = 16 m – 4096 m
RH03 = 8, 16, and 32 $\Omega\text{m}$		THK3 = 1 m – 256 m
RH04 = 1.00 $\Omega\text{m}$		

Figure 2. Model parameters for the four-layer models analyzed with regard to the uncertainty estimates of the resistive third layer.

**RHO2:** RHO2 is well resolved for  $\text{THK2} > 100$  m. This result is independent of  $\text{THK3}$  and varies little with  $\text{RHO3} = 8, 16$ , and  $32 \Omega\text{m}$ . The transition from being undetermined to being well determined is abrupt. The limiting value of 100 m is determined by the smallest Tx-Rx separation.

**RHO3:** RHO3 is undetermined for  $\text{THK3} < 64$  m. For  $\text{THK3} > 64$  m it is determined the best for  $\text{THK2} \approx 200$  m. For  $\text{THK2}$  smaller or larger than 200 m, the relative uncertainty of RHO3 increases. This result also varies very little with RHO3 and can be understood by considering that it is clear that  $\text{THK3}$  has to be above a certain threshold before RHO3 can be determined. Furthermore, if  $\text{THK2}$  is too large, the third layer lies deeper and becomes more poorly determined. The increased uncertainty for small  $\text{THK2}$  comes from the fact that RHO2 becomes more poorly determined and interferes with the determination of RHO3.

**THK2:** As for RHO2,  $\text{THK2}$  is poorly determined for  $\text{THK2} > 100$  m. When  $\text{THK2}$  becomes too large, the uncertainty increases because the lower boundary of the layer lies too deep to be resolved. Also, it is seen that  $\text{THK3}$  needs to be above a certain threshold to make  $\text{THK2}$  well resolved.

Comparing this threshold for  $\text{RHO3} = 8, 16$ , and  $32 \Omega\text{m}$  it is seen that the threshold is determined by the resistance of the third layer: If  $\text{RHO3}$  becomes twice as large, the threshold of  $\text{THK3}$  becomes twice as small. The threshold is around  $\text{RES3} = 128 \Omega\text{m}^2$ .

**THK3:**  $\text{THK3}$  is determined much in the same way as RHO3 and for the same reasons.

**RES2:** The resistance of the second layer,  $\text{RES2}$ , is as well determined as the poorer of the resistivity and thickness (i.e., it follows  $\text{THK2}$ ). There is no particular equivalence involved.

**RES3:** Unlike the situation for  $\text{RES2}$ , the resistance of the third layer,  $\text{RES3}$ , is better determined than either RHO3 or  $\text{THK3}$ . This is an expression of the high-resistivity equivalence, though the combination of the galvanic and inductive aspect of the data ensures that there is some determination. Assuming that the third layer is hydrocarbons, this is good news because, to a certain extent,  $\text{RES3}$  expresses the total amount of hydrocarbons. The total amount is thus determined better than the resistivity or the thickness of the layer. Comparing the templates for the three values of  $\text{RHO3}$ , it is seen that  $\text{RES3}$  becomes better determined, as  $\text{RHO3}$  becomes higher (i.e., the limiting determination is determined by  $\text{RES3}$ ).

500-m water depth, 0.05, 0.15, and 0.25 Hz, inline data

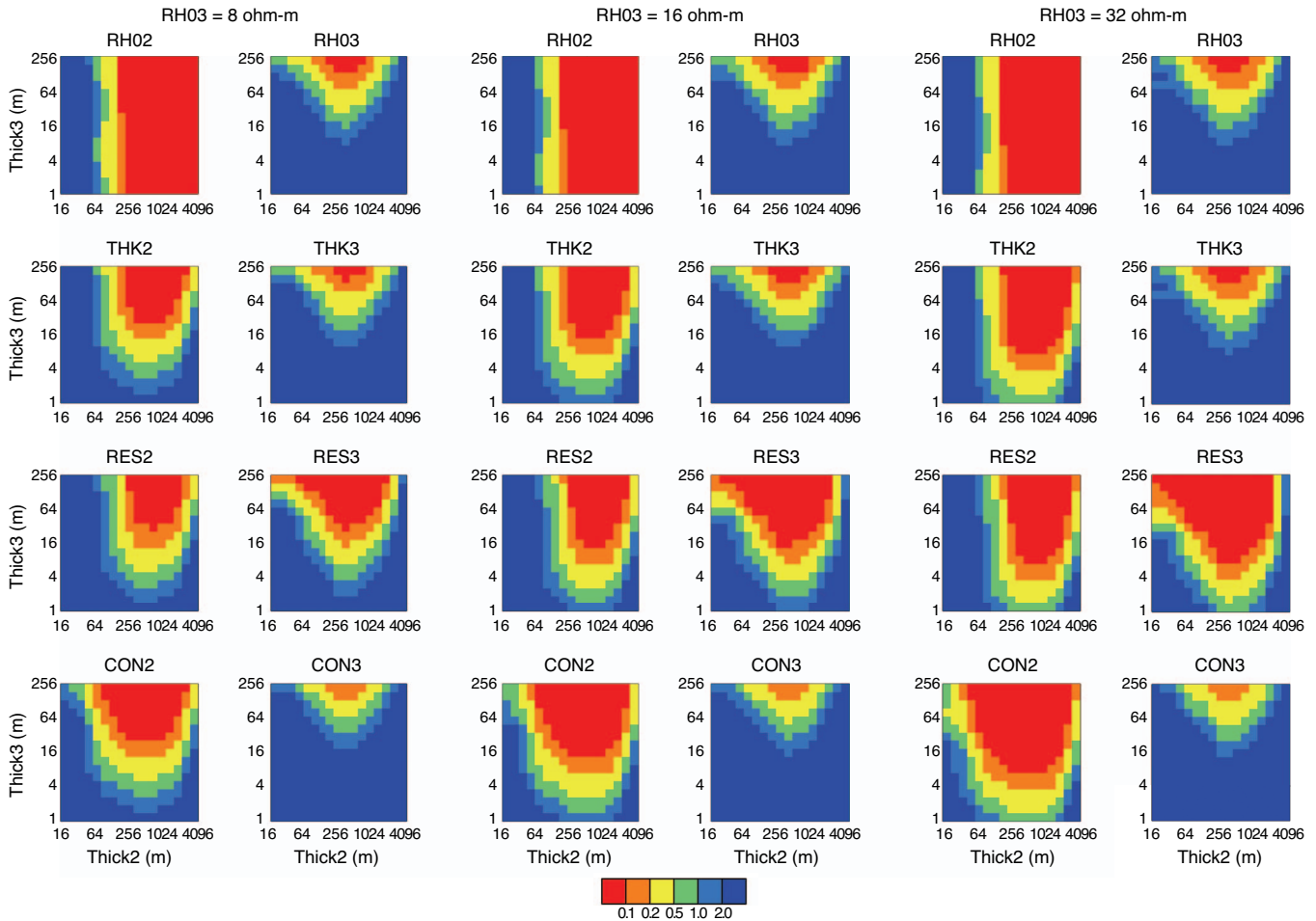


Figure 3. Analyses of the relative uncertainty of the resistivity (RHO), thickness (THK), resistance (RES), and conductivity (CON) of the second and third layers, only inline data. See text for the details of the assumptions behind the analyses.

CON2: We see that CON2 is somewhat better determined than THK2: There is good determination also for smaller THK2 where neither RHO2 nor THK2 are determined. As seen with THK2, the threshold for the good determination depends on RES3.

CON3: CON3 is the poorest determined parameters of the third layer and changes very little with RHO3 or water depth.

For the determination of RHO3 and THK3, a rule can be formulated by inspecting their templates. The area of good determination of RHO3 (a relative uncertainty less than 0.2) lies above two diagonal lines where THK3 is large and THK2 is neither too small nor too large. This area is approximately defined by the equations

$$\text{THK2} \cdot \text{THK3} > 23,000 \text{ m}^2 \quad \text{and} \quad \text{THK2}/\text{THK3} < 8. \quad (10)$$

THK2, RES2, CON2, and RES3 become better determined when RES3 increases: The area of good determination in the templates moves down toward smaller THK3. This shows that RES3 determines the limiting value of good determination. The area of good determination of RES3 (a relative uncertainty less than 0.2) is approximately defined by the equations

$$\begin{aligned} \text{THK2} \cdot \text{RES3} &> 65,500 \Omega \text{m}^3 \quad \text{and} \\ \text{THK2}/\text{RES3} &< 2.8 \text{ S/m}. \end{aligned} \quad (11)$$

### Adding broadside data

Only inline data were included in the above analyses. We added broadside data to analyze the possible beneficial effects on the parameter uncertainties. Similar analyses have been carried out on a combined data set consisting of the same inline data as above, plus an additional data set recorded for a Tx profile on a line parallel to the inline profile, but offset 4 km. The data set now consists of the electric field parallel to the tow directions and the perpendicular magnetic field for the two Tx profile lines. Noise was ascribed to the new data using the same noise model as above.

The results for a water depth of 500 m are seen in Figure 4. The effect is a smaller relative parameter uncertainty estimate on the parameters RHO2, RHO3, THK2, and THK3 — thereby on RES2, RES3, CON2, and CON3 — for values of THK2 smaller than 64 m and for high values of THK3 (the upper left corner of the templates). The decrease in relative parameter uncertainty estimate is more pronounced the higher RHO3 is and moves toward smaller THK3, i.e. it appears that the improvement depends on RES3.

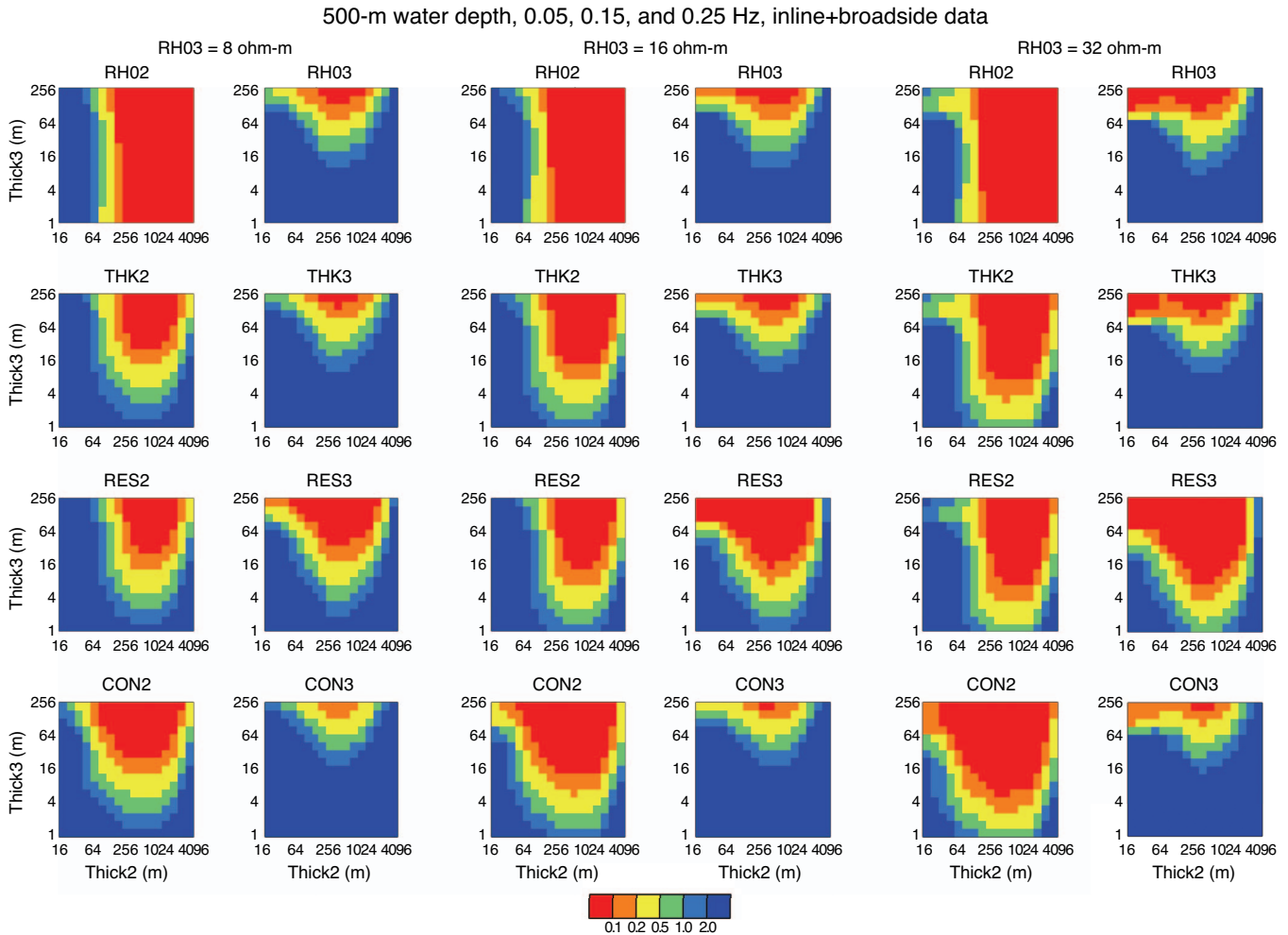


Figure 4. Analyses of the relative uncertainty of the resistivity (RHO), thickness (THK), resistance (RES), and conductivity (CON) of the second and third layers both inline and broadside data. See text for the details of the assumptions behind the analyses.

For water depths above 500 m (not shown) the improvement gradually decreases to be almost absent at a depth of 5 km for the parameters RHO2, RHO3, THK2, and THK3; while it persists to some extent for the combined parameters RES2, RES3, CON2, and CON3.

### Analysis and prior information

Systematic analyses of model parameter uncertainty estimates makes it possible to estimate how well model parameters are determined for a wide variety of models. However, it is also relevant when trying to estimate the potential benefits of including prior information on model parameters.

If a model parameter is well determined there is little use for prior information. For poorly resolved model parameters, prior information of almost any quality will be an asset. For parameter uncertainty in the middle range, prior information is only useful if it can be acquired with an uncertainty of the same order as or smaller than the uncertainty of the model parameter in question. Analyses therefore provide a threshold for the usefulness and relevance of prior information and a means to relate the costs of acquiring the prior information to its usefulness.

As an example, if our four-layer model is a relevant description of the situation, we can see from the analyses in Figure 3 that for  $\text{THK2} > 100$  m, RHO2 is well determined and prior information on RHO2 (e.g., from logging) would contribute little to the determination of the model. The situation is different for THK2: For inline data THK2 is poorly determined when THK3 is small, and prior information on the depth to the resistive target (e.g., from seismic sections) would be very useful. However, as shown in the previous analyses, including broadside data in the analyses does contribute to the determination of THK2 for small THK2, in which case there would be less need for prior information.

The situation differs for 2D and 3D inversion with finely discretized models. The resistivity of any single model cell is generally poorly resolved and any prior information will be useful.

## INVERSION OF THE CSEM DATA SET

### Intow, outtow, and laterally shifted data

One of the characteristics of 1D inversion of CSEM data is that for every receiver there is an intow sounding, where the Tx is moving towards the Rx, and an outtow sounding, where the Tx is moving away from the Rx. These two soundings are sensitive to the resistivity structure to either side of the Rx, meaning that they must be inverted separately, and there will be two model sections of concatenated 1D inversion models: one for intow and one for outtow soundings. We inverted the field data to obtain both an intow and an outtow model section of concatenated 1D inversion models.

Alternatively, data for a given Tx and Rx position can be ascribed a lateral position at the midpoint between Tx and Rx, assuming this is the mass center of the sensitivity distribution for that Tx-Rx pair. The laterally shifted data can be binned in intervals along the profile to construct a sounding combined from both the intow and the outtow data sets by including all data whose lateral position fall inside the interval. We chose binning intervals of 375 m — approximately half the distance between two neighboring Rx's — which gave a total of 100 bins. It is clear that the bins outside the line of receivers have fewer data the further away from the outermost receiver they are, consequently the resolution of the soundings will steadily de-

crease away from the receiver interval of the profile. After binning, the soundings are inverted, including data of all three frequencies.

The inversion procedure has been tested on theoretical data using any combination of amplitudes and phases of the electric and magnetic field, however, because of limited space we will not show these preliminary investigations, or the intow and outtow model sections, but only the model section of the laterally shifted and binned data. However, for every CSEM profile, both intow, outtow, and laterally shifted sections should be produced.

The lateral shifting followed by binning is an obvious way to prepare the data set for 1D inversion and has been used for other geophysical methods. However, the assumption that the lateral focus point of the sensitivity distribution lies midway between Tx and Rx is actually not true. It assumes that the model is 1D which generally is not the case. For the galvanic contribution to the data, the sensitivity becomes higher the closer you get to the electrodes of the Tx or Rx. Though it is a point of symmetry, the midpoint is actually the point of lowest sensitivity. The galvanic contribution is sensitive to the resistivity close to the Tx or Rx, if there are inhomogeneities close to the seabed they will influence the measurements strongly. For an intow or outtow sounding, the Rx is kept constant and only the Tx moves, while for the laterally binned data, both the Tx and the Rx moves within a data set. That means that the binned data may be more influenced by near-seafloor inhomogeneities and thereby more inconsistent with a 1D assumption.

### Initial models for the inversion

A 1D model does not take lateral changes in water resistivity into account, so an initial value for the water resistivity of  $0.255 \Omega\text{m}$  has been chosen based on the measurements along the profile. The resistivity of the water is a parameter free to vary in the inversion; as long as the initial value of the water resistivity is approximately correct the inversion will find the best average value for the data set.

The water depth for the binned soundings has been interpolated between the water depths of the receivers. For soundings outside the receiver interval, a linear extrapolation has been used; in our particular case it is in reasonable accordance with the seafloor topography.

Prior information on the water layer resistivity and thickness proved to be of absolutely no significance for the determination of the model parameters, so it was omitted in this particular case. Our analyses have shown that water depths up to 2 km are well determined in 1D inversion; only from water depths in excess of 5 km does the prior information contribute significantly to the determination of the water parameters.

### Results of the inversion

Inversion has been done for all binned soundings with both a multilayer model (MLM) with 20 layers and fixed layer boundaries and with two-, three-, four-, and five-layer models. The results of inversion with MLM and four-layer models are shown as model sections in Figures 5 and 6, respectively. The models have been plotted to a depth of 6 km below the sea surface; there is very poor determination below that depth as can be seen from the analyses in Figure 3. The color scale for the resistivities shows higher resistivities in red and lower resistivities in blue. The scale is the same for both model sections.

The MLM model section (Figure 5) was inverted with  $L_1$ -norm optimization. The section can be divided roughly into the water layer, a fairly conductive first subbottom layer with a resistivity of

1–2  $\Omega\text{m}$ , a resistive depth interval between 1850–2400 m depth with a resistivity roughly between 5 and 20  $\Omega\text{m}$  and a mostly conductive, but more inhomogeneous, bottom layer.

The outermost 5 km of soundings at either end give an inferior determination of the model parameters and cannot be trusted. The central soundings between profile coordinates 5 and 23 km give coherent results with a consistent resistive third layer, while soundings at the profiles ends are more erratic. Below the model section is a plot of the data residual. Typically, the residual is just around 1, meaning that on average data are fitted to the noise level ascribed to the data. This shows that there is consistency between the noise model and the assumption that data can be inverted with 1D models. Computation time for the inversion of the 100 soundings with multilayer models is about one hour (860 MHz CPU).

When inverting with the  $L_1$ -norm, the models appear blocky and will indicate the number of main units in the section. In this case, four layers can be identified with a resistive third layer consistently appearing in the central parts of the profile. However, because the layer boundaries are fixed, the precise location of the layer boundaries and the resistivities may vary from the true values and the values found in few-layer inversions.

Through inversion with a four-layer model it is possible to quantify the depth to, and the resistivity and thickness of, a resistive layer at depth. The model section is seen in Figure 6, such a layer is found at a depth of  $\approx 1500$  m below the seafloor. The residuals are typically around 2. Again, the central soundings between coordinates 5 and 23 km are fairly coherent with a consistent resistive third layer and more erratic soundings at the ends of the profile.

Below the residual plot is the analysis section, where the relative uncertainty of the layer resistivities and thicknesses, and the depths to layer boundaries, are color coded in rows, one for each parameter.

Looking at the analysis section, it is seen that most model parameters are well determined with a relative uncertainty of less than 0.1. The resistivity of the third layer is fair to poorly determined, as is its thickness. The layer is too thin to be well resolved. The depth of the layer below the seafloor, (i.e., the thickness of the second layer) is well determined. Computation time for the inversion of the 100 soundings with four-layer models is about 20 minutes (860 MHz CPU).

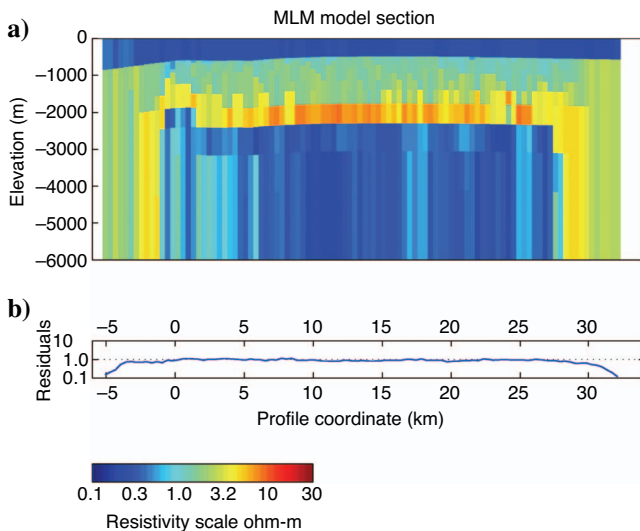


Figure 5. Model section for the MLM inversions. (a) The inverted models, (b) the residuals, and the color bar for resistivity.

## The resistive layer at depth

The most interesting feature of the model sections is that a resistive layer is found at depth, indicating the possible presence of hydrocarbons. As mentioned before, the resistivity and thickness of the resistive third layer are subject to the high resistivity equivalence modified by the resolution enhancement of inductive contribution of the data. In the analysis section of Figure 6, it is seen that they are fair to poorly determined, in the interval from 19 to 22 km they are better determined because of the increased thickness. However, the resistance of the layer (i.e., the product of resistivity and thickness) is much better determined.

Figure 7 shows the resistance of the third layer including its relative uncertainty. In the well-resolved central part of the profile, the resistance of the layer is 2–3.5  $\text{k}\Omega\text{m}^2$  with a relative uncertainty in-

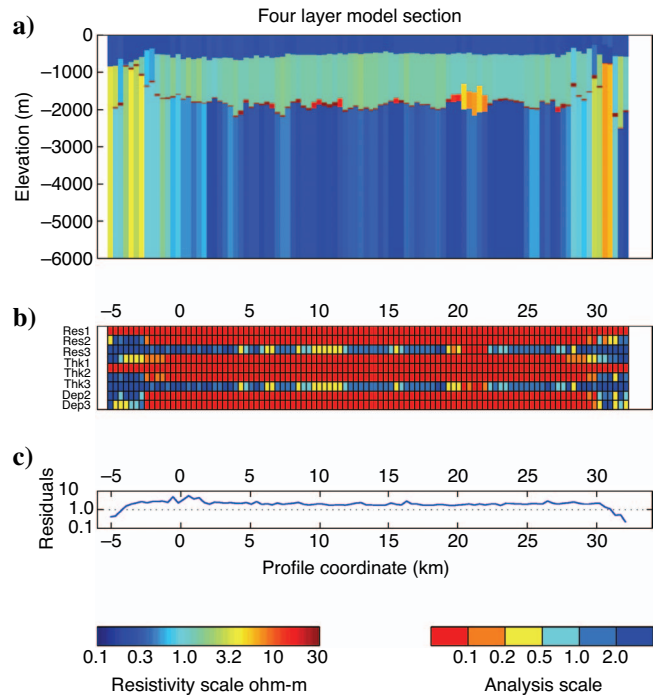


Figure 6. Model section for the four-layer inversions. (a) The inverted models, (b) the analysis section, (c) the residuals, and the color bars for resistivity and analysis.

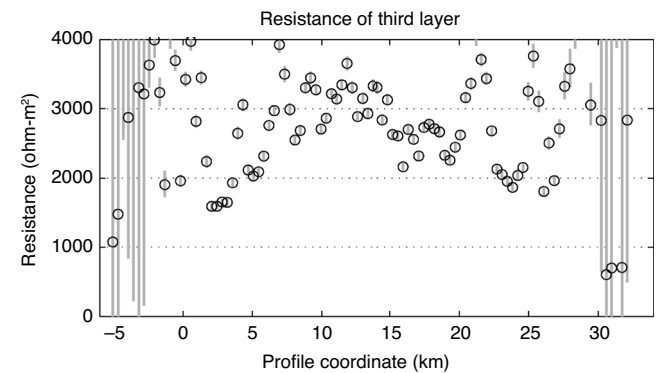


Figure 7. Resistance of the resistive layer (black circles) with error bars (gray lines).

terval 0.05–0.10, while at the ends of the profile, the uncertainty becomes large and the value cannot be trusted. The overall tendency is that resistance increases from 1.5 to 3.5  $\text{k}\Omega\text{m}^2$  in the interval of 0–10 km and varies between 2–3.5  $\text{k}\Omega\text{m}^2$  between 10 and 25 km. The short-distance variability of the resistance is higher than the estimated uncertainty. This most likely can be attributed to the inconsis-

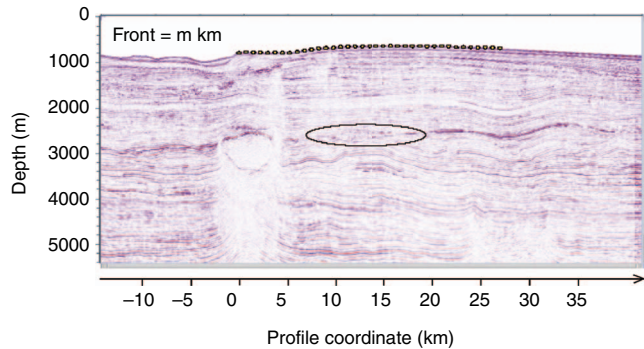


Figure 8. A seismic section showing the CSEM line of receivers. The oval encircles the area where hydrocarbons have been found.

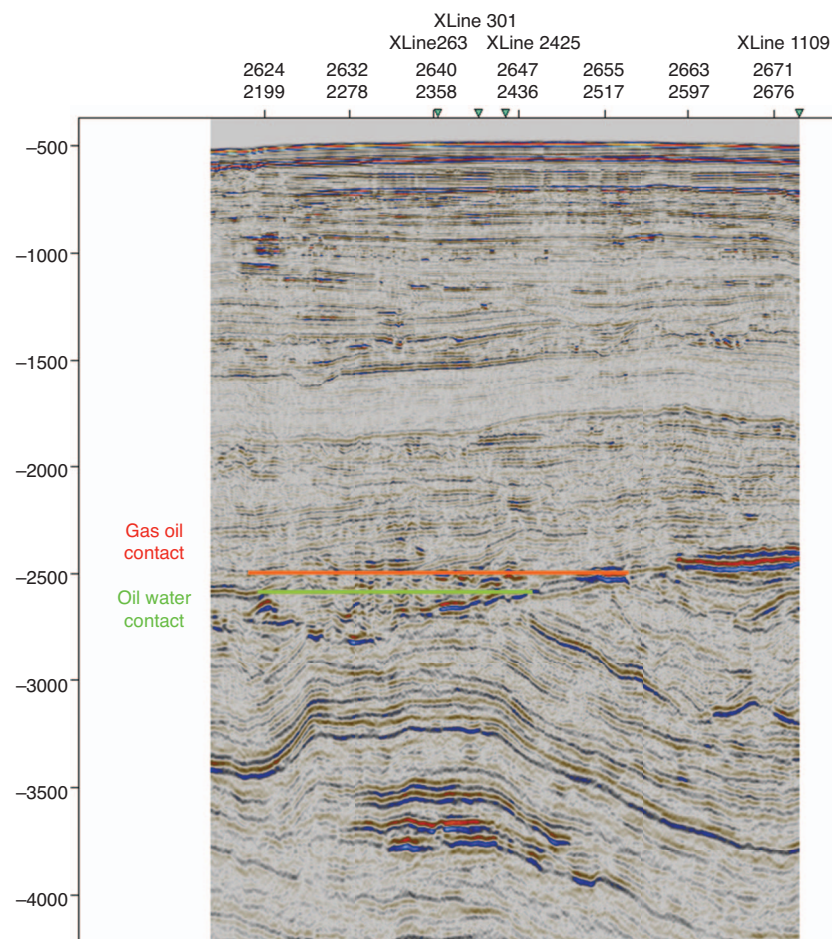


Figure 9. Detail of the seismic section from Figure 6 showing the gas/oil contact and the oil/water contact.

tency between the data and the assumption of one-dimensionality, and/or to inconsistencies between the data in the binned soundings, and/or between the three frequencies.

### Comparison with seismics

A seismic section from the area is shown in Figure 8. The prospect has been drilled, an oval encircles the area where hydrocarbons have been found. The fields contain both oil and gas, a 44-m oil rim underlies a gas cap of at least 70 m. The prospect is situated in an Early Miocene age canyon system in the Mauritanian region filled with several sequences of deep water, mid-slope submarine channels, and turbidite sands. The net to gross of the canyon system ranges from 15%–45%, the seismic line vicinity averages about 25%. Reservoir properties are good to excellent.

The encircled area in Figure 8 corresponds to the profile interval from  $\approx 7$  to  $\approx 20$  km where a consistent resistive layer has been found. The depth to hydrocarbons in the seismic section is  $\approx 2000$  m, in reasonable accordance with the depth found from the 1D inversions of 1500–1600 m. However, the analysis shows that the relative depth uncertainty of the resistive layer below the seafloor is smaller than 0.10, which does not quite cover the actual discrepancy. This most likely can be attributed to the same causes as men-

tioned above: inconsistency between the data and the assumption of one-dimensionality, and/or inconsistencies between the data in the binned soundings, and/or between the three frequencies. Figure 9 shows a detail of the seismic section in Figure 8, indicating the gas/oil contact and the oil/water contact.

It is interesting to compare the average resistivity within the oil/gas column with results from resistivity logging. According to the drilling results, there is an oil/gas column of at least 114 m, with a resistance of 2–3.5  $\text{k}\Omega\text{m}^2$  the average resistivity must be roughly 20–30  $\Omega\text{m}$ . The logs give a value of 20  $\Omega\text{m}$  in two fairly thin sections with values in the range of 3–10  $\Omega\text{m}$  in the reservoir. The mean resistivity value in a 200-m interval around the reservoir sands is about 3.5  $\Omega\text{m}$ . Because the logs measure the horizontal resistivity, the apparent inconsistency can be only explained by a significant anisotropy and a higher effective vertical resistivity, which is not unexpected for layered turbiditic sediments.

### DISCUSSION

Our noise model is an empirical description of the apparent noise of our actual data set using a model with a few pragmatically determined parameters. However, it is likely that our noise model — with different parameters — would describe other data sets. It would be better if the contractor delivered noise figures estimated directly from the measurements. However, the transformation to frequency domain scrambles the noise measures of the time-domain data used in the transformation, so measured noise might not be a significant improvement. If data were inverted in the time domain, noise estimates obtained in the

field would be directly applicable. This speaks for a time domain inversion, which also has been recommended to alleviate the problems encountered in shallow water investigations (Wright et al., 2001).

Keeping a constant Tx height above the seafloor is a challenging task, particularly in deep water regions. Our analysis of varying Tx height effects emphasizes the need for an accurate tracking of the Tx during measurements so that it becomes possible to include the actual position in the inversion of data.

Today, much effort is going into the development of advanced 2D and 3D inversion procedures to solve the shortcomings of 1D inversion. Because it is expensive to collect a data set that justifies 3D inversion, it is important to develop cost-effective data collection methodologies. To get an overall picture of the field (Hesthammer and Boulaenko, 2005), a sound field practice could be to perform an initial investigation with sparsely distributed receiver units. Because ship time is expensive, it is paramount to have a quick way to assess the initially collected data in order to enable an informed decision about subsequent data collection. The present approach to inversion of CSEM data with 1D models could be such a tool. After data have been collected, stacked, and transformed to the frequency domain, 1D inversion could be completed in a few hours by an experienced person.

Resolution in 2D and 3D models will always be poorer than what is found in the 1D analysis. 1D analyses will display the best case scenario and can be readily performed in a short time for a large variety of models.

## CONCLUSIONS

It is possible to invert the field example data sets with 1D models using the noise model and the inversion procedures outlined in this paper. The data quality of the field example is apparently good and the lateral homogeneity is sufficient to fit the data with 1D models.

The noise model seems sound, and data are inverted to residuals close to 1. Thus, the noise levels are reasonable and a 1D model is not too inconsistent with the actual setting.

The uncertainty of most model parameters in the field example is small. It seems safe to conclude that there is a resistive layer at a depth of around 1500–1600 m below the seafloor with a resistance of 2–3.5 k $\Omega$ m². The resistivity of the layer is not generally well resolved. However, the depth of burial is well resolved, as is the resistance. In relation to hydrocarbon potential, the presence of the resistive layer is a positive indication and is in reasonable accordance with the seismic results, where hydrocarbons were found at a depth of  $\approx$  2000 m.

An analysis of model parameter uncertainty is readily realizable with 1D models and reveals the basic capabilities and shortcomings of the CSEM method. In our four-layer analyses we find that, for a given resistivity of a buried resistive layer, the parameter uncertainty primarily depends on the thickness of the layer itself and of the overburden. We have formulated quantitative rules for products and ratios of these parameters that permit a small uncertainty for the parameters of the resistive layer. The higher the resistance is, the depth of burial and the resistance of a resistive layer are better determined.

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