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GEOPHYSICS

Self-Modulation of Seismic Waves in the Subsurface Soil

O. V. Pavlenko

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Nonlinear effects of modulation instability of seismic waves during their propagation in subsurface soils are studied in numerical experiments. The results show that self-modulation and generation of subharmonics take place at high amplitudes of seismic waves in cases of their resonance intensification in soil layers and precede the chaotic stage in the case of further increase in amplitudes. Generation of these effects is related to dispersion of seismic wave propagation velocities in subsurface soils due to their nonlinear response.

The nonlinearity of soils during strong earthquakes remains a debatable issue in seismology. Interactions and self-effects of seismic waves (see, for instance, [1, 2] and others), attenuation of resonance frequency of soil layer oscillations, an increase in nonlinear absorption, and a decrease in elastic moduli (see [3, 4] and others) were studied in field, laboratory, and numerical experiments and based on earthquake records. This article presents results of the numerical simulation of modulation instability observed during intense seismic wave propagation in soil layers. We used numerical models of soil behavior in fault zones of strong earthquakes based on the records of vertical groups [5].

In particular, models of soil behavior in fault zones of the 2000 Tottori (Japan) earthquake ($M_W = 6.7$) were constructed; i.e., the stress and strain excited by strong motions in the upper ~100-m soil layer were determined [6]. The obtained vertical distribution of stress and strain in layers at station TTRH02, the nearest to the fault plane (~2 km), are presented in [6, 7]. The profile at TTRH02 consists of a sandy–clayey sequence (V_S ~ 210 m/s) in the upper 9-m layer. The lower layer (down to ~100 m) is composed of denser layers with V_S gradually increasing with depth to ~790 m/s per 100 m. Subsurface soils are partially water-saturated (V_P ~ 860 m/s), resulting in a noticeable intensification of oscillations on the surface. Stress and strain were assessed in 10 consecutive 1.5-s intervals corresponding to 15 s of strong motion [6].

To assess the degree of nonlinearity of soil behavior, numerical models were tested by harmonic signals with a 3-Hz frequency [8]. Stress–strain relationships obtained for each 1.5-s interval were used for calculating the propagation of testing signals in soil layers. The results of testing (Fig. 1) demonstrate that high harmonics of basic frequency are generated in the course of propagation of harmonic signals in soils. As a rule, the harmonics are of an odd order (3rd, 5th, and others). Harmonics of an even order (2nd, 4th, and others) are generated along with harmonics of the odd order only during the highest intensity of the input signal (intervals 4, 6, 7). Similar results were obtained when testing other soil profiles [9]. However, subharmonics of the basic frequency (intervals 3, 5) are also generated in intervals with the highest intensity of oscillations when testing the soil profile at station TTRH02.

To elucidate conditions of subharmonic generation, the propagation of harmonic signals in the profile at station TTRH02 was studied in greater detail. Figure 2 illustrates the results of testing the TTRH02 profile by signals with a 3-Hz frequency. The input signal of amplitude $V_0 = 0.01$ m/s yielded a 3-Hz signal with several even and odd high harmonics. Increase in amplitude V_0 to 0.011 m/s also fostered an increase in the working interval of the stress–strain relationship and appearance of an inflection point. The output signal became amplitude-modulated, and its spectrum demonstrated subharmonics of type $\frac{nf}{3}$: (1, 5, 7, 11, 13, 17, 19 Hz, and others). Upon further increasing V_0, modulation was retained initially at $V_0 = 0.012$ m/s. Subsequently, the input signal resembled noise at $V_0 = 0.013$ m/s (Fig. 2). The experiment was repeated with a 6-Hz frequency of the input signal, and a slight modulation effect was

Joint Institute of Physics of the Earth, Russian Academy of Sciences, Bol'shaya Gruzinskaya ul. 10, Moscow, 123810 Russia; e-mail: olga@ifz.ru

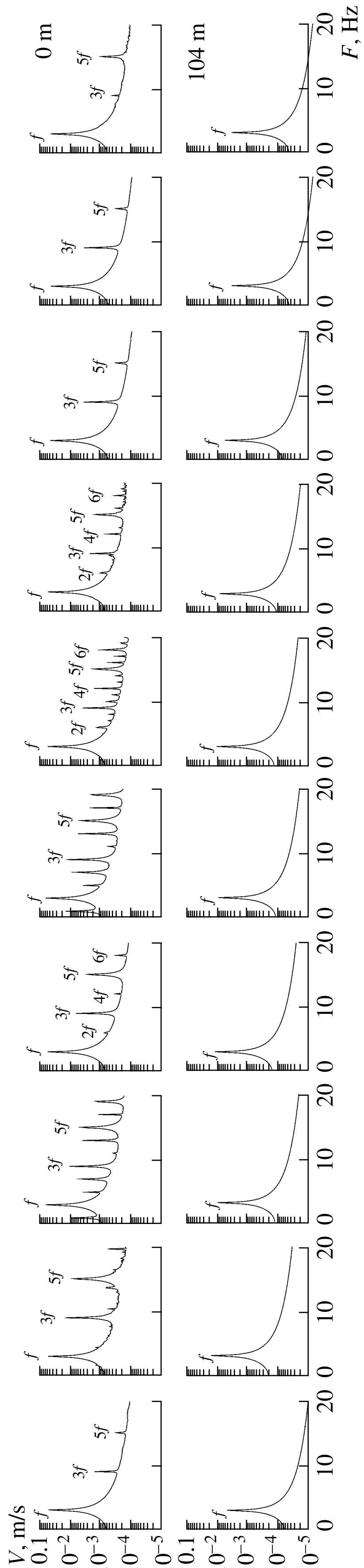

Fig. 1. Results of testing the soil profile at station TTRH02 by harmonic signals ($f = 3$ Hz): spectra of testing signals (below) and signals on the surface (above) in 10 successive 1.5-s intervals (15 s of strong motions).

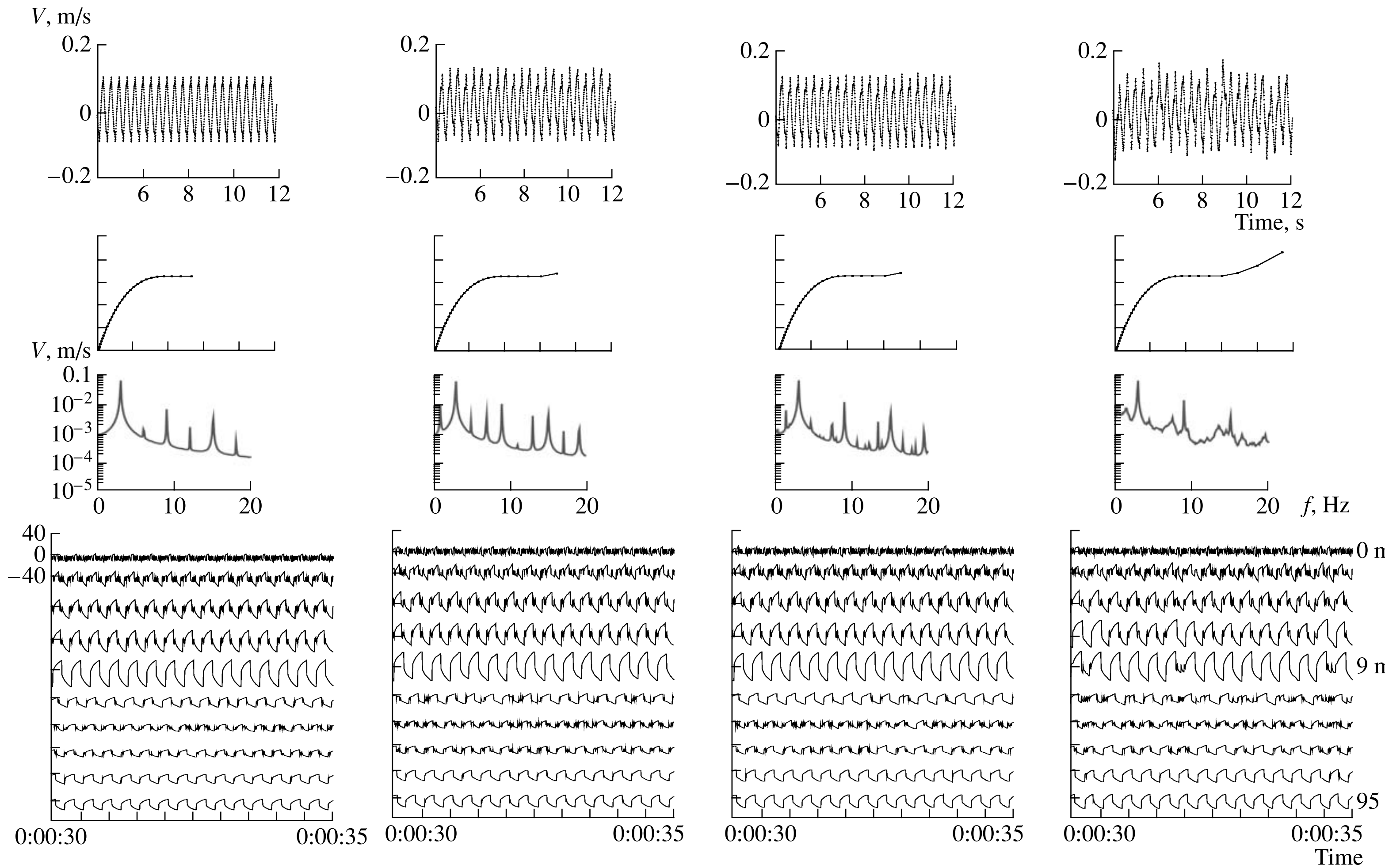

Fig. 2. Self-modulation and chaotic character of the harmonic signal with 3-Hz frequency and amplitudes V_0 = 0.010, 0.011, 0.012, and 0.013 m/s in the soil profile at station TTRH02. From top to bottom: calculated oscillations on the surface, working intervals of stress–strain relationships in the upper 9-m interval, spectra of signals on the surface, schematic view of oscillations in soil layers (upper and lower rows are related to surface and 100-m depth, respectively; oscillations in upper layers are presented in more detail; maximal amplitudes of oscillations correspond to the depth of ~9 m).

revealed at the input signal amplitude of ~0.03–0.05 m/s. Oscillations became chaotic when the amplitude was increased to 0.1 m/s.

Figure 3a demonstrates stress and strain in soil layers in the TTRH02 profile at different depths excited by propagating harmonic signals ($V_0 = 0.01$ m/s, frequency 1–21 Hz). The 3-Hz frequency is resonance frequency f_{res} of this soil profile, and it shows resonance amplification of oscillations in the upper ~9 m (f_{res} decreases due to nonlinearity of the soil response, while f_{res} is equal to ~210/36 = 5.8 Hz in the linear case). The nonlinearity of stress–strain relationship leads to noticeable decrease in the effective propagation velocity of waves of the 3-Hz resonance frequency compared to waves of other frequencies, i.e., velocity dispersion. Conditions for the development of self-modulation appear for harmonics with resonance frequencies.

The phenomenon of harmonic wave self-modulation in nonlinear media with dispersion has been studied sufficiently [10]. Owing to the dispersion, high harmonics generated due to nonlinearity are asynchronous with the wave of basic frequency and, therefore, do not grow, whereas oscillations at the output appear to be amplitude-modulated. Modulation is caused by parametric instability leading to the appearance of ghost reflections with frequencies close to the carrier frequency. Modulation instability is possible only at a certain relation between nonlinearity signs and dispersion of the group velocity (Lighthill condition), owing to which nonlinear distortion caused by synchronism compensates the linear dispersion asynchronism [10].

Figure 3b demonstrates stresses and strains excited in soil layers in profile TTRH02 by testing signals with different V_0 values at $f = 3$ Hz. Hysteresis characteristics in the upper 9 m of soil, where deformations are the greatest, mainly determine the mode and spectrum composition of oscillations on the surface: at $V_0 = 0.012$ m/s, their spectrum becomes more complicated. Upon further increase in the amplitude, the oscillations become chaotic. The chaotic character corresponds to large working intervals of stress–strain relationships and sets of closed loops of hysteresis curves at different depths. It is likely that extension of the working intervals of stress–strain relationships beyond the inflection point also contributes to fast complication and chaotization of oscillations. Changes in the shape of hysteresis curves correspond to variations in the trajectories of oscillations of medium particles in the phase space, i.e., to variations in phase portraits of the system (bifurcations). Transition to stochasticity via a series of bifurcations of period doubling is typical of dissipative nonlinear systems [10]. The described experiments exhibit period trebling, especially prominent in the layer with the maximal oscillation amplitude of ~9 m (Fig. 2), which is likely to be related to the prevalence of cubic (instead of quadratic) nonlinearity in the soil response. Transition to stochastic oscillations takes place in the case of increase in V_0, i.e., increase in the number of nonlinear interactions and formation of mixed harmonics. A sufficiently high intensity of the input signal leads to increase in the number of harmonics and chaotization in an avalanche manner, and the output signal spectrum becomes continuous.

Phenomena of self-modulation and recovery of Fermi–Pasta–Ulam waveforms in seismic fields were studied by Dimitriu in field experiments [11]. The spatial evolution of harmonic signals emitted by a vibrator within the frequency band 7–14.65 Hz was investigated at a distance of 10–70 m from the vibrator. The V_0 value on the surface near the vibrator was 0.00015–0.006 m/s. The recovery of Fermi–Pasta–Ulam waveforms was observed at low V_0 values. We noted a complete recovery of initial waveforms (both envelope and carrier waves) over a short distance, probably due to strong nonlinearity of subsurface soils. We also recorded a substantial dependence of observed effects on the signal frequency and a slight dependence on the amplitude. For the highest frequency of 14.65 Hz, only weak signs of instability were observed and the most remarkable signs were recorded for $f = 10$ Hz [11].

The observed phenomena can be explained from the viewpoint of the nonlinear theory of wave propagation. The Shredinger nonlinear equation [12]

$$iA_t + bA_{xx} + c|A|^2A + dA_x = 0$$

describes three main classes of the phenomena observed during quasi-harmonic wave propagation in nonlinear media with dispersion: modulation instability (self-modulation), stationary waves of envelopes (including solitons), and periodic (in space and time) recovery of a slightly modulated wave (approaching a periodic sequence of solitons). In this equation, A is the complex amplitude of an envelope of propagating wave of type $\exp[i(kx - \omega t)]$; real constants $b = \frac{d^2\omega}{dk^2}$, c, and d determine the value and sign of dispersion, cubic nonlinearity, and absorption, respectively. The sign of bc determines the process evolution: if $bc < 0$, the wave is stable and solitons do not form, and instability develops if $bc > 0$.

In experiments carried out by Dimitriu, the minimum of the group velocity of wave propagation corresponds to 10–12 Hz and the explanation given is confirmed by the experimental results. In our numerical experiments (Figs. 1–3), the appearance of the velocity dispersion in the medium is related to the nonlinearity of the soil response; in this case, the minimum of wave propagation velocity also corresponds to the range of resonance frequencies and the condition of modulation instability $bc > 0$ is fulfilled. As in Dimitriu's experiments, the effects of modulation and subharmonic generation show a direct correlation with the amplitude of the testing signal.

The experiments described here were also carried out with other soil profiles. Self-modulation and gener-

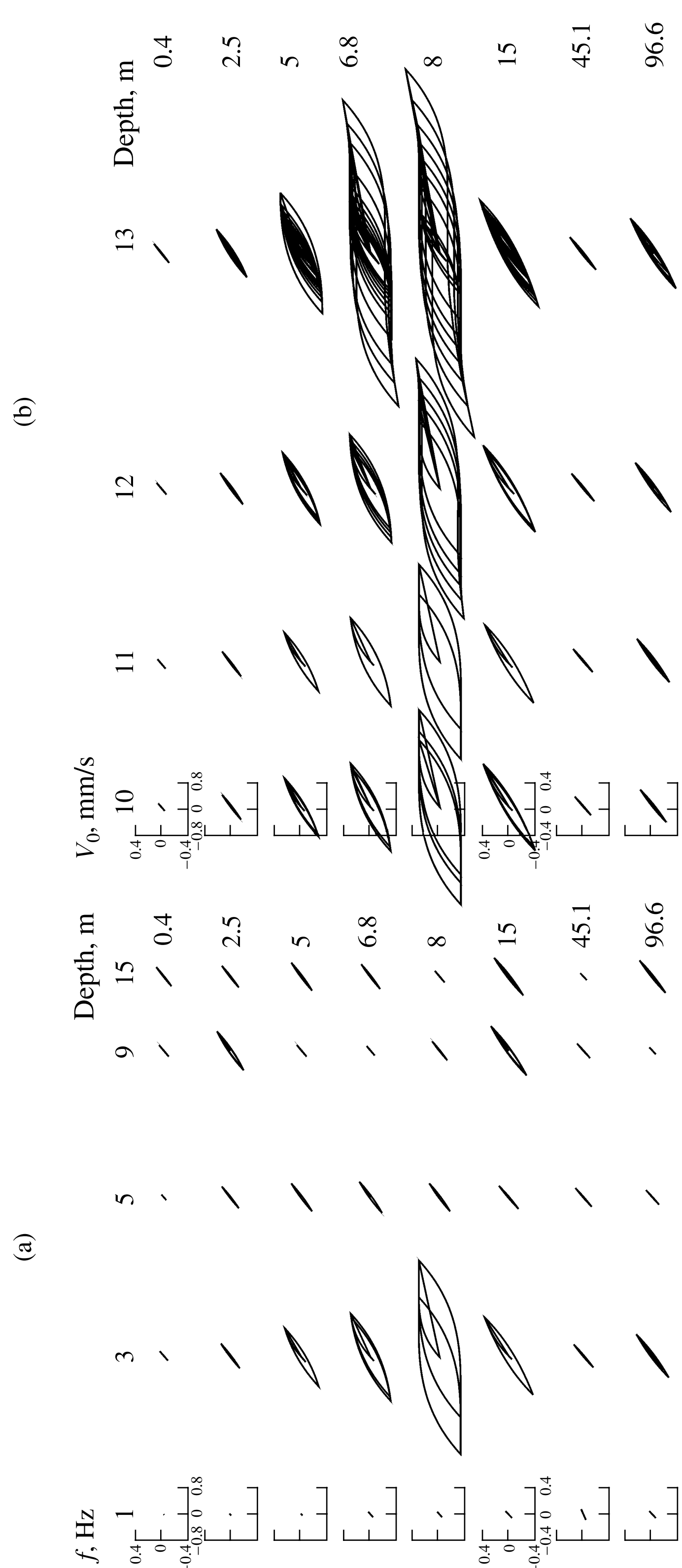

Fig. 3. Stress and strain excited in soil layers: (a) at different frequencies of the input signal, $V_0 = 0.010$ m/s, and (b) at different amplitudes of the input signal, $f = 3$ Hz.

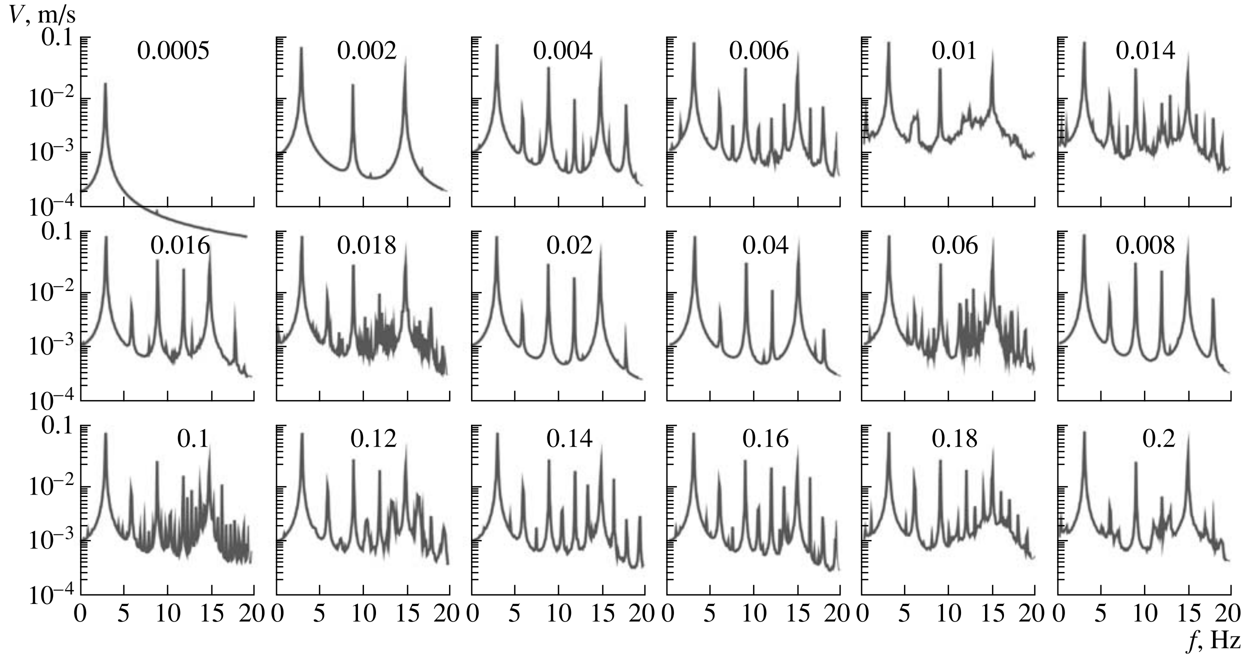

Fig. 4. Spectra of oscillations on the surface of the soil profile in Port Island at different amplitudes of the input harmonic signal ($f = 3$ Hz). Amplitudes of input signals are shown above (m/s).

ation of harmonics were observed in each case for harmonics characterized by resonance intensification of oscillations. We investigated profiles without seismic boundaries with velocities V_S gradually increasing with depth. We also studied profiles composed of the upper loose layer and the lower (more dense) layer with V_S increasing with depth. The soil profile in Port Island is an example of a complicated section (systems with many degrees of freedom). A model of its behavior in the 1995 in Kobe (Japan) earthquake was reported in [5]. The profile includes five groups of layers with different properties: artificial soil above the groundwater level (0–13 m), the same below the groundwater level (13–17 m), alluvial clay (17–28 m), alluvial sand (28–34 m), and dense layers of gravel and clay (deeper than 34 m). Figure 4 illustrates the results of testing this soil profile by the harmonic signal with a 3-Hz frequency of the resonance frequency band (1–4 Hz) of the upper liquefied layer. The soil response is essentially nonlinear even at low V_0. Upon increasing V_0, its nonlinear distortions are intensified. This is followed by the successive appearance of high harmonics of odd orders (3rd, 5th, …), high harmonics of even orders (2nd, 4th, 6th, …), and subharmonics of type $\frac{nf}{3}$ in the soil response spectra. Finally, the oscillations become chaotic (Fig. 4, upper row, 0.01 m/s). Upon further increasing V_0, we observe a phenomenon similar to the recovery state: the system periodically passes from the state of subharmonic generation with subsequent chaotization to the state of generation of high odd and even harmonics until the achievement of the limiting amplitude of $V_0 \sim 0.525$ m/s corresponding to the ultimate soil rigidity.

The increase in V_0 augments stress and strain in the upper layers. Liquefied soils of the upper group are more susceptible to changes in amplitudes. Growth of stress and strain in one of the layer groups entails variations in the whole vertical distribution of stress and strain in layers, and, consequently, variations in oscillation spectra on the surface (Fig. 4). At $V_0 \sim 0.006$ m/s, subharmonics appear for the first time in the signal spectrum on the surface. At higher V_0 values, different soil layers become resonant and the oscillation spectrum on the surface changes qualitatively. Oscillations are modulated at resonance frequencies, and signal spectra on the surface give an idea of the complicated behavior of the system with many degrees of freedom (Fig. 4). One can see subharmonics of types $\frac{nf}{3}$ ($V_0 \sim$ 0.004, 0.006, …) or $\frac{nf}{2}$ ($V_0 \sim 0.018$, 0.06, …) and both types in some cases. Types of both subharmonics and high harmonics are likely to be determined by forms of stress–strain relationships in the soil layers: liquefied soils in Port Island exhibit nonlinearity of the even order [9]. Subharmonics of type $\frac{nf}{2}$ appear in spectra of output signals only in this case. Such subharmonics are absent in spectra of output signals of other studied soil profiles, which lack nonlinearity of the even order.

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