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Unified scaling theory for distributions of temporal and spatial characteristics in seismology

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Abstract

The paper describes the unified scaling theory for distribution functions of temporal and spatial characteristics in seismology. It is based on the scaling of seismological characteristics calculated for various energy–spatial–temporal intervals. The common mathematical methods for the scaling of distribution functions are developed. The means to test possibility of such scaling are found as well. The relationship between the unified scaling theory and other present scaling approaches is determined. The theory is applied to two characteristics of different seismoactive regions. The first characteristic is the waiting time between earthquakes ΔT, the second one is a new space parameter $\Delta D_{\min}$, which is the minimum distance of a current seismic event to the nearest (in space) neighbor in an energy–spatial–temporal interval. The distribution of the characteristics ΔT and $\Delta D_{\min}$ allows estimating the time interval to the next earthquake and the distance of the following earthquake from previous earthquakes. Thus, these characteristics are very important for seismic hazard estimations. Scaling of distributions functions is proven to be successful for $\Delta D_{\min}$ in all energy–spatial–temporal intervals and for ΔT with variations of energy/magnitude range. The distribution function of ΔT for various time domains was stable in only 60% of the cases, and near to unstable for spatial variations.

Keywords: Earthquake; Structure of seismicity; Waiting times; Scaling of distributions

1. Introduction

The structure of seismicity (registered sequence of seismic events) is a general characteristic required to forecast strong seismic events and to understand physical processes in the earth's crust. To analyze it an energy–spatial–temporal interval — ESTI (E_I, ΔE_I; X_I, ΔX_I, Y_I, ΔY_I, Z_I, ΔZ_I; T_I, ΔT_I) must be fixed. Instead of the energy parameters E_I and ΔE_I magnitude parameters M_I and ΔM_I can be used, $\log_{10} E(J)=\alpha M+\beta$. But seismicity is significantly inhomogeneous from one ESTI to another (Sadovsky et al., 1987; Smalley et al., 1987; Turcotte, 1992).

At present, self-similarity of the seismic/failure process in the earth's crust/rocks on different scales from micro-level to strong earthquakes is a very popular idea (Sadovsky et al., 1987; Rykunov et al., 1987; Turcotte, 1992; Kagan, 1994; Kuksenko et al., 1996; Ponomarev et al., 1997; Ulomov, 1998; German and Mansurov, 2002; Baiesi and Paczuski, 2004). This idea is closely related to concepts of self-organized criticality (Bak, 1996) and reveals the possibility to find statistical relations between energy, temporal and spatial characteristics of seismicity for different ESTI.

Probably the largest number of investigations in this field is related to the magnitude (seismic moment, radiated energy) distribution function of seismic events. The simplest version of the distribution corresponds to the

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Gutenberg–Richter law and is actively used for long-term earthquake forecasting (e.g. Ulomov et al., 1999). The universality of parameters of this distribution was demonstrated for different regions (e.g. Kagan, 1999), but local changes of them are useful for strong earthquake forecasting (Sobolev et al., 1991).

Other old research area is the investigation of waiting times distributions between two successive earthquakes. The traditional approach is graphical (without statistical tests) comparing empirical distribution functions or empirical density functions with some theoretical functions for certain regions and one specific ESTI (e.g. Rikitake, 1976; Nishenko and Buland, 1987; Correig et al., 1997; Wang and Kuo, 1998). But such investigations have a limited significance because the variation of large numbers of parameters (parameters of ESTI) was not considered and a question about the universality of their results is still unsolved. Last investigations (German, 2002; Corral, 2003, 2004a,b; German, 2005) consider the distribution of waiting times with variations of different parameters.

In German (2002) the general statement about universality of the waiting times distribution with scale parameter, which is a function of ESTI, was formulated and methods to check this universality were proposed. At the same time the unified scaling law for distributions, which are mixtures of the waiting times distributions in spatial cells, was described in Bak et al. (2002). Corral (2003, 2004a,b) and Davidsen and Goltz (2004) continue investigations in this field. This law is closely related to the assumption about universality of the waiting times distribution (German, 2002). Another popular fields are theoretical investigation of time series with some assumptions (Altmann and Kantz, 2005; Molchan, 2005) and application of nonlinear analysis to experimental data (Correig et al., 1997; Matcharashvili et al., 2000, 2002).

The present paper deals with the following question: is temporal and spatial structure of seismicity (distributions of temporal and spatial characteristics) for different energy (magnitude) ranges, for different time domains or for different spatial areas stable or not? To answer this question the unified scaling theory for characteristics of seismicity, which covers different approaches to the distribution scaling, is developed. Its application to two characteristics is analyzed. The first one is waiting times ΔT of earthquakes; the second one is a new space parameter $\Delta D_{\min}$, which is the minimum distance from a current seismic event to the nearest (in space, not in time) neighbor in ESTI. The distributions of the characteristics ΔT and $\Delta D_{\min}$ allows to estimate the time interval to the next earthquake and the distance of the next earthquake from the prior earthquake. They are very important for seismic hazard estimations.

2. Basic statement of the unified scaling theory

Let V be a stochastic characteristic/variable considered. The basic statement of the unified scaling theory is: seismicity (seismic/failure process) in one ESTI is a scaled version of seismicity in another ESTI. This similarity is not absolute, but statistical: a distribution function (accumulated function) F of the characteristic V is:

$$F(v|\mathrm{ESTI}) = F(v) = F_0(v/{<}v{>}') \quad (1)$$

or for the corresponding probability density function

$$f(v|\mathrm{ESTI}) = f(v) = f_0(v/{<}v{>}')/{<}v{>}', \quad (2)$$

where F_0 and f_0 are constant functions, variable v corresponds to the stochastic characteristic $V$and $<v>'$ is a scale parameter of the distribution.

According to Eq. (1) the distribution function F for fixed ESTI is a scaled version of the function F_0 (Fig. 1). The statement (1) for the stochastic variable V can also be written as:

$$V = {<}v{>}'V_0 \quad (3)$$

where V_0 is a base stochastic variable with the distribution F_0, which is V for ESTI with scale parameter $<v>'=1$. It means that the distribution function of the stochastic value $V/<v>'=V_0$ is stable and equal to F_0.

Eqs. (1) and (2) mean that the distribution function F belongs to a scale family of distributions (Cox and Oakes, 1984). Note that there is no restriction on shape of the base distribution F_0.

The probably most popular distributions that belong to the scale family of distributions are the exponential, gamma, Weibull and not truncated power-law distributions. They are actively used to fit distributions of the energy, temporal and spatial characteristics. In Kagan (1999) for example it was shown that the gamma distribution fits

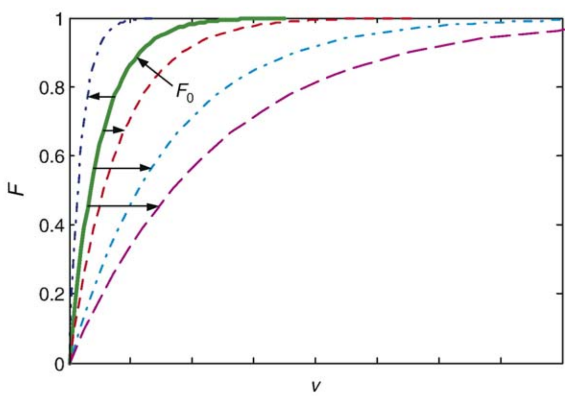

Fig. 1. Scaling of the distribution function F_0.

well a distribution of magnitude (size–frequency relation) for various seismoactive regions. In German (2002) and late in Corral (2003) Eqs. (1), (2) were used to analyze the waiting times ΔT. Furthermore in German (2002, 2005) it was shown that the distribution of ΔT for different catalogues of seismicity (natural and induced) and for different scale levels is well fitted by the Weibull distribution (it is a little bit better than the gamma distribution). Contrary, in Corral (2003) the gamma distribution is considered as the best approximation.

Sometimes (Nishenko and Buland, 1987, etc.) the lognormal distribution is used to fit ΔT distributions, but it does not belong to the scale family of distributions and it cannot be used for scaling. The same problem exists for the application of the truncated power-law distribution to the seismic moment-frequency/energy-frequency relation: to scale the distribution function the scale parameter and cutoff magnitude must be changed.

3. Scale parameter

A calculation of the mean from both parts of Eq. (3) shows that the mean $<V>$ of V in ESTI is proportional to the scale parameter $<v>'$:

$$<V> = <v>'<V_0> = \text{const } <v>', \quad (4)$$

therefore it is possible to scale the function F_0 to obtain $F(v)$ with the scale parameter

$$<v>' = \text{const}<V>. \quad (5)$$

Thus a distribution of the dimensionless stochastic variable $V/<V>$ ($\Delta T/<\Delta T>$ and $\Delta D_{min}/<\Delta D_{min}>$) must be stable (see Eq. (3)). To check this property it is necessary to visualize the distributions of $V/<V>$. They must coincide with each other, but of course some stochastic deviations are possible. Unfortunately in case of V corresponds to the radiated energy E or magnitude M this approach is not valid because it is not possible to record very weak events to estimate $<E>$ or $<M>$ correctly.

Note to obtain Eq. (5) we do not need any assumptions about $<v>'$, even such common like in (Corral, 2003, 2004a) where for ΔT the scale parameter $<v>'$ is inverse mean rate of seismic activity in ESTI. The last statement can be easily obtained from Eq. (5): for $\Delta T <v>' = \text{const} <\Delta T>$, but $<\Delta T> = 1/(N_{ESTI}/\Delta T_I)$, where N_{ESTI} is a number of events in ESTI.

4. Results of scaling and discussion

Let us illustrate the scaling of the characteristics distributions for some seismoactive regions (Table 1). Corresponding catalogues: Southern California earthquake data center catalog, The catalogue of Toktogul region earthquakes, Special catalogue of earthquakes of the Northern Eurasia are described in (Southern..., Database..., Special...) and are available on the Internet. The homogeneous areas are considered, error in earthquakes location in most cases does not exceed 5 km. The catalogue (Special...) does not include foreshocks and aftershocks, which have been removed using the methods described in (Molchan and Dmitrieva, 1992).

The analyses of distribution functions of $\Delta T/<\Delta T>$ and $\Delta D_{min}/<\Delta D_{min}>$ was carried out for different ESTI with variations of parameters E_I and ΔE_I; T_I and ΔT_I; X_I, Y_I and $\Delta L_I = \Delta X_I = \Delta Y_I$. For example, in case of variation T_I and ΔT_I the whole time interval was divided into n subintervals with constant range ΔT_I, but with different

Table 1
Characteristics of considered regions

Region	Southern California	Toktogul	Kamchatka	Kamchatka–Kurils
Analyzed period, years	01.01.1984...10.12.2000	1965...1991	1962...1990	1962...1990
Location	32.5...36.0° N.lat. 120.5...115.0° W.lon.	39.8...42.7° N.lat. 69.9...74.5° E.lon.	51...58° N.lat. 154...165° E.lon.	Area with apexes, (° N.lat.;° E.lon.): (42; 140), (42; 153), (57; 167), (57; 158), (48; 151), (43; 140)
Number of events	278,072	8830	6126	9502
Magnitude of events	0...7.3	−0.43...5.5	3.5...8.0	3.5...8.2
Magnitude completeness threshold M_{th}	2.4	2.2	3.5	5.0
Number of events with magnitude more than M_{th}	28,271	2477	6126	8208

T_I. Such division was carried out for different n (n=1, 2, …,16). As a result 136 subintervals were obtained. For each obtained subinterval the empirical distribution function (EDF) of $V/<V>$ was generated. According to the basic statement of the unified scaling theory these EDFs must be close to each other. The same procedure was realized for spatial subintervals (grid with $\Delta L_I=\Delta X_I=\Delta Y_I$) and for energy intervals (grid with constant ΔM_I or with constant ratio $(E_I+\Delta E_I)/E_I$). The number of spatial subintervals is 1496 because of the variation of not two, but three parameters: X_I, Y_I, ΔL_I. The first examination of the distributions of $\Delta T/<\Delta T>$ and $\Delta D_{\min}/<\Delta D_{\min}>$ shows that the exponential distribution can be considered as a very rough approximation of them.

A comparison of the empirical distribution functions was carried out through application of the Smirnov test (Pugachov, 1984). According to it if the maximum deviation ΔF of the one EDF with number n_1 of elements from another one with number n_2 of elements increases to a critical value $\Delta F_{\max}=S_\alpha\sqrt{1/n_1+1/n_2}$, where S_α is the critical value for confidence level α (for α=0.05 $S_{0.05}\approx 1.36$; $S_{0.001}\approx 1.95$ and $S_{0.0001}\approx 2.18$) then the hypothesis about coincidence of the distribution functions is rejected. Otherwise the deviation between EDFs can be considered as not significant for this confidence level.

For all regions EDFs containing at least 200 elements were analyzed. The confidence level α=0.05 was used, thus typical values of $\Delta F_{\max}$ are: $\Delta F_{\max}\approx 0.14$ for $n_1=n_2=200$ and $\Delta F_{\max}\approx 0.10$ for $n_1=200$ and $n_2\approx\infty$.

Note, a graphical comparison of the density distribution functions is not so informative because it is not clear which deviation is significant and which one is not. Moreover, construction of density functions is always a result of some assumptions (choice of intervals in the histogram method, choice of the kernel function and smoothing factor in the kernel method etc.). The most popular method for the comparison of the distribution density functions (histograms) with each other is the χ^2 test. But unfortunately it does not deliver such clear graphical evidence as the Smirnov test.

The analysis of the distribution functions of $\Delta D_{\min}/<\Delta D_{\min}>$ demonstrates that they are stable for all subintervals in each analyzed region. Typical figures are showed in Fig. 2a, c, e. Distributions of $\Delta D_{\min}/<\Delta D_{\min}>$ have a very important feature: small values of $\Delta d_{\min}$ cannot be determined more precisely than the precision of the earthquake location by the seismic network in the considered region. In Fig. 2a, c, e small markers correspond to $\Delta d_{\min}$=5 km, and a part of EDFs left of them can be not representative. This fact can be a reason of strong deviations between EDFs in this domain and deviations for small $\Delta d_{\min}$ like in Fig. 2c and can be used for the estimation of the earthquake location precision.

The distribution function of $\Delta T/<\Delta T>$ shows a more complicated behavior. Variations of E_I and ΔE_I demonstrate that the distributions are stable enough for each region (for example Fig. 2b). This fact is an extension of the results of German (2002), where the stability of waited times distributions for different E_I was demonstrated by the analysis of some catalogues of natural and induced seismicity.

Some distributions with variation of T_I and ΔT_I have a strong deviation (for example Fig. 2d), but a typical distribution can be found. It includes about 60% of distributions for different subintervals (in case of Kamchatka it includes nearly 100%). The analysis of subintervals for the distributions with strong deviations showed that they correspond to intervals containing strong events. Thus, large numbers of aftershocks can be a reason for such deviations (mentioned deviations occur due to a large percentage of small waiting times Δt, which are typical for aftershocks) (see also Matcharashvili et al., 2002).

The distribution function of $\Delta T/<\Delta T>$ is the most unstable one for variations of X_I, Y_I, ΔL_I. It is stable enough only for Toktogul and for all other regions it is significantly unstable (for example Fig. 2f), but even in this case a group of close distributions can be found. The probable cause for this is the different seismic/tectonic regime in different spatial areas.

5. Accelerated life model

A simple mathematical transformation of Eq. (3) allows to obtain another form of the main statement of the unified scaling theory. In Cox and Oakes (1984) it is named the accelerated life model:

$$\log V=\mu_0+\log(<v>')+\varepsilon, \tag{6}$$

where $\mu_0=<\log(V_0)>$ and ε is a stochastic variable with zero mean and a distribution which is independent from the scale parameter $<v>'$.

According to Eq. (6) the mean of log V

$$<\log V>=\mu_0+\log(<v>') \tag{7}$$

and the standard deviation

$$\sigma(\log V)=\text{const} \tag{8}$$

(Cox and Oakes, 1984). Eq. (7) can also be rewritten as

$$<v>'=10^{-\mu_0}10^{<\log V>}=\text{const}\ 10^{<\log V>}. \tag{9}$$

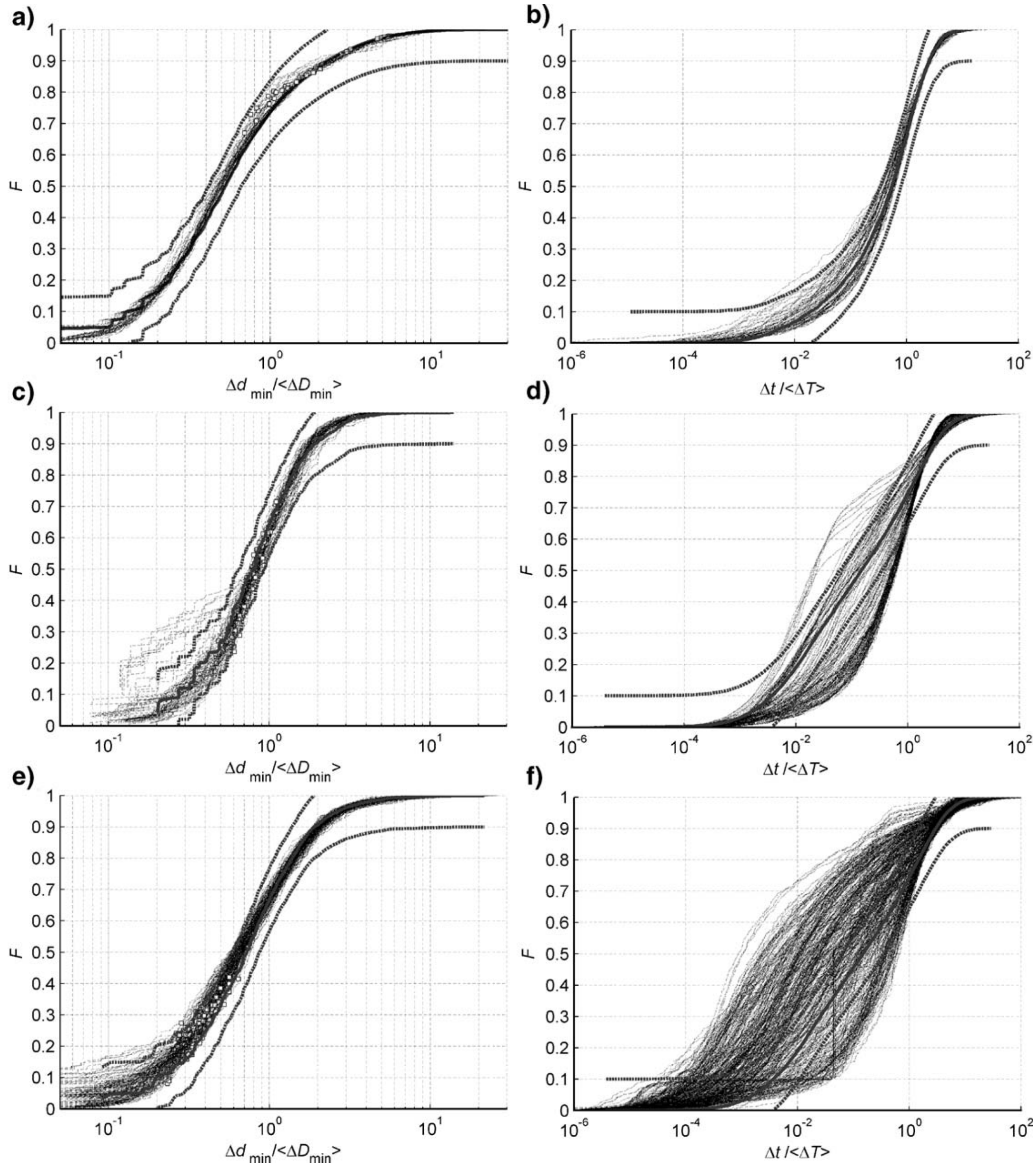

Fig. 2. Scaled empirical distribution functions of waiting times ΔT with the scale parameter equal to $<\Delta T>$ (b, d, f) and distances $\Delta D_{\min}$ with the scale parameter equal to $<\Delta D_{\min}>$ (a, c, e) for Southern California (a, d, f); Toktogul (c); Kamchatka (e); Kamchatka–Kurils (b) with variation of E_I and ΔE_I (a, b); T_I and ΔT_I (c, d); X_I, Y_I and ΔL_I (e, f). The bold line is the rescaled distribution function for the whole region and the bold dashed line is a confidence interval line with $\Delta F_{\max} \approx 0.10$; the dotted lines are rescaled distribution functions for all subintervals. The number of elements in each subinterval is not less than 200.

Relationships (8) and (9) are very useful to check the basic scaling statement (1): whether the stochastic variables are scaled versions of each other or not. Eq. (8) means that the standard deviation of log V is constant for all ESTIs. And Eq. (9) implies that the value $10^{<\log V>}$ can be used for the scaling of distribution functions of V, as $<V>$ was used before. Furthermore, according to Eqs. (5) and (9) $10^{<\log V>}$ and $<V>$ are linear functions of the scale parameter $<v>'$, therefore:

$$<V> = \text{const } 10^{<\log V>}. \tag{10}$$

This fact can also be used to check the possibility to scale the stochastic variable V.

The relations (8) and (10) for different EDFs can be easily plotted and this is an advantage of them over the direct comparison of distribution functions. But in most cases it is not easy to decide whether observed deviations from the theoretical behavior are significant or not. The last fact is a major disadvantage of the relations (8) and (10).

Let us test the possibilities of the distributions scaling applying $10^{<\log V>}$ as the scale parameter (Fig. 3a, b) and check the relations (8) , (10) (Fig. 3c–f). Fig. 3a, c, e corresponds to a case of good scaling with $< V >$ as the scale parameter (see Fig. 2b), and Fig. 3b, d, f to a bad one (see Fig. 2f). It is evident that the application of $10^{<\log V>}$ as the scale parameter does not change the quality of scaling significantly. The analysis of the empirical relations corresponded to Eqs. (8) and (10) confirms obtained conclusions about the possibility of scaling. Thus it can be seen that for Fig. 3c scattering of the standard deviation is much less then for Fig. 3d. In case of Fig. 3e a linear

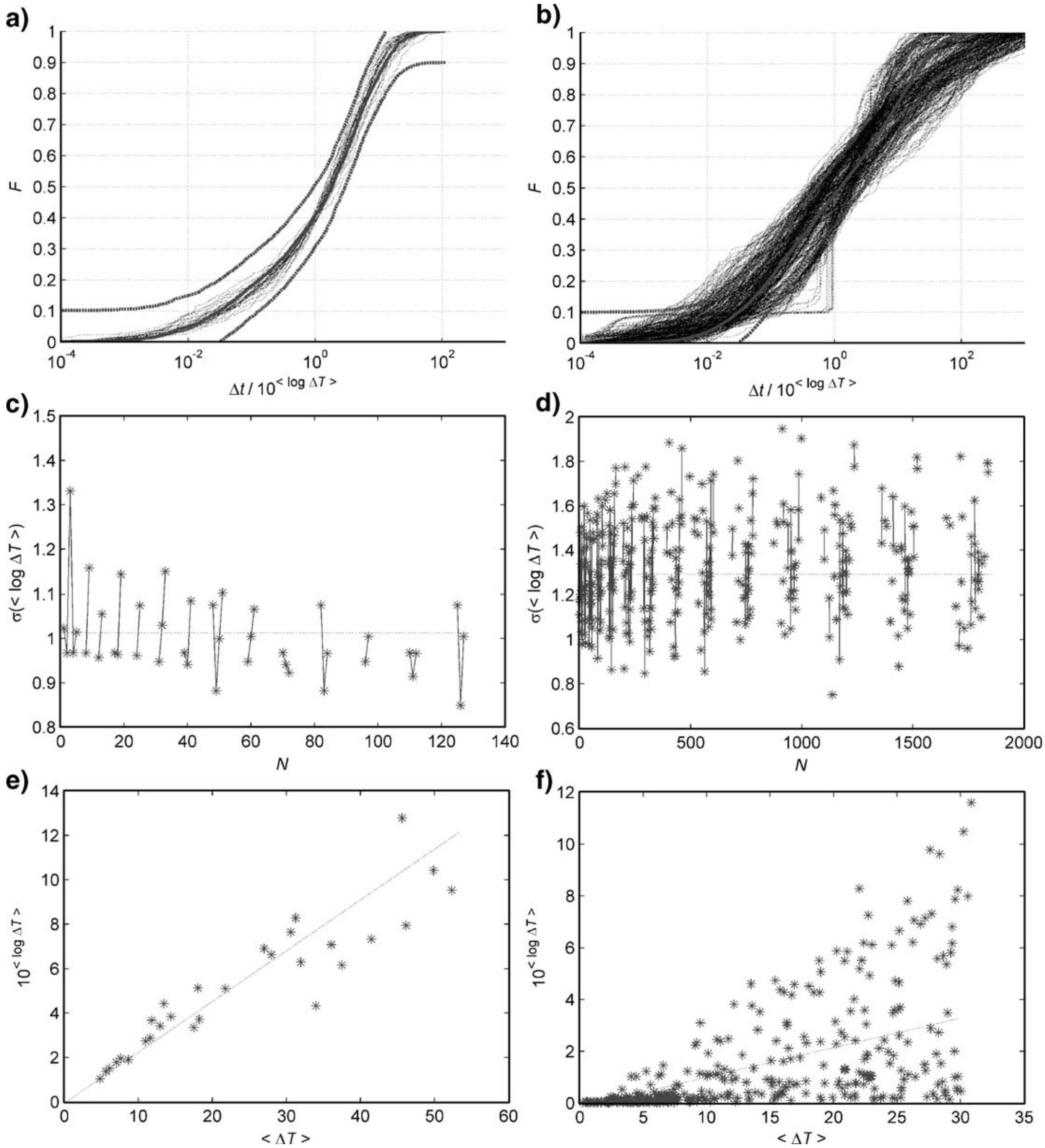

Fig. 3. Verification of the accelerated life model for the waiting times ΔT for Kamchatka–Kurils region with variation of E_I and ΔE_I (a, c, e) and for Southern California with variation of X_I, Y_I, ΔL_I (b, d, f). Days are used as units for ΔT. a, b — Scaled distribution functions of ΔT, $10^{<\log\Delta T>}$ was used as the scale parameter. c, d — Relation of standard deviation of log ΔT with subinterval number N, and the average $\sigma(\log \Delta T)$ (dot line). e, f — Relation between two estimations of the scale parameter: $10^{<\log\Delta T>}$and $< \Delta T >$, and the linear corresponding relation (dot line).

relation can be seen, but in case of Fig. 3f a functional relationship between $10^{<\log V>}$ and $< V >$ is not observed.

6. The Gutenberg–Richter law and scaling

A division of the spatial area of the fixed ESTI into n cells with size ΔL_c allows to determine the new relation for average $\Delta D_{\min}$ in the i-th cell: $< \Delta D_{\min\, i} >= C(\Delta L_c^2/N_i)^{d_1}$, where C is constant for the considered region, $<N> = \sum_i^n N_i/n$ is the average number of events in the cells and $d_1 \approx 2.5$. At the same time it is also obvious that $< \Delta T_i >= \Delta T_I/N_i$.

Thus, in case the stochastic variable V is the waiting times ΔT or the distances $\Delta D_{\min}$, it is possible to find the relationship between the average characteristic $< V_i >$ in the i-th cell and a number of events N_i in the cell:

$$<V_i> = 1/(WN_i^k), \;\; W = \text{const.} \tag{11}$$

Eq. (11) for ΔT and for $n=1$ is $< \Delta T >= \Delta T_I/N_{\mathrm{ESTI}}$. It allows to obtain the scaling as in Corral (2004a), where the value $\Delta T_I/N_{\mathrm{ESTI}}$ was used as the scale parameter.

Eq. (11) is also very useful because N_i is related to parameters of ESTI via the Gutenberg–Richter law:

$$\begin{aligned} N_{\mathrm{ESTI}} &= A(X_I, Y_I, Z_I, T_I)10^{bM_I}(1-10^{b\Delta M_I})\Delta T_I \\ &= A(X_I, Y_I, Z_I, T_I)E_I^{1-\gamma} \\ &\quad \times (1-[(E_I+\Delta E_I)/E_I]^{1-\gamma})\Delta T_I, \end{aligned}$$

where $A(X_I, Y_I, Z_I, T_I)$ is the activity factor. In most cases $\Delta E_I = \text{const} \cdot E_I$ is used. The law can also be written in a generalized form (Ponomarev et al., 1997; Baiesi and Paczuski, 2004):

$$\begin{aligned} <N> &= A_1(X_I, Y_I, Z_I, T_I)10^{bM_I}(1-10^{b\Delta M_I})\Delta L_c^d \Delta T_I \\ &= A_1(X_I, Y_I, Z_I, T_I)E_I^{1-\gamma} \\ &\quad \times (1-[(E_I+\Delta E_I)/E_I]^{1-\gamma})\Delta L_c^d \Delta T_I \end{aligned} \tag{12}$$

Thus it is possible to write the scaling parameter $< \nu >'$ through parameters of ESTI.

Let us consider one simple but important case: let us fix in a region all parameters of ESTI with the exception of M_I. In this case the scale parameter $< \nu >'$ for ΔT is proportional to $< \Delta T >$ and hence to 10^{-bM_I}, then according to Eq. (7):

$$<\log \Delta T> = \mu - bM_I, \tag{13}$$

where μ is constant. The last Eq. (13) together with Eq. (8) (constant standard deviation of log ΔT) allows to find the alternative estimation of parameter b of the Gutenberg–Richter law by ordinal linear least squares method (see also German, 2005). They can also be used to determine the magnitude completeness threshold in the region in case they are considered for all registered magnitudes. For this purpose similar relationships for $\Delta D_{\min}$ can be used. Fig. 4 confirms the theory that the linear part of log V starts from the magnitude completeness threshold.

7. Another approaches for scaling

Another approach for the scaling of variable V (Bak et al., 2002; Corral, 2003, 2004a,b; Davidsen and Goltz,

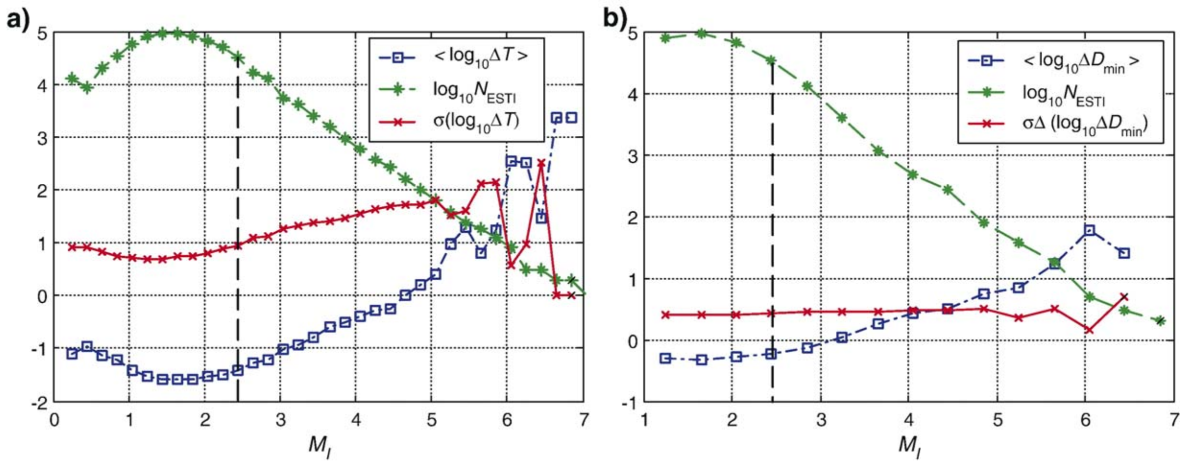

Fig. 4. Application of the scaling of the distributions of ΔT (a) and $\Delta D_{\min}$ (b) to determine the parameter b of the Gutenberg–Richter law, the magnitude completeness threshold for Southern California. Asterisks — the Gutenberg–Richter plot; squares — relation of $< \log V >$ versus magnitude M_I; crosses — standard deviation of log V versus M_I. The vertical dashed line corresponds to magnitude completeness threshold $M_{\mathrm{th}}=2.4$. $\Delta M_I=0.5$. Units of ΔT and $\Delta D_{\min}$ are days and kilometers correspondingly.

2004) is the covering of the spatial area of ESTI with a grid made out of cells of the size $\Delta L_c \times \Delta L_c$ (variations of temporal grid with cells size ΔT_c or an energy grid with ΔE_c can also be considered) and scaling of the probability density function $f_\Sigma(v)$, which is the a mixture of the probability density functions of V in each cells.

Let's assume a characteristic V_i in the i-th cell has a probability density function $f_i(v)$, $< V_i >= 1/(WN_i^k)$, and let $\varphi = 1/(W < N >^k)$ and $R_i = N_i / < N >$. Thus $f_\Sigma(v)$ is a mixture of the probability density functions $f_i(v)$ in cells and

$$\begin{aligned} f_\Sigma(v) &= \sum\{N_i f_i(v)\}/\sum N_i = \sum\{(N_i/<N_i>)f_i(v)\}/n \\ &= \sum\{(N_i/<N>)(WN_i^k)f_0(v\ WN_i^k)\}/n \\ &= W<N>^k \sum\{(N_i/<N>)^{k+1} f_0((N_i/<N>)^k W<N>^k v)\}/n \\ &= \varphi^{-1}\sum\{R_i^{k+1} f_0(R_i^k v/\varphi)\}/n \end{aligned}$$

The last sum is just an average value or mean of $\{\varphi^{-1}R_i^{k+1}f_0(R_i^k v/\varphi)\}$, which is a function of i or R_i. Furthermore a type of the density function $f_r(r)$ of the variable R_i is stable for each kind of grid with a density function $f_r(r)$, what was demonstrated for Southern California by Corral (2003). For a big n it is possible to change the summation to integration and obtain the following equation (the range of R_i can be estimated as $[0, \infty)$):

$$f_\Sigma(v) = \varphi^{-1}\int_0^\infty r^{k+1} f_0(r^k v/\varphi) f_r(r)dr = \varphi^{-1} f_{0C}(v/\varphi).$$

The last result means that it is possible to scale a mixture of the probability density functions $f_\Sigma(v)$. In this case the scale parameter is $\varphi = 1/(W < N >^k)$.

For the characteristic ΔT parameter $\varphi = \Delta T_I / < N >$, such scaling was used in Corral (2004b); and for $\Delta D_{\min}$ parameter $\varphi = C(\Delta L_I^2 / <N>)^{d_1}$. Again, the scale parameter can be described by parameters M_I, ΔM_I; ΔL_I; T_I, ΔT_I, because $<N>$ is related with them through the Gutenberg–Richter law (1). The scaling of characteristic ΔT with $\varphi = 10^{b\Delta M_I}/\Delta L_I^d$ was considered in Corral (2003).

The relationship between the scaling of Bak et al. (2002) $f_\Sigma(v) = f_{0B}(v/\varphi)/v$ and the unified scaling theory is also determined:

$$\begin{aligned} f_\Sigma(v) &= \varphi^{-1} f_{0C}(v/\varphi) = v^{-1}(v/\varphi) f_{0C}(v/\varphi) \\ &= v^{-1} f_{0B}(v/\varphi), \\ f_{0B}(v/\varphi) &= (v/\varphi) f_{0C}(v/\varphi). \end{aligned}$$

Now it is absolutely clear that to obtain decay factor Δt^{-1} in Bak et al. (2002) (where v is Δt) it is not necessary to use the Omori law.

8. Conclusion

The idea about self-similarity of seismic/failure processes assumes that the earthquake occurrence is governed by universal scale-invariant distributions. More general statement was suggested as a basic hypothesis of the unified scaling theory: a distribution function of the characteristic considered in a fixed energy–spatial–temporal interval (ESTI) is a scaled version of the base function (it is not necessary that the base function corresponds to power law distribution). Mathematical analyses demonstrate that the average value of the characteristic can be used as a scale parameter for its distribution.

Two characteristics have been used to test the theory: the first one is the waiting times of earthquakes ΔT, the second one is a new $\Delta D_{\min}$ (the minimum distance from a current seismic event to the nearest neighbor in ESTI). These characteristics are very important for seismic hazard estimations.

The application of the theory to seismicity of four various seismoactive regions (Southern California, Toktogul, Kamchtka, Kamchatka–Kurils region) demonstrates that scaling was successful for $\Delta D_{\min}$ of all ESTI and for ΔT at variations of energy/magnitude range. The distribution function type of ΔT at variations in the time domain was stable only in 60% of cases, and near to unstable for spatial variations.

Some additional methods, based on the accelerated life model, were proposed to check the possibility of distribution scaling. They confirm results obtained by direct comparison of scaled distributions of the characteristics. These methods can also be used to determine the magnitude completeness threshold of the region.

Scaling of mixture of the probability density functions of the characteristic for cells of spatial (or temporal, or energy) grid is considered. The relationship between the unified scaling theory and other present approaches for scaling has been determined as well.

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