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**Article** *in* Journal of Volcanology and Seismology · January 2001



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*Volc. Seis.*, 2001, Vol. 22, pp. 651-661
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# Solitary Waves on a Crustal Fault

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*(Received May 22, 1998)*

A microphysical model is proposed for deformation of granular materials to describe time-dependent slip on a fault, resulting in a "perturbed" sine-Gordon equation. The fault walls interact by friction during relative displacement, the friction being simulated by introducing an effective viscosity of a crushed geomaterial in the fault zone. An exact analytical solution in the form of a solitary wave can be derived, when it is assumed that roughness grains are distributed in a periodic manner along the fault and that the drag exerted by the grain contact surfaces is proportional to the square of shear flow velocity in the parting between the fault surfaces. The dynamical parameters of slow steady slip and rapid failure-causing slip found from the model are in agreement with the observation data.

## INTRODUCTION

Rock failure in crustal faults is due to the relative displacement of fault walls. It occurs either continuously (steady slip) or in jumps (intermittent slip), and is frequently accompanied by a tectonic earthquake. Cataclastic materials inside faults are less resistant to shear than is unfractured rock, and the penetration of fluids imparts special tectonic and seismic properties to faults [17]. Intermittent slip is unsteady gliding involving friction of the surfaces in contact that depends in a complicated manner on the mechanical properties of the rough walls and on the presence and mineral composition of the crushed material in the contact zone [10].

Identification of the causes and conditions that change the succession of the modes of slow steady and intermittent failure-causing slip is essential for the prediction of large earthquakes that occur on the faults. Equally important is to model the movement on the earthquake fault itself which proceeds by shear and may be caused by inhomogeneous friction [8].

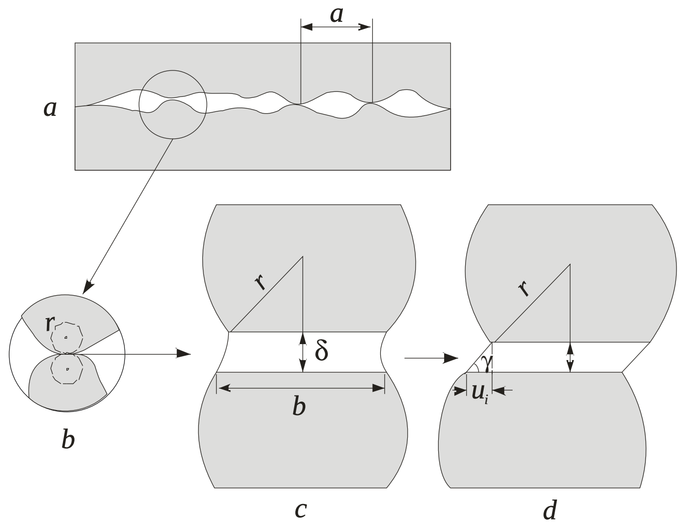


**Figure 1** A sketch showing interacting fault walls and changes in the contact of grains during shear: *a* – contact of realistic walls; *b-d* – changes in grain geometry.

Unsteady slip models [19], [21], [24], [25], [26], [28] assume the transition of steady slip to intermittent slip to be controlled by the friction coefficient which is functionally related to the velocity of the slip, the thickness of the crushed material parameters, temperature, pore pressure, porosity and critical displacement. Variation in the relative velocity of the fault wall movement is assumed by many models to be the leading factor controlling the friction coefficient: increasing velocity tends to make the contact softer, so the friction is reduced and the system passes to an unsteady state.

The present study assumes the change of slip mode to be controlled by a friction parameter which is a function of grain geometry (radius, diameter of the circular contact, distance between grain centers), but chiefly of the viscosity of the intergranular material and its thickness. The microphysical model for strain during shearing movement (intergranular gliding) in the fault zone leads to a "perturbed" sine-Gordon equation. The classical sine-Gordon equation was derived recently during the mathematical modeling of seismic and strain processes due to the rotations of grains (blocks) [5], [6], [16] and intergranular slipping [1], [2].

## A MICROPHYSICAL MODEL FOR STEADY-STATE SLIP

To a first approximation we assume the grains of radius $r$ to be positioned periodically along the fault length. The distance between the grain centers is $a$, the diameter of the circular contact between grains is $b$, and the thickness of a viscous intergranular layer is $\delta$ (Fig. 1). The balance of forces acting on an individual grain can be written in the form

$$F_a = F_i + F_r + F_{tect}\ ,$$

where $F_i$ is the force of interaction between an individual grain and those in the underlying layer; $F_r$ is the force of resistance, and $F_{tect}$ denotes tectonic forces.

The assumption of a periodic grain arrangement makes it possible to write down an expression for the total potential energy of the roughness grain system in the form

$$U = \sum_i m_i gh\left(1 - \cos 2p\,\frac{u_i}{a}\right) + \frac{1}{2} D_{ti} \sum_i \left(u_{i+1} - u_i\right)^2, \tag{1}$$

where $u_i$ is the displacement of the $i_{th}$ grain, $D_{ti}$ the tangential contact rigidity, and $m_i gh$, the maximum potential mechanical energy of an individual grain of mass $m_i$ whose center lies on the line CD at a distance of $h$ from the line AB which connects the grain centers on the adjacent fault wall (Fig. 2). When the upper wall is slipping relative to the lower, the value of $h$ varies, i.e., the grain changes position relative to a fixed level, which by definition means a change in the system's potential mechanical energy. Adding the phase $\pi$ to the argument of the harmonic term in (1) will by no means affect the final form of the equation of motion, but provides a clearer view of cyclic variation in the potential mechanical energy $U_{i1} = m_i gh\,(l - \cos(2\pi u_\mathrm{i}/a - \pi))$. If the grain is at rest (Fig. 2, *a*), then $u_i = 0$ and $U_{i1} = 2m_i gh$ (Fig. 2, *b*). When the grain is slipping along the surface of the grains in the lower wall, its potential mechanical energy declines during descent and grows during ascent, taking on the lowest value $U_{i1} = U_{min} = 0$ at $u_i = a/2$ (Figs. 2, *b* and 2, *c*). When, on the other hand, $u_i = a/4$, $3a/4$, then $U_{i1} = m_i gh$, and $U_{i1}$ takes on the largest value again when the grain has been displaced by a distance $a$. The second term on the right-hand of (1) denotes potential elastic energy and its derivative, the shear strain.

To sum up, the function $U$ is a sum of the potential mechanical and potential elastic energy of an individual grain on the slip surface in the fault, i.e., has a well-defined physical meaning. The force of interaction between an individual grain and the underlying layer has the form $F_i = -\partial U_i/\partial u_i = -2\pi A/a \sin(2\pi u_i/a) + D_{ti}(u_{i+1} - 2u_i + u_{i-1})$.

The force of resistance $F_r$ arising from the relative displacement of two grains with smooth parallel surfaces of contact is due to the flow of the intervening film with a variable thickness [3] $F_r = 3/2\pi r^2 \mu/\delta d\delta/dt$, where $\mu$ is the viscosity of the film material. The film thickness expressed in terms of grain displacement is equal to $\delta \approx 2ru_i/b$, because geometrical constructions (Figs. 1*c* and 1*d*) give the result $\tan\gamma = (r - \delta/2)/(u_i + b/2) \approx \delta/u_i$. For this reason the force of resistance is $F_r = 3\pi r^3 \mu/\delta b \partial u_i/\partial t$.

Long-wave approximation when used without taking the source displacement $F_{tect}$ into account gives the result that grain dynamics is governed by the equation (with $i$ omitted)

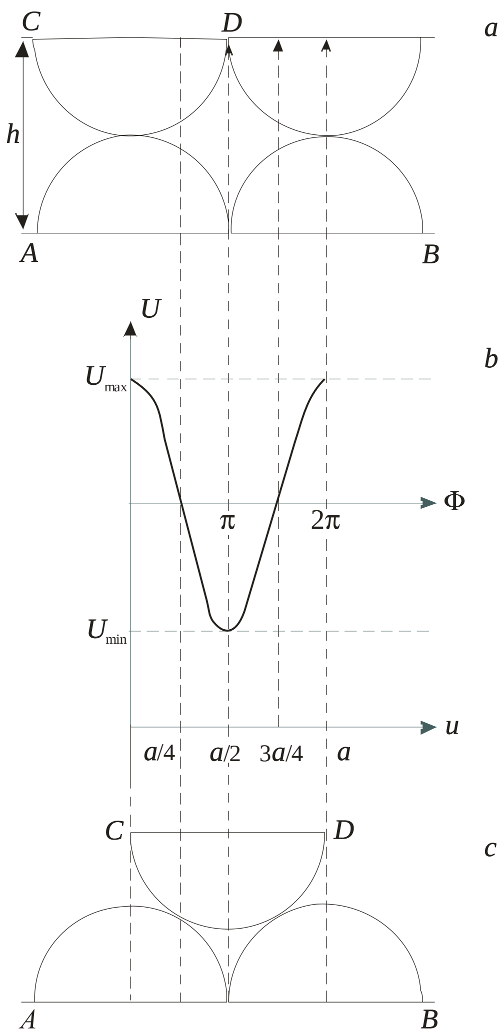


**Figure 2** Slipping of roughness grains (*a*, *c*) and variation in the potential mechanical energy of grains (*b*).

$$m\frac{\partial^2 u}{\partial t^2} = a^2 D_t \frac{\partial^2 u}{\partial x^2} - 2p\,\frac{A}{a}\sin 2p\,\frac{u}{a} - \frac{3pr^2}{bd}\,m\,\frac{\partial u}{\partial t}. \tag{2}$$

It should be noted that the ratio of the nonlinear to the elastic term is $\Gamma \approx 2\pi^2 rpg\,\sin(2\pi u_0/a)x^2/3D_i u_0$ to within the order of magnitude. This estimate shows that nonlinear effects must be felt for $\Gamma \geq 1$, when (2) contains no dissipation term. Taking the case of typical rock parameters (to be indicated below), this occurs when $x \geq x_0 = 0.1$-1 m (depending on the amount of displacement). The nonlinear effects accumulate at $x \approx x_0$, while the elastic and the nonlinear term become quantities of the same order.

Introduction of the new variable $\Phi = 2\pi u/a$ and the passage in (2) to the dimensionless coordinates $\xi = \pi x/ap$, $\eta = \pi\omega_0 t/p$, $p^2 = a^2 D_t/4A$, $\omega_0^2 = D_t/m$ with the subsequent addition of the dimensionless specific tectonic force $\sigma = F_{tect}/m_i g$ (tectonic force per unit grain weight) yields a "perturbed" sine-Gordon equation with the source $\sigma$:

$$\frac{\partial^2\Phi}{\partial x^2} - \frac{\partial^2\Phi}{\partial h^2} = \sin\Phi + a\,\frac{\partial\Phi}{\partial h} - s\,, \tag{3}$$

where $a = \dfrac{9}{8p}\dfrac{am}{bdr_s(gh)^{1/2}}$, where $\rho_s$ is the density of the grain material.

The shear flow of the intergranular material on the contacts of real grains during relative displacement is different from the Couette plane flow. Studies into the effects of roughness in a tube on its resistance showed that the "pulling" flow around the roughness protuberances reduced the braking action of the tube surface to the drag of a bluff body, this being proportional to the square of the fluid particle velocity [13]. It is therefore quite reasonable to make the assumption $F_r \sim (\partial\Phi/\partial\eta)^2$. Putting $\alpha = \alpha|\partial\Phi/\partial\eta|$, we can rewrite (3) in the form

$$\frac{\partial^2\Phi}{\partial x^2} - \frac{\partial^2\Phi}{\partial h^2} = \sin\Phi + a_0\left(\frac{\partial\Phi}{\partial h}\right)^2 - s\,, \tag{4}$$

Equation (4) has the same structure as that which describes the dynamics of an individual fluxon in a Josephson transmission line involving dissipation and a source of energy [18]. Solving (4) in the form of a traveling wave $\Phi = \Phi(\tau) = \Phi(\xi-\beta\eta)$ (where $\beta = (n^2-l)^{1/2}/n$ is dimensionless velocity, and $n$ is a separation constant which is greater than one [14]), one gets [18]

$$\Phi = \arcsin s_0 + 2\arcsin\left\{ cn\left[\frac{1}{k}\left(\frac{s_0}{2a_0 b^2}\right)^{1/2}(t - t_0);\ k\right]\right\}, \tag{5}$$

$$k = \left(\frac{2s_0}{s + s_0}\right)^{1/2}, \quad s_0 = \frac{2a_0^2}{\left[(1-b^2)^2 + 4a_0^2 b^2\right]^{1/2}}.$$

As $\sigma \to \sigma_0$, the absolute value of the elliptic function $k \to 1$, so that the extreme nonlinear case arises in which the periodic waves described by Jacobian elliptic functions become traveling solitary waves:

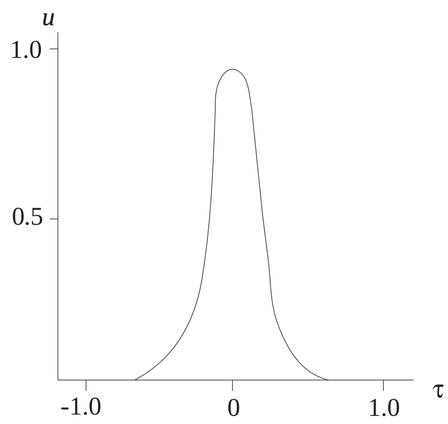


**Figure 3** Profile of a solitary wave ν (*x*, *t*).

$$\Phi = \arcsin s_0 + 4\,\mathrm{arctg}\left[\exp\left\{\left(\frac{s_0}{2a_0 b^2}\right)^{1/2}(t - t_0)\right\}\right]. \quad (6)$$

Equating the arbitrary constant to zero $\tau_0 = 0$ and passing again to the parameters of the original equation (2), we can write down the solution (6) and its derivative $\nu(x, t) = \partial u/\partial t$ in the form

$$u(x,t) = \frac{a}{2p}\arcsin\left[\frac{2a_0(n^2-1)}{\{1+4a_0^2n^2(n^2-1)\}^{1/2}}\right] + \frac{2a}{p}\,\mathrm{arctg}\left[\exp\left\{\frac{x-V_a t}{\Delta}\right\}\right], \quad (7)$$

$$n(x,t) = \frac{2n(gh)^{1/2}}{aw_0}V_a\,\mathrm{sech}\left\{\frac{x-V_a t}{\Delta}\right\}, \quad (8)$$

$$V_a = a\left(\frac{D_t}{m}\right)^{1/2}\frac{(n^2-1)^{1/2}}{n[1+4a_0^2n^2(n^2-1)]^{1/4}}, \quad (9)$$

$$\Delta = \frac{a^2 w_0}{2pn(gh)^{1/2}}. \quad (10)$$

## MECHANISM RESPONSIBLE FOR CHANGES IN SLIPPING MODE OR IN THE VELOCITY OF SOLITARY WAVES

The profile of ground particle velocity ν(*x*, *t*) on the fault surface has the form of a soliton (8) (Fig. 3) traveling along the fault length at velocity $V_\alpha$ (9). When $\alpha_0 >>1$, (9) becomes

$$V_a = \left[ \frac{D_t}{3m} g^{1/2} \frac{abdh^{1/2}}{r^3} \right]^{1/2} \frac{\left(n^2 - 1\right)^{1/2}}{n^2}. \tag{11}$$

When the friction parameter $\alpha_0 \to 0$, then (9) assumes the form

$$V = \frac{a}{2r} \left( \frac{3D_t}{prr_s} \right)^{1/2} \frac{\left(n^2 - 1\right)^{1/2}}{n}. \tag{12}$$

With values that are typical of sandstone [29]: $D_t$=2×10$^6$ *N/m;* $g$ = 10 m/s$^2$; $r$=2×10$^{-4}$ m; $b$= 10$^{-4}$ m; δ= 10$^{-5}$; $\rho_s$= 2.65×10$^3$ kg/m$^3$ and the viscosity of the intergranular material declining from 10$^{18}$ Pa·s (sand and clay) to 10$^{-3}$ Pa·s, the velocity of the solitary wave increases from 10$^{-9}$ to 10$^2$ m/s (Fig. 4). The velocity of displacement (8) is a function of the friction parameter (which would be more natural with a constant load), and not conversely, as is assumed in many models of unsteady slip. The friction parameter $\alpha_0$ controls the change of slip modes: as $\alpha_0 \to 0$, the soliton velocity and amplitude ν(*x*, *t*) sharply increase. On the one hand, the velocity ν depends on the condition of the contact, i.e., on the parameter $\alpha_0$, on the other, the growth of ν must weaken the contact itself. From (8) it follows that, when $V_\alpha$ is small, so is ν, and what is observed is a steady slip without any appreciable weakening of the contact. When the velocity $V_\alpha$ is comparatively large (10$^2$ to 10$^3$ m/s), one gets ν ~ 0.1-1 m/s and a dramatic increase in the displacement $u(x, t)$. A solitary wave "loosens up" the contact, which leads to the displacement of the fault walls under a constant load, i.e., to a dynamic movement. It thus appears that slip on a crustal fault is controlled by the velocity of a solitary waves $V_\alpha$. The solitary waves described above are similar in nature to slip waves observed in a rock specimen involving a fracture that appears before failure [21].

Experiments show [22] that a small change in water content (about 8%) will cause the dynamic viscosity of a clay suspension to decrease from 10$^8$ to 10$^4$ Pa·s. Consequently, the presence of fluid films along a fault at points of roughnesses in contact may give rise to intermittent slip. This state of affairs can arise, when moisture has penetrated into the fault, e.g., due to hydraulic fracturing at the contact of the blocks [11], [23] or to dehydration of clay minerals and silicates under vigorous shear strain [12]. In both of these cases a substantial lowering in the viscosity of the material that makes up the layer between the fault walls sharply diminishes $\alpha_0$ and causes a change in the slip mode. The mechanism of intermittent slip due to the presence of water can operate down to depths of about 100 km, because the minerals at greater depths are completely devoid of water [10].

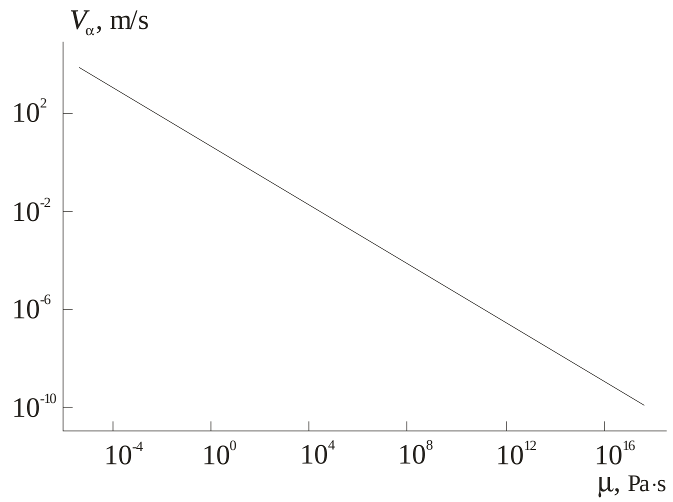


**Figure 4** The velocity of solitary wave $V_a$ as a function of viscosity of intergranular material μ.

The velocity of the solitary wave $V_a$ (Fig. 4) coincides with the rate of seismicity migration ($10^{-3}$ to $10^{-2}$ m/s) or with the rupture velocity at earthquake faults ($10^2$ to $10^3$ m/s) for certain values of viscosity in the intergranular material. This demonstrates that our model and its solutions can be used to explain certain features of the seismic process [4], e.g., in a Benioff zone (a set of hypocenters).

## CORRELATION BETWEEN THEORETICAL DYNAMIC SLIP PARAMETERS AND OBSERVATIONS

The mechanical properties of contacts in faults depend on the amount and condition of the geomaterial that fills the contact [7]. For this reason it is especially important to choose correct geometrical characteristics of grains (grain radius, the diameter of a circular contact, distance between grain centers), viscosity, and the thickness of the intergranular material. It would be reasonable to assume that the δ layer thickness is at least one or two orders of magnitude greater than the particle size in this layer. According to [9], the original grains (roughness grains in our case) are reduced to much smaller sizes during recrystallization and mylonitization, and ultramylonites may contain quartzite grains as small as $10^{-6} - 10^{-5}$ m embedded in ultra-fine-grained material. The grain size of crushed fractions of secondary quartz, calcite, kaolinite, etc., abundant in sedimentary formations, is $10^{-6}$ to $10^{-7}$ m [20]. Thus, δ should not be smaller than $10^{-6}$ m, the value used in our calculations (Table 1). The increase in δ by an order of magnitude requires the viscosity of the intergranular material to be increased by an order of magnitude as well to preserve $V_a$. However, the

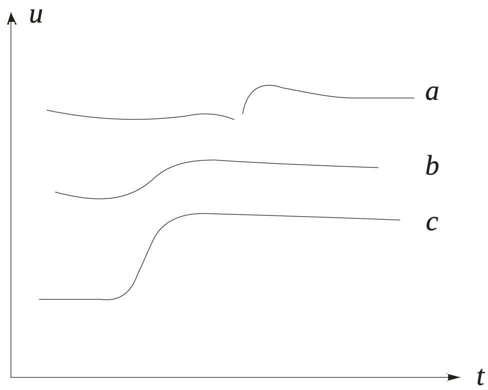


**Figure 5** Displacement vs. time: *a* - laboratory measurements during an inhomogeneous fracture [15]; *b* - variation of relative displacement in granite [27]; *c* - solution of a sine-Gordon equation.

main cause of a change in $V_\alpha$ lies in the choice of a viscosity value for the intergranular material (Fig. 4). The least viscous (clay, silty clay) and low-viscous rocks (sand-clay and limestone-marl sequences) have viscosities in the range $2 \times 10^{14}$ to $3\times 10^{18}$Pa·s [20].

**Table 1** Velocity of solitary wave in relation to values of the model parameters during steady slipping.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Viscosity of intergranular material, $\mu$ (Pa · s) | $10^6$ | $10^8$ | $10^8$ | $10^8$ | $10^9$ | $10^9$ | $10^{10}$ | $10^{10}$ | $10^{10}$ | $10^{12}$ | $10^{10}$ |
| Thickness of intergranular material, $\delta$(m) | $10^{-6}$ | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ | $10^{-6}$ | $10^{-5}$ | $10^{-6}$ | $10^{-5}$ |
| Contact stiffness, $D_t$ (MN/m) | $10^{-2}$ | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| Grain radius, $r$ (mm) | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 0.2 | 0.2 | 0.2 | 0.2 |
| Wave velocity, $V_\alpha \cdot 10^4$ (m/s) | 2 | 2.5 | 8 | 20 | 0.8 | 2.5 | 2.7 | 0.5 | 1.4 | 0.05 | 0.14 |

The above range was used to calculate dynamic slip parameters on faults. The resulting values of particle velocity, the velocity of solitary waves, and dynamic slip are comparable with the observed data:

1. The function $u(x, t)$ which is the solution of equation (7) describing grain dynamics has a shape similar to that of observed displacement (Fig. 5).

2. The calculated velocities $V_\alpha$ of solitary waves at continuous slipping are close to the velocities of strain waves (see Table 1).

3. The velocity of solitary waves controls the slip behavior on the fault. With velocities of $10^2$ to $10^3$ m/s the amplitude of slip velocity $\nu_{max}$ reaches values around

1 m/s, the displacements being 0.1–1 m, the values consistent with field measurements near faults [10].

4. A change in the slip mode from stationary to intermittent is due to an abrupt decline in the viscosity of the intergranular material, which may occur, e.g., when water flows into the fault. Events like this are not unlikely: at different degrees of the defluidization of the fault blocks their contact (a fault) can experience hydraulic fracturing with the fluid flowing into the fault [11], [23]. The resulting substantial decline in the viscosity of the geomaterial on the fault may trigger a seismicity increase.

## CONCLUSION

This microphysical model of earth deformation due to shear in granular material containing a viscous intergranular material leads to a "perturbed" sine-Gordon equation and provides a satisfactory description of the mechanism responsible for changes in the slip mode on a fault. Calculations of some parameters pertaining to steady slip on faults and earthquake rupture are consistent with observed values. The velocity of solitary waves on a fault is controlled by the viscosity of the material filling the space between roughness grains. The properties of surfaces in contact (periodic arrangement of roughness grains) and of intergranular films make the deformation nonlinear. A slow slip on a fault may indicate the approach of an unsteady slip propagating at great velocities, i.e., a tectonic earthquake, the viscous fluid controlling the earthquake dynamics.

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