
An approach to interpretation of step drawdown tests

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Abstract To understand the behaviour of drawdown with discharge under site-specific conditions, a step drawdown test (SDT) was conducted on a tubewell in alluvium near Delhi, India, and the data were analyzed to find the value of well efficiency. The entrance velocity of groundwater into the well screen was computed to see if it exceeded the safety limit of 0.03 m/s. Reynold's number (Re) was also calculated to study the nature of flow at the well screen. The decrease in efficiency of the well at higher discharge was attributed to an increase in turbulence and curvature of the flow path of water. An alternative, more efficient approach has been put forward, using a spreadsheet programme to fit a polynomial trend line on a plot of drawdown versus discharge and deriving a polynomial trend line equation. This equation predicts the behaviour of drawdown with discharge under site-specific conditions. The calculated efficiency of the tubewell can, at best, be regarded as a reflection of head loss on account of the laminar flow from the aquifer.

Résumé Afin de comprendre le comportement du rabattement par rapport au débit d'exhaure dans un contexte spécifique, un pompage d'essai par paliers enchaînés a été effectué dans un forage en contexte alluvial, à proximité de Delhi (Inde); les données ont été analysées dans le but de quantifier l'efficacité du puits. La vitesse d'entrée de l'eau souterraine à travers la crépine a été évaluée pour déterminer si elle excédait la limite de sécurité de 0.03 m/s, et le calcul du nombre de Reynolds a permis d'étudier le régime d'écoulement à travers la crépine. La diminution de l'efficacité du puits pour les plus hauts débits a été attribuée à l'augmentation des turbulences et à la courbature des lignes de courant. Une approche alternative

plus efficace a été mise en avant: elle utilise un tableur pour ajuster une courbe de tendance polynomiale sur la représentation graphique rabattement/débit et en tirer une équation caractéristique polynomiale. Cette équation anticipe l'évolution du rabattement avec le débit dans le contexte donné. L'efficacité du forage calculée peut, au mieux, être considérée comme le reflet des pertes de charge liées aux écoulements laminaires depuis l'aquifère.

Resumen Para entender el comportamiento del descenso con la descarga bajo condiciones específicas del sitio se llevó a cabo una prueba de descenso escalonada en un pozo entubado en aluvión cerca de Delhi, India, y los datos se analizaron para encontrar el valor de la eficiencia del pozo.

Se calculó la velocidad de entrada del agua subterránea a la malla del pozo para ver si se excedía el límite de seguridad de 0.03 m/seg y se calculó el número de Reynolds para estudiar la naturaleza del flujo en la malla del pozo.

El decremento en eficiencia del pozo a descargas elevadas se atribuyó a un incremento en turbulencia y curvatura de la trayectoria de flujo del agua. Se propone un enfoque alternativo más eficiente usando un programa de hoja electrónica para ajustar una línea de tendencia polinomial en una gráfica de descenso versus descarga y se deriva una ecuación de tendencia de línea polinomial. Esta ecuación predice el comportamiento de descenso versus descarga bajo condiciones específicas del sitio. La eficiencia calculada del pozo entubado puede, cuando mucho, considerarse como un reflejo de la pérdida de presión por el flujo laminar del acuífero.

Keywords Step drawdown test · Well efficiency · Entrance velocity · Trend line · Tubewell design

Introduction

One of the main objectives of hydrogeologists working in alluvium areas with good groundwater potentiality is to construct and design tubewells for extraction of subsurface water. Once the well has been designed and put in the desired place an evaluation of the structure is done. The design of the tubewell becomes difficult on account of the groundwater quality variation with depth and in order to avoid anthropogenic pollution from the surface. The same well either alone or with a set of observation wells can be used

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for the evaluation of aquifer characteristics. The aquifer parameters themselves are of more use for scientific/academic purposes than for commercial purposes. From the perspective of a groundwater user, the performance of the abstraction structure and the relationship between discharge and drawdown of the tubewell is more important. In order to make a structure ideal from a hydraulics viewpoint and at the same time considering the quality and pollution aspects, one comes to a compromise point based on harmonization of different interests.

While working in the field and analyzing data from various pumping tests, the following questions come to mind:

1. In a given set of conditions, how does drawdown behave with different levels of discharge?
2. Does the well efficiency calculated using the conventional well performance tests truly reflect the performance of the well?

Experimentation

Thinking in this direction a six-step step drawdown test (SDT) was conducted on a tubewell at Mayur Vihar in the National Capital Territory (NCT) of Delhi, India. The subsurface formation as deciphered during the course of drilling consists of sand particles of different sizes mixed with some silt and a little clay down to the depth of nearly 55 m below ground level. Below this depth, clay predominates in the formation with little silt and Kankar (local term for small calcareous concretions mixed with some small gravels). The aquifer can be said to have unconfined conditions. On the basis of the lithological characteristics and the geophysical log of the borehole, a suitable configuration of blank and slotted pipe was lowered in the borehole to convert it into a production well. The Johnson's slotted

pipe with 30% open area of 254-mm diameter was used. The slotted pipe was installed at depth ranges of 21–25 m, 29–31 m, 34–38 m and 39.5–43.5 m below ground level (mbgl). The initial depth to water level was 4.22 mbgl.

Step drawdown test and the alternative approach

In order to understand the behaviour of drawdown with discharge and to make a comment on the performance of the well and its design a six-step test of 50-min duration each was planned and executed. The raw data for drawdown and time are given in Table 1. From this table the value of drawdown was plotted against time on semi-log paper for the six steps (Fig. 1). From this the observed drawdown for each step was taken and the corresponding discharges for each step were noted (Table 2). Then it was decided to use Jacob's (1947) equation modified by Rorabaugh (1953) for analysis of data. The Rorabaugh's general equation is given below:

$$s = BQ + CQ^p \quad (1)$$

where s is the drawdown, B is the formation loss coefficient, C is the well loss coefficient, Q is the discharge, p is a variable having value in the range of 1.5 to 3.5.

The objective of data analysis now focuses on finding B , C and p values in the above equation. If Hantush Bier-schenk's method (given in Kruseman and de Ridder 1990) is followed, it assumes $p=2$ and then plots specific drawdown and discharge data for each step to find B and C . In Rorabaugh's approach (given in Kruseman and de Ridder 1990) p varies in the range of 1.5–3.5 and the approach involves considerable computation and graphical plotting. The plot of specific drawdown and discharge data for the

Table 1 The data of drawdown (m) and time (min) for the six-step test

Step-1 ^a		Step-2 ^b		Step-3 ^c		Step-4 ^d		Step-5 ^e		Step-6 ^f			
Time	Drawdown	Time	Drawdown	Time	Drawdown	Time	Drawdown	Time	Drawdown	Time	Drawdown		
1	1.47	16	4.6	55	5.61	105	6.68	160	6.93	210	7.95	260	8.51
2	3.05	18	4.58	60	5.99	110	6.82	170	7.27	220	7.96	270	8.45
3	3.16	20	4.53	65	5.97	115	6.72	180	7.3	230	8.1	280	8.75
4	3.58	22	4.54	70	6.05	120	6.92	190	7.45	240	8.1	290	8.4
5	3.73	24	4.54	75	6.02	125	6.98	200	7.54	250	8.2	300	8.45
6	3.97	26	4.57	80	6.02	130	6.82						
7	3.97	28	4.93	85	5.93	135	6.9						
8	4.16	30	5.02	90	6.13	140	6.96						
9	4.12	35	5.06	95	6.13	145	7.06						
10	4.38	40	5.03	100	6.19	150	6.93						
12	4.39	45	5.04										
14	4.71	50	5.19										

^aStep-1, Discharge 700 l/min (0.0117 m³/s)

^bStep-2, Discharge 771 l/min (0.0129 m³/s)

^cStep-3, Discharge 852 l/min (0.0142 m³/s)

^dStep-4, Discharge 920 l/min (0.0153 m³/s)

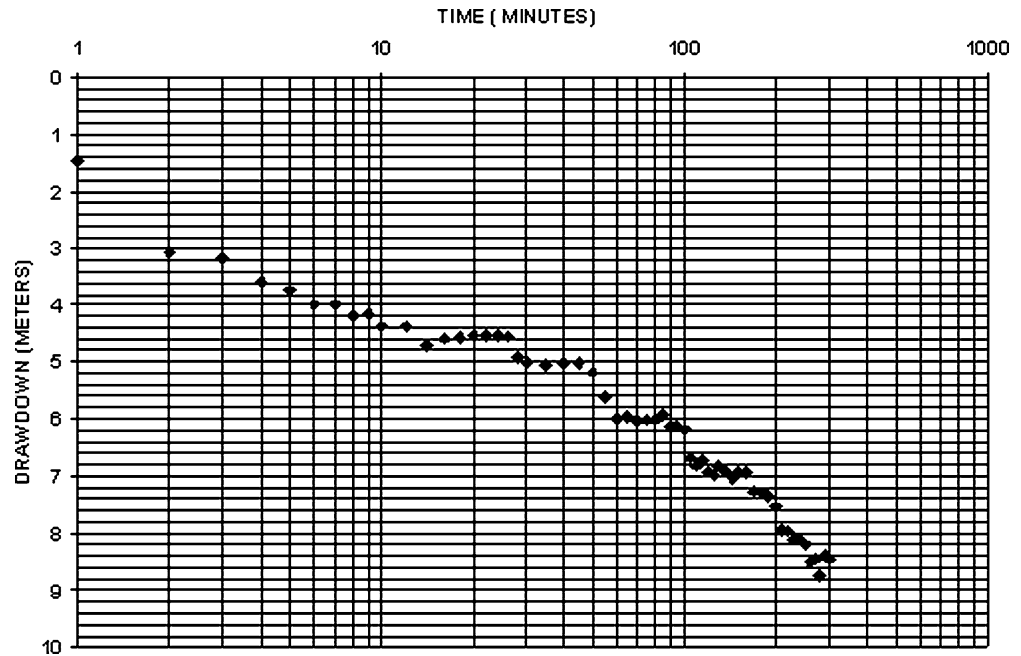
^eStep-5, Discharge 973 l/min (0.0162 m³/s)

^fStep-6, Discharge 1,026 l/min (0.0171 m³/s)

Table 2 Data for all the six steps taking the *B* and *C* values from the polynomial trend line equation of Fig. 3

Discharge (m ³ /s)	Drawdown		<i>B</i>	<i>C</i>	<i>BQ</i>	<i>CQ</i> ²	<i>(BQ + CQ</i> ²) Calculated	Well efficiency <i>BQ/(BQ + CQ</i> ²)
	Observed (m)	Specific (s/m ²)						
0.0117	5	427.4	328.6	10272	3.8	1.4	5.2	0.7308
0.0129	6.1	472.9	328.6	10272	4.2	1.7	5.9	0.7119
0.0142	6.9	485.9	328.6	10272	4.7	2.1	6.8	0.6912
0.0153	7.45	486.9	328.6	10272	5	2.4	7.4	0.6757
0.0162	8.05	496.9	328.6	10272	5.3	2.7	8	0.6625
0.0171	8.5	497.1	328.6	10272	5.6	3	8.6	0.6512

Fig. 1 Semi-log plot of time versus drawdown



six steps did not show a linear trend (Fig. 2). At this stage it was decided that a trend line-fitting approach should be applied to the plot of drawdown and discharge. Microsoft Excel (1997 and onwards version have been used here) has an option of fitting a polynomial trend line into the scatter plot of points. This gives a polynomial trend line equation which approximates to Rorabaugh’s general Eq. (1) with *p*=2. The plot with a polynomial trend line fitted into points is shown in Fig. 3. After rearranging the trend line equation the *B*, *C* and *p* values can be directly found from it.

The approach mentioned above skips the process of plotting specific drawdown and discharge and finding slopes etc., and at the same time it avoids the complex computation and plotting of Rorabaugh’s approach. In fact only one plot of drawdown and discharge suffices to find *B*, *C* and *p* values. These *B*, *C* and *p* values are used to find the well efficiency and attain the calculated drawdown values. It has been observed in this case study that the calculated and observed drawdown values show good concordancy.

The analysis of the data here shows that the efficiency of the tubewell decreases as the discharge increases (Table 2). In order to account for this the nature of flow

at the well screen has been examined by ascertaining Reynold’s number and at the same time entrance velocity to the well screen has been ascertained. The two factors are separately dealt within the subheadings given below.

Entrance velocity

Entrance velocity of water into the screen of the well is an important criterion in deciding the safety and longevity of the structure. The entrance velocity of water moving into the screen should not exceed 0.03 m/s. At this velocity, the friction loss in the screen opening will be negligible and the rates of incrustation and corrosion will be minimum (Driscoll 1986). In the light of the above fact the entrance velocity at different discharge rates were calculated. The calculations are given in Table 3. The data reveal that the entrance velocities calculated for the different discharge levels are well within the permissible limit of 0.03 m/s (Driscoll 1986) prescribed for safety of the structure. In order to understand the nature of flow in the vicinity of the well Reynold’s number was calculated.

Fig. 2 Plot of specific drawdown and discharge data

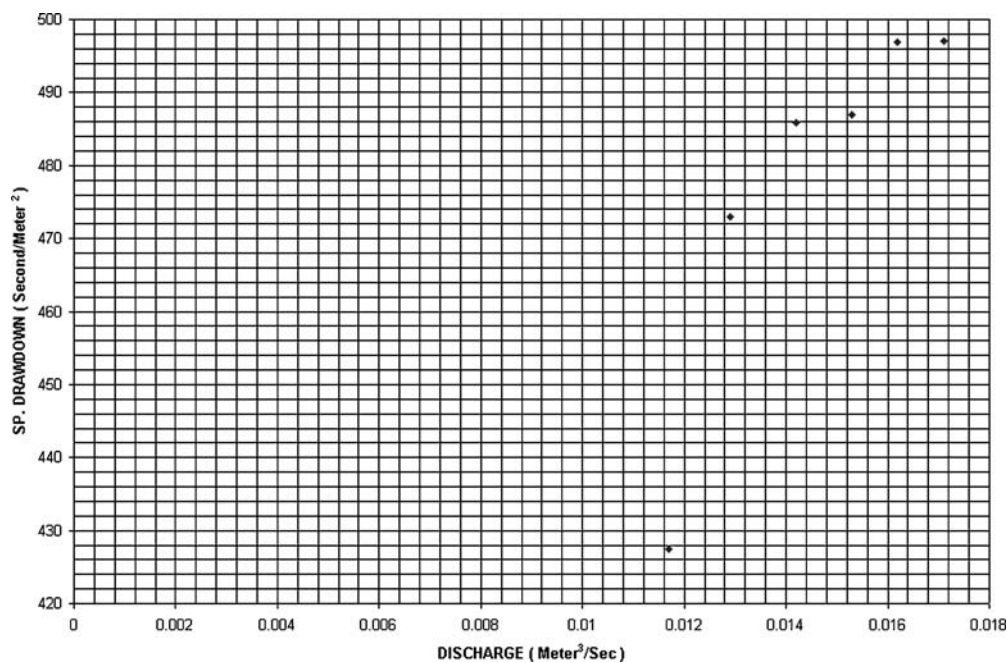


Fig. 3 Discharge versus drawdown plot with a polynomial trend line

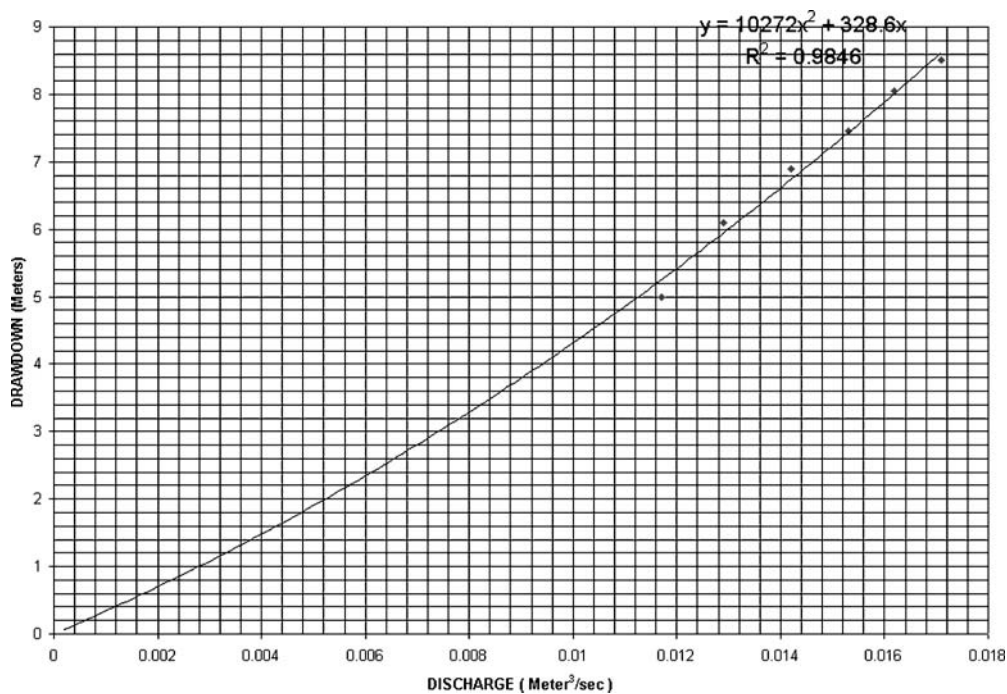
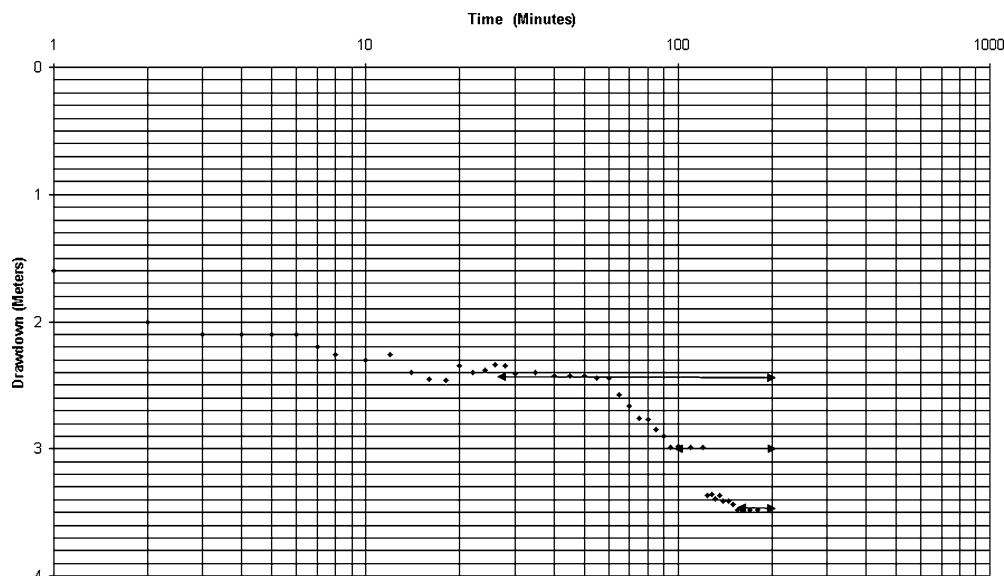


Table 3 Entrance velocity of water for the different discharges

Steps	Discharge (m³/s)	Radius (m)	Total surface area of screen (m²)	Total open area (m²)	Entrance velocity (m/s)
1	0.0117	0.127	11.176	3.3528	0.0035
2	0.0129	0.127	11.176	3.3528	0.0038
3	0.0142	0.127	11.176	3.3528	0.0042
4	0.0153	0.127	11.176	3.3528	0.0046
5	0.0162	0.127	11.176	3.3528	0.0048
6	0.0171	0.127	11.176	3.3528	0.0051

The total area was calculated on the assumption of 30% open area in the slot used

Fig. 4 Semi-log plot of time versus drawdown, Production Well no. 2, Palla area



Calculation of Reynold's number

Reynold's number is an important criterion to distinguish between the laminar and turbulent flow. Therefore it was calculated for the different steps. The formula used is given below:

$$Re = \frac{(\text{density of fluid} \times \text{magnitude of its velocity} \times \text{boundary to fluid flow})}{\text{viscosity of fluid}}$$

Two approaches for fixing the boundary to the fluid flow were taken. In the first approach the boundary to the fluid flow was assumed to be the length from well bottom to the pumping water level. The second approach was based on the assumption of the boundary length being the total length of the screen. The values calculated on the two assumptions are given in Table 4.

The onset of turbulence in a fluid flow may take place anywhere in the range $1,000 < Re < 3,000$. The study of the above results reveal that the Reynold's number on both assumptions are much higher than the critical limit set for the onset of turbulence. One thing has to be also kept in mind that these observations of Reynold's number are at the face of the screen. Though the entrance velocity is well within the critical limit of 0.03 m/s (Driscoll 1986), the flow into the tubewell is of turbulent nature. This flow, even though of turbulent nature, does not harm the structure in any way.

General applicability of the proposed approach

After having arrived at an alternative (efficient and accurate) approach for analysis of the step drawdown test data during the course of above mentioned experimentation and

data analysis, it would be appropriate at this stage to demonstrate the general applicability of the proposed approach by applying it to analysis of some more step drawdown test data. Thus three more step drawdown test datasets from the Palla area (located in the flood plain of River Yamuna) of NCT Delhi were analyzed with the help of the proposed approach. The raw data are given in Table 5. The semi-log plot of time versus drawdown for Production Well no. 2 is shown in Fig. 4, for Production Well no. 5 is shown in Fig. 5 and for Production Well no. 60 is shown in Fig. 6. The value of the observed drawdown (stabilized drawdown during the later part of each step) for each step with corresponding discharge for the individual production wells was taken from Table 6 and plotted on normal scale. The plot of drawdown versus discharge with a polynomial trend line fitted into the plot of points (equations of the polynomial trend line is also displayed in the figures) for Production Well no. 2 is shown in Fig. 7, for Production Well no. 5 is

Table 4 Reynold's number at different entrance velocities and at different discharge levels

Steps	E. Vel (m/s)	Density (kg/m ³)	Dd (m)	B.L (m) Asmptn1	B.L (m) Asmptn2	Viscosity (N s/m ²)	Re-1 Asmptn1	Re-2 Asmptn2
1	0.00349	1000	5	41.5	14	0.001	144835	48860
2	0.00385	1000	5.8	40.7	14	0.001	156695	53900
3	0.00424	1000	6.6	39.9	14	0.001	169176	59360
4	0.00456	1000	7	39.5	14	0.001	180120	63840
5	0.00483	1000	7.5	39	14	0.001	188370	67620
6	0.0051	1000	7.7	38.8	14	0.001	197880	71400

E.Vel Entrance velocity, B.L Boundary length, *Re* Reynold's number, Asmptn assumption, Dd Drawdown, Ns/m² Newton seconds per m²

Table 5 The raw data (drawdown (m) and time (min)) for individual production wells used for testing general applicability of the proposed approach

	Step-1		Step-2		Step-3		
	Time	Drawdown	Time	Drawdown	Time	Drawdown	
Production Well no. 2 Palla Area, Initial depth to water level 5. 23 m below ground level ^a							
1	1.6	20	2.35	65	2.58	124	3.37
2	2.0	22	2.4	70	2.66	128	3.36
3	2.1	24	2.38	75	2.76	132	3.39
4	2.1	26	2.34	80	2.77	136	3.37
5	2.1	28	2.35	85	2.85	140	3.41
6	2.1	30	2.41	90	2.90	145	3.41
7	2.2	35	2.4	95	2.99	150	3.44
8	2.26	40	2.43	100	2.98	155	3.48
10	2.3	45	2.43	110	2.99	160	3.48
12	2.26	50	2.43	120	2.99	170	3.48
14	2.4	55	2.44			180	3.48
16	2.45	60	2.44				
18	2.46						
Production Well no. 5 Palla Area, Initial depth to water level 4. 34 m below ground level ^b							
1	5.34	20	5.82	55	7.44	110	9.1
2	5.37	22	5.79	60	7.61	120	9.21
3	5.44	24	5.89	65	7.54	130	9.43
4	5.47	26	5.85	70	7.59	140	9.41
5	5.6	28	5.79	75	7.73	150	9.44
6	5.65	30	5.89	80	7.65		
7	5.76	32	5.85	85	7.66		
8	5.65	36	5.87	90	7.67		
10	5.86	38	5.87	95	7.76		
12	5.82	40	5.91	100	7.76		
14	5.81	45	5.94				
16	5.82	50	5.94				
18	5.91						
Production Well no. 60 Palla Area, Initial depth to water level 5. 44 m below ground level ^c							
1	4.44	14	4.89	54	6.19	104	7.31
2	4.84	16	4.87	58	6.18	108	7.45
3	4.94	18	4.87	62	6.2	112	7.47
4	4.82	20	4.88	66	6.26	116	7.45
5	4.82	25	4.81	70	6.27	120	7.58
6	4.82	30	4.94	75	6.31	125	7.64
7	4.83	35	4.95	80	6.36	130	7.54
8	4.84	40	4.97	85	6.38	135	7.54
10	4.73	45	5.03	90	6.36	140	7.62
12	4.80	50	5.04	95	6.38	145	7.61
				100	6.39	150	7.62

Minor logical correction in a few of the observations were done on account of human factor, use of steel tapes, prevailing field conditions etc.

^aStep-1, Discharge 1,734 l/min (0.0289 m³/s), Step-2, Discharge 2,104 l/min (0.03507 m³/s), Step-3, Discharge 2,407 l/min (0.04012 m³/s)

^bStep-1, Discharge 1,923 l/min (0.03205 m³/s), Step-2, Discharge 2,339 l/min (0.03898 m³/s), Step-3, Discharge 2,680 l/min (0.04467 m³/s)

^cStep-1, Discharge 1,923 l/min (0.03205 m³/s), Step-2, Discharge 2339 l/min (0.03898 m³/s), Step-3, Discharge 2,680 l/min (0.04467 m³/s)

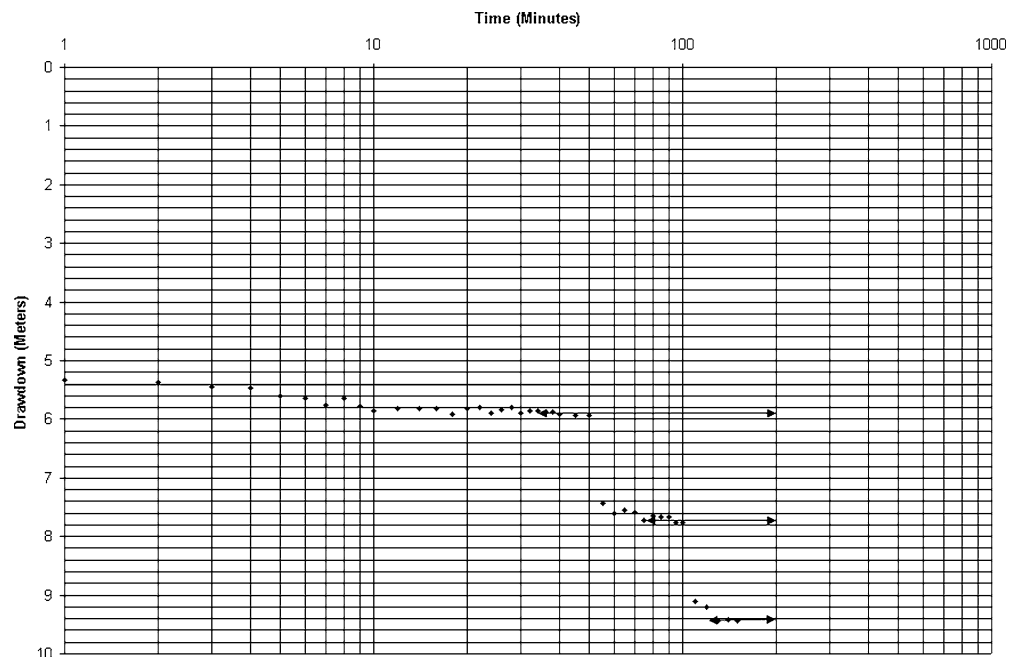
shown in Fig. 8 and for Production Well no. 60 is shown in Fig. 9. The formation loss coefficient (B) and well loss coefficient (C) values were obtained from the polynomial trend line equation for individual production wells: these values, along with the calculated drawdown and the calculated well efficiency, are given in Table 6. From this table it becomes apparent that there is very good concordancy between the observed and the calculated drawdown to the extent that they match each other. Thus the general applicability of the approach is proved to a great extent.

Summary and conclusions

The design of tubewells in alluvial areas becomes difficult on account of the quality variation with depth and in order to avoid anthropogenic pollution from the surface. From the perspective of the groundwater user, the performance of an abstraction structure and the relationship between discharge and drawdown of the tubewell is important. The relationship between discharge and drawdown of a tubewell can be ascertained by a step drawdown test. Thus a six-step

Table 6 Data analysis for the different steps of individual production wells taking B and C values from polynomial trend line equation of respective production wells

Production Well no./Step	Discharge (m ³ /s)	Observed drawdown (m)	Formation loss coefficient B	Well loss coefficient C	BQ	CQ^2	$(BQ+CQ^2)$ Calculated drawdown (m)	Well efficiency $BQ/(BQ+CQ^2)$	Well efficiency (%)
Production Well no. 2									
2/Step-1	0.0289	2.44	78.345	208.16	2.26	0.17	2.43	0.93	93
2/Step-2	0.03507	3	78.345	208.16	2.75	0.26	3.01	0.91	91
2/Step-3	0.04012	3.48	78.345	208.16	3.14	0.34	3.48	0.9	90
Production Well no. 5									
5/Step-1	0.03205	5.94	121.57	1989.2	3.9	2.04	5.94	0.66	66
5/Step-2	0.03898	7.76	121.57	1989.2	4.74	3.02	7.76	0.61	61
5/Step-3	0.04467	9.4	121.57	1989.2	5.43	3.97	9.4	0.58	58
Production Well no. 60									
60/Step-1	0.03205	5.04	124.42	1022.6	3.99	1.05	5.04	0.79	79
60/Step-2	0.03898	6.4	124.42	1022.6	4.85	1.55	6.4	0.76	76
60/Step-3	0.04467	7.6	124.42	1022.6	5.56	2.04	7.6	0.73	73

Fig. 5 Semi-log plot of time versus drawdown, Production Well no. 5, Palla area

pumping test was performed on a tubewell at Mayur Vihar. Using Hantush Bierschenk's method the plot of specific drawdown and discharge did not show a linear relationship. As such a simple alternative approach was used. On the plot of drawdown and discharge a polynomial trend line was fitted. The equation of the polynomial trend line approximated to the general equation of Rorabaugh (1953). By rearranging the equation the unknown parameters B , C and p can be noted directly from it. This skips the process of analyzing the data by plotting specific drawdown and discharge in Hantush Bierschenk's method and avoids the complex computation and plotting in the Rorabaugh's method. Thus it is a quick, efficient and accurate method of finding the values of unknown parameters B , C and p in Rorabaugh's equation.

The entrance velocity calculated for different discharge levels was well within the critical limit of 0.03 m/s. The

flow into the well screen is of turbulent nature from the first step discharge itself. The decrease in the efficiency with increase in discharge can be thought of from the perspective of increase in turbulence on account of the increase in discharge and the fact that as the drawdown increases the curvature of flow path increases leading to greater head loss. The well efficiency calculated can be regarded as a reflection of head loss on account of the laminar flow from the aquifer.

The general applicability of the approach is shown with the help of three more production well datasets and the effectiveness of the approach is proven. Once having established the general applicability of the approach it would be appropriate here to explain the need for this experimental set up and the utility of step drawdown tests.

As mentioned earlier the test was conducted to understand the behaviour of drawdown with discharge under

Fig. 6 Semi-log plot of time versus drawdown, Production Well no. 60, Palla area

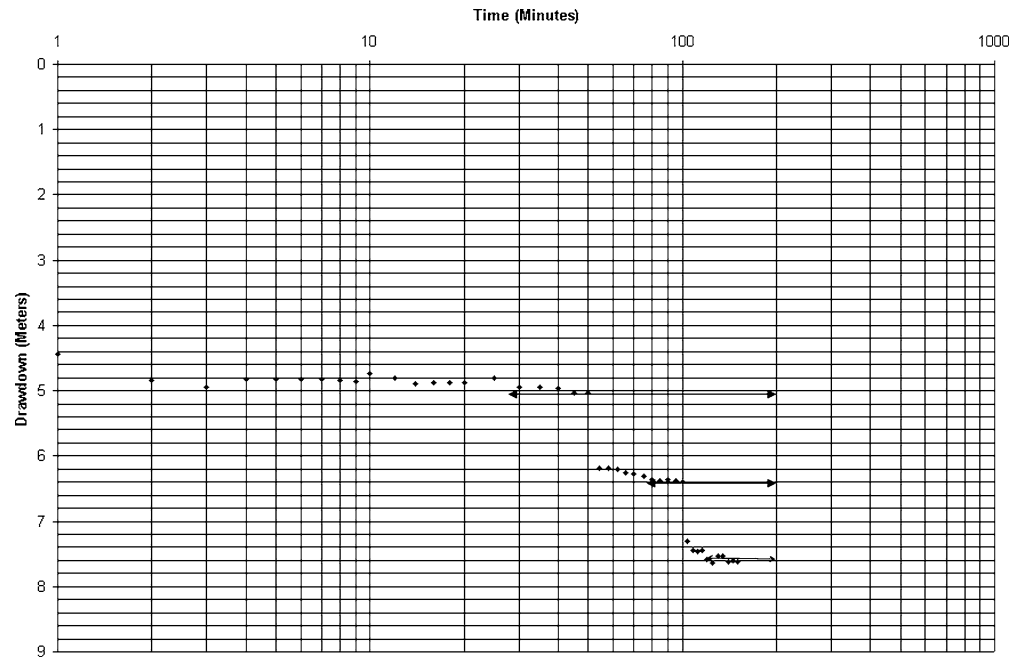
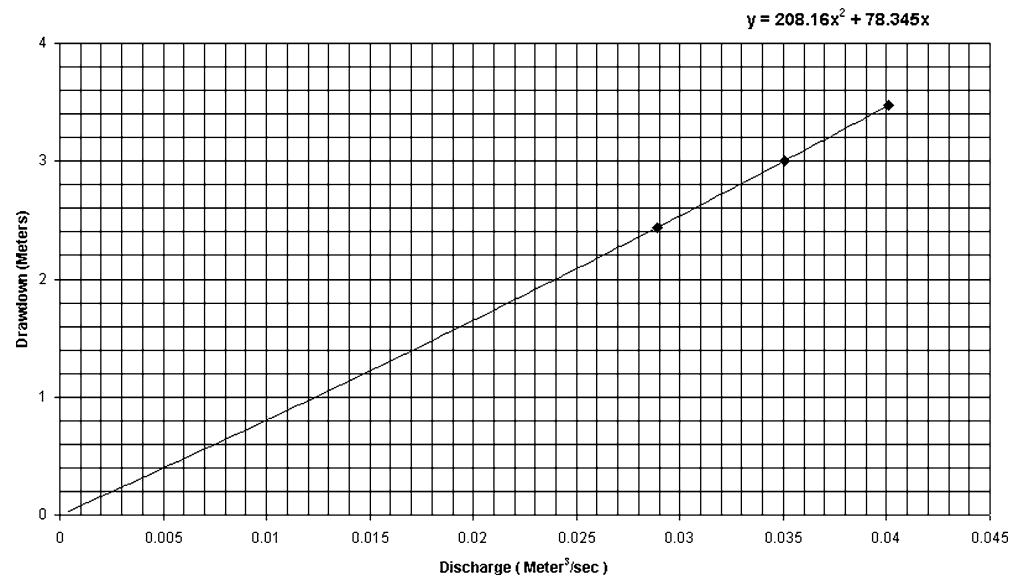


Fig. 7 Discharge versus drawdown plot with a polynomial trend line, Production Well no. 2, Palla area



site-specific conditions. The second issue was that very often well efficiency values calculated by interpretation of the step drawdown test data used to be over relied upon as an index reflecting the appropriateness of tubewell design and the well performance (in terms of production, design sustainability etc.). Here it has been shown that well efficiency calculated is not a unique indicator of the well performance but as mentioned earlier it is only a reflection of head loss on account of the laminar flow from the aquifer. In a nutshell, the reliance on well efficiency calculated, as an indicator of the well performance, should not be overemphasized.

With regards to the utility of the step drawdown tests in thickly saturated unconfined aquifers (the type of area of the experimental set up used in this study), note that the production wells constructed are mostly partially penetrating the

aquifer. With a single production well, understanding the relationship between discharge and drawdown is difficult without using Jacob's (1947) equation subsequently modified by Rorabaugh (1953). Even in an experimental set up where an observation well is used, the relationship between the discharge and the drawdown is given with the help of estimated aquifer parameters, arrived at by complex mathematical computations involving a lot of variables. It must be emphasized here that in commercial well construction practices observation wells are not constructed on account of cost escalation and often space availability constraints. Moreover, Kruseman and de Ridder (1990) mention that the extra head loss induced by partial penetration of a well is included in the aquifer loss component (BQ) of the Jacob's (1947) equation (Eq. (1)). Solving Jacob's (1947) equation

Fig. 8 Discharge versus drawdown plot with a polynomial trend line, Production Well no. 5, Palla area

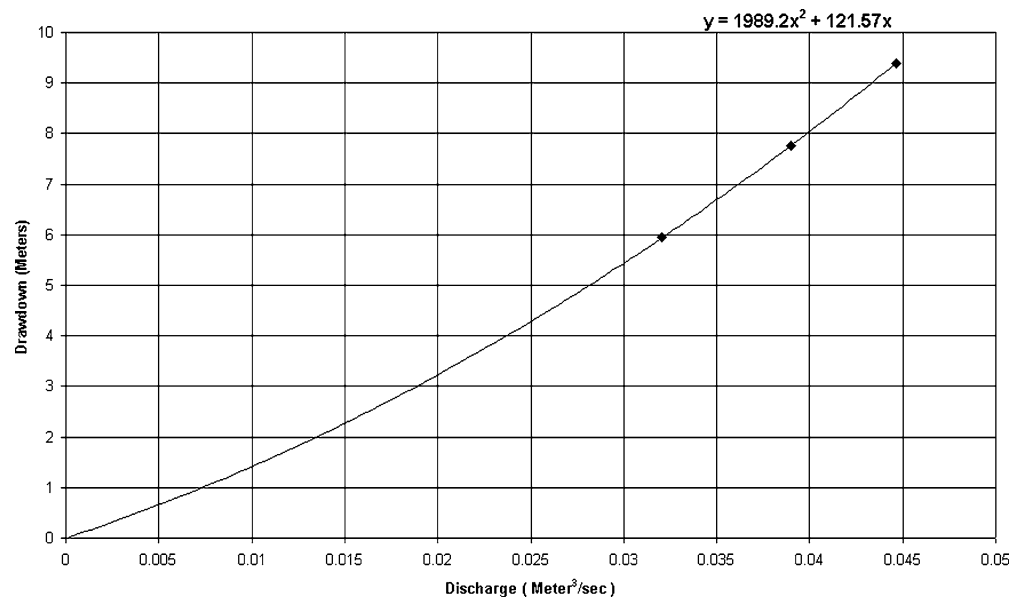
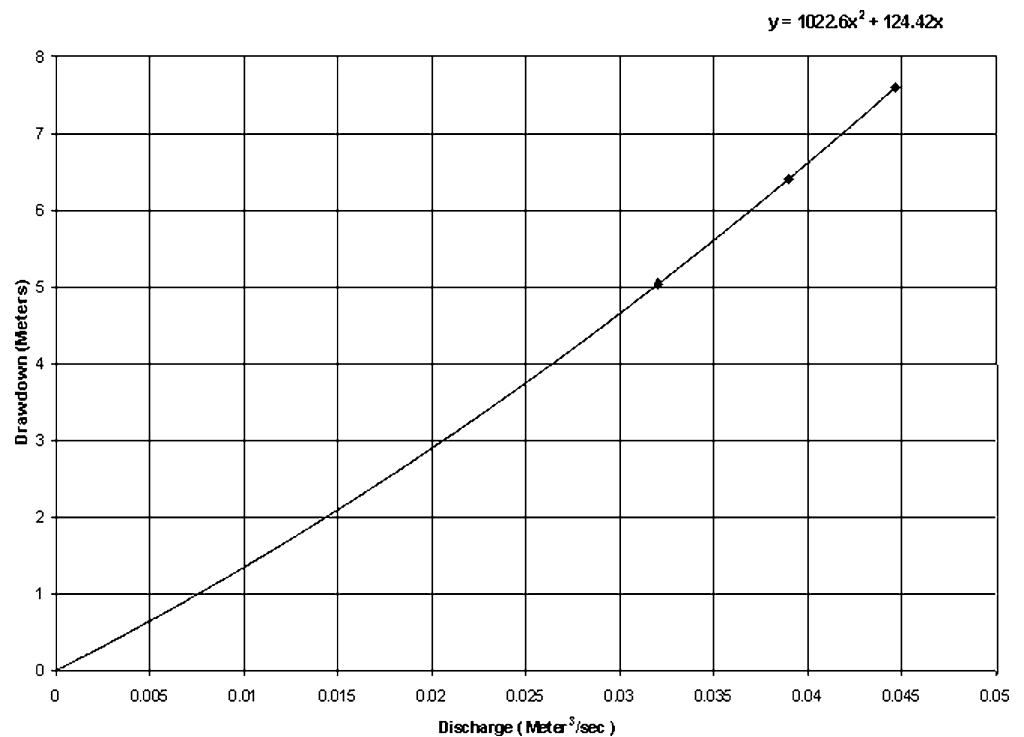


Fig. 9 Discharge versus drawdown plot with a polynomial trend line, Production Well no. 60, Palla area



and even the one modified by Rorabaugh (1953) by the proposed approach is easy and accurate and provides a simple approach for understanding the relationship between discharge and drawdown under site-specific conditions. Further, the loss in head on account of the turbulent flow and the design component of a tubewell can only be ascertained with the help of Jacob's (1947) equation modified by Rorabaugh (1953). Furthermore as mentioned in Kruseman and de Ridder (1990) the above-mentioned equation can be used to predict drawdown inside the well for any realistic discharge and then one can use this relationship between drawdown and discharge to choose empirically an optimum

yield for the well. In analysis of the step drawdown test data, observation well data are not required and thus the proposed approach is a simple, cheap and time-saving method for obtaining the relationship between discharge and drawdown.

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References

- Driscoll FG (1986) Ground water and wells, 2nd edn. Johnson Division, St. Paul, Minnesota
- Jacob CE (1947) Drawdown test to determine effective radius of artesian wells. *Trans Am Soc Civil Eng* 112:1047–1070
- Kruseman GP, de Ridder NA (1990) Analysis and evaluation of pumping test data, 2nd edn. International Institute for Land Reclamation and Improvement, The Netherlands
- Rorabaugh MJ (1953) Graphical and theoretical analysis of step-drawdown test of artesian well. *Proc Am Soc Civil Eng* 79:23