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GEOPHYSICS

Quasi-Stationary Form of the Spectrum of Intense Seismic Waves at the Surface of the Ground

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Transformations of spectra of intense seismic waves propagating in ground columns are studied in numerical experiments. It was found that transformation of spectra to the form $E(f) \sim f^{-k}$ in the ground occurs due to nonlinear (hysteresis) energy dissipation and redistribution of the energy of oscillations in the spectrum by means of interaction between waves. Emphasizing the low-frequency components agrees with the Manley–Row relations of decay instability, according to which energy transport of low-frequency oscillations to high-frequency oscillations is hampered, but decay of high-frequency oscillations and transport of their energy to low-frequency oscillations is facilitated.

The authors of papers [1, 2] devoted to accelograms of earthquakes in 1995 in Kobe and 2000 in Tottori (Japan), which were recorded by borehole seismic groups, developed numerical models of the behavior of ground layers in situ; i.e., they estimated the stresses and strains excited by strong motions in the upper ~100 m of the ground. The numerical models describe the behavior of ground layers in ten 1.5-s time intervals during 15 strong motions. The authors of [3] used these models in numerical experiments to study the propagation of intense noise (transverse) waves in the ground layers. The obtained results characterize the statistical phenomena observed during the propagation of intense seismic waves in nonlinear media (the near-surface ground). The interactions between individual spectral components lead to multiplication of spectral lines, nonlinear widening of spectra, and the appearance of a constant component of the seismic field.

Figure 1 illustrates transformation of the spectra of intense seismic waves propagating in the ground layers: the results of testing the numerical models are presented for the behavior of the ground in Port Island and the SJK site (a conventional name of the location of vertical groups) during the Kobe earthquake 1995 by the Gaussian white noise (the duration of test signals was chosen to be sufficiently large (~40 min) to obtain smooth spectra; their intensity was chosen so that the stresses and strains excited in the ground coincided on average with the stresses and strains generated during earthquakes to estimate the degree of the response of the ground to earthquakes). The results are presented for five 1.5-s intervals (first, third, …) among ten intervals studied. It is seen in the figure that the spectra of oscillations at the surface take a form close to $E(f) \sim f^{-k}$. In the layers at intermediate depths (16 and 32 m in Port Island; 24.9 m in SJK), the forms of spectra demonstrate a resonance intensification of individual components. Interaction between spectral components takes place along with the resonance intensification. High- and low-frequency combination harmonics are formed. The energy of the propagating seismic waves is redistributed over the spectral band. The spectral peaks are smoothed, and the resulting spectrum of the signal at the surface of the ground tends to obtain the limiting form $E(f) \sim f^{-k}$. Relative deformations in the ground at both points reached $\sim 10^{-3}$ in the surface layers and $\sim 10^{-5}$ at depths.

Table 1 presents the estimates of coefficient k obtained in the numerical modeling for the spectrum of oscillation velocity $V(t)$ in five 1.5-s intervals (corresponding to the intervals in Fig. 1), as well as estimates of k based on velocigrams for Kobe earthquakes in 1995. In Port Island at the station closest to the fracture plane (2 km from the fracture plane), dilution of the ground was observed during the earthquake. The nonlinear distortions of the propagating signals are large. All spectra of signals at the surface obtain a smoothed form close to the limiting one $E(f) \sim f^{-k}$ (Fig. 1). As seen from the table, coefficient k changes only slightly in time with the development of ground dilution (intervals 7 and 9), but remains within 1.13–1.44. In SJK (6 km from the fracture plane), the nonlinearity of ground behavior is slightly smaller than in Port Island and variations in k are more notable. If signal spectra at

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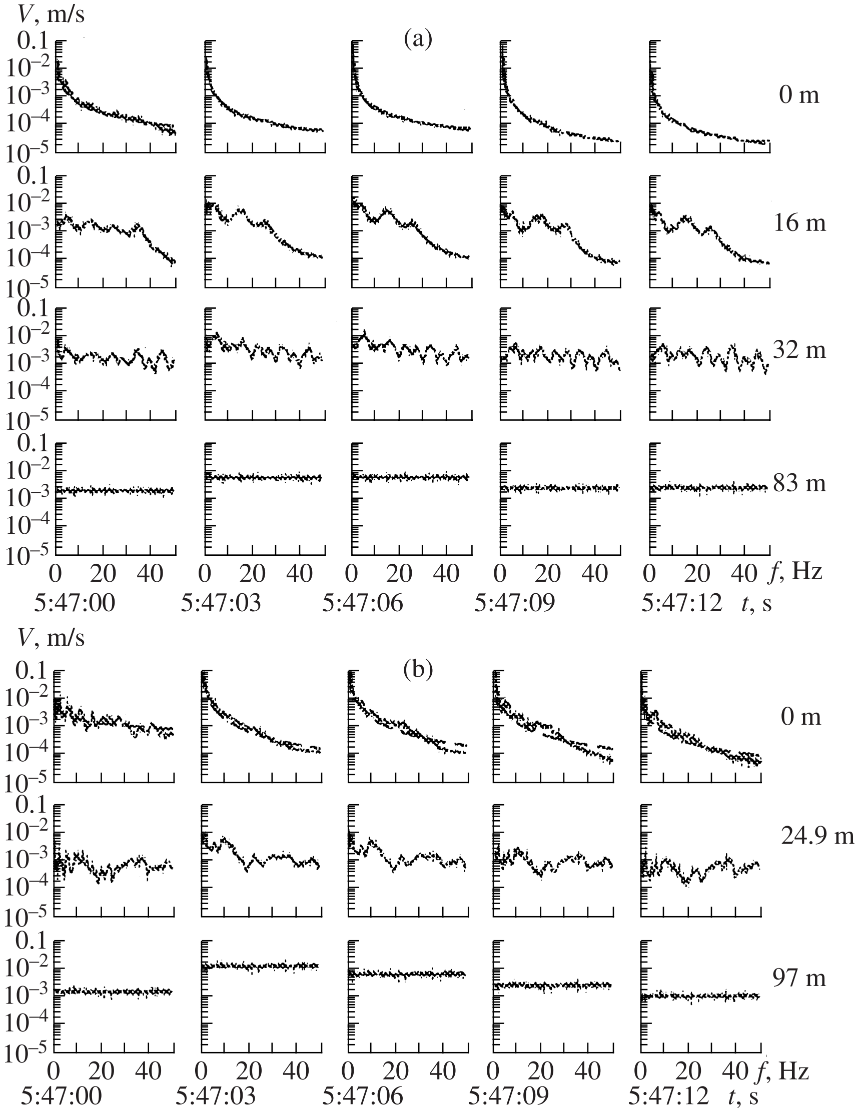

Fig. 1. Spectra of test signals of the Gaussian white noise in ground layers at the depths of the location of recording instruments of borehole groups in Port Island (a) and SJK (b) at five 1.5-s intervals during strong motions. Component E–W (east–west).

the surface can be approximated sufficiently exactly by the dependence $E(f) \sim f^{-k}$, the values of k are close to 1.3–1.4. In SJK, intervals with a sufficiently high (third interval) level of the oscillations are characterized by $k \sim 1.26$–1.51, which is close to the values of k in Port Island. The estimates of k calculated from the velocigrams for the Kobe earthquake are presented in the last column of the table. In general, they are close to the values obtained by numerical modeling.

The analysis of real accelograms shows an increase in the low-frequency components and smoothing of spectral peaks in the oscillations at the surface. This is clearly seen in the examples of accelogram spectra during the earthquake in 1995 in Kobe, Port Island, and SJK (Fig. 2). The spectra signals at the surface and at ~100 m differ notably, which evidences significant nonlinearity in the response of the ground. Thus, the input signals of any spectral composition are transformed at the surface to the signals with spectra $E(f) \sim f^{-k}$; i.e., the information about the spectral composition of waves propagating from the earthquake source is either completely or partly lost as a result of nonlinear distortions of seismic waves in the near-surface ground.

In acoustics, the description of the transformation of broadband spectra is related to the problem of acoustic turbulence. For simple acoustic waves, the measured statistical characteristics of nonlinear perturbations can be calculated exactly. The general relation for the correlation function of a random process was found for an arbitrary section of a nonlinear medium as a function of the 2D characteristic function of the input signal. Relations were found for a process with normal distribution between the spectral density in the medium and the correlation function at the input [4, 5]. If the input of the system is broadband noise, the anomalously low attenuation of high-frequency components evidences energy transport upward in the spectrum to lower frequencies [6–8]. Energy transport also takes place to the low-frequency part of the spectrum. The spectrum of broadband acoustic noise widens to both the high-frequency and low-frequency parts of the spectrum due to nonlinear interactions [9].

The equilibrium form of the spectral distribution or the universal law of decay at high frequencies is most interesting. According to the data of many researchers, the spectrum decays as $\sim\omega^{-2}$ in the case of strong nonlinearity in the region of the existence of developed shock fronts, because the amplitudes of the harmonics, which make up the rupture, decrease as $\sim\omega^{-1}$ [4]. This dependence was obtained theoretically in the approximation of chaotic phases for the medium without dispersion in the inertial frequency ranges (without sources and sinks of energy) [10].

In the case of the existence of spatially distributed sources, which generate waves in a nonlinear medium, the propagation of these waves is described by nonuniform nonlinear equations, the right part of which is a random function, e.g., Burgers equations or equations of simple waves [9]. By calculating the correlation functions for noise waves for approximate solutions of these equations and spectra of the wave intensity based on these correlation functions, we also obtain that low-frequency components are emphasized in the quasi-stationary spectrum. This is usually explained by magnification of the field scale due to the merging of moving ruptures. High-frequency asymptotics formed by nonlinearity is related to the structure of the front of long-lived shock waves, which has the form ω^{-2}. Sometimes, at small inverse Reynolds numbers Γ ($\Gamma = x_p/x_\partial$, i.e., at small ratios of the characteristic nonlinear x_p and dissipative x_∂ wavelengths), the power asymptotics is transferred to the exponential form $\exp(-\beta\omega)$ [9].

Coefficient *k* estimated for five 1.5-s intervals in numerical experiments and on the basis of Kobe earthquake (EQ) records in 1995 for two horizontal components: east–west (E–W) and north–south (N–S)

Location	Direction	1	3	5	7	9	EQ
Port Island	E–W	1.33	1.35	1.24	1.31	1.28	1.21
	N–S	1.31	1.35	1.28	1.25	1.22	1.39
SJK	E–W	0.31	1.51	1.43	1.3	1.08	1.46
	N–S	0.25	1.26	1.48	1.45	1.42	1.39

As shown in review [9], emphasizing the low-frequency components is a general feature and is physically related to the fact that high-frequency dissipation cannot prevent energy accumulation in the low-frequency range. According to the Manley–Row relations, low-frequency modes do not transfer energy to high-frequency modes, whereas high-frequency modes can decay and transfer the energy to low-frequency

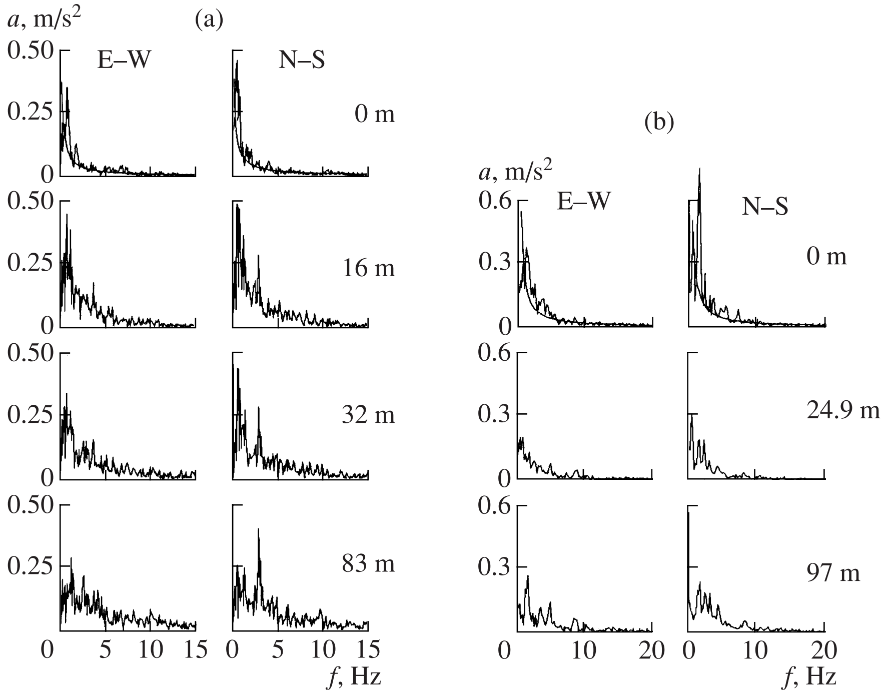

Fig. 2. Spectra of accelograms of the Kobe earthquake in 1995 recorded by borehole groups in Port Island (a) and SJK (b). Components E–W (east–west) and N–S (north–south).

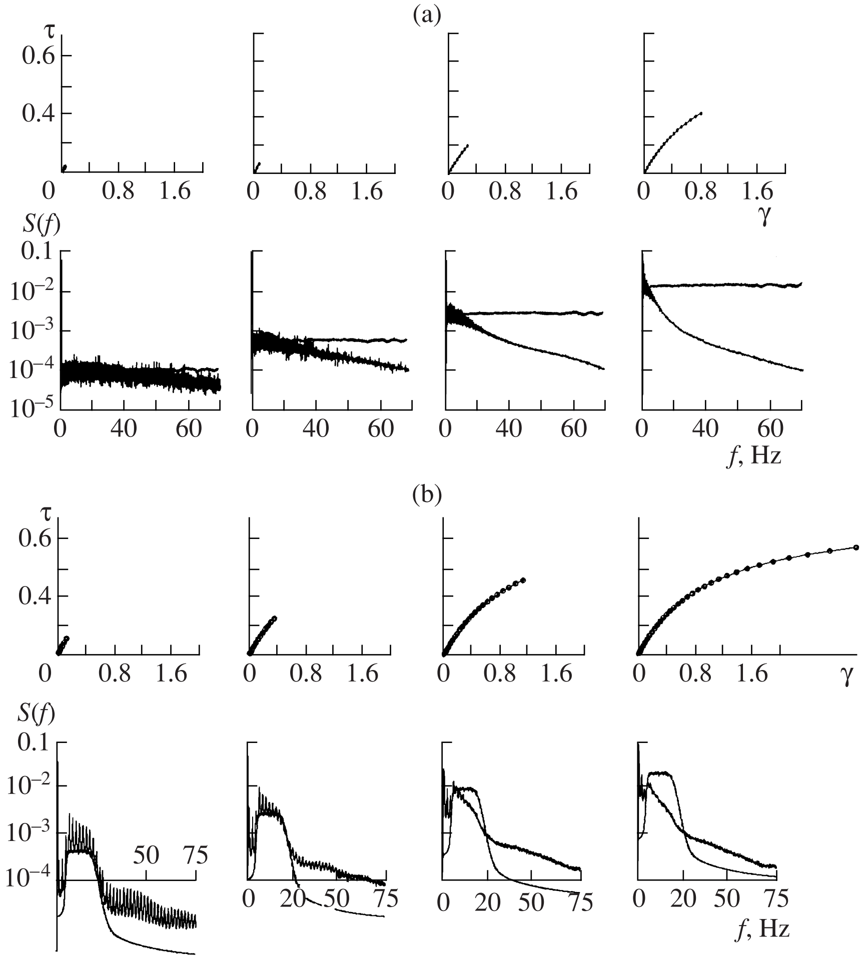

Fig. 3. Linear and nonlinear responses of the simple ground section to (a) Gaussian white noise and (b) narrowband noise of variable intensity. At the top, operation intervals of nonlinear stress–strain dependences are shown (stresses and strains are normalized as described in [12]). At the base, spectra of oscillations at the surface are shown.

modes [11]. Similar regularities are also found for seismic waves.

The mechanisms of spectra transformation for intense seismic waves in the near-surface ground are important for engineering seismology because they are related to the problems of intensification of seismic waves in the ground columns. Intensification depends on the level of seismic forcing, which is related, in particular, to the dependence of absorption on the intensity of seismic waves. According to the theory, nonlinear mechanisms of absorption, which do not depend on the dissipative properties of the medium [9], begin to act when intense waves propagate in nonlinear ground. In seismology, increase in the absorption coefficient has been recorded in sedimentary rocks with increasing intensity of seismic waves [12].

In the subsequent numerical experiments, we studied the transformation of spectra of seismic waves in the near-surface ground. We studied the propagation of broad- and narrow-band noise signals in grounds of different compositions and water saturation levels. Figures 3a and 3b demonstrate the spectra of signals at the surface of linear and nonlinear responses of a simple ground section (200-m of alluvium) with gradual increase of seismic velocities with depth to the Gaussian white noise (a) and narrow-band noise (b) of different intensities. The figures show that, in the case of nonlinear response of the ground with increasing intensity of the input signal, the spectrum of oscillations at the surface gradually decays in the high-frequency direction and takes the form $E(f) \sim f^{-k}$. If the intensity of the input (narrowband) noise signal increases, the spectra of oscillations at the surface also tend to transform as $E(f) \sim f^{-k}$. This form of the spectrum is achieved at strong manifestations of nonlinear response of the ground. The energy from the frequency band of the input signal is transferred to both the high-frequency and the low-frequency spectral ranges. In the case of the linear response, the form of the signal spectrum at the surface does not change (the spectrum of the linear response has been smoothed for better visualization;

periodicity of the spectra is related to reverberation in the ground layer).

The results of modeling of the responses of other ground sections to the Gaussian white noise demonstrate that the amplitudes of low-frequency oscillations during nonlinear response of the ground practically do not change as compared to the linear case, while the amplitudes of the oscillations in the medium- and high-frequency ranges decrease due to nonlinear decay. This agrees with the results in [13], in which the authors used the methods of numerical modeling of accelograms of strong earthquakes and demonstrated that the nonlinear response of the ground decreases oscillations in the medium-frequency range. Low-frequency components are not attenuated, while the high-frequency components slightly increase.

Thus, transformation of spectra to $E(f) \sim f^{-k}$ in the near-surface ground occurs as a result of nonlinear (hysteresis) dissipation and redistribution of the energy of oscillations in the spectra due to the interaction between waves. Emphasizing the low-frequency components agrees with the relations of the Manley–Row decay instability, according to which energy transport of low-frequency oscillations to high-frequency oscillations is hampered, while the decay of high-frequency oscillations and transport of their energy to low-frequency oscillations is facilitated [11].

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