Mathematical Geology, Vol. 36, No. 4, May 2004 (© 2004)

Refinement Indicators for Optimal Selection of Geostatistical Realizations Using the Gradual Deformation Method[1]

Thomas Schaaf,[2] Guy Chavent,[3] and Mokhlès Mezghani[2]

In the analysis of petroleum reservoirs, one of the most challenging problems is to use inverse theory in the search for an optimal parameterization of the reservoir. Generally, scientists approach this problem by computing a sensitivity matrix and then perform a singular value decomposition in order to determine the number of degrees of freedom i.e. the number of independent parameters necessary to specify the configuration of the system. Here we propose a complementary approach: it uses the concept of refinement indicators to select those degrees which have the greatest sensitivity to an objective function quantifying the mismatch between measured and simulated data. We apply this approach to the problem of data integration for petrophysical reservoir charaterization where geoscientists are currently working with multimillion cell geological models. Data integration may be performed by gradually deforming (by a linear combination) a set of these multimillion grid geostatistical realizations during the optimization process. The inversion parameters are then reduced to the number of coefficients of this linear combination. However, there is an infinity of geostatistical realizations to choose from which may not be efficient regarding operational constraints. Following our new approach, we are able through a single objective function evaluation to compute refinement indicators that indicate which realizations might improve the iterative geological model in a significant way. This computation is extremely fast as it implies a single gradient computation through the adjoint state approach and dot products. Using only the most sensitive realizations from a given set, we are able to resolve quicker the optimization problem case. We applied this methodology to the integration of interference test data into 3D geostatistical models.

KEY WORDS: optimization, optimal parameterization, refinement indicators, adjoint state.

INTRODUCTION

The main challenge of reservoir characterization is to obtain the most predictive reservoir model from available observations, either static (e.g. log/core measurements) or dynamic (well test, production data, etc.). Such a model helps

[1]Received 31 March 2003; accepted 27 December 2003.

[2]Department of Reservoir Engineering, Institut Français du Pétrole, 1 et 4, Rueil-Malmaison, France; e-mail: thomas.schaaf@gazdefrance.com

[3]Ceremade, University Paris-Dauphine, Paris, France/Inria-Rocquencourt, Le Chesnay, France.

to efficiently quantify uncertainties and to run risk analysis. To this end, engineers have to complete structural and petrophysical characterizations as both reservoir properties and geometry may have a large influence on the fluid flow pattern. In this study, we do not focus on the structural characterization (e.g. Oldenziel, Van Dithuijzen, and Van Kruijsdijk, 2002; Roggero and Hu, 1998) but rather on the petrophysical characterization of the reservoir. Development of data integration within a stochastic framework for a more efficient uncertainty management has led to a large use of geostatistical tools. Powerful geostatistical simulation algorithms are currently able to generate multimillion-cell geological models. Hereafter, the main challenge is the proper integration of available data (dynamic as well as static data) into the reservoir model without losing the spatial variability and with a reasonable computation time. Concerning the first point, some innovative approaches have been proposed (Mezghani and Roggero, 2001; Schaaf, Mezghani, and Chavent, 2002; Wen and others, 2000) to have an efficient data integration without losing the spatial variability of the initial geological model. The computational time constraint is linked to an optimal parameterization search *i.e.* finding the right number of parameters explaining the data in the quickest way. In this work, the parameterization used for the geological model is based on a linear combination of geostatistical realizations. Rather than picking out randomly those realizations, we propose to compute for a large set of realizations refinement indicators (a single dot product associated to each realization) whose absolute values quantify the sensitivity of each realization with respect to the objective function of the inverse problem. Thus, selecting for the linear combination only the most useful degrees of freedom *i.e.* geostatistical realizations, we are able to speed up the optimization process. Choosing a priori the most promising geostatistical realizations for the iterative optimization process, we get closer to an optimal parameterization in petroleum engineering inverse problems.

PARAMETERIZATION WITH THE GRADUAL DEFORMATION METHOD

The development of history matching within a stochastic framework called for the development of new parameterization techniques able to preserve some initial geostatistical constraints (linked to the prior geological knowledge) throughout the dynamic data integration process.

De Marsily and others (1984) developed the pilot point method involving a gradient based search. More recently was developed the gradual deformation method for performing history matching on stochastic reservoir models (Hu and Blanc, 1998; Roggero and Hu, 1998). It consists in iteratively updating (through an optimization process) a combination of independent realizations of a random

function (representing a petrophysical property for instance) until both static and dynamic data are matched.

Let us consider a stationary Gaussian random function $Z(x)$ with zero mean and unit variance. The gradual deformation methodology consists in writing a new realization z of the random function Z as a linear combination of N independent realizations z_i of Z:

$$z = \sum_{i=1}^{N} \rho_i z_i \; with \sum_{i=1}^{N} \rho_i^2 = 1 \tag{1}$$

The normality constraint $\sum_{i=1}^{N} \rho_i^2 = 1$ is automatically satisfied when spherical coordinates $\theta = \{\theta_1, \ldots, \theta_{N-1}\}$ are used. Hereafter, the new realization z is a function of $(N-1)$ independent gradual deformation parameters $\{\theta_1, \ldots, \theta_{N-1}\}$ which are the inversion parameters for the optimization process. Considering for instance the gradual deformation of $N = 2$ realizations z_1 and z_2, we have a single gradual deformation parameter θ_1:

$$z(\theta_1) = \sum_{i=1}^{2} \rho_i z_i = \underbrace{\cos(\theta_1)}_{=\rho_1} z_1 + \underbrace{\sin(\theta_1)}_{=\rho_2} z_2 \tag{2}$$

This algorithm leads to an efficient parameterization of the geological model thanks to at least three advantages: (1) preservation of the spatial variability; (2) reduction of a high-dimensional optimization problem to a low-dimensional one; (3) smooth variations in the objective function. Currently, there are several extensions of this algorithm: local and structural deformations (Le Ravalec, Noetinger, and Hu, 2000), deformation of non-Gaussian simulations (Hu, 2000), and deformation of sequential simulation models (Hu, Blanc, and Noetinger, 2001).

INDICATORS FOR THE REFINEMENT OF PARAMETERIZATION

Because of operational time constraints, the optimization process based on the updating of gradual deformation parameters θ is generally completed with gradient techniques. The mismatch between the *nd* observations D_{obs} and the simulated data $D_{\text{simul}}(\theta)$ is quantified with an objective function J defined in a weighted least square sense:

$$J(\theta) = \frac{1}{2} \sum_{j=1}^{nd} w_j \left(D_{\text{obs}}^j - D_{\text{simul}}^j(\theta) \right)^2 \tag{3}$$

This leads to a hybrid approach which involves a random search (random picking of the N geostatistical realizations) crossed with gradients techniques. A history matched model is more or less rapidly achieved (based on the realizations that are picked out) what may not be compatible with the operational constraints.

Thus, rather than picking out randomly the N realizations, we propose to select within a set of $N^{\#}$ randomly generated realizations ($N^{\#} \gg N$) the most sensitive (at first order) geostatistical realizations with respect to the objective function J. This approach is based on the concept of refinement indicators and is extremely fast as it is based on discrete adjoint states and dot products. Moreover, it requires a single objective function J evaluation (and thus a single fluid flow simulation) contrary to other approaches (Subbey, Christie, and Sambridge, 2003; Wang and Kovscek, 2002).

The concept of refinement indicators was first introduced by Chavent and Bissel (1998). They addressed the problem of determining a set of degrees of freedom allowing to honor correctly the data while avoiding the pitfall of over-parameterization (i.e. a multiscale adaptative parameterization). Given a class of imbedded parameterization for the problem under concern, a refinement indicator can be associated to each degree of freedom of the next finer parameterization level. A given indicator indicates the effect (at first order) on the optimal data misfit of adding the associated degree of freedom to the current set of parameters. Thus one is able to select only those degrees of freedom giving the strongest decrease of the optimal data misfit. This approach was applied to a problem of hydraulic transmissivities estimation (Ben Ameur, Chavent, and Jaffré, 2002).

We have extended the concept of refinement indicators to the gradual deformation parameterization. In this approach, a (usually small) number N of realizations $z_i \in R^{nm}$ is picked out randomly, where *nm* denotes the number of geostatistical cells (*nm* is a usually large number), and the optimization algorithm is initialized with the gradual deformation parameters $\theta = 0$. This amounts to initialize the weights to $\rho_1 = 1$, $\rho_2 = \ldots = \rho_N = 0$.

The use of refinement indicators reduces from N to 1 or even 0 the number of randomly picked initial geostatistical realizations:

Case 1: Existence of an a Priori Geostatistical Model z_1

Thanks to previous reservoir studies for instance, one may already have a reservoir model which corresponds to an a priori geostatistical realization z_1. We choose the N-1 associated realizations $z_2, \ldots, z_N$ used in the gradual deformation among a large collection of $N^{\#}$-1 ($N^{\#} \gg N$) realizations $z_2, \ldots, z_{N^{\#}}$. The idea there is to draw randomly the realizations $z_2, \ldots, z_{N^{\#}}$ using a large number of seeds (associated to the random number generator of the geostatistical simulation algorithm), and to retain only for the gradual deformation the N-1 ones which correspond to the strongest refinement indicators λ_i, which we define now.

Let $\rho = (\rho_1, \ldots, \rho_{N^\#})$, and consider the constrained optimization problem:

$$\begin{cases} \textit{Find } \rho^* \in R^{N^\#} \textit{ which minimizes the function } J(z) \textit{ with } z = \sum_{i=1}^{N^\#} \rho_i z_i \\ \textit{under the constraints :} \\ \textit{(a)} \ \rho_i = b_i, \, i = 2 \ldots N^\#, \quad \textit{(b)} \ \sum_{i=1}^{N^\#} \rho_i^2 = 1 \end{cases} \tag{4}$$

where b_i, $i = 2, \ldots, N^\#$ are given numbers such that $\sum_{i=2}^{N^\#} b_i^2 < 1$. Any local solution ρ^* of (4) satisfies the associated Lagrange necessary condition: there exists $\lambda^* = (\lambda_2^*, \ldots, \lambda_{N^\#}^*)$ (multipliers for the constraint (a)) and μ^* (multiplier for the constraint (b)) such that:

$$\frac{\partial L}{\partial \rho}(\rho^*, \lambda^*, \mu^*) = 0 \tag{5}$$

where L is the Lagrangian, defined by :

$$\begin{cases} L(\rho, \lambda, \mu) = J\left(\sum_{i=1}^{N^\#} \rho_i z_i\right) + \sum_{i=2}^{N^\#} (\rho_i - b_i)\lambda_i + \left(\sum_{i=1}^{N^\#} \rho_i^2 - 1\right)\mu \\ \forall \rho = (\rho_1, \ldots, \rho_{N^\#}), \forall \lambda = (\lambda_2, \ldots, \lambda_{N^\#}), \forall \mu \end{cases} \tag{6}$$

Equation (5) gives immediately the Lagrange multipliers:

$$\lambda_i^* = -\left\langle \frac{\partial J}{\partial z}(z^*), z_i \right\rangle_{R^{nm}} \quad i = 2, \ldots, N^\#, \ \textit{with } z^* = \sum_{i=1}^{N^\#} \rho_i^* z_i \tag{7}$$

The gradient $\partial J / \partial J(z^*) \in R^{nm}$ corresponds to the derivative of the objective function J with respect to each geostatistical cell of the realization z^*. Of course, the solution ρ^* of (4), — as well as the associated realization z^* and Lagrange multipliers λ^*, μ^*—depend on the right-hand side $b = (b_2, \ldots, b_{N^\#})$ of the constraint (a), so we can denote them by ρ_b^*, z_b^*, λ_b^*, and μ_b^*. The minimum value of the objective function associated to a right-hand side b is then:

$$J_b^* = J\left(z_b^*\right) \tag{8}$$

It is then a known result of constrained optimization, whose proof is recalled in the Appendix, that the Lagrange multiplier λ_i^* coincides with the derivative of the optimal misfit J_b^* with respect to the ith right-hand side b_i of the constraints:

$$\frac{\partial J_b^*}{\partial b_i} = -\lambda_i^* \tag{9}$$

In order to apply this result to our problem, we remark that for the choice $b_2 = b_3 = \ldots = b_{N^\#} = 0$, the admissible set of problem (4) contains only two isolated points $\rho = (\pm 1, 0, \ldots, 0) \in R^{N^\#}$ which are hence local solutions of (4)! So we can apply the above analysis with $\rho^* = (1, 0, \ldots, 0)$ and $z^* = z_1$. If we denote by $J(z^\otimes)$ the optimal misfit when the ith right-hand side b_i is changed from $b_i = 0$ to $b_i = \delta b_i$, we see that at first order:

$$J(z^\otimes) - J(z^*) = -\lambda_i^* \delta b_i \tag{10}$$

So the ith Lagrange multiplier λ_i gives us the sensitivity of the optimal data misfit when unlocking the ith degree of freedom i.e. using the ith realization z_i in the gradual deformation process. We shall call these Lagrange multipliers *refinement indicators*.

In order to select, among the $N^\#$ candidate realizations, the N-1 to be appended to z_1 in order to perform the gradual deformation, we compute the $N^\# - 1$ refinement indicators $\lambda_2, \ldots, \lambda_{N^\#}$ by Equation 7. This is extremely fast, as each λ_i is obtained by a single dot product, once the gradient $\partial J / \partial z(z^*) \in R^{nm}$ has been computed. Refinement indicators are then ranked according to their absolute value, and we select the N-1 geostatistical realizations corresponding to the N-1 refinement indicators of largest absolute value.

Case 2: No a Priori Geostatistical Model

Suppose that we are performing petrophysical characterization through a random function $Y(x)$ of lognormal distribution with mean m and variance σ^2. The function $Y(x)$ is linked to a function $U(x)$ of normal distribution with mean m' and variance σ'^2 through the relation:

$$Y(x) = e^{U(x)} \tag{11}$$

The gradual deformation process is currently performed using a standard normal variable, $Z(x)$, with zero mean and unit variance. Equation (11) then gives:

$$Y(x) = e^{(m' + \sigma' Z(x))} \tag{12}$$

Instead of evaluating the gradient $\partial J / \partial z$ for a given realization z_1 (case 1), we evaluate $\partial J / \partial z$ for the null random function $Z \equiv 0$. This gradient gives the sensitivity of the objective function for a deterministic permeability field of value $e^{m'}$.

In order to choose the N realizations to be used in the gradual deformation, we draw randomly as in case 1 a large number $N^\#$ of geostatistical maps z_i, and

compute the *initialization indicators*:

$$\Lambda_i = \left\langle \frac{\partial J}{\partial z}\left(Z \equiv 0\right), z_i \right\rangle_{R^{nm}}, \quad i = 1, \ldots, N^{\#} \tag{13}$$

By definition of the gradient, we have:

$$J(\delta\rho_i z_i) - J(0) \approx \Lambda_i \delta\rho_i, i = 1, \ldots, N^{\#} \tag{14}$$

Hence realizations z_i with a large $|\Lambda_i|$ are likely to produce a strong decrease of the objective function for a $\delta\rho_i$ of proper sign. So we rank the Λ_i's according to their absolute value, and choose to perform the gradual deformation with the N geostatistical realizations corresponding to the initialization indicators Λ_i with largest absolute value.

COMPUTATION OF THE GRADIENT $\partial J/\partial Z$

Before computing refinement indicators [Eq. (7)] or initialization indicators [Eq. (13)], one has to compute the gradient $\partial J/\partial z$. Let us consider the general flowchart of the forward problem when gradual deformation is used as parameterization of the geological model (Fig. 1).

The four successive steps are:

[1] Gradual deformation of the N realizations z_i which result in the current realization z,
[2] Geological modeling step: one can consider either lognormal permeability fields or facies-based models; conditioning to hard data is currently performed,
[3] Upscaling issue to get the fluid flow simulation model,
[4] Fluid flow simulation and objective function J calculation.

Using the chain rule, we can write the gradient $\partial J/\partial z$ as:

$$\frac{\partial J}{\partial z} = \underbrace{\frac{\partial J}{\partial K}}_{1}\underbrace{\frac{\partial K}{\partial k}}_{2}\underbrace{\frac{\partial k}{\partial z}}_{3} \tag{15}$$

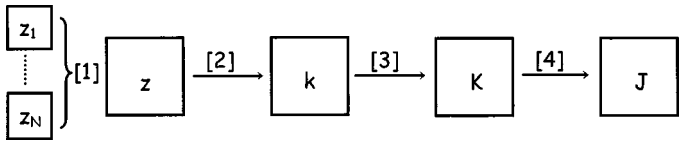

Figure 1. Flowchart of the key issues of the forward problem.

The key point of the proposed methodology is to compute the gradients using the adjoint approach (Chavent, 1974; Sun, 1994). The computation of $\partial J/\partial K$ using a discrete adjoint state is detailed in the Appendix. The second term $\partial K/\partial k$ has also been computed by a discrete adjoint state, but the calculations are not detailed. The third term corresponds to the geological modeling sequence and is easily computed analytically (Schaaf, Mezghani, and Chavent, 2002 for facies-based models; Wen and others, 2000).

We are thus able to compute the gradient $\partial J/\partial z$ for both lognormal and facies-based models (Schaaf, Mezghani, and Chavent, 2003).

With this approach, the additional cost required to compute the gradient $\partial J/\partial z(z^*)$ is similar to the computational cost of one evaluation of J—and, mostly important, independent of the number nm of geostatistical cells, which is very large.

REFINEMENT INDICATORS FOR AN IMPROVED INITIAL GUESS OF PARAMETER VALUES

From Equation (10), we see that the signs of the refinement indicators contain useful information. Suppose that a given refinement indicator has a positive value. If we allocate a positive weight to the associated geostatistical realization, this will tend to decrease the objective function J value (at first order). The same argument stands for negative values of refinement indicator and weight. Thus, when initializing the optimization algorithm, one should allocate to each geostatistical realization a weight of same sign than that of the associated refinement indicator.

Case 1: Existence of an a Priori Geostatistical Model z_1

Except the geostatistical realization z_1, we choose the N-1 realizations $z_2, \ldots, z_N$ used in the gradual deformation process based on the refinement indicators. We have computed $\nabla_z J(z_1)$ and also, by dot products, the components $\lambda_1, \ldots, \lambda_N$ of $\nabla_\rho J((1, 0, \ldots, 0))$. We can then search on the sphere $\sum_{i=1}^{N} \rho_i^2 = 1$ a new initial point in the direction $-\nabla_\rho J$:

- If $\lambda_1 > 0$, J tends to decrease when ρ_1 is increased. Thus, $\rho = (1, 0, \ldots, 0)$ is the best initial reference point,
- If $\lambda_1 < 0$, one can move in the direction $-\nabla_\rho J$ until intersecting again the circle (Fig. 2):

$$\begin{cases} \rho_1 = 1 - \varepsilon\lambda_1,\ \rho_j = 0 - \varepsilon\lambda_j,\ j = 2, \ldots, N \\ \rho_1^2 + \cdots + \rho_N^2 = 1 \end{cases} \Rightarrow \varepsilon = \frac{2\lambda_1}{\lambda_1^2 + \cdots + \lambda_N^2} \tag{16}$$

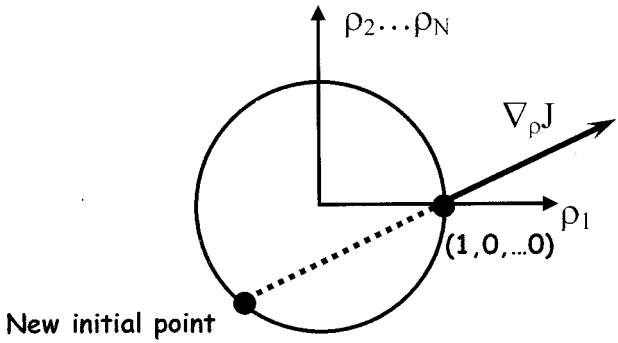

Figure 2. Initialization in the case of an a priori geostatistical model z_1.

Case 2: No a Priori Geostatistical Model

We have computed $\nabla_Z J(Z \equiv 0)$, thus we have $\nabla_\rho J(0, \ldots, 0) = (\Lambda_1, \ldots, \Lambda_N)$. We can initialize the optimization algorithm using the following rules:

$$\begin{cases} \rho_j = 0 - \varepsilon \Lambda_j,\ j = 1, \ldots, N \\ \rho_1^2 + \ldots + \rho_N^2 = 1 \end{cases} \Rightarrow \varepsilon = \frac{1}{\left(\Lambda_1^2 + \ldots + \Lambda_N^2\right)^{1/2}} \tag{17}$$

Equation (17) cannot be applied when conditioning to static well data is performed (through kriging for instance) as we have a deterministic permeability field of value $e^{m'}$ [Eq. (12)] only outside the kriged area.

VALIDATION OF THE METHODOLOGY ON 3D INTERFERENCE TEST CASES

The optimization process is based on the analytical calculation of the sensitivity coefficients i.e. the gradients of the fluid flow simulation results with respect to the parameterization (Mezghani and Roggero, 2001; Schaaf, Mezghani, and Chavent, 2002). Observation data reduce to pressure P as we are considering a single-phase case and parameterization reduces to the gradual deformation parameters $\theta = \{\theta_1, \ldots, \theta_{N-1}\}$. We compute the sensitivity coefficients $\partial P / \partial \theta_i$ using the gradient method (Anterion, Eymard, and Karcher, 1989; Rahon, Blanc, and Guerillot, 1996). The objective of this test case is to match synthetic pressure data simulated with a reference model at the geostatistical simulation scale using the gradual deformation parameterization and the refinement indicators guide to speed up the optimization process.

Synthetic Reservoirs Description

We consider two 3D reservoir models:

- the first one, noted R_1, illustrates the concept of refinement indicators without any upscaling step interference,
- the second one, noted R_2, allows us to test refinement indicators on a multi-million cells geological model.

The synthetic model R_1 is a 3D reservoir containing $141 \times 141 \times 4$ cells of dimension ($\Delta x = 20$ m, $\Delta y = 20$ m, $\Delta z = 5$ m). The permeability field is lognormal with a mean $m = 100$ *mD* and a standard deviation $\sigma = 100$ *mD*. The other petrophysical properties are constant and known: porosity is 30% and total compressibility is 1.10^{-4} bar^{-1}. The variogram is spherical with anisotropic correlation lengths ($l_{c1} = 1000$ m, $l_{c2} = 150$ m and $l_{cz} = 6$ m), the principal direction being 45° clockwise from the Y cartesian axis. At the geostatistical simulation scale, we assume $k_x = k_y$ and $k_z/k_x = 0.1$. The fluid flow simulation is single phase with a viscosity value of 1.1 cP. A production well with a radius of 7.85 cm and no skin is located at the center of the reservoir and perforated over the four layers. Its history comprises a period of constant production rate at 150 m^3/day during 7 days and a buildup period during 55 days. Four observation wells are located around P_1 in a five-spot pattern. These wells are perforated over the four layers too. All five wells are considered for hard data conditioning.

The synthetic model R_2 is a 3D reservoir containing $201 \times 201 \times 50$ cells of dimension ($\Delta x = 20$ m, $\Delta y = 20$ m, $\Delta z = 1$ m). The permeability field is lognormal with a mean $m = 250$ *mD* and a standard deviation $\sigma = 250$ *mD* (Fig. 3(A)). The variogram is spherical with anisotropic correlation lengths ($l_{cx} = 750$ m, $l_{cy} = 150$ m, and $l_{cz} = 6$ m). All other properties are equivalent to those of the model R_1. A production well (noted P_1) with a radius of 7.85 cm and no skin is located at the center of the reservoir and perforated over the 30 upper layers. Its history comprises a period of constant production rate at 600 m^3/day during 7 days and a buildup period during 55 days. Four observation wells (noted O_1, O_2, O_3, and O_4) are located around P_1 in a diamond pattern (Fig. 3(B)). These wells are perforated over the 30 lower layers. Only the production well is considered for hard data conditioning.

Reference and Initial Data

For a given synthetic reservoir model at the geostatistical simulation scale, we compute the synthetic pressure for the five wells using an inhouse monophasic flow simulator. The interference test data include the bottom-hole pressure evolution at the production and observation wells and the pressure derivative evolution at the production well.

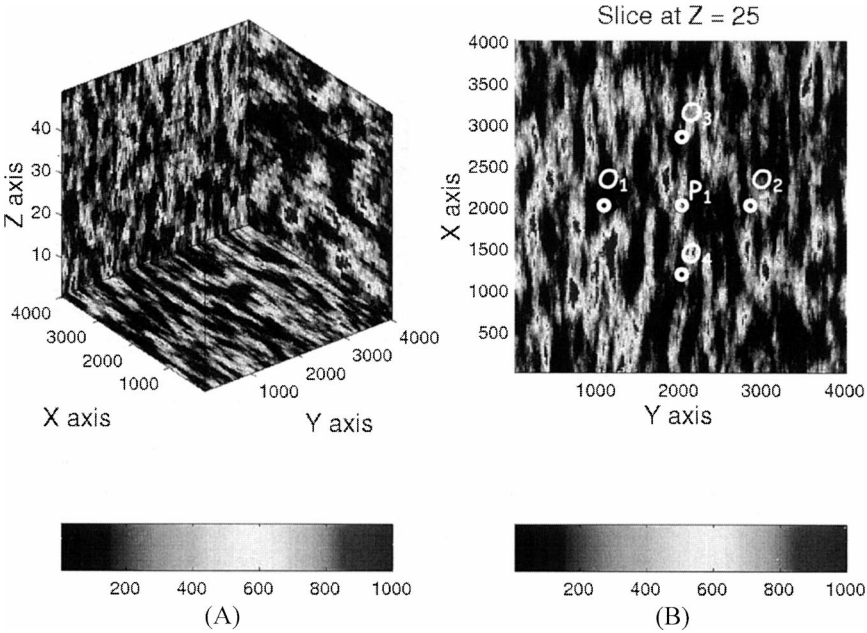

Figure 3. Permeability field for (A), the whole 3D reference model, and (B), the horizontal slice at $Z = 25$.

For the inversion process, the upscaling step of the geological model R_2 results in a flow simulation model composed of:

[1] 43 blocks along the X axis with variable ΔX,
[2] 57 blocks along the Y axis with variable ΔY,
[3] 10 blocks along the Z axis with $\Delta Z = 5$ m.

We use a numerical upscaling technique resolving the pressure field at the geostatistical simulation scale over the upscaled block using no flow boundary conditions *i.e.* a pressure gradient along the equivalent permeability direction calculation and no flow boundaries elsewhere. This calculation is done for each diagonal term of the equivalent permeability tensor so we get three upscaled permeability fields K_X, K_Y, and K_Z (respectively along the X, Y, and Z axis). This simulation model contains $NM = 24{,}510$ blocks. It corresponds to a reduction of around 99% of the initial number of cells.

Inverse Problem Formulation

All structural parameters of the random function $Z(x)$ and petrophysical properties except permeability are assumed to be known. The objective is to

characterize the permeability distribution by adjusting the parameters θ of the gradual deformation.

The inversion process is based on a Gauss–Newton's optimization algorithm. Matching the interference test consists in minimizing the objective function defined by Equation (3). Associated weights are calculated so that each observation has the same weighting in the objective function J calculation.

For the observation wells, both draw-down and buildup pressure variations were considered whereas only the buildup pressures (and associated derivatives) were considered for the production wells.

Refinement and Initialization Indicators Calculation for the Model R_1

Case 1: Existence of an a Priori Geostatistical Model z_1

For a given realization z_1, we may compute many refinement indicators through dot products. Figure 4(A) represents the absolute value of refinement indicators for 1000 geostatistical realizations picked out randomly. To demonstrate the usefulness of refinement indicators, we compare three optimization loops (Fig. 5) based on the gradual deformation of $N = 5$ geostatistical realizations.

The first loop is based on the gradual deformation of the realization z_1 (from which we calculate $\partial J/\partial z(z_1)$, and four realizations picked out randomly. The optimization algorithm is initialized with $\theta = 0$. This was the current

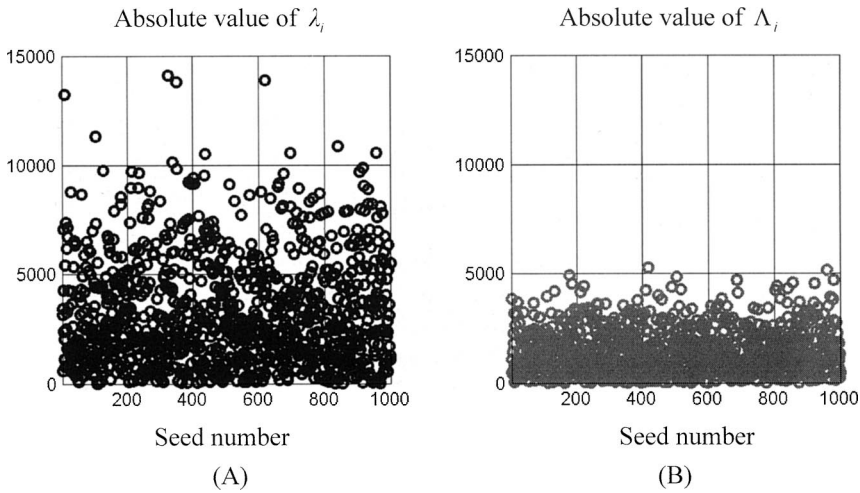

Figure 4. (A), refinement indicators absolute value, and (B), initialization indicators absolute value, for the 1000 considered geostatistical realizations.

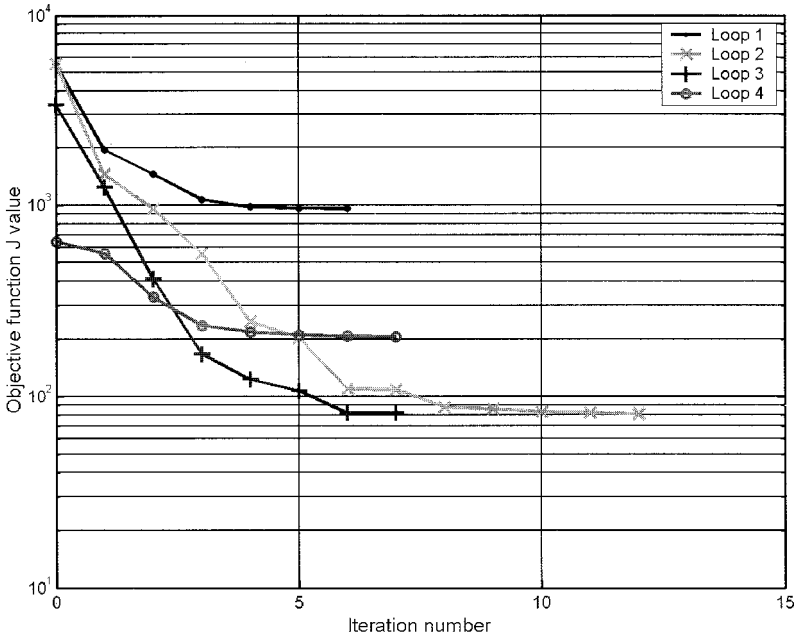

Figure 5. Evolution of the objectives function J for the four considered optimization loops with model R_1.

approach before refinement indicators were extended to the gradual deformation parameterization.

The second loop corresponds to the gradual deformation of the realization z_1 and the four realizations having the greatest sensitivity with respect to the objective function J taken from the considered set of 1000 realizations. The refinement indicators where not used for an improved initial guess of the optimization algorithm: we still set gradual deformation parameters $\theta = 0$.

The third loop is based on the same geostatistical realizations as those of the second loop but the optimization algorithm is initialized using the additional information [Eq. (10)] we get from refinement indicators.

Several key issues have to be underlined:

- For a given iteration number, refinement indicators are associated with a faster and better optimization process (loops 2 and 3 versus loop 1),
- Use of refinement indicators for an improved initial guess of the optimization algorithm (loop 3) gives a lower initial objective function J value, but the remaining optimization process may not be significantly speeded up.

As history matching is not the purpose of this study, we do not perform another optimization loop and we consider that the data integration process is over.

Case 2: No a Priori Geostatistical Model

The alternative approach, based on so called initialization indicators, has been tested. We consider that no a priori geostatistical model z_1 is available. Initialization indicators Λ_i [Eq. (13)] are computed for the same 1000 geostatistical realizations as those considered for the case 1 (Fig. 4(B)). Once again, we compare the proposed approach to a random picking of the geostatistical realizations.

The fourth loop (loop 4, Fig. 5) corresponds to the gradual deformation of the five realizations having the strongest initialization indicators $|\Lambda_i|$ taken from the given set. Initialization indicators cannot be used for an improved initial guess of the optimization algorithm [Eq. (17)] as conditioning to hard data is performed. Through initialization indicators we select the most sensitive geostatistical realizations with respect to the objective function. At this point, we do not have any control on the initial value of the objective function. In order to have a low initial value, we must recall Eq. (14) and choose between the two solutions $\rho = \pm 1, 0, \ldots, 0$ the one having a weight ρ_1 of the opposite sign of Λ_1. As the value of Λ_1 is -5243, the choice $\rho = +1, 0, \ldots, 0$ ends with an initial value of 643 for the objective function. The choice $\rho = -1, 0, \ldots, 0$ would have led to an initial value of 14086 for the objective function.

The proposed method allows us to get a better and faster optimization process compared to the current approach (loop 1, Fig. 5). When initialization indicators are used instead of refinement indicators, the dynamic of the optimization process may lead to a less lower objective function value.

Refinement and Initialization Indicators Calculation for the Model R_2

Case 1: Existence of an a Priori Geostatistical Model z_1

We now test the concept of refinement indicators for multimillion-cell geological models for which an upscaling step is mandatory to perform a fluid flow simulation. Upscaling has a more or less pronounced averaging effect which may affect the refinement indicators' efficiency to select the most sensitive geostatistical realizations.

Once again, we compare optimization loops (Fig. 6) based on the gradual deformation of $N = 5$ geostatistical realizations.

The first loop is based on the gradual deformation of the realization z_1 (from which we calculate $\partial J/\partial z(z_1)$), and four realizations picked out randomly. The optimization algorithm is initialized with $\theta = 0$.

The second loop corresponds to the gradual deformation of the realization z_1 and the four realizations having the greatest sensitivity with respect to the objective function J taken from a set of 1000 realizations. The refinement indicators where

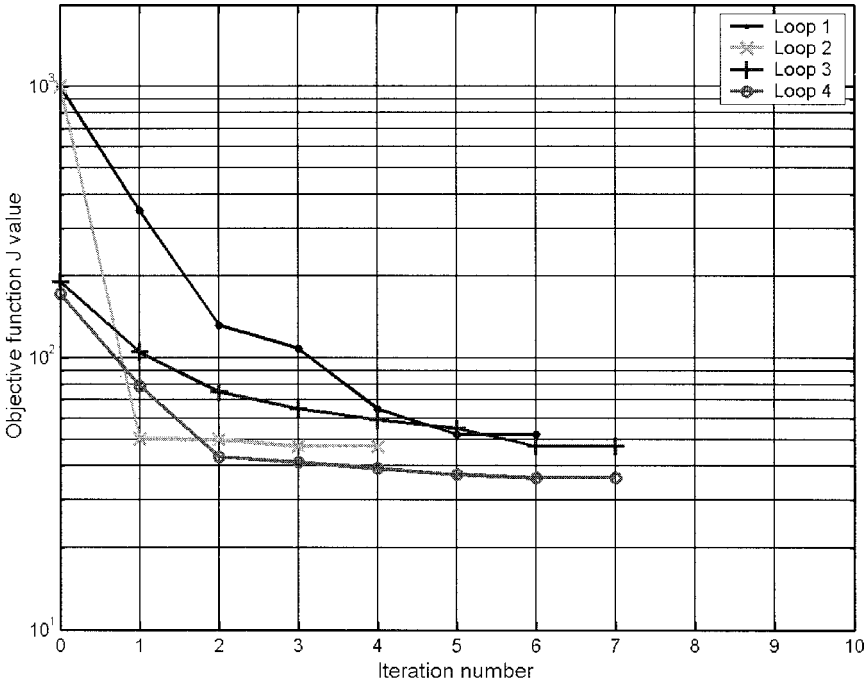

Figure 6. Evolution of the objective function J for the four considered optimization loops with model R_2.

not used for an improved initial guess of the optimization algorithm thus give gradual deformation parameters $\theta = 0$.

The third loop is based on the same geostatistical realizations as those of the second loop but the optimization algorithm is initialized using the additional information [Eq. (10)] we get from refinement indicators.

We notice that the use of indicators for an enhanced initial guess of the optimization algorithm (value of the objective function at iteration 0, loop 3, Fig. 6) is useful but less efficient than an iteration of the optimizer (value of the objective function at iteration 1, loop 2, Fig. 6).

Refinement indicators appear meaningful when a mandatory upscaling step is performed as one is still able to select the most sensitive geostatistical realizations from a given set.

Case 2: No a Priori Geostatistical Model

We compute initialization indicators Λ_i [Eq. (13)] for the same 1000 geostatistical realizations as those considered for the case 1.

The fourth loop (loop 4, Fig. 6) is based on the gradual deformation of the five realizations having the strongest initialization indicators $|\Lambda_i|$ taken from the given set. In order to have a low initial value, we recall Equation (14) and choose between the two solutions $\rho = \pm 1, 0, \ldots, 0$ the one having a weight ρ_1 of the opposite sign of Λ_1. As the value of Λ_1 is $+1267$, the choice $\rho = -1, 0, \ldots, 0$ corresponds to an initial value of 172 for the objective function (the choice $\rho = +1, 0, \ldots, 0$ would have led to an initial value of 915 for the objective function).

Initialization indicators select the most sensitive geostatistical realizations when no a priori geostatistical model is available to get a better and faster optimization process compared to the current approach (loop 1, Fig. 6).

CONCLUSIONS

In this report, we explored the feasibility to get closer to an optimal parameterization for dynamic data integration problems in reservoir engineering. We proposed an approach based on refinement indicators that selects the most sensitive geostatistical realizations from a given set with respect to the objective function of the inverse problem. Selecting only the most useful realizations, we are able to speed up the optimization process and to better constrain the geological model. The refinement indicators calculation is extremely fast (compared to a standard optimization loop timing) as it implies only one objective function evaluation, adjoint state calculations and dot products. Refinement indicators are also useful for an improved initial guess of the optimization algorithm as they contain useful information (at first order) about the objective function behavior.

ACKNOWLEDGMENTS

The authors thank Institut Français du Pétrole for permission to release the results.

REFERENCES

Anterion, F., Eymard, R., and Karcher, B., 1989, Use of parameter gradients for reservoir history matching, SPE Paper 18433: Proceedings of the SPE Symposium on Reservoir Simulation, Houston TX.

Ben Ameur, H., Chavent, G., and Jaffré, J., 2002, Refinement and coarsening indicators for adaptative parameterization: Application to the estimation of hydraulic transmissivities: Inverse Problems, v. 18, p. 775–794.

Chavent, G., 1974, Identification of functional parameter in partial differential equations, *in* Goodson, R. E., and Polis, M., eds., Identification of parameters in distributed Systems, Am. Soc. Mach. Emge., New York, p. 31–48.

Chavent, G., and Bissell, R., 1998, Indicator for the refinement of parameterization: Proceedings of the International Symposium on Inverse Problems in Engineering Mechanics, Nagano, Japan, p. 185–190.

De Marsily, G., Lavedan, C., Boucher, M., and Fasanino, G., 1984, Interpretation of interference tests in a well field using geostatistical techniques to fit the permeability distribution in a reservoir model, *in* Verly, G., David, M., Journel, A. G., and Maréchal, A., eds., Geostatistics for natural resources characterization: Reidel, Dordrecht, The Netherlands, v. 2, p. 831–849.

Hu, L. Y., 2000, Gradual deformation of nongaussian stochastic simulations: *in* Kleingeld, W. J., and Krige, D. G., eds., Geostats 2000, Cape Town, South Africa, Vol. 1, p. 94–103.

Hu, L. Y., and Blanc, G., 1998, Constraining a reservoir facies model to dynamic data using a gradual deformation method, Paper B-01: Proceedings of the 6th European Conference on Mathematics of Oil Recovery (ECMOR VI), 8–11 September 1998, Peebles, Scotland, UK.

Hu, L. Y., Blanc, G., and Noetinger, B., 2001, Gradual deformation and iterative calibration of sequential stochastic simulations: Math. Geol., v. 33, no. 4, p. 475–489.

Le Ravalec, M., Noetinger, B., and Hu, L. Y., 2000, The FFT moving average (FFT-MA) generator: An efficient numerical method for generating and conditioning Gaussian simulation: Math. Geol., v. 32, no. 6, p. 701–723.

Mezghani, M., and Roggero, F., 2001, Combining gradual deformation and upscaling techniques for direct conditioning of fine scale reservoir models to dynamic data, SPE Paper 71334: Proceedings of the SPE Annual Technical Conference and Exhibition, New Orleans, LA.

Oldenziel, T., Van Dithuijzen, R., and Van Kruijsdijk, R., 2002, Geologic parameterization of reservoir model enhances history-match procedure: The Leading Edge, v. 21, no. 6, p. 544–551.

Rahon, D., Blanc, G., and Guerillot, D., 1996, Gradients method constrained by geological bodies for history matching, SPE Paper 36568: Proceedings of the SPE Annual Technical Conference and Exhibition, Denver, CO.

Roggero, F., and Hu, L. Y., 1998, Gradual deformation of continuous geostatistical models for history matching: SPE Paper 49004: Proceedings of the SPE Annual Technical Conference and Exhibition, New Orleans, LA.

Schaaf, T., Mezghani, M., and Chavent, G., 2002, Direct conditioning of fine-scale facies models to dynamic data by combining gradual deformation and numerical upscaling techniques, Paper E-44: Proceedings of the 8th European Conference on Mathematics of Oil Recovery (ECMOR VIII), 3–6 September 2002, Freiberg, Germany.

Schaaf, T., Mezghani, M., and Chavent, G., 2003, In search of an optimal parameterization: An innovative approach to reservoir data integration: SPE Paper 84273: Proceedings of the SPE Annual Technical Conference and Exhibition, Denver, CO.

Subbey, S., Christie, M., and Sambridge, M., 2003, A strategy for rapid quantification of uncertainty in reservoir performance prediction: SPE Paper 79678: Proceedings of the SPE Reservoir Simulation Symposium, Houston, TX.

Sun, N. Z., 1994, Inverse problems in groundwater modeling: Kluwer Academic, Dordrecht, The Netherlands, 337 p.

Wang, Y., and Kovscek, A. R., 2002, A streamline approach for ranking reservoir models that incorporates production history: SPE Paper 77377: Proceedings of the SPE Annual Technical Conference and Exhibition, San Antonio, TX.

Wen, X. H., Tran, T. T., Behrens, R. A., and Gómez-Hernández, J. J., 2000, Production data integration in sand/shale reservoirs using sequential self-calibration and geomorphing: A comparison: SPE Paper 63063: Proceedings of the SPE Annual Technical Conference and Exhibition, Dallas, TX.

APPENDIX

Computation of Lagrange Multipliers λ_i^* — Statement of Equation (9)

Recall the constrained optimization problem (4) and the associated Lagrangian $L(b, \rho, \lambda, \mu)$ defined by Equation (6). b corresponds to the parameters; ρ corresponds to the state variables; λ and μ correspond to the Lagrange multipliers.

We have:

$$J_b = L(b, \rho, \lambda, \mu) \tag{A1}$$

The differential of the Lagrangian is given by:

$$\delta L = \frac{\partial L}{\partial b}\delta b + \frac{\partial L}{\partial \rho}\delta \rho \tag{A2}$$

For Lagrange multipliers verifying the adjoint equations $(\partial L/\partial \rho)\delta\rho = 0, \forall \delta\rho$ [Eq. (5)], we have:

$$\delta L = \frac{\partial L}{\partial b}\delta b \tag{A3}$$

The coefficient of δb_i in the differential δL corresponds to $\partial J_b^*/\partial b_i$. Differentiating Equation (6), we deduce that this coefficient is equal to $-\lambda_i^*$ and we have [Eq. (9)]:

$$\partial J_b^*/\partial b_i = -\lambda_i^* \tag{A4}$$

Adjoint State of the Monophasic Fluid Flow Simulator

The adjoint state method computes the gradients of any real function $h(\alpha, e)$ of state e with respect to parameters α. Contrary to the gradient method which requires a number of auxiliary simulations proportional to the dimension of the parameter space, the adjoint state technique requires only one extra simulation to get the adjoint variables. The computational cost of the gradients $\partial h/\partial \alpha$ is similar to the computational cost of two evaluation of h. This real function $h(\alpha, e)$ may be the objective function [Eq. (3)] $J(K, D_{\text{simul}})$ with parameters $\alpha = K$ corresponding to the permeability field and state variables $e = D_{\text{simul}}$ corresponding to the fluid flow simulation results.

As we are limiting us to a single-phase case, the pressure behavior in a reservoir is governed by the diffusivity equation:

$$\nabla \cdot \left(\frac{K}{\mu} \nabla P \right) = \Phi c_t \frac{\partial P}{\partial t} + Q \tag{B1}$$

where K is the permeability tensor, P the pressure, μ the fluid viscosity, Φ the porosity, c_t the total rock pore and fluid compressibility, and Q the sink/source term. Discretization of Equation (B1) in space and time may be written under matrix form:

$$\left| \begin{array}{l} p_t^{(0)} = Po \\ A_t^{(n)} p_t^{(n-1)}(K) + B(K) p_t^{(n)}(K) = C_t^{(n)} \, for \, n = 1, \ldots, NT \end{array} \right. \tag{B2}$$

where:

- NT is the number of time steps,
- NM the number of blocks of the fluid flow simulation model,
- NP the number of wells,
- $NG = NM + NP$,
- $B(K) \in R^{(NM+NP)} \times R^{(NM+NP)}$ a matrix function of the permeability field K,
- $A_t^{(n)} \in R^{(NM+NP)} \times R^{(NM+NP)}$ a matrix function of the time discretization,
- $p_t^{(n)}(K) \in R^{(NM+NP)}$ the pressure vector at time step n.
- Po the initial reservoir pressure

For clarity, let us forget subscript t and write variable such that vector p^n is there instead of $p_t^{(n)}$. To compute the gradients $\partial J / \partial K$, consider the optimal control problem:

$$P \; : \; \left\{ \begin{array}{l} Find \, K \in R^{NM} \, that \, minimizes \\ J(K) = h(K, p^1(K), \ldots, p^{NT}(K)) \\ under \, the \, constraints \\ \left| \begin{array}{l} p^0 = Po \\ A^n p^{n-1} + B(K) p^n = C^n \end{array} \right. \end{array} \right. \tag{B3}$$

The Lagrangian of the Equation (B3) is:

$$\begin{aligned} L(K, p^1, \ldots, p^{NT}, q^1, \ldots, q^{NT}) &= h(K, p^1, \ldots, p^{NT}) \\ &+ \sum_{n=1}^{NT} \langle A^n p^{n-1} + B p^n - C^n, q^n \rangle_{R^{NG}} \end{aligned} \tag{B4}$$

where K correspond to the parameters, p^i are the state variables, and q^i correspond to the Lagrange multipliers. For Lagrange multipliers verifying the adjoint equations $\sum_{n=1}^{NT} \frac{\partial L}{\partial p^n} \delta p^n = 0, \forall \delta p^n$, we have to solve:

$$\left| \begin{array}{l} q^{NT+1} = 0 \\ A^{n+1} q^{n+1} + B q^n = -\dfrac{\partial h}{\partial p^n} \text{ for } n = NT, \ldots, 1 \end{array} \right. \tag{B5}$$

Equation (B5) corresponds to a time retrograde system of linear equations whereas the pressure equations system is a time forward one. The gradients $\partial J / \partial K$ correspond to the coefficients of δK in δL:

$$\delta L = \frac{\partial L}{\partial K} \delta K + \sum_{n=1}^{NT} \underbrace{\frac{\partial L}{\partial p^n} \delta p^n}_{=0} = \frac{\partial h}{\partial K} \delta K + \sum_{n=1}^{NT} \left\langle \frac{\partial B}{\partial K} p^n, q^n \right\rangle_{R^{NG}} \tag{B6}$$

If there is no explicit dependence between the objective function value and the permeability field, the term $\partial h / \partial K$ is null. For a $\partial J / \partial K$ calculation along a given direction (X, Y, or Z), one has to compute the associated matrix B derivative. Along the X direction for instance, we have:

$$\frac{\partial J}{\partial K_X^j} = \sum_{n=1}^{NT} \left\langle \frac{\partial B}{\partial K_X^j} p^n, q^n \right\rangle_{R^{NG}} \quad \text{for } j = 1, \ldots, NM \tag{B7}$$

Notation

b_i: ith component of the RHS of the constraint (a) of the problem (4), $i = 2, \ldots, N^{\#}$

D_{obs}^j: jth component of the observation vector, $j = 1, \ldots, nd$

D_{simul}^j: jth component of the simulated data vector, $j = 1, \ldots, nd$

J: objective function of the inverse problem defined in a weighted least square sense

$J(z^{\otimes})$: optimal value of the function J when the ith RHS of constraint (a) is changed from $b_i = 0$ to $b_i = \delta b_i$

K: permeability field of the fluid flow simulation model

k: permeability field of the reservoir model at the geostatistical simulation scale

L: Lagrangian of the constrained optimization problem (4)

N: number of realizations considered for the gradual deformation method

$N^{\#}$: number of realizations considered for the refinement indicators calculation
nd: number of observations for the objective function J calculation
nm: dimension of the realization z
P: pressures, results of the fluid flow simulation
$U(x)$: Gaussian random function with mean m' and variance σ'^2
w_j: weight of the jth term of the objective function J, $j = 1, \ldots, nd$
$Y(x)$: lognormal random function
$Z(x)$: standard Gaussian random function
$z(x)$: realization of $Z(x)$ at location x
z_i: ith realization of $Z(x)$
z^*: realization associated to the weights ρ^* for the gradual deformation method
z_b^*: express the dependence on z^* of the RHS of the constraint (a) of the problem (4)
Λ_i: initialization indicator associated to the ith realization z_i
λ: Lagrange multiplier of the constraint (a) of the optimization problem (4)
λ^*: Lagrange multiplier verifying the Lagrange necessary condition defined by Equation (5), so called refinement indicator
λ_b^*: express the dependence on λ^* of the RHS of the constraint (a) of the problem (4)
μ: Lagrange multiplier of the constraint (b) of the optimization problem (4)
μ^*: Lagrange multiplier verifying the Lagrange necessary condition defined by Equation (5)
μ_b^*: express the dependence on μ^* of the RHS of the constraint (a) of the problem (4)
θ_j: jth gradual deformation parameter, $j = 1, \ldots, (N-1)$
ρ^*: local solution of the constrained optimization problem (4)
ρ_b^*: express the dependence on ρ^* of the RHS of the constraint (a) of the problem (4)
ρ_i: weight of the ith realization for the gradual deformation method, $i = 1, \ldots, N$
$\frac{\partial J}{\partial z}(z^*)$: gradient of the objective function J with respect to each cell of the realization z^*
$\langle ., . \rangle$: dot product